Indian Institute of Technology Kanpur

National Programme on Technology Enhanced Learning (NPTEL)

Course Title
Applied Electromagnetics for Engineers

Module – 09
Lumped equivalent circuits of T-lines

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Hello and welcome to the NPTEL MOOC unapplied electromagnetic for engineers this is a module where we will discuss some bits and pieces of transmission line that is still left out with the steady-state considerations steady-state behavior that we are considering. And we begin by trying to find a lumped equivalent circuit of a transmission line; of course we know that the transmission line actually is a distributed circuit okay because it has a certain spatial extent which cannot be ignored.

But in simulation especially when you want to simulate circuit which contains transmission line and other circuit elements the lumped circuit elements in your simulations it is better or, it is useful for you to consider equivalent lumped circuit for a transmission line so that it can be easily simulated along with the other circuit elements so you have a complicated Network then you can reduce that complicated Network which contains transmission lines into a lumped equivalent lumped circuit okay.

There are few advantages of Y you know as I said one would be to help you in the simulation the other is actually more of problem, because if you were to retain the complete transmission line behavior then the simulation of thus those circuit is fairly involved as compared to a simulation of a lump in current circuit, therefore to simplify your simulation and to get some insights which may be reasonably approximate, we normally consider the circuit equivalent of a transmission line this circuit equivalent let me remind you again is only for the case of a steady state behavior at a particular frequency of the source that we are considering.

We are considering sinusoidal sources and for bad cases because the voltage and currents in every place of the transmission line is sinusoidal this equivalent circuit is okay, now let us focus on what this equivalent circuit is and apply the concept of this lump equivalent circuit to understand a small problem okay. To begin with the idea of an equivalent circuit is that.

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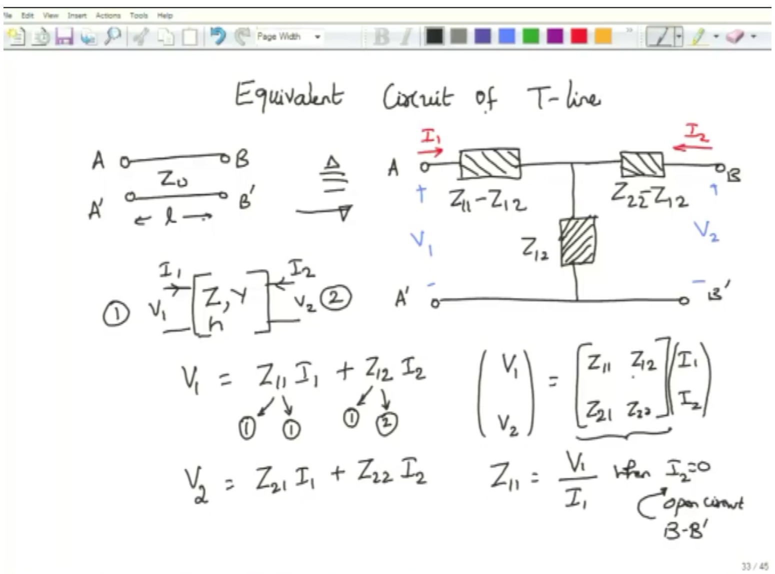

I consider a transmission line okay, which has a certain spatial extent of length L okay. and the transmission line has characteristic impedance of step zero, we consider the two ends of a transmission line to be with the plane a ′ and plane DN B ′what the equivalent circuit will mean is that it can be represented in this particular session where is each of these elements which I have written in the blocks are all the lumped elements okay.

You might rightfully ask if these are lumped elements what will happen to the spatial extent L of the transmission line and we will have an answer in fact we will show that each of these blocks which we have written which we recalling as a lumped equivalent circuit will contain or they will be function of this length L, except that they will turn out to be just some numbers based on whatever the length that we are considering.

To remind you again we are considering only the sinusoidal sources or the sinusoidal steady-state behavior which means my source is actually a favor of a particular frequency Omega right and whatever it has it has a certain amount of value okay .the lower that I am considering will also be ,I mean in what in this one but we will attach load shortly the load that we are considering will also be described by inappropriate value of the complex number which depends on the frequency of the source that we are considering.

