Indian Institute of technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module – 06
Lossy transmission lines and primary constants

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Hello and welcome to NPTEL mooke on applied electromagnetic for engineers I hope you have been finding this lectures useful, so far we have consider only the case of lossless transmission lines which although might be a very good approximation to some cases especially at high frequency transmission lines. Where not really the most general transmission lines that one can think of nor their quite practical admit frequency and low frequency conditions where the line resistance and the dielectric loss of the dielectric that fills the transmission line medium has been neglected.

Of course in a real world dielectrics are not perfect there will be a certain amount of leakage through the dielectric which minifies as a leakage current, and this leakage current flow between the two conductors that are use for the transmission line and constitute a leakage current okay. And this leakage current can be modeled by assuming that the dielectric behaves not just as an ideal capacitor but also as a small amount of conductions do it okay.

So this conductions times the voltage difference between the two conductors of the transmission line will create the leakage current, current being g x v. similarly when I consider a transmission line made out of a conductor of course if I assume a super conductor then the line resistance can be neglected, but in any ordinary metal conductor that I am find for example copper or a gold or you know aluminum wires of whatever those wires that I consider they all have a certain amount of resistance associated with it.

This resistance is also because of the material properties okay as well as by the guided nature of the electromagnetic waves are all propagating, that evaluation of the resistance as well as the conductance I will leave it for a later case where we have thoroughly understood Maxwell equations and the resistance of this conductors goes by the name of skin effect and evaluating skin effect is the major topic that we will later on consider after we have looked at Maxwell equations.

However the circuit model of the transmission line that we were considering the distributor circuit model in which we had modeled the series inductions and the shunt capacitance can be augmented a little bit in order to consider this non you know non ideal effects of series resistance and the shunt conductors. So if I go back to the unit cell that is the kind of one infinity symbol length that I consider in deriving the voltage and current equations okay that unit cell will now have in the series side will have the resistance as well as the inductions and in the shunt side will have the capacitance and a certain amount of shunt conductance.

Instead of working in a general you know Z and T domain we will specialize to the frequency domain behavior which means that all my sources are oscillating at a particular oscillating frequency ω, and therefore I can re place the impudence of an inductor and the shunt susceptance of a capacitor by their equivalent frequency domain reactance values. You must remember or you may recall.

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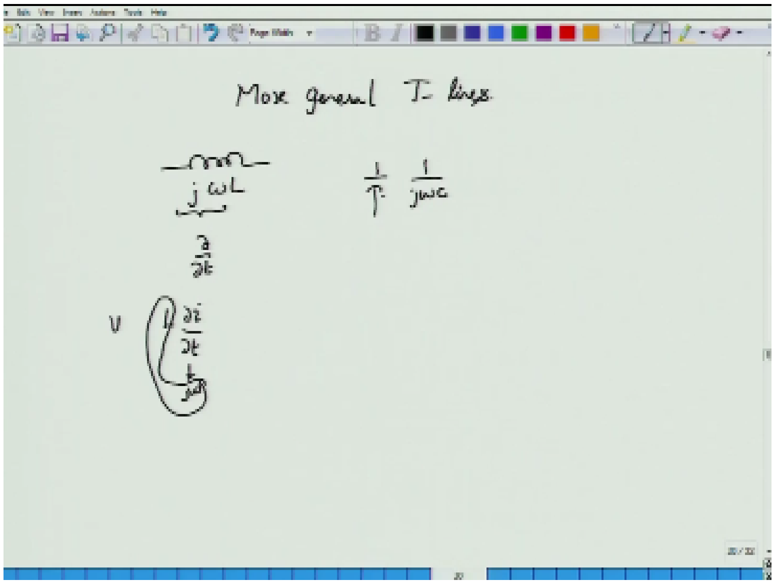

That if I consider and inductor which taking up a voltage or having a current through it at a particular frequency ω the reactance that it presents will be ω l in the complex phase notation this corresponds to j ω l and you remember that this jω is nothing but ∂/ ∂t and it kind of make sense right because the voltage across an inductor will be ldi/ dt and this ∂/ ∂t term can be replace by jω and therefore jω x l will be the complex reactance or the impedance of the inductor.

Similarly for the capacitor you can see that this would be equal to 1/ jω c that would be the equivalent reactance of a capacitor.

