Indian Institute of Technology Kanpur
National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module – 07
When to apply T-line theory?
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Hello and welcome to the NPTEL mooke on applied electromagnetic for engineers, in this module we will consider a few conceptual problems and conceptual questions that are associated with transmission lines, which we might have after going through the previous modules and then we will also consider couple of paradoxes are at least one paradox that I talk you about already and the resolution of this paradox. Let us begin by re visiting the topic of characteristic impedance, what is characteristic impedance of a transmission line? We have defined trans characteristic impedance of a transmission line.

Denoted it by z0 when in most case of that we will be consider are the transmission lines without any losses, so in which case that 0 will be completely real and you can instead of calling it as characteristic impedance you can call as characteristics resistance and denoted by r0 okay. But nevertheless what is this characteristics impedance we have that, so far not offered a definition of characteristics impedance except point out, that when we solves the transmission line equations there is how the voltage is changing and how the current is changing through the transmission line.

It is possible for us to define the ratio of the forward going voltage that is v0 + along the transmission line, and a forward going current i0 + along the transmission line the ratio of these two is what we called as the characteristics impedance right. We could also consider the other scenario I could put my source at you know at one particular end, and then extended transmission line all the wave from z = 0 may be towards z = - ∞ okay. Here one side excites the transmission line that is one side connects the source that will be backward travelling voltage.

Where v0 = - and a backward traveling current i0 – backward in the sense that it is going from z = 0 to z = - ∞, again – of this ratio will tell you the characteristics impedance in a case where we have a finite length transmission line which is what would happen in a in practice, then you will have both forward going as well as backward travelling waves v0 + and v0 – in which case to define characteristics impedance you pick either forwards going voltage and forward going current and take their ratio or pick the backward travelling voltage v0 – and.

Negative of the backward travelling current that is – i0 – and that ratio will again be equal to characteristics impedance, now this is a straight forward definition in literature we will find z0 define by other methods consider for now that I have a source here and z = 0 and then I have a long transmission lines stretching all the ray towards ∞, okay so my source if kept at z = 0 and my transmission line is kept all the way to ∞ okay. Now supposing I consider I chop of the transmission line at a particular point okay, so imagine that this is my transmission line okay I cut my transmission line here.

And replace it with the impedance that impedance let me call as z0 okay, I can of course replace it with whatever impedance that I want, but let me choose a particular value of the impedance such that when I connect the source and excite the transmission line, I do not see any reflection back in other words that impedance with which, the transmission line will be terminated or must be terminated in order to cause no reflections is called as the charactestics impedance okay so this is another definition charactetstics impedance that you would actually find.

Of course mathematically another way in which you define the characteristics impedance is to take the ratio of the line impedance per unit length that is z, and admittance y of that one and then take the ratio of z/y that will also denote the charactestics impedance okay. So regardless of hat definition that you adopt it should be clear that characteristics impedance is actually a quantity that is completely specified by the characteristic constants of the line that is the distributed, constants of R L C and G that corresponds to a given transmission line will determine what is the characteristics impedance.

Usually there is a another question that comes to once mind, that if I consider a loss less transmission line, so why would I use a loss less transmission line obviously I do not want any kind of an energy to be wasted in the transmission line that I connect, between the source and the lot okay. So for that reason I will have to consider loss less line whether it is realize that practice or not it is a different question but at this point consider a loss less transmission line, and what is the characteristic impedance of a loss less transmission line it is given by the ratio of L2C √ of L/C.

Is the characteristic impedance, now you might immediately ask well z0 for a loss less transmission line turned out to be real and real impedance is nothing but resistance, but resistance that we have learned from our basic circuit theory implies decapitation of the power or decapitation of the energy, output a loss less line be considered equivalent to our impedance or equivalent to resistance the answer is quite simple, you have to again go back to the characteristics impedance definition that is the impedance which you seen by the source when it is connected to infinitely long transmission line.