So everything is harmonic or sinusoidal in nature okay, so that is what we actuallymean when we say that we are considering sinusoidal behavior and we want to find out the equivalent circuit okay. In your earlier circuit courses you might have studied certain parameters okay. So these parameters relate the external voltages and the currents so for example ,if I consider this two-port Network okay .with current i1 and i2 entering into the pork and voltages v1 and v2 measured there are a number of parameters that can be used to relate these four variables ,some of the examples are the Z parameters them parameter okay.

Sometimes you might also relate them with the H parameter asks normally done for a transistor okay, for passive circuits which are you know represented by equivalent parameters we normally consider them to the Z or Parameters okay. So if I want to find out red parameter circuit for this I need to relate the voltages on one side so I will have to write down that the voltage that I am seeing from the port one here .so one this is a port one this is a port2 okay. Whatever may be the complex circuit inside I might, I am trying to represent that complex circuit just by giving you the value for v1 i1 v2 and i2and relating them by this z parameters.

In terms of the Z or the impedance parameters we have v1 as z11times i1 they are Z11 one is sometimes called as the sending end impedance okay,or sometimes called as the self impedance of the port one there are all these various names but for us this subscript one and one refer to the fact that voltage here is at port 1current is also at port 1 this is some kind of self impedance that we are considering of course.

V1 is not just equal to z1 1 into i1 but there is second parameter which is Z 1 2 in which case the voltage is at the port 1while the current is from the port 2therefore this would be z1 2 into i2similarly we have one more relationship for v2 where v2 is z2 1 i1 with obviously the subscripts known to you how to interpret them + Z 2 into i2,we can in fact condense this notation bywriting this set of two equations in terms of a matrix relationship .

So I have in this matrixes Z11 that 1 2 Z 2 1 and Z 22 with the currents i1 and i2 then considered as the input okay , just to remind you what it is that 11 is actually the ratio of the voltage measured at port 1to the current that is measured at port1, when the current i2 is set to 0 .So when can current i2 be set to 0 or when we can put I 2 equal to 0 when we open circuit that second plane B 1 B ′ so that no current actually flows through this.

The interpretation of other parameters I will leave it to you they are all very similar either your open circuiting the plain D and B ′ or rather Glenn B D ′ or plain a ′ for sync I 2 equal to 0 or I 1equal to 0 and in the bargain obtaining all the four parameters called the Parameters okay. this vector ammeter is just a set of four numbers for a for the case of network which is reciprocal that is it behaves in the same way as it behaves when you go from BB ′ to an ′,as when you go from a ′ to BB′ okay.

This is satisfied in a simple example of a resistor ,it does not matter which way you connect a resistor on the breadboard right you can label the connect resistors as a and B and then connect it as a B or you can reverse the connections and put them as B it simply does not matter, because a register is a reciprocal device it behaves the same way is to turn around their ports or you turn around the connections it has a symmetry of this particular sort okay these four number four in such a case what I want to tell you was in such a case that one two will be equal to z21.

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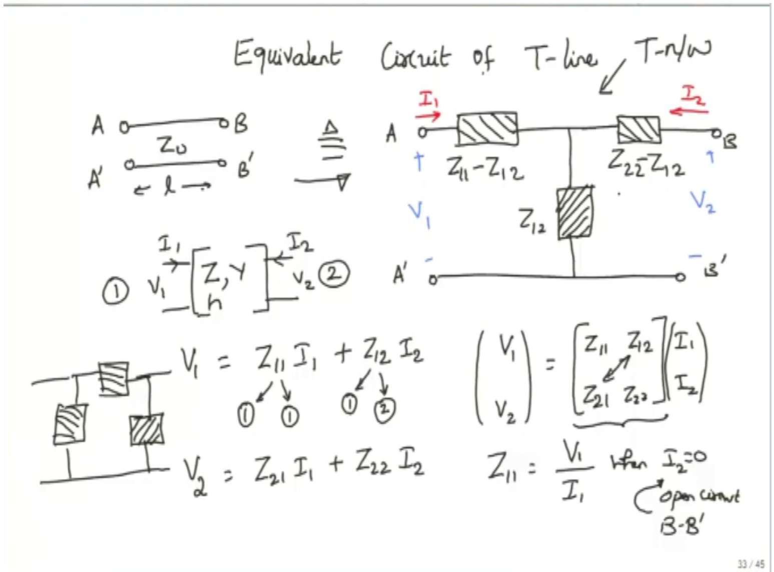

