Fiber- Optic Communication Systems and Techniques
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Lecture - 04
Properties of UWPs (propagation constant, polarization, and Poynting vector)

Hello and welcome to NPTEL MOOC on Fiber Optic Communications Systems and Techniques course.

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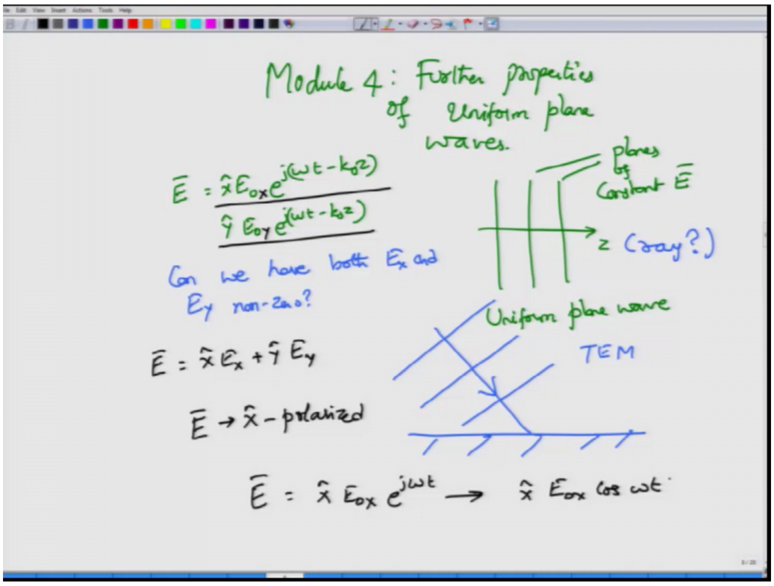

Today is module 4 and we are going to look at Further Properties of Uniform Plane Waves. Recall what we talked about I mean what we define the uniform plane wave as you had a wave which was propagating along x axis. It had an electric field E, either along x axis or along y axis. Both cases are different, but both cases can be independent. I mean they are independent in the sense that you can have E x E y and you know waves propagating along the z axis or you have E y and E x and the wave is propagating along z axis.

We call this as Uniform Plane Waves because as the wave propagates along the z axis, if you were to look at constant planes at some distance, so any constant planes z that you take, the electric field and magnetic field values will be completely independent of x and y, which is a transverse plane, transverse to the direction of the propagation and they only change along z value, right. Of course, they would also change along the time, but at no point they are actually functions of x and y. Therefore, the electric field amplitude and magnetic field amplitudes would remain constant for a constant value of z. This is pictorially shown in this diagram, right.

So, you can see here that I have drawn an axis which is along the z direction and then, I have erected lines indicating that these are constant values of electric field. Of course, if you take those constant plane, such that the electric field value would be the same at these two planes, the distance between these two is called as the wavelength. In fact, when you write down a ray, remember light can also be thought about as a ray. This is what the picture that you actually have in mind, right.

So, I have an interface like this and then, you have a ray of light incident at certain angle, ok. What we actually mean is that the wave is propagating in the direction that I have indicated with this arrow and the electric field and magnetic fields are constant and these are the constant electric fields. Of course, there will be constant magnetic fields also transverse to this one. Such waves are called as Uniform Plane Waves and because we have a situation where electric field is transverse to magnetic field and magnetic field and electric field both are transverse.

Transverse is a very fancy name for perpendicular. So, both electric field and magnetic fields are transverse to the direction of propagation we have what is called as a transverse electromagnetic wave. And, this is exactly what we are introduced to when we talk about light as a ray, but the electromagnetic nature means that there is electric field and magnetic fields and you in the case of a planewave, that we have considered these two will also be further transverse to the direction of propagation.

So, in terms of equations you have x hat E 0; E 0 being the constant along x direction. So, let me modify this and write down this E 0 x and similarly, I will write this as E 0 y. So, for what I have said is that you have an electric field component either along x or along y. Can we have both at the same time? Yes it is possible for us to have both non-zero at the same time, ok. So, when they are non-zero at the same time, the total electric field would simply be the sum of the electric field along x and the electric field along y.

So, where E x is E naught x e power j omega t minus k 0 z and where, y is E 0 y e power j omega t minus k 0 z. So, please note that the value of k 0 and omega are not functions of x and y. They are not functions of the direction in which the electric field is propagating, because this medium is what is called as isotropic medium. So, for an isotropic medium electric field and magnetic field or the properties of the fields do not depend on the direction in which the electric field and magnetic fields are oriented.

