Fiber - Optic Communication Systems and Techniques

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Lecture – 30
Graded index fibers

Hello and welcome to NPTEL MOOC on Fiber Optic Communication Systems and Techniques. In the last two modules, we saw some numerical approaches to understand modes of an optical fiber. And if you look at the previous modules that we discussed in all those modules we emphasized the step index, refractive index, step index profile of the optical fibers.

We choose step index optical fibers, because amongst all the other refractive index profiles that step index profile is very simple refractive index profile, which can be actually solved analytically to obtain the characteristic equation of the modes to obtain the propagation constant, to understand the dispersion characteristics, and to do lot of other things. However, step index profiles are not realistic refractive index profiles, because you cannot get a sudden change in the refractive index when you go from core to the planning right.

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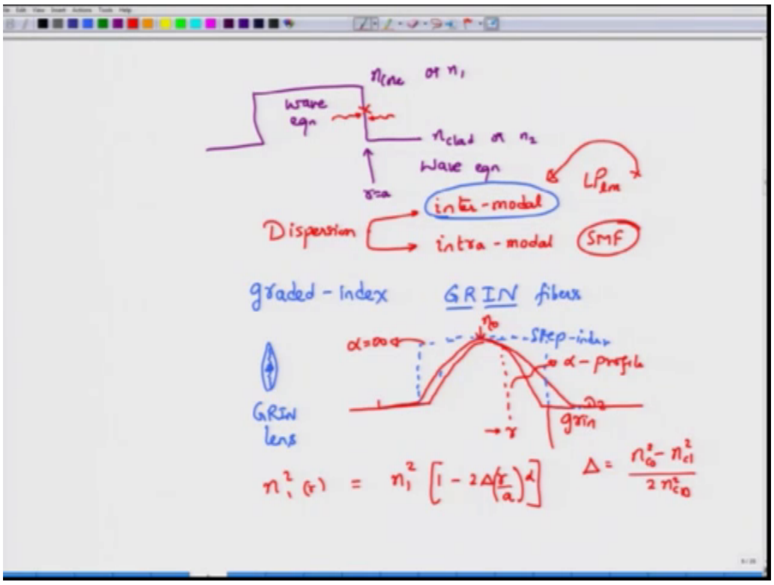

If you remember, what the step index profile would look something like this right. So, you had the refractive index of the core being n core or n 1 that we have written n core or n 1 as we have written. And then you had the cladding refractive index, which we called as n clad or we denoted as n 2. And we said that right at the boundary r equal to a; which corresponds to the core cladding interface. There will be an abrupt change, in the refractive index from core to the cladding right.

And within the core, the refractive index is constant. So, you write down the wave equation or the Helmholtz equation here. You write down the wave equation in the cladding right, or the Helmholtz equation here. And then in order to obtain the characteristic equation which allows you to find out the propagation constant beta of the fiber, you would then match the boundary conditions or you would then match the solutions in the core to the solutions in the clad. And we have seen all of these things in the previous module.

One of the important characteristic of the step index fiber is its dispersion. We can show and we have indeed talked about it that dispersion actually in typical step index optical fibers or in general optical fibers can be classified into two types; one is called as the inter-model dispersion, inter-modal dispersion occurs between two different modes. We recall that the modes are typically represented as L P with subscript l m or nu m, whatever that subscript that we have used.

And there will be different modes right and these different modes talk to each other giving rise to inter-modal dispersion. And of course, we have seen certain expressions for this inter-modal dispersion, and we do not really want to go back to those details, but the idea is that when information is launched into multiple modes; these modes will talk to each other.

And when you actually launch information in the form of an optical pulse, then the result of inter-modal dispersion is that since each mode arrives with its own phase velocity and the group velocity. When you look at the output at the output of the fiber, the pulse would actually have been distorted more, rest of the times it would be just expanded out or rather broadened out; but in rare cases where dispersion is quite strong, you might actually see some kind of distortion as well.

