# Fiber – Optic Communication Systems and Techniques
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## Lecture - 12
## Ray theory of light propagation in optical fibers

Hello and welcome to NPTEL MOOC on Fiber Optic Communication Systems and Techniques. In this module we continue our discussion of light transmission through the fibers, using the ray optics approach and I hope you remember this picture, that I have drawn here.

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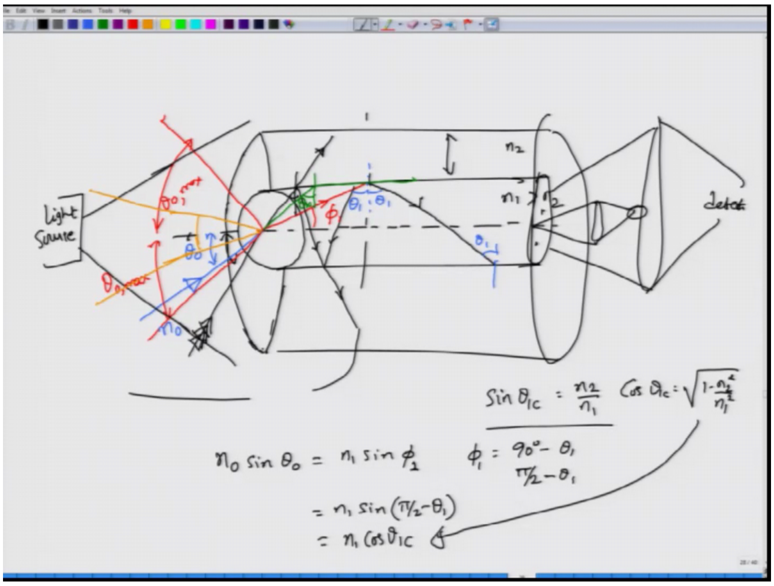

$\sin\theta_{1c} = \frac{n_2}{n_1}$ $\quad \cos\theta_c = \sqrt{1-\frac{n_2^2}{n_1^2}}$

$n_0 \sin\theta_0 = n_1 \sin\phi_1$ $\quad \phi_1 = 90° - \theta_1$

$\pi/2 - \theta_1$

$= n_1 \sin(\pi/2 - \theta_1)$

$= n_1 \cos\theta_{1c}$

Where we had light incident from n 0 which is coming from outside, making an angle of theta 0 with respect to the axis of the fiber, which is the normal to this optical fiber and air interface, once its hits this core or the optical fiber end phase of the optical fiber. There it gets refracted into the cladding and once the angel of incident theta 1 at the core cladding interface is greater than or equal to critical angle, equal to is not really the correct way to look at, because then the light would be propagating along the boundary.

So, you want the light to make an angle theta 1, which is actually greater than the critical angle. So, that this light ray can be reflected off, but even if you were to just assume at the boundary level kind of a thing, that the minimum value of theta 1 required for total internal reflection, would be the critical angle then it is easily calculated.

So, the sin theta 1 c, which is the critical angle sin of the critical angle for the core and cladding interface is of course, given by n 2 by n 1, please remember that n 1 is greater than n 2, because core always has a higher refractive index than the cladding for this type of fibers ok.

So, you have sin theta 1 c equals n 2 by n 1, but if I go back to this part of the figure and then applies Snell's law at the end phase of the fiber. So, I will have n 0 sin theta 0 being equal to n 1 sin phi 1, but from this geometry you know from the red line, the blue line, and this black line that simple triangle. That you have we can see that phi 1 is basically 90 minus theta 1 where 90 is basically 90 degrees actually so, 90 degrees minus theta 1, or phi by 2 minus theta 1 in case your expressing them in terms of radiance ok.

So, this is phi by 2 minus theta 1 let us say ok. Now, I can substitute for phi 1 into this expression and rewrite sin phi 1 as sin of pi by 2 minus theta 1. And then using the sin a minus b formula, which is sin a cos b minus cos a sin b. So, this is essentially cos of theta 1. So, this becomes n 1 cos theta 1.

