Fiber- Optic Communication Systems and Techniques

Prof. Pradeep Kumar K
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 02
Review of Maxwell's Equations

Hello and welcome to NPTEL course on Fiber Optic Communication Systems and Techniques. This is our second module and this is the module where we actually begin the study of fiber optic communication systems. In the last module you might have seen the system of fiber optic communications as it would look today and you would have seen that there are lot of elements in that right, but the basic fact is that information in an optical fiber is conveyed by transmitting light and modulating one of the properties of the light.

So, how exactly do we generate light is a subject of lasers and LEDs to which we will come sometime later in the course, but how do we actually describe light first of all is the topic that we would consider now. This is important because understanding the properties of light or light waves as you know you would shortly see is critical to understand the behavior of light inside an optical fiber. So, with that in mind let us actually start our study of fiber optic communication systems by looking at optical fibers itself. So, once we understand the fibers, we then go and understand the sources light sources that is lasers and LEDs.

And once we understand lasers and LEDs then we go and understand the detectors and once we have all these 3 basic components of a optical communication system or a fiber optic communication system understood, then we will study how to modulate, how to efficiently transmit light. If there are problems in fiber transmission that is light as it propagates through the fiber what are we supposed to do to overcome those problems and make our communication systems better in terms of very high data rate, very low bit error rate ok.

Let me begin by recapitulating something that you already know of light of course, can be modeled in many different ways and I am using the word model because no one really knows exactly what light is, you know you have certain physical phenomena and then you develop your own understanding of that physical phenomena by abstract means which we call as models.

For example, you must all know Ohms law you have a resistor and then if you apply a certain voltage across this resistor then there will be a certain current through it right and you must have done many experiments, but if you actually look at a resistor you will see that it is a physical piece of you know device right and what you have captured is mathematically the relationship between the voltage current and the resistance that this particular resistor is offering.

So, you would never understand or you do not want to complicate your understanding by knowing exactly what material this resistor is made out of, how exactly is this resistor connected and how is it that some someone has guaranteed that this register gives you say 20 kilo Ohms of resistance right. So, those are the finer details to which you do not want to get into it perhaps does not serve your purpose, but on the, but to understand this resistor and it is behavior in circuits it is enough to understand Ohms law.

So, what you have is a physical device and what you used to understand, what is going on when you connect multiple such resistors is the abstract model of the resistors. So, much in the same way you can think of light and various phenomena that involves light right. So, you take light pass it through a lens, you take light you know and then pass it through a cavity or to take light and then reflect it off of a mirror, then light you take 2 different kind of light sources or you take the same light source and split it into 2 parts and then inter make them interfere with each other, these are all the phenomena which involves light.

And depending on the phenomena that is involved and the simplicity of the explanations that are required you can think of light as rays. So, when you take a piece of pen like this and then take a piece of you know or you take a bucket of water immerse this piece of I mean immerse this pen inside the water to describe what happens, what you see that the pen appears to be bent right you do not want to include lot of details you can think of and arrive at conclusions which are basically correct by treating light as rays.

But there are situations where you cannot just treat light as rays. So, this actually happens when you start thinking about light propagation inside an optical fiber ok, there you need much more sophisticated model which we call as electromagnetic model of light. So, what we claim is that light is an electromagnetic wave and electromagnetic waves are studied by means of certain equations and certain theory known as electromagnetic theory and this equations which describe electromagnetic theory are due to Maxwell and many other researchers in electromagnetic.

And we use Maxwell's theory of electromagnetic waves to understand light, please remember that this electromagnetic theory can explain many phenomena which involves light where light appears to act like a wave ok. However, there are phenomena where light appears to act like a particle for example, in the photoelectric effect which we would not really be able to treat it you know when we think of it as an electromagnetic wave.

So, there are cases where you hand you handle light as a wave handle light as a ray and then handle light as a particle or particle like behavior called as photon behavior and that goes to the quantum theory of light. So, you have these different models not because you are just interested in coming up with different models, these models serve the purpose of explaining the observed phenomena in the most appropriate way ok.