Because on a infinitely long transmission line because on a infinitely long transmission line I only have forward going voltage and forward going current my source is kept it z = 0, so only have forward going voltage and forward going current and that ratio is the characteristic impedance, no for a loss less transmission line suppose imagine that you launch a certain amount of pulse energy inbuilt or a lat launch loss a pulse on to the transmission line, this pulse begins to propagate from the z = 0 and keeps going all the way up to ∞. When would the pulse return back to the source after the infinite amount of time?

So in effect this pulse that you have launched on the transmission line moves all the way to ∞ and never comes back, so for as long as the as far as the source is concerned is energy that is represented by the pulse is simply lot. It is like a black whole the transmission line is acting like a black whole because it is not going to give you any return pulse or return energy, but you might object well this is the case for an ∞ long transmission line and in practice you do not have An infinitely long transmission line.

And you will be write, so consider a finite length loss less transmission line what are the characteristics impedance of this one, this is the impedance with which you have to terminate the line right. So that there are no reflections so if I take a finite length transmission line terminate in it is own characteristic impedance I obviously when I send a pulse that pulse is completely absorbed into that notes and 0 that you have connected and no pulse comes back because line terminated with its own characteristic impedance.

Will give you more reflections that is another definition of a charactestics impedance right, so as far a source is concerned again the energy is represented by this pulse is completely lost because it is not getting anything back okay, so it is completely lost and the case is almost I mean the case is identical to that of an infinitely long transmission line. So because of these reasons we represent the characteristics impedance of a transmission line can be represented by a resistance or by a pure impedance pure real impedance okay, let us come back to another paradox that we talked about this paradox was the possibility that.

If I connect my source onto an open circuited transmission line there is a possibility that my v0 + have could have become larger inside it could have become ∞, when I consider a particular length of a transmission line if I consider a λ by 4 length of a transmission line and connect a source to it, and monitor the output voltage on the oscilloscope and do the calculations in order to find out what is v0 + that v0 + would turn out to be ∞ of course in practice you do not find infinities if that you could have found it would have been very nice you could have drawn power kind of perpetual motion.

You could have performed without you know spending any amount of energy which is clearly not possible, so what is the resolution for the paradox I told you that the resolution for this paradox happens either because the line is loss, okay or even if the line is lossless then for that you consider will have an internal resistance RS okay. We will do a short calculation I will leave the steps for you to verify that one later in your leisure time but follow the argument to resolve this paradox, incidentally this paradox is also sometimes called as Ferranti paradox and that was something that was observed in practice not ∞.

But a larger voltage than the launched voltage was observed in early power transmission systems, so what is this paradox again I connect a source okay, so I take my source which could be a sinusoidal voltage.

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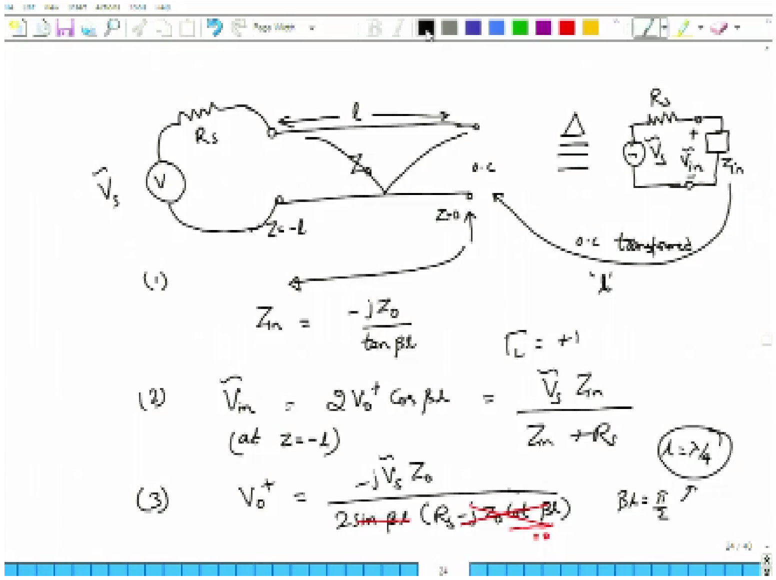

And I represent that source voltage by the phaser, please note this over bar that is denoting the phaser voltage to this I connect or rather this voltage source has an internal resistance of RF which it must write, which is then connected onto a transmission line which is uniform and loss less at this end I open circuit it instead of denoting what is the length of the transmission line to a particular number let me consider the general case and you guys can verify what would happen if I keep changing the value of L, formally to calculate v= + I know that I have to find out the equivalent circuit.