So you do not have to represent by four numbers only three numbers are sufficient to them represent ,one more thing this Z parameters as Said are four or three numbers depending on the network that you are considering but these parameters themselves does not give you a unique circuit representation the representation .That I have shown here in this picture is called as a T network hopefully the reason is quite simple it is called as T Network because it looks like a e right so there is that there is this block there is a second block here and then there is a third block this block that one tree disconnected to these two blocks by common node.

so this is a T Network mean you can always obtain another network which is called as a highly network you minute the blocks with actually be in this way of course the individual values of this pie I mean the blocks will not be the same for the pie and the T Network there is a definite relationship between the two if you are remind it's something of the star and Delta transformation then I think that’s what I am trying to tell you that right so this can be considered as a star Network I if I remember correctly and this can be considered as a delta network or a PI Network .

There is a one-to-one relationship between the T and the PI Network but the actual numbers that are there will not be the same this is quite obvious that coming back to the network that I am considering want to find out what is the T Network and here is a small exercise for all of you who are watching this module.

There is v1 here v2 here in the planes a a crime and BB ′ but the blocks that I have represented are not straight awayz11 or z12 kind of a thing it is this block which is connecting which is to which the current i1 is flowing I am connected to the plane a ′ actually is given by z11 _ z12 the block that is connecting to plane B ′ is z22_ z12 the block that is the standing block of here is that one2 what I want you to confirm is that by applying Klan KCL appropriately to these two loads.

You should be able to show that this network is exact equivalent or exactly represent this particular matrix relationship where V is equal to Z into Z being the set parameter matrix so this you can consider it to be a small exercise this is just to show that you can represent these numbers in any form that you want as long as you are satisfying the original relationships of B Z and I okay.

How do I find out these individual numbers that one dead one two again I do not have to find that tone because the transmission line is exactly symmetrical I can turn around BT a D ′ to a ′ and nothing will happen to the behavior of this class online so what I am to find out are only three parameters z11 z12 and z2.

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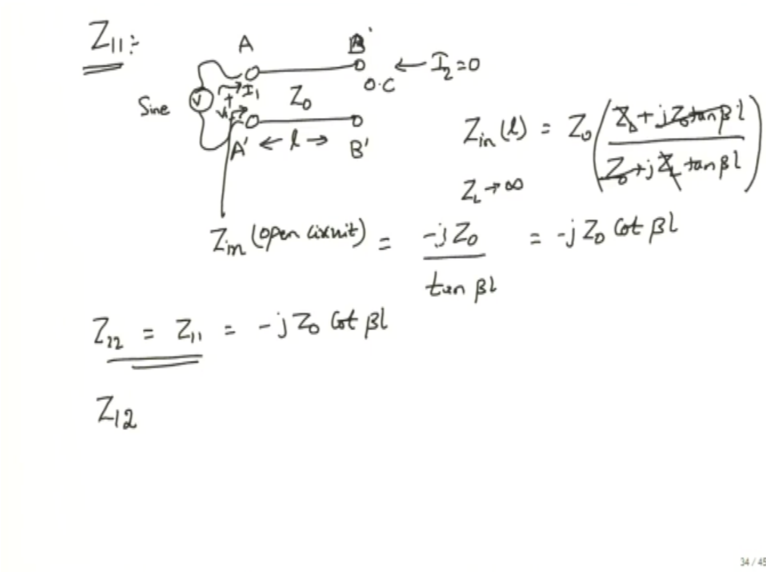

So let us see how to find out will first consider z11 this is fairly easy to find right how do I find that one will you consider the transmission line here okay .which has some a and cape sorry plane a ′ and B D ′ and then you have to open circuit this particular port that is BB ′ plane will be open circuited so that your current I 2 flows through this one.. However here you have to connect a certain voltage source which again will be a sinusoidal voltage source here so that you can drive some current i1 into this particular terminal a ′ plane okay.