So, having said that we now go on to electromagnetic theory I will not describe electromagnetic theory from the first principles, I assume that you have all done a course on electromagnetic theory in your earlier semesters and we will use only some of the aspects of that electromagnetic theory, do not worry we are not going to use Coulombs law we are not going to use you know I mean you are not going to solve problems using Coulombs law or Biot Savart's law or anything like that.

We stick to reasonably basic electro magnetics that we need and most of these electromagnetics happens to be use of Maxwell's equations and manipulation of these Maxwell's equations as you will see. So, having said that what are Maxwell's equations these Maxwell's equations describe almost every known property of electromagnetic waves right from d c that is 0 frequency all the way to optical frequencies.

Just for the record optical frequencies are in the range of terahertz and more whereas, microwaves electromagnetic waves are in the range of few gigahertz 10 to few gigahertz of range and then there are of course, various other frequencies which are important and this entire thing is called as the electromagnetic spectrum of which optical is at the higher frequency range and those are the things that we are quite interested in and it is interesting that all of this phenomena which is at a wide frequency scale is described by a very few equations supported by certain material properties.

There are 4 equations traditionally known as Maxwell's equations and couple of additional equations describes the properties of the materials and these 4 equations connect 4 electromagnetic quantities, what are those electromagnetic quantities? You have what is called as the electric field intensity.

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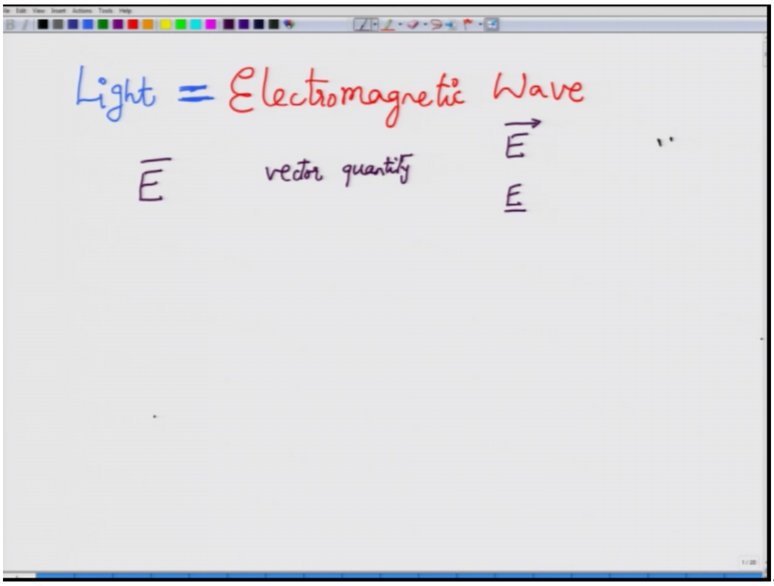

And you can see that after writing electric field E which is denoted by the symbol E I put a bar over here, this bar indicates that this is a vector quantity ok. This is my notation of using a vector you might see people using different notations for example, writing an arrow is considered another equivalent way of talking about a vector some books or some literature uses an under bar to denote a vector these are all equivalent representations and these vectors are of course, very different from scalars in the sense that vectors have direction.

So, these electromagnetic waves which I use by point putting a bar overhead is what is called as a vector and it has of course, direction and this electric field vector or electric field intensity actually can depend on the various, I mean it can be different at various points.