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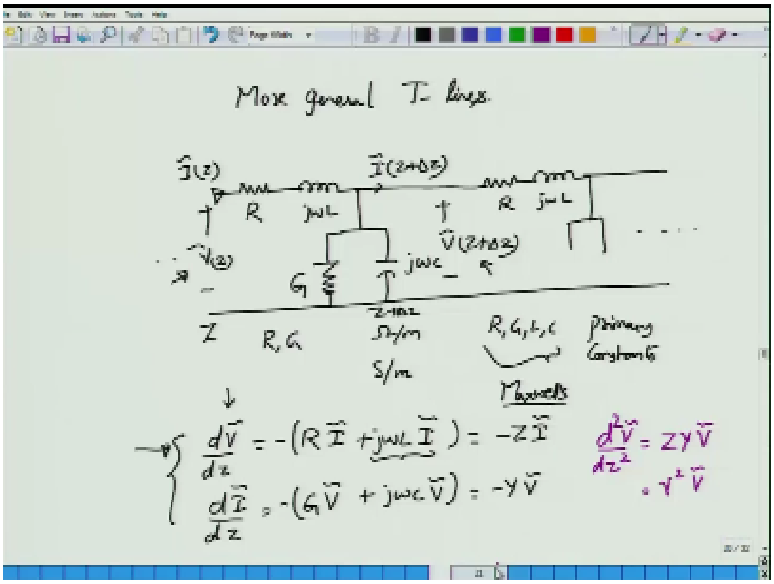

So the unit length or the unit cell of the transmission line if you want to say so will now be different in that it will include the series resistance and the inductance of j ω l reactance and in the shunt case will include a capacitance of j ω c, and then the admittance or the conductance of this particular dielectric loss that is being model will be denoted by g. so this happens to be one unit cell of course in the adjoining unit cell you will again have as R you will have jω l and then you will have capacitor and conductor and so on and so for this would never end.

As before R and G are parameters okay R will be measured in Ω/m and g is measured in s/m which is the unit of conductance and these are per unit length quantities, together R G l and C are called as primary constants of a transmission line. Sometimes also called as distributed constants of a transmission line as before evaluating them requires knowledge of electromagnetic theory which will come after this particular transmission line chapter okay.

So we need to know Maxwell equations to find out g and C but for now we will assume that someone else has calculated this Rg and L and C and they given it to S, and our idea would be to try and find out what would be the voltage is sent currents, to that effect again I consider two planes one at Z and the other plane at Z + δZ okay. So I consider one more plane at Z + δZ and find out the voltage passer here which is be of set, the voltage passer here will be v(z + δz).

There will be some current which would be the current passer IZ and the current here would be I' z + δ z, I can go through the derivation again but I am go to leave this and the small exercise to you to show that the way the voltage passer changes along the transmission lien the voltage difference between v (z) here and v(z+δz) over here will be given by $R\tilde{I}$ + j ω l x $\tilde{I}$ this part is familiar due this is actually coming from ∂/ ∂tl, ∂/ ∂t (I) okay.

Whereas this R x I is simply the omic drop across this resistance R, similarly we have another equation for the current passer which is d I / dz and that would be given by g x v, g be in the conductance, conductance in to voltage will be current plus jω c x v / sorry there is a – sign all over the place j I forgot okay. So there will be a –sign here.

I can even simplify this one and also confuse you a little bit by writing this as – z x I and writing this as – y x v where clearly z is given by R + j ω l and this is called as the series impedance per unit length and Y is the admittance which is series admittance per unit length given by g + j ω c there is a lot of z is going around 1z that is there is the direction of propagation this capital z is series impedance we also use z to denote the impedance at any particular line of the transmission line.

So we have to little bit careful in understating this notations okay, usually the context in which this symbols are used makes it very clear as to what quantity we are talking about. Coming back to this set of equations these are tanks to the passer notation there ordinary differential equations and they can be quite easily solved for example I can solve for the voltage V by differentiating this expression here.

If I differentiate the first expression for the voltage I get d^2 v /dz ^{2}, and that would be equal to –z I am assuming that z is independent of z right, this is the uniform lossless transmission line in to di/ dz but di/ dz is nothing but –y x v, so if you actually solve this I am if you make a second order differential equation for this you wil end up with an equation which says $d^2 \tilde{v} / dz^2$ how the voltage is changing along the transmission line is given by zy x v okay.

You can substitute for z x y and it turns out that it should be one complex number multiplied by another complex number therefore I can write this as some γ^2 x v, where γ will be complex number called as.