So, if in a simpler case that I have an electric field which is entirely along the x direction, then I call this wave as x polarized wave; polarization in this context means orientation. So, if I have electric field, you know take the first equation in this equation, there is no y term. And, further if I assume that I am actually considering a case where k 0 z is equal to 0, then I have the electric field given by x hat E 0 x, E 0 x being the amplitude of this component and then, I have e power j omega t. Of course, in the real case this would be x hat E 0 x cos omega t.

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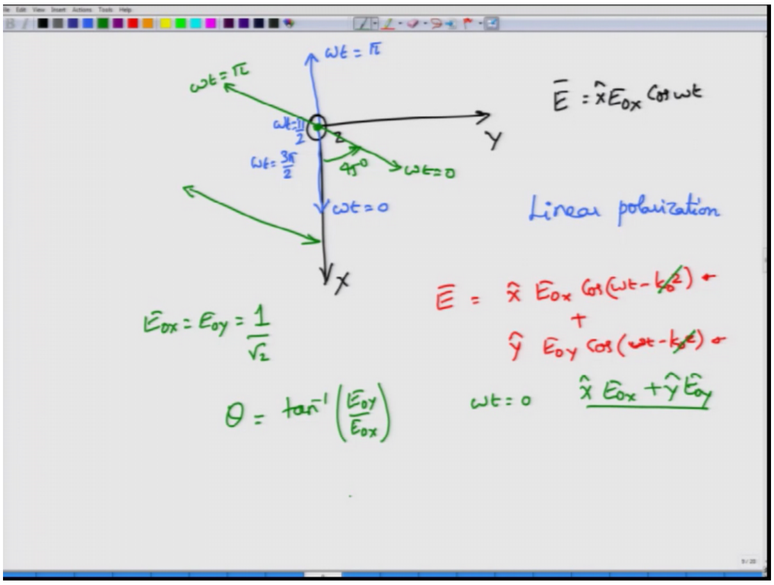

Now, let me actually do something. I will write down two lines. This is the plane that I am considering and z axis would be coming out of this one, right in the sense that I am actually looking towards the direction in which the wave is approaching me. So, the wave is approaching me and I am looking at this particular plane. So, one of them will be x direction. This would be the y direction and you know that if you go along from x to y, the direction should point towards z, right. So, if you just put them down like this and look at it from the top, you should have an x direction and y direction and x cross y should point in the direction along the wave, right.

So, you have this coordinate system. Now, what I am trying to do here is, I go back to the electric field E which was basically E 0 x, right. So, this was an x directed field E 0 x cos omega t. So, now let me try and write down where this electric field would be oriented at different times and because of cosine nature, I am actually, it is actually easy to do because I need to consider only 4 values for omega t to get a full picture, right. So, I mean 2 is also ok, but 4 seems to be a reasonable number. First omega t is equal to 0. So, when omega t is equal to 0, cosine of 0 is 1 and the electric field will be directed entirely along the x axis with an amplitude of E 0 x. So, this is what you actually obtained at omega t equal to 0.

What will happen at omega t equal to pi by 2? At omega t equal to pi by 2 cosine is going through 0. So, the electric field will actually be 0, right that is at omega t equal to pi by 2. Now, what will happen at omega t is equal to pi? Pi means cos of pi cos of pi is minus 1. So, the electric field will be directed along minus x axis, right with an amplitude of E 0 x again, right. So, it will be directed along minus x axis at omega t is equal to pi and at omega t is equal to 3 pi by 2 cosine is going through 0 again. So, you have the value I mean the electric field having a value of 0 at omega t is equal to 3 pi by 2.

So, what you have just seen is that no matter what time you consider, the electric field component would be oscillating along the x direction itself, right. So, the electric field is oscillating along the x direction itself. Naturally if you repeat this problem by assuming that E 0 x is 0, but E 0 y is not 0, then you will see that the field is oscillating only along y axis. So, you have an x axis alone f oscillating or y axis alone oscillating. Such oscillation along a particular line is called as linear polarization and the wave is said to be linearly polarized along x or along y.

How about this case, where we consider the electric field to have both x component? So, I have omega t minus remember it is k 0 z plus y hat E 0 y cos omega t minus k 0 z, ok. So, you can show independently that E x or the first equation will be solution of Maxwell's wave equation. The second one is independently the solution of Maxwell's equation. The sum of these two will also be the solution simply because of superposition, right. So, we have equations which are linear and therefore, the sum is also a solution. Now, let us consider again the same kind of a scenario. I have omega t and I am going to consider k 0 z equal to 0 which means their constants and I am going to change the value of omega t.