For this the equivalent circuit would have the resistance Rs and the source phaser voltage V connected to the impedance that is seen by the source this impedance reading is nothing but open circuited transformed over the distance L, or the length of the transmission line L okay the voltage across this v in is the voltage v in which we would like to calculate, we already know that an open circuited transmission line would transform into an equivalent impedance so this is the first step right. So I find out what is the impedance Z in of the open circuit transforms to a length L if given by – jZ0 / tan β into L is that 0 is the characteristic impedance of the transmission line.

So this is the input impedance that I have then once I found what is that in it is very easy for me to find out what is the input phasor, V_{in} this is the phaser that would appear across the transmission line terminal okay, cross this equivalent circuit from which we will calculate what is V0 + V0 + being the amplitude of the forward growing voltage, so clearly this V in we already know this is nothing but the voltage that would exist here and I know that voltage would go to a maximum and then it would follow this kind of a magnitude would follow us magnitude of Cos β L kind of acting.

So this voltage I know this is measured at Z = - L at which point, we are connecting the load so this is z = 0 for the load position open circuit and z = - L is where I am connecting the source okay, so this VN is given by 2 V - 0 + Cos βL I hope that you are not wondering why I got this expression here γ L = + 1 γL being the load reflection coefficient okay, now solving this you know or equating the voltage of four this is the voltage V in but from the equivalent circuit what I get from the equivalent circuit this must be VS bar Z in / Z in + Rs.

Rs correct you can solve this I believe this solution as an exercise for you to verify and get V0 + s – jvs bar z0 divided by 2 Sin βL Rs - J that 0 Cos βL cot is nothing but 1 by tan okay, now this is an expression for v - 0 plus let us see if our earlier choice of M equal to λ by 4 will lead to some sort of a ∞ to appear okay, consider two cases I will consider two cases one case will be to try and make β equal to π by 2 βL equal to π by 2 corresponds to the case of n equal to λ by 4 so when you substitute β L equal to π by 2.

Notice what happens yes Sin of βL will become 1 because Sin π by 2 but what is caught βL I know that pan of π by 2 goes to ∞ therefore this cot βL must go to 0, so which means this entire term is gone right and this term has become equal to 1 and I just have this - jvs bar z0 by 2 Rs and if I only retain the magnitude the magnitude of this voltage is given by.

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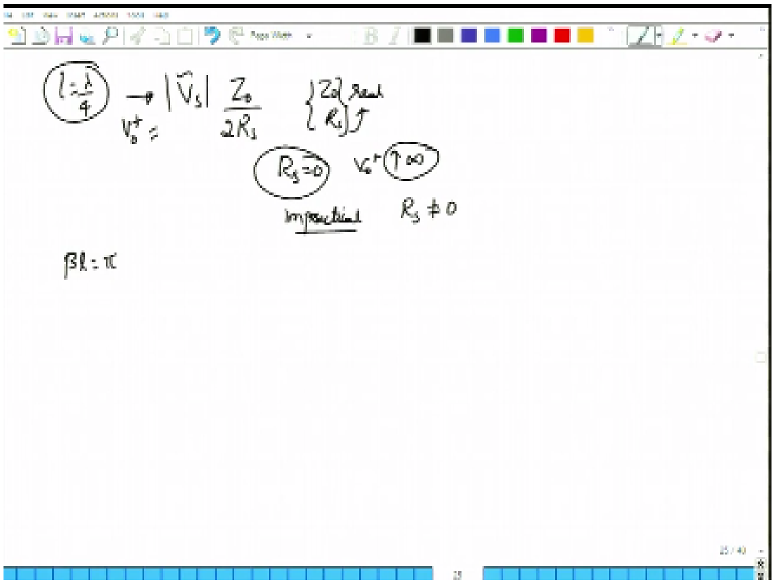