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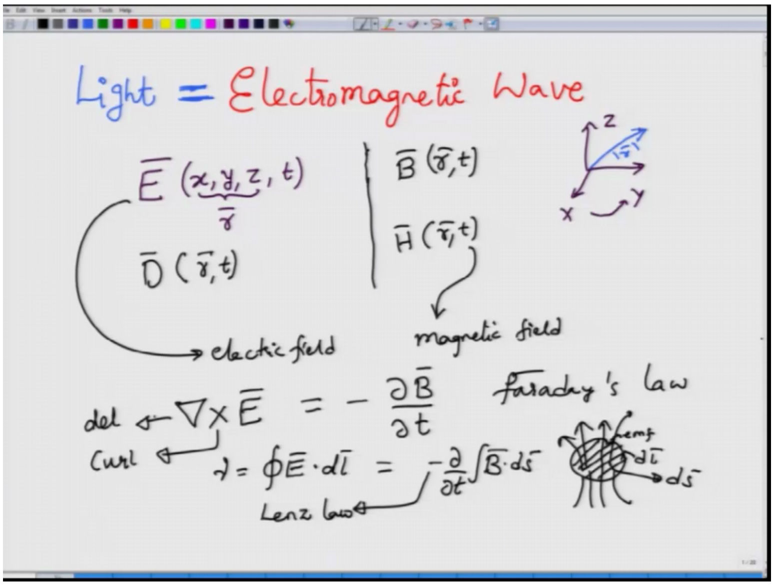

Therefore, we generally denote them of course, after choosing a certain coordinate system in this case I have chosen the Cartesian coordinate system which has 3 perpendicular axis you know this very well these are the x y and z axis and this is the. So, called right handed coordinate system in the sense that if you rotate from x to y your fingers the thumb finger would actually point along the z direction.

So, this electric field intensity or electric field as we would abbrev you know call it for short depends not only on the space or you know the points x y z, it also in general depends on time of course, when it does not depend on time you go to what is called as electrostatic phenomena, but when it is dependent on time you go to what is called as an electro dynamic, you see static time equal to or it is independent of time dynamic where things change with time.

So, it is our dynamic field quantities and we use a shorthand notation instead of writing x y z, we use a shorthand notation to denote this by a vector point a r ok, r is the position vector of space. So, for example, if this is r then, this is the position vector drawn from the origin, the distance from the origin to the point here will be the length of this vector and this is the position vector that we can specify. So, any point every point can actually be thought of as being connected to the origin through a line of appropriate length and therefore, you can write electric field in as a function of these points where we are using this vector notation.

So, these are the position vectors and then you are talking about this position vector and time dependent field quantity ok. So, Maxwell's equations do not only talk about electric fields, you also have what is called as a displacement or electric flux density vector, even this flux vector is dependent on both r and t. Then you have the magnetic field quantities which are b the magnetic flux density sometimes it is called or magnetic field it is called in some text. So, does not matter what it is so, we denote them by the letter B and then you have magnetic field intensity which we denote by H.

So, in my ordinary usage I will call this as electric field just to simplify you know instead of talking every time that this is electric field intensity, intensity I will just call this as electric field, similarly I will call H as magnetic field ok, please note that all these field quantities are dependent both on position as well as on time. So, in general they are time and space varying quantities they vary with space they vary with time.

So, how are these equations connect I mean how are these quantities connected, well one of the laws which is originally due to Faraday tells you that the time rate of magnetic field is the reason why you actually have an electromotive force or it induces an electromotive force. So, if you take a certain experimental situation where the magnetic flux density, which is obtained by integrating this B over whatever the area and if this magnetic flux is changing with respect to time right, then that induces certain e m f in the electric wire that it would be there.

So, the exact details of these equations I am assuming that you would have you know read in electromagnetics courses therefore, I am not showing you I am not going into lot of details here the objective is to just write down Maxwell's equation and point to couple of interesting I mean important relationships and then move on to consider the manipulation of these equations ok. So, I am not going to the physical experiments which led to these equations I am hope hoping that you would have done all that in the electromagnetic theory courses earlier.

So, this equation which was originally due to Faraday is called as Maxwell Faraday law or Faraday's law just for simplicity and the form that, I have used is what is called as the differential form. So, in the differential form of writing, you have these operators which are called as point operators and the special symbol, which is inverted triangle is what I call as a del operator ok.

So, this is an operator and this particular cross is denoting what is called as a curl the corresponding integral version of this law which in some sense is much more general goes something like this, you have an electric field integrated over the length or the closed loop which for example, could be made out of a simple wire ok. So, the closed loopness is indicated by this integral I mean by this circle symbol on the integral ok.