So, first let me assume omega t equal to 0. So, when I assume omega t equal to 0, what will happen to this expression? So, at omega t equal to 0 you have x hat E 0 x plus y hat E 0. If I have been given the values of E 0 x and E 0 y, then it is possible for me to use that values and then, point out a vector at omega t equal to 0 in the direction given by this vector, ok. So, this is the direction of the electric field, the total electric field at omega t equal to 0. For simplicity let me assume E 0 x equals E 0 y equals 1 or rather 1 by root 2. For you we will see why 1 by root 2 later on. So, you have this 1 by root 2.

So, the direction of the electric field will be at I mean at an angle of 45 degrees as measured with respect to x axis, ok. This would be the case at omega t equal to 0. What about omega t equal to pi by 2? In that case, the electric field will actually be 0 because cosine of pi by 2 will make this one go to 0. So, electric field is at the origin and at omega t equal to pi, the sins of E 0 x and E 0 y or the direction of E 0 x or the E x and E y will be reversed which means that along the same line, but directed in the opposite direction, right. So, this would be the case at omega t equal to pi and at 3 pi by 2 omega t equal to 3 pi by 2, the field would have gone back to 0.

Now, what you have seen is that the wave is linearly polarized because it is oscillating along a line, but that direction or the angle at which this is oscillating is 45 degrees. In general that direction or the angle at which this would be oscillating a linearly polarized wave would be at an angle of tan inverse of E 0 y by E 0 x, right. So, depending on the amplitude, your field component would be polarized in a line, but at an angle which is given by tan inverse of E 0 y by E 0 x. So, these are still linearly polarized wave except at an angle of theta.

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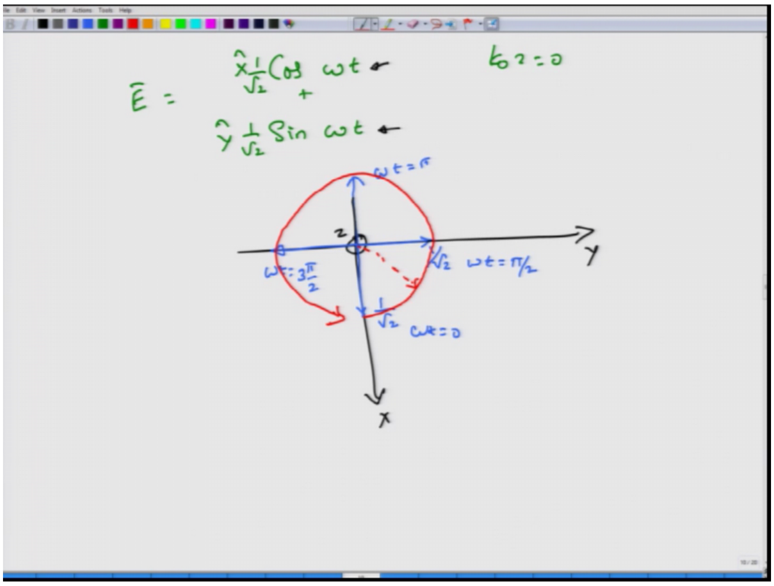

Now, let us do something else. It is possible for us to show that in addition to cosine type of solutions, even sin is a function of I mean sin of omega t minus k 0 z is a solution of wave equation and what is the difference between cosine and a sin cosine and a sin are different by of amount of phase which is equal to 90 degrees, right. So, one of them is 90 degree phase shifted version of the other one. So, cosine and sin are 90 degree phase shifted. If I assume at sum k 0 z equal to 0, the electric field components are equal in amplitude, but have a cosine and a sin you know dependence with respect to omega t minus k 0 z, then the total electric field would be 1 by root 2 x hat cosine omega t plus 1 by root 2 y hat sin omega t as I have written here.

Now, what is the polarization of this wave? Let us look at this again. I have x and y axis. Please remember you are looking from the top and the wave is approaching you, ok. So, this is z axis. The z axis is coming out of this particular plane at omega t equal to 0. What will be the situation at omega t equal to 0? Only the x component will be non-zero whereas, the y component will be 0 because sin 0 is 0. So, I have electric field directed along x axis with an amplitude of 1 by root 2, but omega t equal to pi by 2, the x component will be 0, the y component of the electric field will be along the y axis of course and will also have an amplitude of 1 by root 2.