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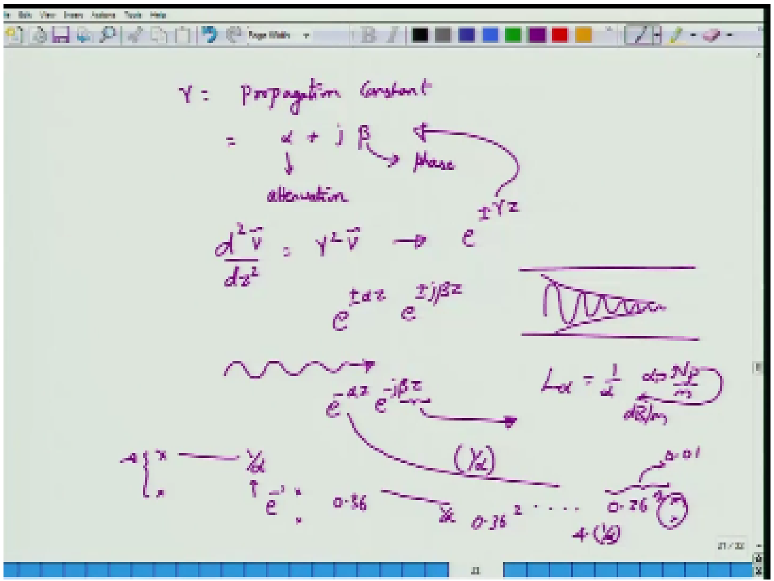

The complex propagation constant okay, the complex constant will have two terms I can al\ways write a complex number as some real number and an imaginary number this real number is called as the attenuation constant or attenuation coefficient and this would be the phase constant or the phase coefficient. Again the equation that we have consider is d2v / dz2 = γ 2 x v the solution of this equation will be in the form of $e^{\pm \gamma x\ z}$ by substituting for γ as α + jβ, you will see that the solutions would be of the form $e^{\pm \alpha z}$, $e^{\pm j\beta z}$.

For a way that is propagating in the forward direction if I chose a minus sign here the voltage passer can be consider as $e^{-\alpha z}$ $e^{-j\beta z}$, the phase continuously gets delayed that is the wave gets delayed as it propagates along the transmission line. In addition the amplitude of the wave also d case, the rate at which amplitude d case is given by 1/ α or rather α it is the rate at which this is given by the amplitude dk and this –βz will tell you that voltage wave is actually harmonically oscillating.

So if you look at the voltage all the transmission line here the voltage along the line would change something like this okay so along the line you would see that the amplitude continuously d case, and the voltage is actually still oscillating at an appropriate frequency and the phase constant of β. If you start at one particular point on the transmission line and then move a distance of 1/ α you know that the value would have drop, the value of the voltage of this point would be just e^{-1} times whatever the voltage that we started out here okay, this is the voltage that we started out at this point times e^{-1} which is about 0.36 times the value would have dropped.

If I go to another 1/α the voltage would have dropped by 0.36^2 if I go four times the value would be dropped to 0.364 of the initial value at 0.364 is approximately 0.01, so you can say that the voltage after 4 1/α turns of propagation length would have dropped by 99% of the original value which means that the remaining value is almost 0 right, if v 0 is reasonably okay the voltage of this point after 4 x 1/ α will be very, very small compare to the voltage amplitude at the initial or whatever point along the z as you consider.

This 1/ α therefore is called as the characteristic or the attenuation length of the transmission line and it is given by 1/α, α is a unit that is measured in nipper per meter okay, α is an attenuation constant that is measured in nipper per meter but in practice this is also given in db/m the conversion from nipper db I will leave this as an exercise to you in the text book it is very quite clearly given to you and I would refer you to text book of there. Okay to summarize what we just did.

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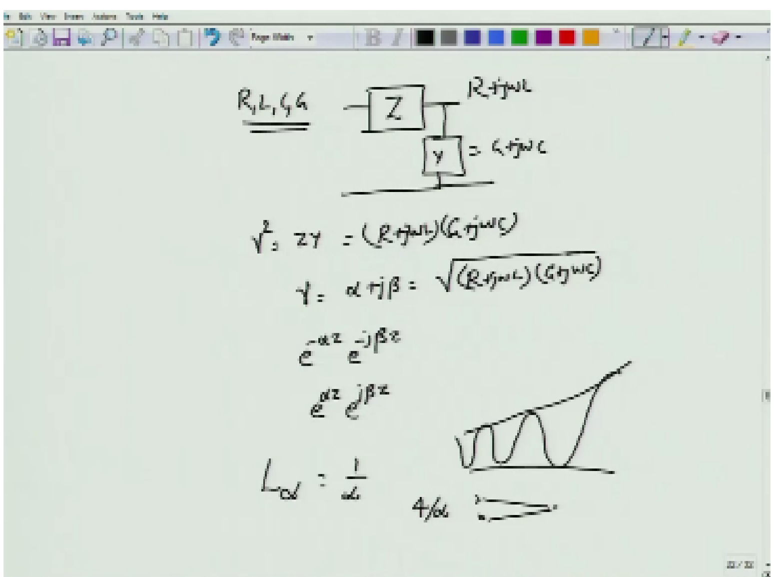

We consider the more general transmission line wherein we had rlc and g all terms related to the transmission line where present it resulted in the transmission line having a units cell with the series impedance and a conductance or the admittance of y, y was given by g + j ω c z is R + j ωl,

γ = or rather γ^2 was z x y which is r + j ω l x g + j ω c the complex propagation γ which is α + j β is given by √r + j ω l x g + j ω c.