Now, you might be able to eliminate inter-modal dispersion, but there is another dispersion called as intra-modal dispersion. This intra-modal dispersion is a peculiar characteristic only to single mode fibers, because in single mode fibers there is only one mode of course, we do know that that mode is degenerate in the sense that there are two polarizations, but if you look at, if you do not worry about the polarization aspect, then there is only a single mode. But, because the propagation constant even for that single mode is a non-linear function of the frequency omega, we will see that the pulse will still be distorted; which we call as the intra-modal dispersion.

In the first few generations of fiber optic communications systems, people tended to use multimode fibers, because they assume that when you have many number of modes, information can be you know applied on to many different modes with each mode carrying potentially independent information. And therefore, you can increase the information carrying capacity of the optical fiber. However, with the advent of single mode fibers and the associated advantage of single mode fibers, the multimode fibers have now been reduced; only to those situations where the fiber links are shorter such as Ethernet; or maybe from say within building to building connection or within the same building, one floor to another floor connection or kind of a thing.

So, when the when the when the fiber links are shorter, and you do not really want to spend too much on the single mode fiber and the associated lasers, you can get away with using a led or something; then you prefer using a multimode fiber. Even when you prefer using a multimode fiber, you still have to understand that there will be intra-modal dispersion which will limit the information carrying capacity. And we see that in step index fibers this inter-modal dispersion is quite strong, and as we have seen in the earlier modules.

Now, what people did was to actually modify the refractive index profile, they actually came up with different refractive index profile; which could minimize this inter-modal dispersion. And one of that which is quite popular is called as a graded index profile ok. And the fibers which have this graded index are called as GRIN; in GR stands for Graded, IN stands for Index ok, these are called as GRIN fibers.

GRIN as such is also used in lenses, rods and other kinds of applications, the classical optical applications. And for example, this lens can be called as a GRIN lens, because the refractive index here will be higher; and the refractive index actually goes or become smaller, as you go to the edges of this particular lens. So, this lens is sometimes called as

GRIN lens, although the terminology is not quite popular, but grin stands for graded index.

And in the optical fiber, the graded index profiles actually can be generated by a certain equation or can be modeled by an equation. For example, this is one of the graded index profile which are plotted against or in comparison with the step-index profile. So, the blue color that we have plotted is the step-index profile. And then red color this one that we have plotted is called as the graded index profile.

Of course, the grading which is how it actually starts off the profile start with a maximum value of n 0 at the center or maybe n c 0 at the center. And then reaches down to n to at the cladding, and of course, we are assuming that in the cladding, the refractive index is going to be constant although there could be changes even in the cladding refractive index as well; but it starts off with n 0, and then it slowly changes over from n 0 to n 2.

The point where it happens is what we would call as r equal to a; so that way this is not really a correct representation, the corresponding step index profile should actually look something like this, not the earlier way, so now this is alright, but the important point is that you can start with this value of n 0 at the center.

And then come down to the value of n 2, and this change happening that r equal to a, but you can also have different fall off rates. For example, this is one way of falling off ok, and these different rates of fall off is graded or is captured by what is called as the grading profile, which unfortunately uses the same letter alpha that we have used for attenuation as well.

So, please forgive that because this use of alpha is fairly standard the fiber optic literature. So, we are going to use this term alpha order symbol alpha, to denote the grading profile, hopefully the context will make it clear which one we are talking about.

And the refractive index is now as you can see is not constant at any distance r inside the core of course in the cladding we are assuming it to be constant. And in fact, you can write down the equation for the refractive index in the core, which let us say call it as n 1 square, and we will make it as a function of r to explicitly show that this is a function of r itself.

And this n 1 square of r can be written as, n 1 square minus 2 delta r by a, where a is the radius of the core raise to the power alpha, where alpha would correspond to the grading profile. In fact, we can show that the step index grading profile corresponds to alpha equal to infinity or alpha being very very large that it can be kind of approximated to infinity.

Delta of course, is what we have defined earlier or maybe we will define it slightly, differently in this case, which we will define as n core square minus n clad square divided by 2 n core square. So, we will define this delta in this manner or rather delta as in this particular manner which of course, can be approximated as well, which we do not need to worry about it.