And whatever the quantity that you have E dot d l inside from when you integrate it over is what is called as the e m f and sometimes this e m f is denoted by this particular letter nu ok. So, this denotes an e m f over the entire loop in the static case when there are no magnetic fields involved this e m f over the entire loop will be equal to 0 this is your familiar KVL equation ok. So, there are no sources magnetic field is not changing and then in as you pass along the loop you would see that the total e m f induced would be 0.

So, there might be some elements where the e m f is increasing, but there will be some elements where the e m f will be decreasing with the net result that in the static condition the e m f will be equal to 0, but not so, when there is a magnetic flux. So, for example, if this is a wire and just put the dot to ends to measure the e m f here if there are magnetic flux lines which are changing with respect to time ok. Then this changing magnetic field or changing magnetic flux induces a nonzero e m f and the amount of that induced is given by this particular equation.

So, this is the change in magnetic flux. So, this around the loop will be d l and the area that this is covering the wire or the mesh is covering or the loop is covering is the surface area d s. So, when you integrate the magnetic field over that surface you will get the flux and the time rate of change of this flux the negative time rate of change of this flux is the amount of e m f induced and this minus sign comes because what is called as Lenz law ok.

So, please refer to electromagnetic theory courses to get a better understanding of these equations ok. So, we will now write only the differential forms the integral forms you can look at the textbooks and then you know you will be able to write them out. So, that was one equation which connected E and B.

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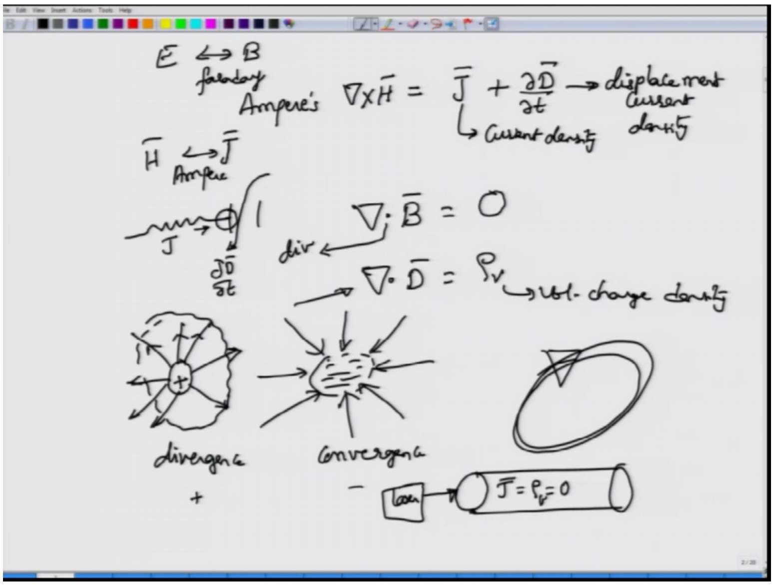

So, E connected to B via Faraday ok, what is the other equation. So, consider del cross H the same curl operation, but now performed on the magnetic field H before Maxwell this was equal to J, where J is called as the current density. So, if you have a source of current you know producing battery for example, connected to a wire producing this current then there is a certain current density.

So, you before Maxwell came, you know to understand all these things this was known as Amperes law Amperes law connect H to J before Maxwell ok. So, this was Ampere, but what Maxwell observed after looking at these equations are that these equations are not consistent with what comes out from these relationships. So, he added a term to make it consistent and called that term as displacement current.

So, he called this as displacement current or this is displacement current density this kind of completes the current law for example, what it means is that, you might have an regular conduction current density J, but when you connect a capacitor clearly there are no free electrons inside this capacitor right. So, how would current then continue so, if you try and apply KCL to this node then KCL seems to fail because there is an incoming current, but there is no outgoing current.

But what Maxwell did is that, he said it cannot happen that way there is current beyond this or there is current in the capacitor and that current is what we call as displacement current which is given by del d by del t. So, I hope you are able to see all these things and what Maxwell did to complete Amperes law is now called as Ampere Maxwell law and you can see that ampere Maxwell law connects H with J and D.