But as I have told you if fiber to look at this ray of light which is incident on the or which is there at the core cladding interface, just at the critical angle ok, when theta 1 becomes equal to theta 1 c sin theta 1 c is n 2 by n 1 and then I will make theta 1 as theta 1 c ok, for whatever the value of theta 0 that is this happened and clearly if sin theta 1 c is n 2 by n 1 cos theta 1 c is square root of 1 minus n 2 square by n 1 square.

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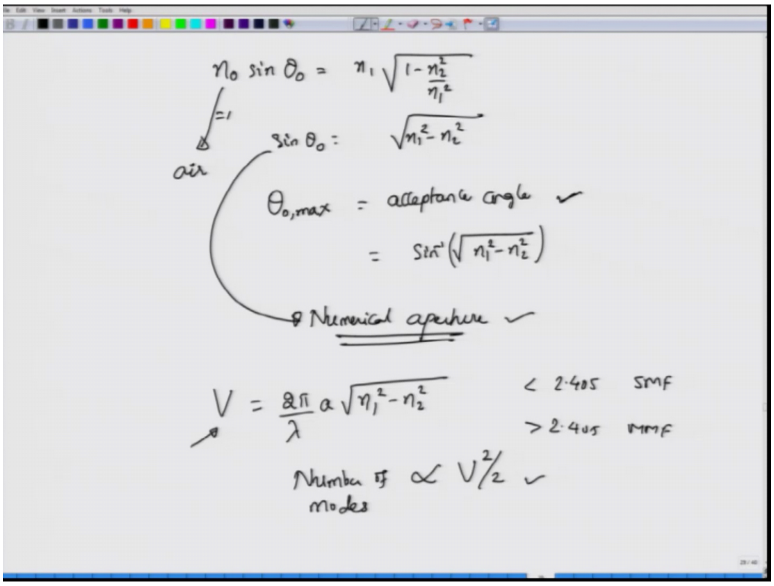

$n_0 \sin\theta_0 = n_1\sqrt{1-\frac{n_2^2}{n_1^2}}$

$=1$

air

$\sin\theta_0 = \sqrt{n_1^2-n_2^2}$

$\theta_{0,max}$ = acceptance angle ✓

$= \sin^{-1}\left(\sqrt{n_1^2-n_2^2}\right)$

Numerical aperture ✓

$V = \frac{2\pi}{\lambda}a\sqrt{n_1^2-n_2^2}$

$< 2.405$ SMF

$> 2.405$ MMF

Number of modes $\propto V^2/2$ ✓

So, I can put that into this expression and write n 0 sin theta 0 equals n 1 square root of 1 minus n 2 square by n 1 square, so, I can rewrite this one. So, I will have n 1 square minus n 2 square under root and, then n 1 comes out and cancels off with this n 1 usually the outside medium we take to be air. So, we take it to be air so, that n 0 actually is equal to 1, because the outside medium I have told you is air.

So, what I have is a relationship which governs the angle of incidents to the optical fiber end phase, or the input of the optical fiber the sin of that angle of incidence, must be equal to n 1 square minus n 2 square at the critical value. Because if I now consider an angle theta 0, which is greater than this maximum angle c. The blue line or let us actually rewrite the lines ok.

So, let us say I have a red line which is the maximum angle theta 0 that I can make ok, which I will call as theta 0 max such that once it goes into the fiber, it just barely makes an angle of critical angle here ok. And then light would continue to propagate just on the interface between core and cladding ok.

So, this is the extreme condition if I were to send in light, which is mark by this particular ray ok. So, this is mark by this ray, this ray because it is falling outside this angle would go into the fiber at an angle, which is actually less than critical angle.

And therefore, would not really propagate inside the fiber, but kind of major part of it goes on to the cladding side only some part gets reflected, but again most of the part goes in the cladding some part gets reflected, because of multiple reflections it keeps losing energy, because every time it hits the core cladding interface it kind of loose us energy.