The question is given this refractive index profile, how are we going to solve the equations. It turns out that the solution of you know graded index fibers or the fibers with graded index profiles is not so simple, because only in a very special case of alpha do we actually have analytical solutions.

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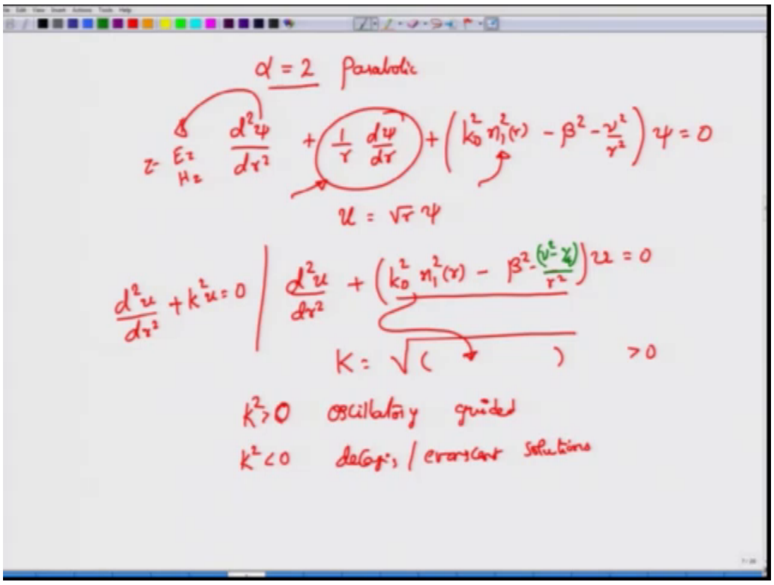

And that special case happens to be what is called as parabolic grading or parabolic index profile, because for that case alpha is equal to 2. In a certain approximate sense, the equation that you will be solving would look something like this, the where psi corresponds to the z component of the electric field or the magnetic field. So, this would correspond to E z or H z have denoted that one by just using this psi.

And the equation would look very similar to the equation that we have written for the step index profile. Unfortunately, the equation is not so simple because of the presence of a r dependent refractive index. Of course you are still assuming that you want the azimuthal dependence to be in the form of nu, and the propagation constant beta is what you are trying to find out. So, this would be the equation, but it turns out that this equation cannot be solved except for the case of alpha equal to 2.

We are not going to solve the equations, but I will tell you what really happens in this case; by making a certain change of variable you can actually rewrite an equation, within the in terms of another variable called u, compared to the variable psi that we have used previously. And in that case you can take this as a simple exercise, what we will actually find out is this fellow that this term 1 by r d psi by d are actually goes away. And then what you have is a simplified expression, this term would still remain as it is, but now the equation is written not for psi, but for u, and this is a simple second order equation.

Of course, simple meaning that this term is not there, but the equation is not so simple in the sense that n 1 is actually a function of r. So, this is approximately the equation, there are lot of approximations that I have gone into writing this equation, but this is a fairly good approximate equation that we can actually use to understand, and find out the value of beta. Now, this equation should remind you of something, so let us say d square u by d r square plus K square u equal to 0 is what we can write this one; where K of course will be square root of this term in the brackets right.

So, it would be square root of this term in the bracket. And when K square is a positive constant, then you will have; so when K square is positive constant, then you will have oscillatory kind of solutions, which means these are the guided solutions that you are looking for. And when K square happens to be less than 0; then you will have exponential solutions or decaying or what we call as evanescent solutions.

It turns out that not the in the entire core you will have the oscillatory or the guided solution, because the condition for K to be greater than 0 means that this term in the bracket has to be greater than 0, and if you simply sketch, all those values.