Vs bar magnitude into Z0 by 2 Rs I am assuming that Z0 is real I am also assuming that Rs is just a resistive load internal resistance as resistive source internal resistance and therefore Rs is also real, so what we see is that for the case of L equal to λ by 4 our output of our v - 0 + voltage is actually equal to a finite quantity which is given by magnitude of the source voltage Z0 by 2 Rs, now when you put Rs equal to 0 obviously V0 + goes to ∞ but clearly this is not a practical situation because all sources in practice have some internal resistance for which you have to account for.

So in a in a real case Rs is never equal to 0 it could be low but it could never be 0 and therefore v-0 + could be large but it could not go to ∞ okay, consider another case for example we might consider βL equal to π if I go back to this expression here I know that Sin of π.

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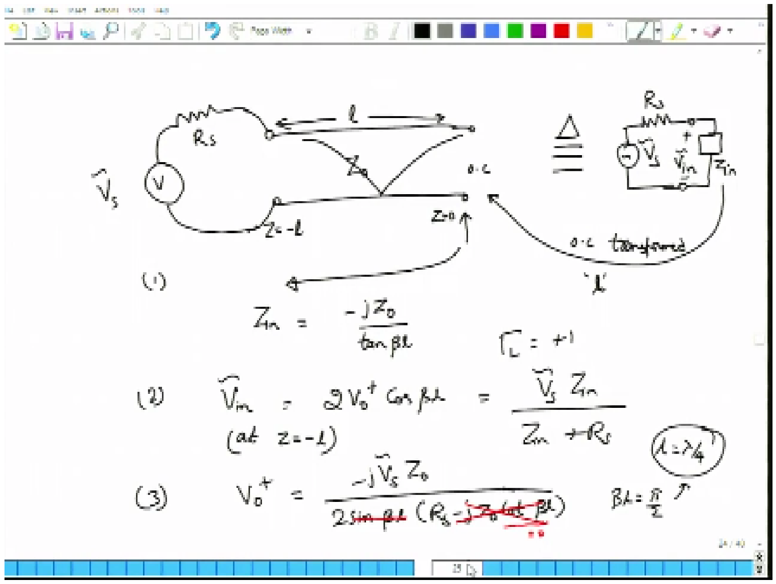

Must turn out to be 0, but then the resolution for this is to completely expand this okay, not just leave it as Sin βL, but to expand it and you get caught of beta L and there is a Sin β L, so Cos βL what would happen cot is nothing but Cos βL by Sin βL but when βL is equal to π cot becomes ∞, so there is an ∞ here there is a 0 here that to have to become finite, so when you expand it through the brackets open up the bracket and expand it this term will actually go to 0 this term will actually be this term will go to 0 this term will go to 1 and what you get again verify this case.

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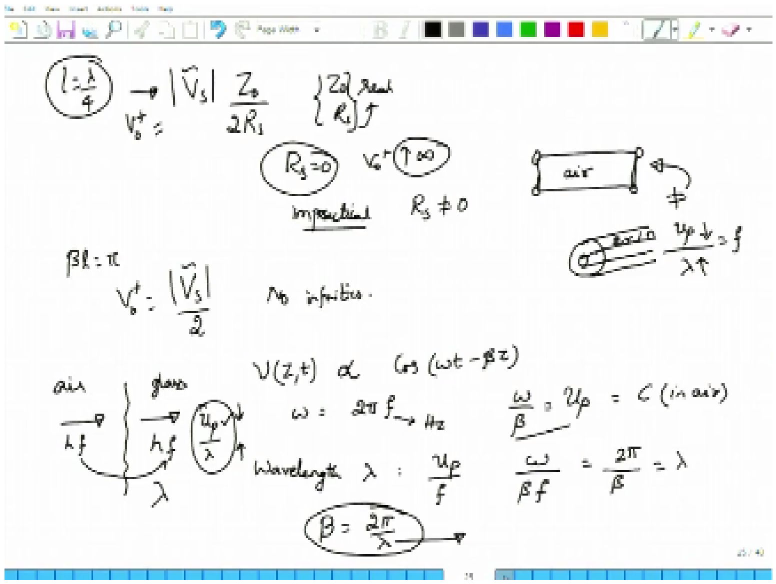