So you type certain current i1 and you have a certain voltage V here and what we are trying to find out if the ratio of this V or everyone to the current i1, that is simply nothing but finding the input impedance of a transmission line which has been terminated in open circuit okay, And has a length of L with characteristic impedance of z0, if you remember what is the input impedance formula when in for a length Transmission line was given by Z 0 into L. Which is the load impedance + GZ0 and β L in the numerator divided by just 0 + j z l tan β and so this is what the impedance transformation formula that we actually derived in one of the previous module and the ribbon is open circuited the other.

Tends to infinity in the numerator ZL will be remaining and in the denominator the dell by Del tan β L will be present the 0 and J 0 tan β L terms are gone okay and verbal cancel from both sides so the input impedance of an open circuited line which is of length L is given by _ JZ 0 divided by tan betel or simply _ JZ 0 cot β into Lcotthe cotangent that I am considering within argument of β L because the lines symmetric z22 is exactly equal to z11 which means this is also equal ter_ JZ 0 cot β into L okay.

So we have found these two now what we need too is to find what is the input impedance ,now we have found out z11 andz22 because of the symmetry what remains to be found is just read one to you order to do this one let us go back to the equivalent circuit. this is the equivalent circuit that we had drawn it's little clumsy slight but pleaseforgive that what you can observe is that if I were to sort of short circuit here but if I were to short circuit this one over here and then find out what is the equivalent input impedance right, Icon do that then I can do the same set off condition on the transmission line itself.

That is I will short-circuit the transmission line here find the input impedance that in obviously if these two circuits have to be equivalent the value of Z being that I find here must be exactly equal to the value of Z in then find from this left hand side circuit let us do that here you see what is that in that can be obtained once you short this BB ′ plane you have the 2 Z 1 2 block in parallel with Z 1to a block and then addict in serious with z11 _ z12 okay so you have read in is equal to z22 _ z12 which is the blocks that was there this would been parallel with read one to this entire thing will be in series with z11 _z12 you can find out what would be this relationship and solve for bed one two you might be little surprised Here I am trying to calculate that one two but Iam trying to calculate that that one two from Z in I do not know what is that going to at this point.

I don't know whatisas well however I have an operational procedure as I said I can short-circuit the transmission line and then find out what if they didn't equate dibs or in oruse the value of this Durden into the second equation .so I will leave this verification that after solving this you know above expression for that one two you will actually see that this is equal to square root of Z 1 1 _ Z ok x sub2 - y already know what is there - - I know what is that 1 1 I don't know that already I will find out that once I know all these three parameters it is very easy for me to find out what if that one- so how do I find out where is that into you are considering Z in of short-circuited mode right so we have short-circuited the load.

The transmission line again has a length of and what is the input impedance for this case go back to this impedance transformation formula here you substitute Z L equal to 0 so in the numerator this is 0 and in the denominator the term corresponding to JLtan β L will be equal to 0 and what you are left with after canceling these not that appears in both numerator and denominator this would be equal to+ J Z naught and β into L .now we have all the three parameters that are necessary you can plug that one inside this equation find out what is up 1/2and in fact you can see that this third one - turns out to be after you substitute for all these values it turnout to be _ J z0 cosecant of into L.

I hope you remember the cosecant formula right so secant was 1 by co cosecant is actually 1 by sine and again would like you to verify this using the basic trigonometric relationships know we have everything that is necessary you can easily go back and construct this T equivalent circuit you know what is that 1 this is _ J they are not caught β L you know z1 2 which is_ J said not cosecant of β L you put the _ J cosecant of β L hereandz22 again it would be - j judge not caught beetle that one - a mirror you know subtract the two simplify it and you will be able to write down the Equivalent circuit.