There are 2 additional laws known as Gauss's law for electro I mean electric fields and Gauss's law for magnetic field gausses law for magnetic field tells you that divergence of B is equal to 0, now divergence is denoted by del dot ok. So, this dot denotes divergence and divergence of B equal to 0 simply means that there are no sources or sinks to magnetic field lines right and the same case for electric field tells you that in general in matter it is a D field, that you need to consider and divergence of that D field is equal to whatever the volume charge that is enclosed by the region.

So, you can see this picture out there rho v corresponds to volume charge density. So, these are all the density quantities because we are considering the differential form. So, what it means is that if I actually have a positive charge right, then electric field vectors all come out of this. So, the electric field vectors are so, it does not have to be a single positive charge it could be positive I mean it could be charge density, but if it is positive then the electric field lines or the D field lines actually come out of this particular charge region right.

And then when you enclose this certain region around this and then evaluate the divergence of D, then it will tell you exactly the amount of charge that is enclosed of course. There are certain mathematical difficulties which I am pretty sure you must have studied them in electromagnetic theory if not you can go back to that, but these equations are actually quite valid and what it is, what it says is that, it is not just that del dot D is equal to rho v acting like positive charges having the sources, if you have negative charge region a region which is negatively charged then electric field lines can actually terminate on that right.

So, in one case you have a convergence of field lines and in one case you have a divergence of field lines if you think of convergence as negative divergence, then one equation itself is sufficient to denote divergence you use a positive value denote convergence you use a negative value. So, del dot D equals rho v, when rho v is positive means that electric field lines are all diverging away from a particular region which encloses the charge and when del dot D is equal to minus rho v then it means that field lines are all converging to a particular point or to the region where that is charges are enclosed.

So, you have divergence and then you have convergence what gausses law for magnetic field tells you is that, there are no magnetic charges because if there were magnetic charges it would be possible for us to actually put a magnetic charge and have the magnetic field lines come out of that and put a negative charge and then have magnetic field lines come into it right, but there are no positive magnetic charges or sometimes called as North Pole separately and South Pole separately. So, I do not have an ability to separate a North Pole from South Pole ok.

So, in that cases the field lines are always going to be continuous right. So, the field lines are never going to have a point where they are starting and where they are converging. So, these field lines are all continuous and if these field lines originally were 0 at some point in time, then they will actually turn out to be 0 at all the times. In fact, the divergence turns out to be 0 all the time. So, these are the 4 Maxwell's equations which are very important which we will use in order to study whatever the light propagation inside the fiber and other optical elements.

We have so, far been talking about these vectors; obviously, the knowledge of J and rho v needs to be known these are what are called as source vectors and source charge density, because unless someone gives me the current distribution or the charge distribution which act as the sources to generate E H B and D. I would not be able to solve these equations, but there are situations where these field quantities may be taken to be 0.

So, for example, think of this you have an antenna where there is a certain current distribution and this current distribution when it is changing with time can give rise to changing electric field and magnetic field right and because the medium is free space I am assuming the medium to be free space, there is no real distinction between D and E right and no distinction between B and H I will come to that in a moment, but we can just get away by using 2 vectors E and H to describe the entire system out, there in free space right. So, you have an antenna where currents are changing and this changing current distribution creates a changing electric field and magnetic field, which then radiates away right.

And then now you imagine that you are far away from the antenna and you have a receiving antenna over there. So, in the receiving antenna you know you have this antenna pointed to the region where there are these radiating fields and some of that field can be intercepted by the receiving antenna and collected there in between region there are no currents right. So, if you want to describe the propagation of electric field and magnetic field the radiating electric and magnetic field in the region between the 2 transmitters I mean between the 2 antennas there is no J and rho v there right, but these J and rho v are required at the transmitter side the transmitter antenna side to actually generate E and H.

Now when it comes to the receiver side the receiver has intercepted some of the radiating fields and that electric and magnetic fields that impinge on the receiving antenna create the current distribution ok. So, current distribution is necessary to create E and H and E and H ones at the receiving antenna create a current distribution of course, not the same current distribution, but an equivalent current distribution. So, that you can then understand what happens to the fields converted back into the current ok.