That the v-0 + voltage magnitude of this V0 + would be magnitude of Vs divided by 2 okay so you see again that there are no infinities, let me introduce another parameter we have not really talked about this parameter we have assumed that the voltages that I am seeing on the transmission line would be a travelling voltage wave right of the form cause, so should be of the form Cos ωt - β into Z okay. Already I have told you what is ω is nothing but frequency which frequency of the source which is given by 2π x f ω.

Is the angular frequency measured in radians per second F is the frequency that is measured in Hertz, an older notation for F was in cycles per second but the modern SI unit for this is Hertz, okay we have also seen what is the ratio of ω by β ω by β is the phase velocity and this phase velocity will be equal to C in air correct in vacuum or in air the phase velocity will be equal to C okay, now there is another parameter that we normally talk of which is called as the wavelength okay this wavelength is denoted by λ and it is actually given by the phase velocity of the traveling voltage which would be UT right.

/ SF being the frequency you can substitute for expression of UP from this one, so you have the phase velocity being given by ω by β so this would be ω by β into F but ω is nothing but 2 π into F, so divide this fellow by β into F clearly F from numerator and denominator cancel with each other and what you end up is that wavelength λ is given by 2 π by β alternatively, you can also write β as 2 π by λ. Please note that this λ inside a material will change because λ is dependent on you P, so this is one of the other things that you should remember suppose I consider air and I consider glass okay.

If I launch light or if I think of light and a photon then the energy of this photon would be some H into F, where H is the Planck's constant okay what would be the energy of the photon inside glass most people would think that the energy would be different but actually the energy would always be the same whether you are working in air or whether this photon is propagating in glass, so the energy of this photon will be the same H of s in glass in other words the frequency when it goes from air to glass the frequency of the photon does not really change it does not change what does change is the wavelength λ okay.

Because inside glass your phase velocity changes and the ratio of phase velocity to λ must remain constant because the frequency must remain constant therefore if the phase velocity decreases lambda actually increases why am I bringing up this photon picture is because if I consider a transmission line which maybe you know is a coaxial transmission line, so this is a parallel wire transmission line which is filled with air as a insulating medium in between but on a coaxial cable I can actually fill this region between the inner core and the outer or the second copper enclosure by some material whose permittivity would be epsilon R, or the refractive index will be and we will talk about this epsilon R.

And n what it means is that, the phase velocity here will not be the same as the phase velocity in the air but that is alright for us because phase, velocity usually reduces when I consider an insulating medium but my corresponding λ actually increases, there by heating the ratio to be constant and equal to F so I hope you understand the significance of this wavelength finally, we have talked so much about transmission line but we are not answered we will answer this question.

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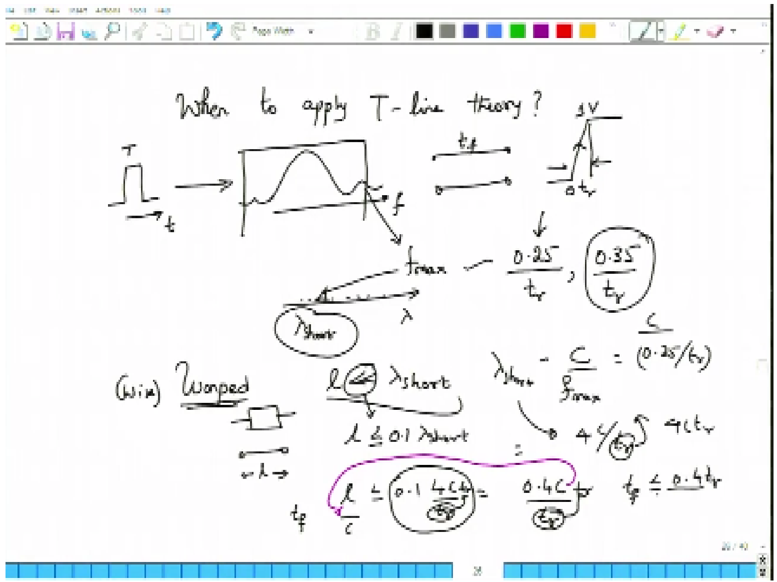