And therefore, this black ray of light would not really propagate for a very long distance. So, we have a maximum value for theta 0 and, this maximum value of theta 0 is called as the acceptance angle of the fiber, for single mode fiber this is given by square root of n 1 square minus n 2 square ok. The sin of this one is given therefore; this acceptance angle is inverse of sin square root of n 1 square minus n 2 square.

Now, this is very interesting right and this kind of gives us the first parameter to look for when I am going to use of optical fiber for, different applications having this theta 0 max that is acceptance angle much larger, allows me to actually use a light source, either laser or something, which would typically be sending of light in this manner.

So, if I have this angle of incidence which is or if I have this acceptance angle to be quiet wide and of course, acceptance angel is actually an acceptance cone in the sense that you have a same theta 0 max on to the other side as well light is in the form of this cone, or this acceptance angle actually makes a cone kind of a thing.

So, having a larger value means that I am able to tap in or couple more light from the light source ok, because if I have a fiber whose acceptance angle is very small. So, let us say this is the fiber I am running out of colours to right here. So, I am hoping that this will be visible to you. So, if the acceptance angle of a hypothetical fiber is just this orange cone right.

Then you can very clearly see that, we will not be able to get as much light as we would have obtain with the fiber which has a larger acceptance angel. This box in the other way round as well. So, I go to the receiver side ok. So, I am now at the receiver side light is coming out of the optical fiber oh in this manner, if the fiber has a larger acceptance angel, then the fiber will come out with the larger cone and, then I can either put a lens and then converge it and then put this on to a detector ok.

But if light comes out over a very small you know region or a very small cone, then I would not able to couple much of the light into the detector. So, most of the light is actually sitting inside here, if the acceptance angle is small. So, acceptance angle on one hand tells you how much light can be coupled into the fiber, on the other hand acceptance angle also tells you, how much light can be delivered to the detector ok.

And if you want to have a high efficiency coupling system, you want to be able to capture much of the light that comes from the light source, and be also able to deliver much of that light into the detector ok. So, right away there is a trade off in a standard single mode optical fiber is acceptance angle is actually quite small. So, coupling light is not very efficient. So, one has to device methods to efficiently couple light into the standard single mode fibers, but the trade off here is that those single mode fibers have only one mode and therefore, eliminate dispersion based on intermodal dispersion, which is present in the multi mode fibers we are going to discuss all these dispersion concepts later on.

So, do not worry about it right now, but you just understand that having multiple modes for long distance communication is not always good, because it can destroy information in the form of intermodal dispersion ok, it can distort information ok. So, for that purpose so, if you want to carry light over very very long distances distances across thousands of kilometres with very minimal distortion, then you want single mode fibers which come with a smaller acceptance angle.

However, if I am working with the biomedical you know I am going to make a prob to detect something on the skin, or just deliver light on the skin then; obviously, I need a fiber with the larger core area or fiber with a larger acceptance angle and delivery angle, why because I have to gather light that is you know incident, or that is reflected from the tissue and as much light has to be gathered, because that reflected light itself is usually quite weak ok.

So, you want to be able to collect as much light and also to deliver as much light ok. So, if you were to go for that application having fiber with the larger reflect, I mean larger acceptance angle is very important and then those fibers it is usually the multimode fibers which are use, because these multimode fibers have a larger acceptance angle ok.

So, anyway so, you have these acceptance angle thing that we talked about the sin of this angle is what is normally called as the numerical aperture ok. So, you want a fiber to have a very good numerical aperture, and again in a single mode fiber the numerical aperture is usually half of that of the multimode fibers ok.

So, this is fairly standard analysis that I have done. So, I have in fact assumed a step index fiber, because I am nor really looked at the graded index fiber and defined the numerical aperture accordingly for that one, but this should give you the basic idea of what this numerical aperture is all about ok. So, you want a fiber with larger numerical aperture, but you then have to trade off that with the other things.