So, but there are regions in between where J is 0 rho v is 0 a very similar situation occurs in an optical fiber as well. So, I will have a source so, maybe a laser or something I couple light into the optical fiber, but inside the optical fiber I do not have any J or any rho I do not place any magnetic, I mean I do not place any electric charge in there nor do I place the current density, I do not put a wire in the optical fiber and then you know connect that wire to a potential source ok. So, inside the optical fiber, I can assume that both J and rho v can be equal to 0 rho v corresponds of course, to only free charges J corresponds to the current density.

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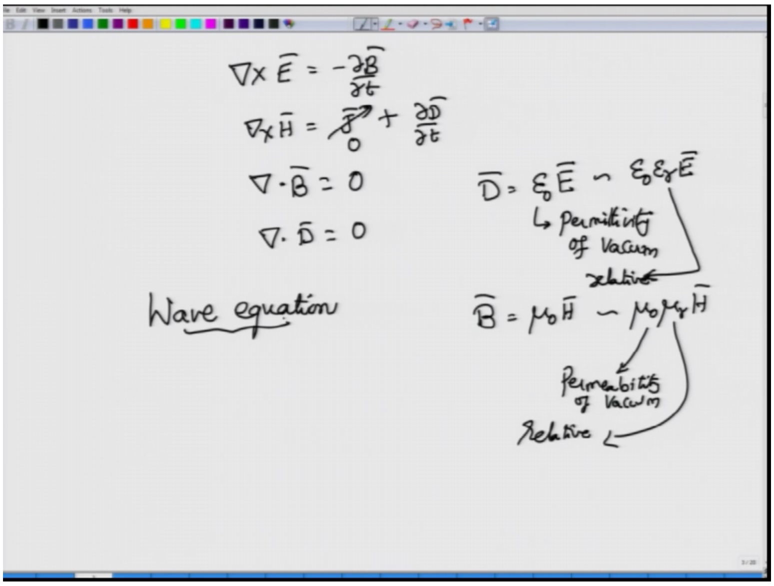

So, let me collect all these Maxwell's equations for you one is del cross E equals minus del B by del t, then you have del cross H equals J which I will assume to be 0 because I am interested in considering the situation, where there are no free charges and there are no free currents of course, they have to be there at some point in the space because they are the ones which generate E and H and additional fields and then I have del dot B.

So, I have dell dot B equals 0 and then I also assume del dot D equal to 0, in general D and E are related by the material properties, if it is free space then D is equal to epsilon 0 times E this is called as epsilon, epsilon 0 or epsilon naught and this is the permittivity of the medium, permittivity of the medium tells you how susceptible the medium is to the electric fields. If it is easily susceptible then the permittivity will be very high, but epsilon naught is the permittivity of vacuum ok.

In the material medium you can rewrite this one as epsilon 0, epsilon r, E where epsilon r is called as the relative permittivity, relative referring to vacuum ok. There are more detailed D and E relationships which we will some of them we will see in later parts of the course ok. Similarly your B is related to H by mu 0 H for the vacuum case and for the non vacuum case it is very complicated. But one possible way of representing them is to write down with the relative mu r, mu 0 is called as permeability and mu r is called as relative permeability, relative again with respect to vacuum ok.

So, from the tables you can find out what is the value of epsilon 0 and mu 0 this is a small exercise for you. So, that you can go back and take a look at your electromagnetic textbooks ok. Now that we have all these quantities you know all these equations written down and the relationship between D and E vectors, B and H vectors known we can now manipulate these equations and arrive at very interesting equation called as wave equation ok, which will then allow us to describe how waves can be expressed first in terms of either E or H and how exactly this wave would propagate in free space, in optical fibers where ever you want to.

We will of course, first restrict ourselves to free space propagation and then introduce what happens I mean then introduce boundaries and study how light behaves at the boundary because that is the basis of understanding propagation of light inside the fiber ok. So, thank you very much we will meet in the next module.