In much more detail with all mathematical ideas later on but we have kind of not really told you if you know when to apply this transmission line theory, so when do I need to consider a particular wire as a transmission line or when do I treat this as a simple wire I will just give you a 5 - minute idea on when we consider transmission line theory to be applicable to the wires that we are considering in the experimental setup or in the actual setting up of the system electronic system you basically understand one thing right.

So when we consider a particular piece of wire as a wire we are not concerned about the time that is required for the voltage at one end of the wire to travel to the other end, where even if I consider this as my wire which is connecting say one integrated circuit here and another integrated circuit at this point, of course it would not be like this but imagine that way when the IC changes its state maybe it goes from 0 to 1 changing the voltage level from 0 to 3. 3 volt if the load has to correspondingly change if the load is just kind of a you knows inverter maybe for example.

Then the inverter input voltage must change accordingly right, so that the output of the inverter must change, but this wire which I have connected which is the interconnect that I am talking about or the wire that I am considering, if the voltage change is immediately available at the input end of the next IC then this wire can be treated as a wire because it has not introduced any delay it has not changed the shape of the pulse that is that is being transmitted or kind of the voltage that is being transmitted.

So we consider as long as the velocity or as long as the delay introduced by this piece of wire to be very small compared to the rise time of the change of the state of the voltage source that is connected to one end, then we usually do not consider the wire as a transmission line but imagine that my IC is changing and it is changing or it is changing its state in about one nano second whereas the delay introduced by this wire because of the length of the wire would or the insulating material that I amusing which slows down the velocity the delay introduced by this wire may be about 10 nanoseconds.

So while the voltage at one end is changing from 0 to 3.3 volt the voltage does not the voltage end does not immediately come over here because there is a considerable delay here. And that change will be visible to the load side only after an nano second right, so that so the load will take some amount of time to see what is the change that is happening and while this is happening is the in no integrated circuit changes from 0 to 3.3.

Then falls back from 3.3 to 0 then this short duration pulse will not be able to be seen by the load side immediately okay, so that is essentially when you have to consider transmission line so in other words since we are talking of pulses and a pulse we will have a typical frequency spectrum right and let us say for our purposes this is the frequency spectrum that I am interested, so this is along the F axis this is the time axis and it said this is my maximum frequency of interest okay but I can also imagine that instead of talking about the frequency.

I can talk about the wavelength then corresponding to this F max there will be a shortest wavelength over here, okay we say that an element is in the lump regime when the length of the element is very small compared to the shortest wavelength of interest, so if I consider this particular pulse of some duration T then you know there will be a certain F max that one can consider and corresponding to that F max there will be a shortest wavelength of course there will be other wavelengths also here, but this is the shortest wavelength that you are considering and this the shortest wavelength happens to be much larger compared to the length of the element.

So if this is my element that I am considering or just a simple wire lead that I am considering if this length happens to be much smaller than the shortest wavelength, then we call this as a lumped element okay, if there an estimate for the maximum frequency well yes the typical estimate for this maximum frequency is 0.25 by tr there are a few conservative estimates which tell you that it must be 0.35 by TR depending on your application, you can pick this particular number I will just pick 0.25 by TR as the estimate for the bandwidth of the pulse that I am considering okay.