We have also discussed the V parameter right so, we know that V parameter is given by 2 pi by lambda times a square root of n 1 square minus n 2 square and we have said that, if this is less than 2.405, you have a single mode fiber, if it is greater than 2.405, then you have a multimode fiber ok. So, how exactly this works out I am going to tell you that, but in case you know the V number of a given fiber, then approximately the number of modes that the fiber does support is you know given by V square by 2.

So, when you have a V value of say about 10 then the number of mode that you can support is about 50 which is very very large ok. So, this is what you have about some of the important parameters of the optical fiber. So, when you go to market and buy an optical fiber you are looking for a numerical aperture, you are looking for its acceptance angle and of course, the V number of that one which is determined by the geometry of the optical fiber, itself what is its core radius and things like that.

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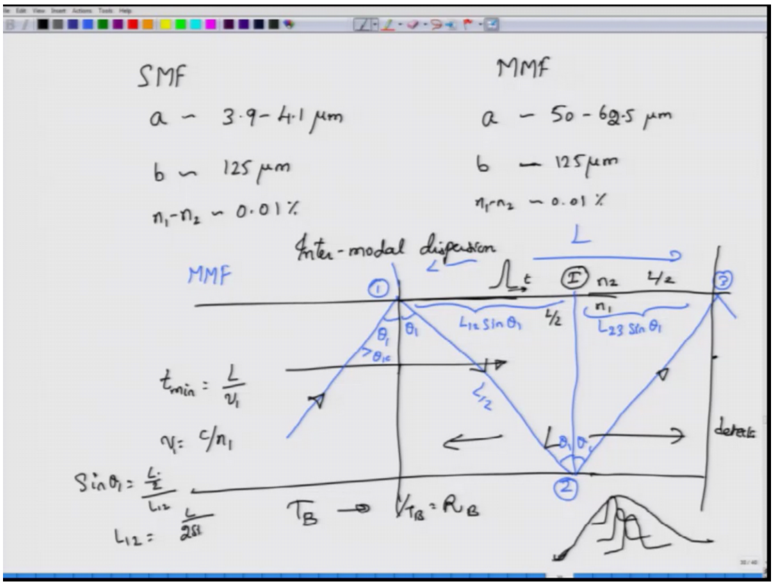

SMF

$a \sim 3.9 - 4.1\ \mu m$

$b \sim 125\ \mu m$

$n_1 - n_2 \sim 0.01\%$

MMF

$a - 50 - 62.5\ \mu m$

$b - 125\ \mu m$

$n_1 - n_2 - 0.01\%$

Inter-modal dispersion

$t_{min} = \frac{L}{v_1}$

$v_1 = c/n_1$

$\sin\theta_1 = \frac{L/2}{L_{12}}$

$L_{12} = \frac{L}{2\sin\theta_1}$

$T_B \rightarrow 1/T_B = R_B$

For standard single mode fiber the core radius a goes anywhere between 3.9 to 4.1 micrometre the cladding radius b is usually standardise to about 125 micrometre ok.

For a multimode fiber and the refractive index difference n 1 minus n 2 is about 0.01, or maybe sometimes less than that percentage ok. So, which means that n 1 and n 2 are very very close to each other almost like there equal, but there is a very small difference between the 2 ok.

For a multimode fiber on the other hand the a core radius is actually quite large, it is anywhere between 50 to 62.5 micrometre and the cladding is still standardized at 125 micro metre radius and, then you have all those core coating and other things, n 1 minus n 2 can still be kept the same, remember I have produced I mean I have kept n 1 minus n 2 same, but to increase the V number I have increase the core radius.

So, I have traded off or I have actually kept this constant, this is anyway constant, but then increased a to support the multi mode fibers ok, but you can also find out n 1 minus n 2 to be slightly larger in a multimode fiber. So, that you can support even more modes, if you wish but the problem with this multimode fiber is that, this multimode fibers are going to distort signal.

Because of an important distortion mechanism called intermodal dispersion ok, what is this intermodal dispersion? Suppose consider you know light incident at an angle, which is say exactly equal to theta 1 c this is core this is cladding I am only showing you the core cladding interface ok.