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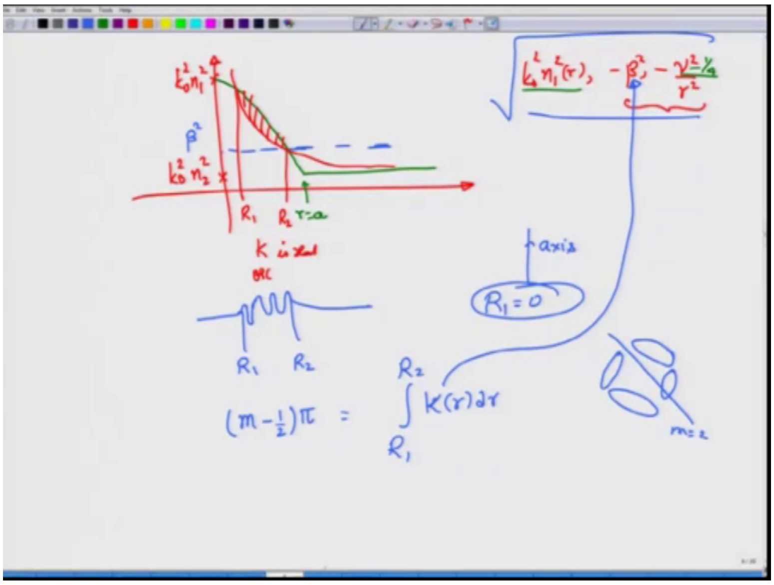

Starting with say K 0 square and c 0 square, where n c 0 or n 1 square would be the peak value. So, here if you go back and look at this one, I have written the graded profile here right. So, I have written this as n 1 square, pardon my little confusing notation here; n 1 in this particular case would stand for the max value of the refractive index in the core. And if you look at the refractive index, the limit for beta it would always lie between two things for the guided case, it should lie between K 0 square and n 2 square.

Now, if you look at the individual terms K 0 square n 1 square of r, then you have minus beta square; and then you have finally, minus nu square by r square, the equation when you write down in this way turns out to be not mu, but turns out to be nu minus nu square or nu square minus 1 by 4 ok. So, this is what it is.

So, if you go back and look at that term right, nu square minus 1 by 4 divided by r square and when you plot all of them, this is what you will look this is what you will find. So, K 0 square n 1 square would start off with a max value at r equal to 0; and then gradually go change and then become this constant value when you reach the flooding. And the point where this happens, is where the core cladding boundary is situated where r is equal to a.

Whereas when you look at the other solutions beta would of course, be a constant. So, this would actually be some value of beta or beta square if you would like. And if you look at this nu square minus 1 by 4 divided by r square, at r equal to 0 this is very large right. And then it actually drops down right. To when it drops down, it also gets up shifted by a constant value of beta square.

So, the result is that the corresponding value for this sum of this beta square and nu square minus 1 by 4 r square would look something like this ok. And it is in this shaded region with the values say capital R 1 and capital R 2 that K is real, and then you have a oscillatory solution, outside you have an exponentially decaying solution. So, here inside you will have oscillatory solution and outside you will have the decaying solutions. And these points where, there is a where within the region; where there is an oscillatory solution are called as the turning points of this particular profile.

Of course, the actual turning points have to be found out by solving the equation for K, and when you solve the equation for K, you will see that of course, there are various things that can happen, but one of the things that will happen is you will find out two turning points. And this is valid for any value of nu not equal to 0 with say nu equal to 1, 2, 3 and so on, but when nu equal to 0; then the turning point R 1 will be equal to 0, meaning that the entire core region close to the core region or the first turning point R 1 will never have a value apart, I mean outside the core, but it actually extends all the way to the axis of the fiber.

So, this is how you would actually go and solve of course, you want equation for obtaining the value of beta. And we can find out, an approximate equation that will allow you to find out the value of beta, and that approximate equation I am not going to derive that one; but this approximate equation is actually the integral between the turning points R 1 and R 2 which of course, different further on how the value of K itself is dependent. K if you recall, is given by square root of this particular term.

So, only for the parabolic case one can actually solve this equation completely. And this m stands for the number of half cycles along the given radius. For example, this is number of half cycle; so in this case m is equal to 2, and you can obtain that value. And once you put down this equation and solve it, this will implicitly contain the value of beta which you can find out.