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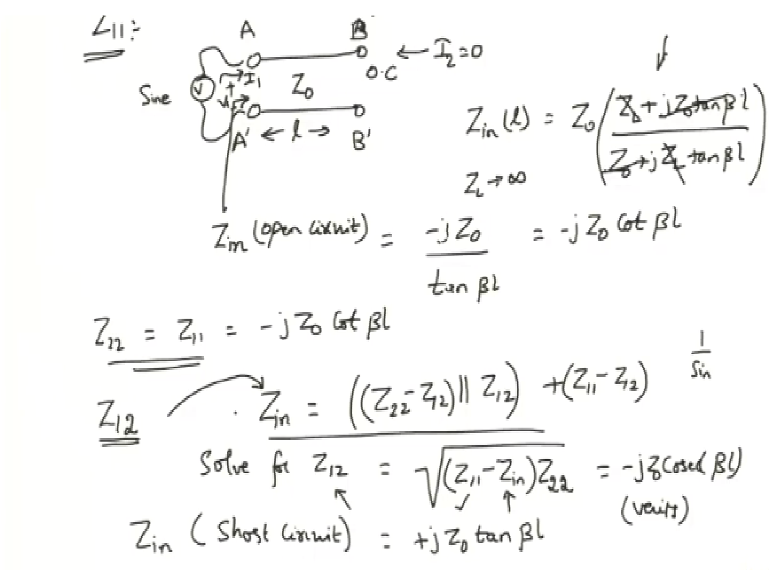

What is the use as this let us consider a very familiar situation ok we will learn more about this one when we talk of connecting Iscor ICS driving loads inductive and capacitive loads but at this point don't worry about the genesis of the problem that let us see how we can apply the knowledge of T equivalent circuit in order to solve the following problem let's consider a printed circuit board on which there is one particular integrated circuit this could be a logic circuit may be an inverter or a NAND gate it could be right now it could be an innovator or a NAND gate this would be driving another load the node could be an interface logic or an interface pursuit could be just an connector that I am considering or connecting to this printed circuit board in order to take the output from the IC one ,or it could be one more I see for example I might betraying to find the clock okay.

Or maybe creating a clock by connecting two inverters in series and the connection between one IC to the other IC happens wherever a wire is length again I cannot ignore and this can be a micro strip line for example acting like a transmission line let us consider that this has an impedance of Z 0 again lossless and the input impedance of the IC 2 is Z 2whereas the output impedance of IC 1 is 1 okay.

The two of course could be complex depending on what kind of input impedance that IC is presenting, but we will consider two cases in one case the magnitude of Z 2 is much smaller than the magnitude of Z 0, another words the impedance of the transmission line is much higher than the load to which it is connected threnody impedance for the transmission line is the input impedance of IC – okay.

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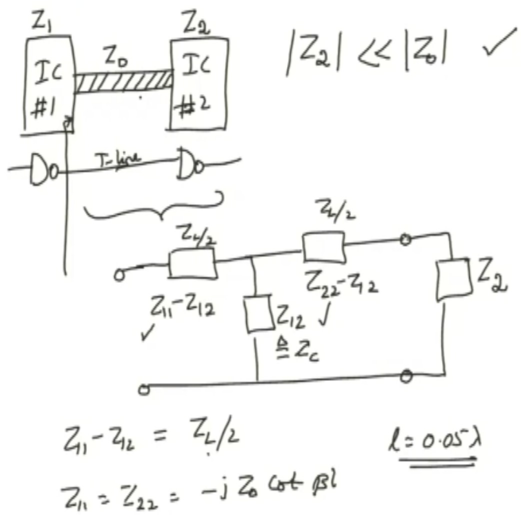

So let us consider this case I will leave the other case for you to worry about okay what we want to know is what is the equivalent circuit in this particular scenario so seen from IC 1, what should be the rest of the circuit to be simplified, again we will go to the approximate T equivalent circuit with these three blocks that I have and then substitute this as transmission line equivalent circuit to which we are going to connect the second I see whose input impedance acts as load impedance for this transmission line and this has a value of z2 okay.