So this bandwidth that I am considering will have the maximum frequency of 0.25 by TR, where TR is the rise time of the pulse, so rise time being same ad 0 to 1 volt in the normalized sense whatever the time that is required for the voltage, to change or the pulse to change is called the rise time. And let us say this is my rise time and this is of course my transmission line that I am connecting there is a time delay introduced by the transmission line or let me call as this as time-of-flight that is required, so as I said if TF is much smaller than T R then this can be considered to be a wire.

So lumped means I consider this to be a wire so the length L must be very small compared to the shortest wavelength but I can consider you know in engineering notation that L much less is actually meaning to be less than or equal to 0.1 times the quantity, so when I say that you know a person's height it is very small compared to B person's height my engineering notation is that the height of a is only about 0.1 times the height of B of course that case, is not very nice analogy but you think of this case okay, so left an early mean far less is less than or equal to 0.1okay so this is when I consider it to be a lumped regime.

So whenever my length is much smaller than 0.1 λ short I consider this to be a lumped regime but I know what is λ should I know what is the maximum frequency assuming for, now that I am working in a or you know the phase velocities see the corresponding λ short happens to be C divided by maximum frequency which happens to be in this particular case 0.25 by TR right so this would be C divided by 0.25 by TR, since 1 by 0.25will go to four times see by TR is the shortest wavelength and L being less than or equal to 0.1 times 4 3 by TR this right hand side quantity is nothing but 0.4C by TR.

But length is L right if I divide this length by or other so I have 0.4 C by TR, so if I take this C here okay which is the speed to the denominator of the left hand side then I get L by C which happens to be the time of flight from one end of the wire to the other end of the wire, so my condition for lump regime is that TF must be less than or equal to 0.4 divided by or rather 0.4into TF. So this is not I guess I made as mall mistake over here this should have been 0.4 C into TF so this should actually have been point 4 C into T are in the numerator so this is actually for seat here so please correct this one so I get TF to be less than or equal to 0.4TR.

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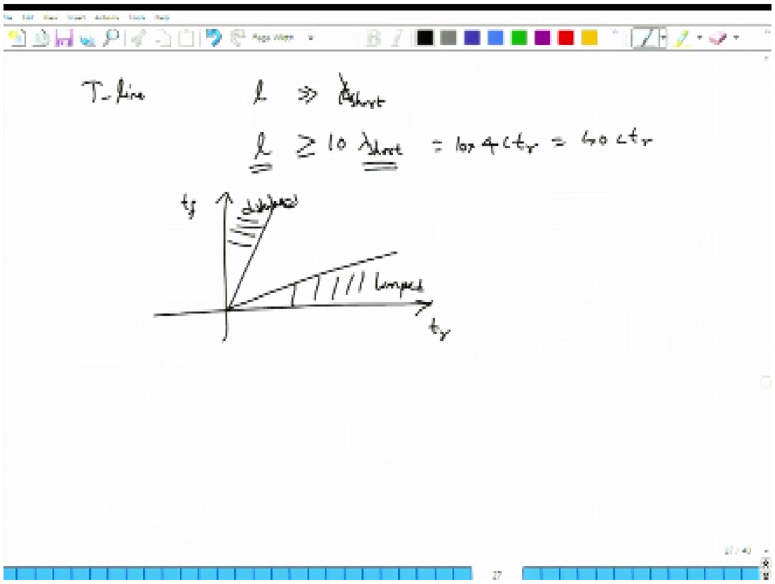

On the other hand my condition for something to be considered as a distributed or a T line effect would be the other way around that is I consider some wire to be you know important in terms of the transmission line, effect when the length of this wire will be greater than or equal to mean much greater than λ short okay. But what is much greater than can be interpreted as 10 times the quantity on the right hand side therefore when L is much larger than 10 times λ short but I also know what is lambda short lambda short is, nothing but 4 C into TR into 10 will give me 40C into T are.

So when the length of the wire is greater than 40 times ctr then I consider it to be a distributed regime, so I can actually plot both TR and TF on this particular axis and you see that this region corresponds to lumped region and this region corresponds to distributed region with this we stop here and consider other affects in the next class thank you very much.

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