So, light is incident at an angle theta 1 c and clearly for you know theta 1 c is the angle made with respect to this normal of course, so because of angle being at theta 1 c the ray of light actually starts to propagate just along the core cladding surface ok. So, if there is an information sitting here maybe in the form of a small pulse, then this pulse will be propagated this pulse is of course, with respect to time.

So, this is a light ray incident and this light ray is actually a pulse of light, that is incident at this angle theta 1 c and this pulse of light will travel through this fiber ok, because this is a multimode fiber when you incident light, you normally cannot just incident light at an angle only at theta 1 c or at a specific angle each of these angles which are allowed ok.

Will correspond to different modes so, this will be the fundamental mode, this will be the next mode this will be next mode and so, on and so forth ok, or rather the fundamental mode comes from the other side, but anyway do not worry about that. So, you have this next ray of light which is also carrying the same impulse or which is carrying a pulse ok, incident and now this fellow starts to propagate and it would arrive at the output of the fiber.

So, if this is my output of the fiber that, I am considering, it would arrive at this point with the slightly different velocity ok, or let us not consider this as the output, what I am going to consider is to actually let this ray of light go all the way back I am going to do them something like this.

So, this is not very nice in the sense that I am considering the angles not correctly, but please forgive me this is just to for illustration purposes. The blue light is also propagating carrying an impulse ok, this also propagating and carrying an impulse, but at a different velocity than the other rays of light right.

So, what you actually have seen is that if I were to consider this multimode fibers ok, in this multimode fibers one starts to one couples light into this different modes, identical information identical pulse to this different modes, each of those rays of light which carry this pulses arrived, or travel through a fiber ok, with different velocities ok.

And because the travelling with different velocities at the output, what you would see is one pulse coming here, another pulse coming after sometime another pulse coming to after sometime and, when you place a detector at the output of the fiber, what you get is essentially one big pulse, or one broadened pulse which would then limit, how quickly you can transmit this pulses again.

If you imagine each pulse carrying an information at a certain rate so, that is 1 pulse every T B seconds let us say having a bit rate of 1 by T B or bit rate of R B ok, then you have a constraint or how quickly you can transmit this, pluses because you have to give a certain time, or you to understand that when you transmit a pulse over a duration T B, then the pulse duration can actually increase because of intermodal dispersion, and that can play or that can limit the rate at which you are transmitting information ok.

If you want to estimate what is no roughly the limit of this bit rate because of intermodal dispersion, you can consider two rays ok, one ray which is incident at the critical angle ok, And then the other ray which is kind of incident straight up through the fiber ok.

So, if I consider fiber of length L the ray of light, which is travelling along the axis let us say, will be reaching the receiver in a time frame of or the minimum time of L by v ok, where v is basically c by or v v 1 let us say v 1, because the velocity of light inside the core is basically c by n 1 ok, but what about the ray of light that is incident at an angle.

So, if I have a ray of light incident at an angle, then it will cover the same length L in a different time. So, for example, if my angle of incidence is theta 1, this angle is also clearly theta 1, the overall distance here is L, but this distance L so, this is the half distance. So, this is theta 1 again, this is theta 1 again and the length over which it is actually propagated right is given by in terms of this hypotenuse, this would be given by so, if I call this as a say 0.1 0.2 the length of 0.1 and 2 which I will call as L 1 2 is this one.

So, in terms of L 1 2 this distance is basically L 1 2 sin of theta 1, because sin theta 1 is opposite by hypotenuse right. So, I mean going by the basic trigonometry of this was. So, this length is L 1 2 sin theta 2 and let us say I call this part has 3, or this point has 3, then this would be L 2 3 same angle sin theta 1, because it has totally internally reflected.

So, theta 1 is of course, greater than theta 1 c please understand that 1, or keep that in mind. If I consider the distance 1 2 3 and call that entire distances has L, then that distance L will be the sum of these two distances.