So here you will have a source + red one's the output impedance that can be connected later on in case you are interested so let us not connect this anything right now let us just try to simplify the side of the circuit in order to find what is that I see one is seeing as the rest of the circuit these blocks are from the T equivalent circuit of the transmission line so this is redone - this is the grunion - red one -this is red - 2 _ Z 1 - because the gunman is same as Z 2 - we can define this red 1 _ Z 1 2 as some Z L by2. so that I have in this one there is all by 2 as that this is just a way of denoting they said 1 _ Z 1 2 okay.

So this is the Del by 2 and said 1 2 let us denote it by let's see later on you will see that these subscripts L and Actually represent inductive and capacitive loads or capacitivereactances which is what we will discover later on okay. we will discover the reason what is this double by 2 that I need to find out for that I need to find out that 1 1 _ Z 1 2 I know that is wedge 1 1 in fact that 1 is equal to Z 2 which is _ J that not caught β L okay.

If I consider length to be equal to 0.05 lambda so please note that this is actually a very small length of the line that we are considering although we say that this is a transmission line okay. The length of the line that we are considering is only a fraction of lambda therefore this must almost be a lumped circuit and in fact this is the region where these lumped equivalent circuits actually work very well.

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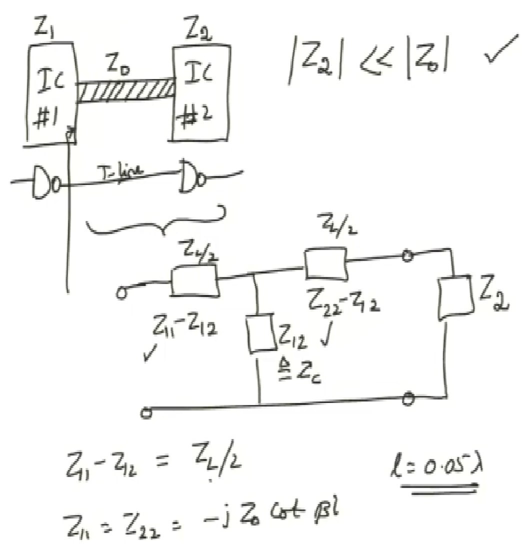

Then lambda Line circuit works very well when the transmission line length is very small compared to the wavelength of the source that you are connecting we are connecting. coming back if I consider lengths of point zero five lambda here I can find out what would be β Redcap we do not really need to do anything here and substituting for β and L here data in terms of lambda because β is nothing but 2 pi by lambda say substitute evaluate this cot and what you get is _ j 3.0 7.0 so this is the value that you get for z11 and z12we can find out what is z12 as well I will leave this as an exercise remember this is _ j h0 cosecant of β L. So which turns out to be _ J3.24 into Z zero okay, so this is the advantage so what is Z L now Z L by 2.

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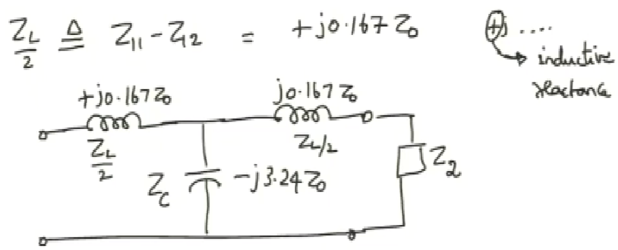

It is actually this is just a desk or diverting that quantity of z11 _ z12this turns out to be + j 0.167z0 and you notice that we have a + Something term over here and this + Indicates that for this particular frequency or this particular wavelength, the impedance or this block is actually acting like a inductor giving rise to an inductive reactance okay. So you have an inductive reactance so this is what this completes all of our equivalent circuits the equivalent circuit would be like an inductor with a value of + J 0.167Z o as del by 2 so this is the Z Lay 2 that we consider and the reactance of z12 which was _ J 3.24 sub 0 is equivalent to a capacitive reactance or capacitive element.