The solutions where that we considered for the positive travelling wave was $e^{-\alpha z}$ $e^{-j\beta z}$ if I consider z to be negative that for the wave that is propagating in the left or the backward direction these science can be reversed I have $e^{-\alpha z}$ but please note that z has to be negative for this and then I will have $e^{-j\beta z}$, z is still positive in your coordinate system you cannot have, so you can still considering to be $e^{\alpha z}$ except that the amplitude here would be larger and the amplitude actually8 drops as you go along z.

So yu have to carefully consider which coordinate system that you are using, we frequently do not really worry about this backward forward things because the context kind of makes it clear and the science are taken appropriately as we would like it. The attenuation length or the characteristic length l α is given by 1/α and you can kind of be pretty much sure that that by four times 1/α most of the voltage wave would have dropped to vary small amount of the initial voltage.

So it would have dropped to very small amount of the initial voltage, there is one other thing that we have not talked about for this transmission line and that happens to be the characteristic impedance.

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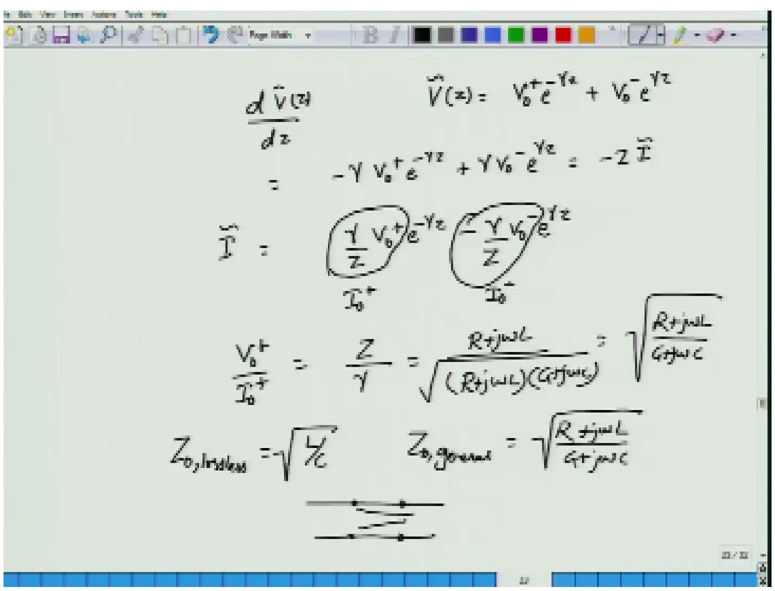

To get to the characteristic impedance let us we first write v as v0 + $e^{-\gamma z}$ + v0 - $e^{\gamma z}$ right, so this is how the total voltage passer can be written and the voltage phase if you differentiate with respect to z what you end up will be –γ v0 + $e^{-\gamma z}$ + γ v0 - $e^{\gamma z}$ but this is nothing but –z x Ĩ ,so this is nothing but –z x Ĩ I can write by taking out this z on to the left hand side and rearranging the equation I can write I to be equal to – γ/ - z, so minus and minus sign cancel with each other, giving you γ/z and v0 + $e^{-\gamma z}$ and then – γ / z v0 - $e^{\gamma z}$.

I can consider this as the amplitude i0 + I can consider this as the amplitude i0- and then take the ratio of v0 + to i0 + th8is ratio turns out to be z / γ, z is nothing but r + j ω l, γ is r + j ωl x g + j ωc okay √ and this would be equal to √ r+ jωl / g + jωc, you can actually get the same relationship by taking –v0 -/ i0- which will again give you γ/z or and there is the admittance and the impedance as z / γ.

In contrast to the lossless case where z0 for the lossless case was poorly real quantity l/c this time you see that in this particular case which can be consider as the more general transmission line case the characteristic impedance happens to be complex right. So this characteristic impedance being complex means that not only the voltage value over at one particular z will be different from the voltage at this point right in terms of phase but then the amplitude also will be different usually the amplitude kind of d case therefore the voltage you start propagating along the transmission line near towards the load will not only reduce in amplitude but will also change in phase okay.

These also introduce couple of additional factors that we will consider in the next few modules, so we stop at this point oaky and we consider few problem solving in the next session and also address a couple of miss constructions that normally is form when we are discussing transmission lines. Thank you very much.

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