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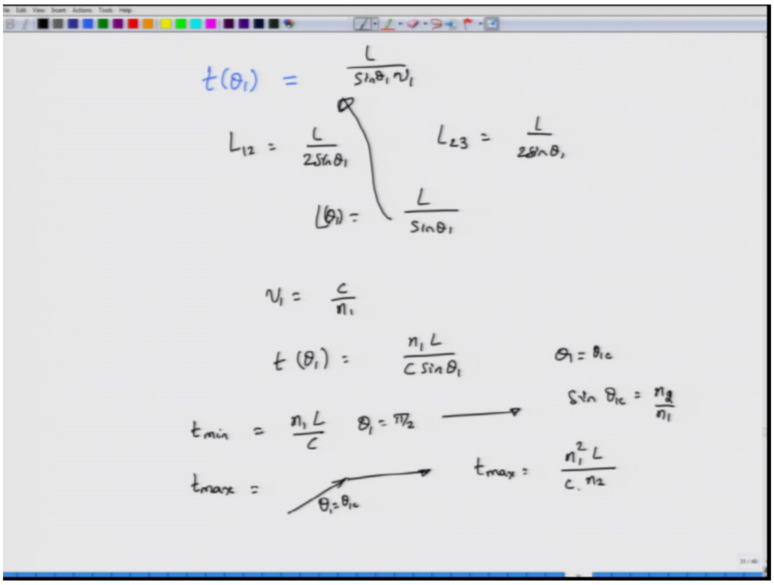

$$t(\theta_1) = \frac{L}{\sin\theta_1 v_1}$$

$$L_{12} = \frac{L}{2\sin\theta_1} \qquad L_{23} = \frac{L}{2\sin\theta_1}$$

$$L(\theta_1) = \frac{L}{\sin\theta_1}$$

$$v_1 = \frac{c}{n_1}$$

$$t(\theta_1) = \frac{n_1 L}{c\sin\theta_1} \qquad \theta_1 = \theta_{1c}$$

$$\sin\theta_{1c} = \frac{n_2}{n_1}$$

$$t_{min} = \frac{n_1 L}{c} \quad \theta_1 = \pi/2$$

$$t_{max} = \qquad \theta_1 = \theta_{1c} \qquad t_{max} = \frac{n_1^2 L}{c \cdot n_2}$$

And the velocity with which for the time delay that is now taken by this ray of light, which is actually making an angle of theta 1 is given by L 1 to sin theta 1 plus L two 3 sin theta. So, this one is sin theta so, this is L 1 2 sin theta 1 L 2 3 sin theta 2. So, for the ray that you know is given by this blue line, if I assume that the distance between 1 2 3 is L.

And there is a symmetric symmetry out there so, that this length which is going from one to say, some intermediate point I these are not very nicely labelled, but please excuse that. So, the length from 1 2 I if I call that has L by 2 and, then the length from I 2 3 is another L by 2. Then the actual length that the ray propagates to get from 1 2 3 is this length L 1 2 plus L 2 3 right.

And what is this length L 1 2 in terms of L by 2 and sin theta, I know that sin theta 1 is given by the intermediate length 1 2, I from this triangle ok. So, from this triangle that I have it is given by L by 2 divided by this length L 1 2. So, L 1 2 is L by 2 sin theta 1 ok. Clearly L 1 2 is L by 2 sin theta 1.

Similarly L 2 3 will also be L by 2sin theta 1, the total length of propagation will be for an angle theta 1 I mean as a function of angle theta 1 is given by L by sin theta 1 ok. And therefore, the time taken for the ray of light, which is actual propagating along this blue line making an angle of theta 1 with respect to the normal core cladding interface is given by L by sin theta 1 and it actually does this in a time span of or in a with a velocity v 1.

Therefore, the time taken is given by L by sin theta one times v 1 and, I know that v 1 is given by c by n 1, which is the phase velocity of the ray of light inside the core therefore, the time taken for theta 1, or ray of light which makes an anger of theta 1 is given by n 1 L by C sin theta 1. Clearly the t min condition that we derive that is the minimum time taken by the ray of light which is propagating horizontally ok, because horizontal means theta 1 equals pi by 2 is valid over here.