So, it is not as easy to find this value of beta for any general arbitrary refractive index profile. And for that case you are better off solving them in a commercial solver or you can write your own code, and actually solve numerically to obtain the value of beta. One final point about this graded index fiber is that the number of modes guided by this one is given V square by 4.

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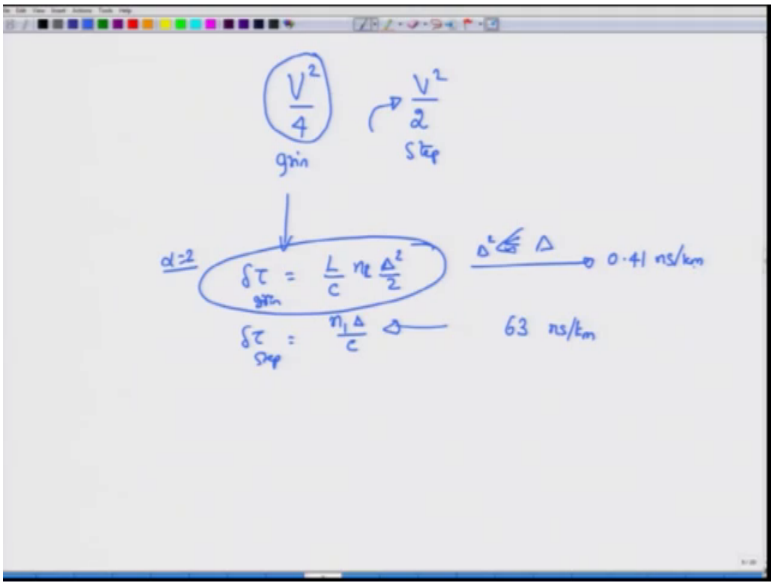

Whereas, the number of modes guided by the step index fiber or step index profile is V square by 2. So, this is for the step index, and this is for the graded index. So, you can see that the graded index fiber actually carries half the modes as that off the step index fiber, and also its power handling capacity is smaller, because not all the power is coupling into it; only half the power compared to step index fiber gets coupled to the greater index fiber when the two fibers are the same parameters or the same geometrical values.

And the numerical aperture of the graded index fibers are also quite difficult to calculate, but in general they are actually smaller compared to that of the step index fiber. So, the advantage of step index fiber is the higher power handling capacity as well as higher numerical aperture which may be useful in many cases. Whereas GRINs trade off the power handling capacity and the numerical aperture, but they are advantages in terms of reduced modal group delay or if or the inter-modal dispersion.

If at I would not solve the equation, but I will give you the value of or I will give you an expression for the modal delay because of this graded index profile and only for the case where alpha equal to 2. It turns out that this is given by n 1, where n 1 is the maximum value of the core at the center times del square by 2.

Whereas, the same value for step index fiber is n 1 delta by c. And, because delta is usually quite small, delta square is much smaller compared to value of delta itself. And therefore, the group velocity dispersion for the GRIN or the modal delay for the GRIN fibers is actually quite small compared to the same case of the step index fiber.

So, this is what is very important and this is what I wanted to tell you; that for a standard single mode fiber, if the group velocity at least in the older fiber used to be in the range of some 63 Nano second per kilometer for the step index fiber. Whereas, that has been significantly brought down by the use of graded index fibers to 0.41 Nano second per kilometer. So, this was principally the advantage of graded index fibers.

Then there are additional type of refractive index profiles people used to tailor the dispersion, so that one can actually shift the 0 dispersion wavelength from the standard 1300 nanometer to 15 nanometer, where the attenuation of the fiber is less. And they also tailor the refractive index profile of the fiber, in order to play around with the dispersion to perhaps make it flatter, in the so called dispersion flattened fibers.

And all these designs are not trivial designs they have complicated refractive index profiles. And as I have told you many times, these complicated refractive index profiles seldom have analytical solutions, but they can be solved reasonably with the numerical methods one of which we discussed using COMSOL in the previous module.

Thank you very much.