So this was equal to _ J 3.24 into Z 0 then finally I have one more inductor or inductive type reactance which is again ZL by 2j 0.167z0 ,so this is the equivalent circuit of the transmission line to which I can now connect the load dead to here okay. I can connect the load Z to and I am now trying to weight or trying to find out what would be the simplification further can perform, if I don't want to perform the simplification I can leave this circuit as it is connect whatever the source that I want to connect the source would represent the IC one which is driving the second circuit along with the transmission line and you know whatever the result that you get will be okay.

However for this particular case where we are assuming that the impedance of the characteristic impedance of the transmission line is much largercompared to the load to which it disconnected which is that the two of the second I see then let us see what kind of a simplification we will be able to make out okay.

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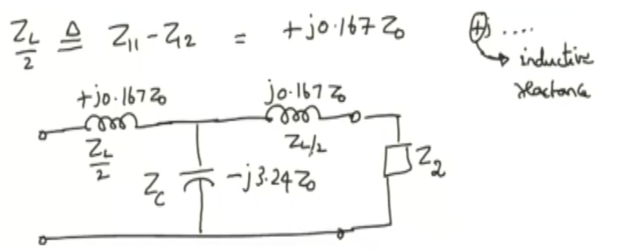

You can see that there are two elements over here this vet to inmagnitude is supposed to be very small in magnitude with respect to Z zero therefore when you add these two terms you can kind of neglect this z2 right you can kind of neglect this z2 and when you further so many neglect that one what you get is one inductor or inductive type of reactants of 0.167 the knots connected in series with _ J3.24 Z naught okay.

So this is what you are going to get and you can find out what would be the parallel combination of these two since this is about half of it I am not finding the exact value here, but I will leave this as an exercise to you to use your calculator to find out what would-be the equivalent circuit here okay.

once you find the equivalent circuit you can then add to this circuit what would happen and you know and then you can obtain the full equivalent circuit over here and as I said I will leave this as an exercise because these are just a few simple things that you can actually workout and when you do and work that particular thing out what you get will be so since this is very large this is very small will give you the qualitative answer the qualitative answer would be that the impedance of the parallel combination will be very close to this that in series with this one would Bethe total impedance.

And essentially what you will see will be that of a inductor okay so essentially what you will see that that would be of an inductor and this would be very nearly double of what you get first Z L so this actually is the impedance that you would see connected down to the low impedance Z to that we have connected to the Isis 2 or this is their impedance Z 2 the lesson for all this let us summarize all that what we have been doing so far. the lesson here is that if I consider a transmission line of very small length but very high input or very high characteristic impedance and connect that to a load of very low impedance then the equivalent circuit or equivalent reactance or impedance that Iam going to find will be that of a inductor.

It will act almost like an inductor so this is something that you should remember a short piece of transmission line with very high characteristic impedance compared to the load impedance to which it is connected will behave like an inductor if we ask the other question that what happens when I consider a piece of transmission line with characteristic impedance that is very small compared to the load impedance over here you can do that calculation also very easily.

Because in that series combination the load impedance dominates that load and short small capacitive reactance of to the parallel combination will turn out to be the capacitive reactance to which you can add a little bit of the inductive reactance that anyway will be smaller you will see that the net result when z0 is much smaller in magnitude compared to z2 of the second. I see that short piece of transmission line will act like a capacitor.

And it will produce or it will present a capacitive reactance to the rest of the circuit to the to the IC 1 this short piece of transmission line with very low characteristic impedance connected to

Avery high impedance mode looks like capacitor ok we will see couple of additional relationships between the input impedance and the various conditions short circuited and open circuited in the next module for now we will leave at this point to ponder over that what would be the PI equivalent circuit of this transmission line I will leave this as an exercise to you thank you very much.