So, when theta 1 is equal to pi by 2, you have n 1 L by C which is exactly what the expression for t min that we actually found out right. So, if you combined these two equations you will see that what we have obtained is the same equation. So, you have n 1 L by C when theta 1 equals phi by 2 and the ray of light is travelling in this manner ok.

The maximum time is taken by that ray, which actually travels just at the angle of critical angle. So, when your angle is just at the critical angle, then it would take maximum time duration for it to propagate and that can be obtained by letting theta 1 equals theta 1 c and recognising that sin theta 1 c is basically given by n 2 by n 1 ok.

So, the maximum time is now taken by the ray given by n 1 L divided by C sin theta 1 C is basically n 2 by n 1. So, therefore, n 1 goes on to the top I think so, you will have n 1 square L by C n 2 this is the maximum time duration.

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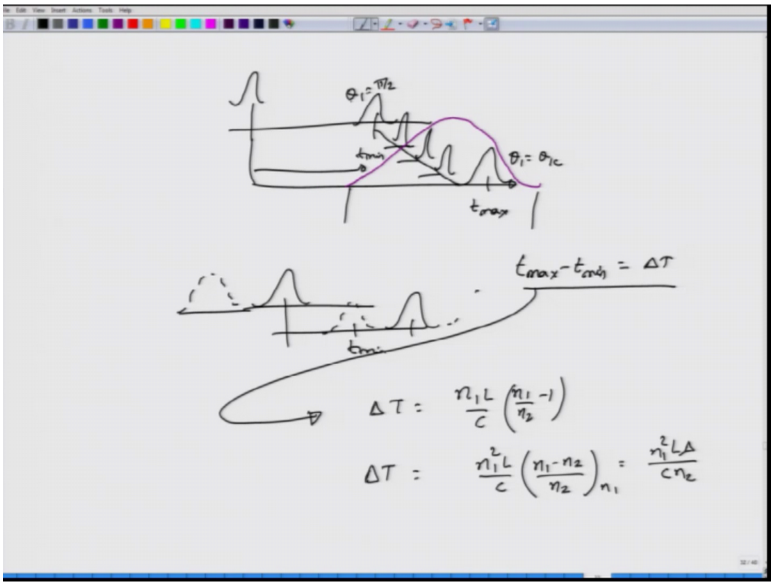

$$t_{max} - t_{min} = \Delta T$$

$$\Delta T = \frac{n_1 L}{c}\left(\frac{n_1}{n_2} - 1\right)$$

$$\Delta T = \frac{n_1^2 L}{c}\left(\frac{n_1 - n_2}{n_2}\right)_{n_1} = \frac{n_1^2 L \Delta}{c n_2}$$

So, I being at no 0 or at whatever the time which is taken minimum time of arrival as measured from 0 axis and, then you have another one which is arriving ok, with the max angle of or with the max time of t max. And there will be all this intermediate pulses so, corresponding to different angles of theta 1 between this is for theta 1 equals pi by 2 and this for theta 1 equals theta 1 c, for all angles between them they would all be arriving at different times which are between t min and t max.

So, your overall pulse would actually be at least of this broad end value. So, you have seen that. So, you I mean you have just seen that the pulse essentially gets broadened and if I had only a single mode of propagation, then I could just transmit write after this arrives at the receiver t min, then I could transmit another pulse ok.

So, maybe another pulse follows here so, pulse would come you know arrived at the same time t min. So, it would be possible for me to send multiple pulses, I mean pulses at a much faster rate, because the pulses are not getting broadened ok, we are ignoring other types here, but the pulses are not getting broadened.

But because of this broadening nature I have to wait for a quite a bit of a time, at least I have to wait for a time duration of t max minus t min is the amount by which it is broadening. So, I have to wait until this time delta t, before another pulse can be launched. So, because this t max minus t min which is essentially determine in the amount of pulse broadening is coming because of multiple modes information carried same information carried by multiple modes of the fiber.

This type of a dispersion which was very wide spread in the 19 17s fibers is called as intermodal dispersion, intermodal means dispersion or the difference in the velocity giving rise to pulse broadening because of multiple modes in a multimode fiber.

So, this is the pulse broadening that you are going to obtain and, if and you can actually substitute for the expressions of t max and t min and find out the expression for delta t, which turns out to be n 1 L by C n 1 by n 2 minus 1. So, I am going to rewrite this one as n 1 L by C n 1 minus n 2 divided by n 2 ok. So, this is the expression for the amount of pulse broadening delta t.

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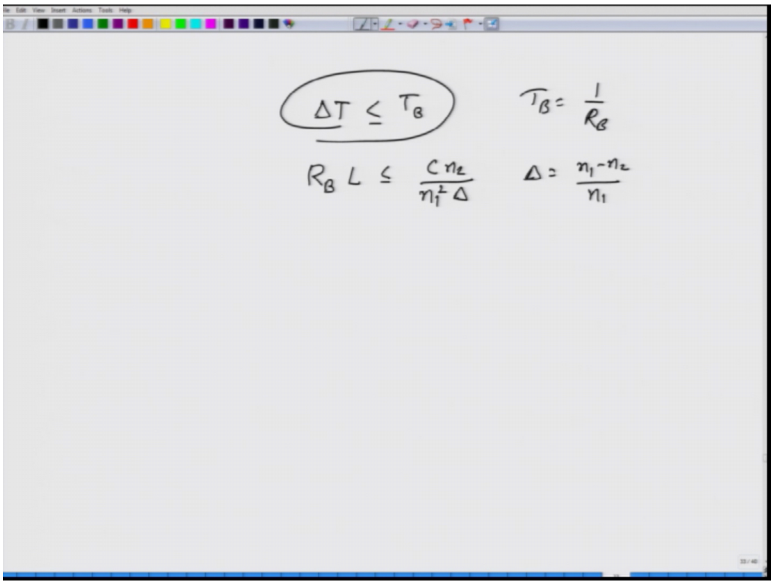

$$\boxed{\underline{\Delta T} \leq T_B} \qquad T_B = \frac{1}{R_B}$$

$$R_B L \leq \frac{C n_2}{n_1^2 \Delta} \qquad \Delta = \frac{n_1 - n_2}{n_1}$$

And this pulse broadening eventually have to be in such a way that, you the pulses broadened by less than say amount of T B which is the pulse, or the bits slot that I am sending out. So, if this delta T is less than or equal to T B, then 1 I mean this is one criteria of course so, if I let that the pulse broadening be less than the time slot, or equal to 1 time slot, then I will be able to communicate at this particular rate ok.

So, if I impose this condition, then I will actually get and since I know that T B is basically 1 by R B, you can rewrite these conditions in terms of R B L being less than or equal to C n 2 divided by n 1 square delta, where delta is usually given by n 1 minus n 2 by n 1 ok.

So, you can go back to even this expression multiply by n 1 on both sides and therefore, this becomes n 1 square and what you have is n 1 square L delta by C n 2 ok. So, this would be the expression for delta t and because I need to restrict my pulse broadening 2 less then the time period T B, that will itself impose you know the amount of the it will impose a limit on the amount of or the rate at which I can transmit pulses and hence information ok.

So, this product R B times L where R B is the bit rate and, L is the length of the fiber, or the length of propagation is very important. So, this is called as distance bandwidth limitation or bit rate length product ok.

So, for the fibers this bit rate length product is reasonably large, but for a copper this bit rate length will be very very small and of course, that explains one of the reasons why we switched over from copper to optical fibers ok, we will do a more rigorous analysis of dispersion in the coming modules.

But for now it is important to understand whatever that has been done is only based on the ray theory approach, we have not really looked at how electrics fields are distributed inside the fiber and, how they propagate what essentially happens at the core cladding interface, to do all that, we need to look at electromagnetic wave nature of light and use Maxwell’s equations to study light propagation inside the fiber.

Thank you very much.