Since between omega t equal to 0 to omega t equal to pi by 2, the field line has changed from x to y, right and at omega t equal to pi, you will see that sin of pi will be 0, but cosine of pi will be minus 1. So, the direction of the electric field will be along minus x axis at omega t equal to pi and at omega t equal to 3 pi by 2, this would be directed along minus y direction. So, what is actually happened is the electric fields started off at omega t equal to 0 at you know lined up along the x axis, then it changed its position. So, you can put for example, omega t equal to pi by 4 and realized that at omega t equal to pi by 4, the amplitude of this one would be 1 by root 2 along x axis plus 1 by root 2 along y axis which means that in terms of magnitude or the length, they would actually be equal, right.

So, if you now join all this tip, right so if you join the tip, you would see that the tip of the electric field vector is rotating in which direction is this tip rotating. It is rotating in the clockwise direction. So, if you now go clockwise direction and then, look at your thumb, the thumb is pointing in the direction of the propagation. So, since you have used a right hand to get to the z axis and this is a polarization this is said to be right circularly polarized circular because at every point on this circle, the magnitude of the electric field which is the value of that electric field at different values of time will actually be constant and because we have used a right hand thumb, I mean right hand to point to the direction of the propagation, this is called as a right handed polarized wave or simply called as right circularly polarized wave, ok.

Now, you can show I leave this as an exercise to you. What would be a left circularly polarized wave? So, left circularly polarized wave would actually started to omega t equal to say maybe along x axis, but at omega t equal to pi, it would be directed along minus y. Therefore, you need to go with your left hand and point the left hand thumb along the direction of propagation and then, you would obtain left circularly polarized wave and the equation for that one would be 1 by root 2 x hat cos omega t minus 1 by root 2 y hat sin omega t, ok. In fact, we have a simple notation for all this polarization. We write the electric field as you know given the amplitudes E x and E y. So, instead of E 0 x E 0 y will write this as E x and E y, sorry and call this as a two-dimensional vector.

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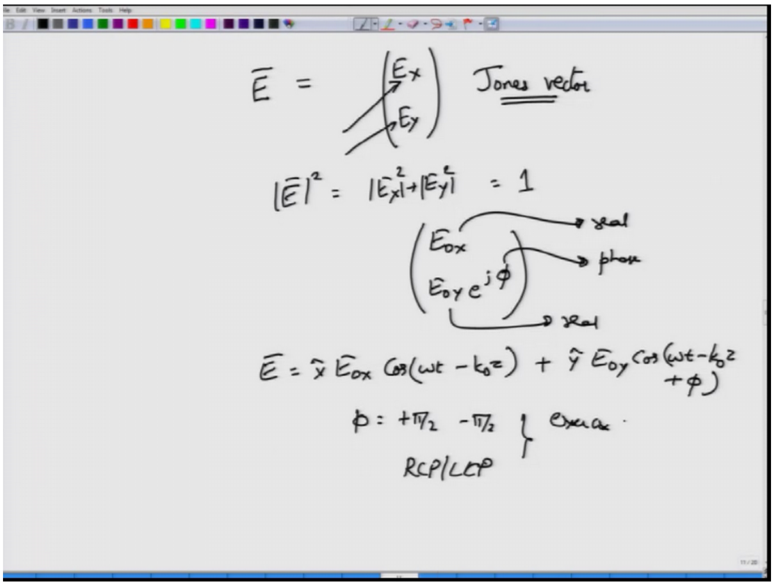

So, this is a two-dimensional vector E x and E y. So, this vector which is a two-dimensional vector is called as a Jones vector, and this is used to describe polarization of the electric field and we note that E x and E y are themselves complex, ok. We also have considered a situation where the magnitude of E square which is given by E x square plus E y square magnitude, of course would be equal to 1. This is just to normalize the intensity or normalize the power in electric field. We will talk about power sometime later, but this Jones vector is a very popular method for as to denote polarization.

So, you can in fact write down if you are not really happy with this one. You can write down this in a slightly different way. So, you can write this as E 0 x E 0 y E power j phi where you can consider E 0 x to be real, E 0 y to be also real and all the complex thing comes by having E power j phi. So, this phi is called as the phase difference. So, it is possible for you to even go and write it something like this and then, the electric field would be given by E 0 x cosine omega t minus k 0 z right plus and this of course would be along x axis and then, you have y axis field which is E 0 y cosine omega t minus k 0 z plus phi and this phi controls what type of polarization we have.

So, when phi is equal to 0, you have a linearly polarized wave and when phi is equal to plus pi by 2 or minus pi by 2, you will have right circularly polarized wave or left circularly polarized wave, I am not sure which one is which one. It is just a simple algebra for you to figure this out. So, I leave this as an exercise to you.

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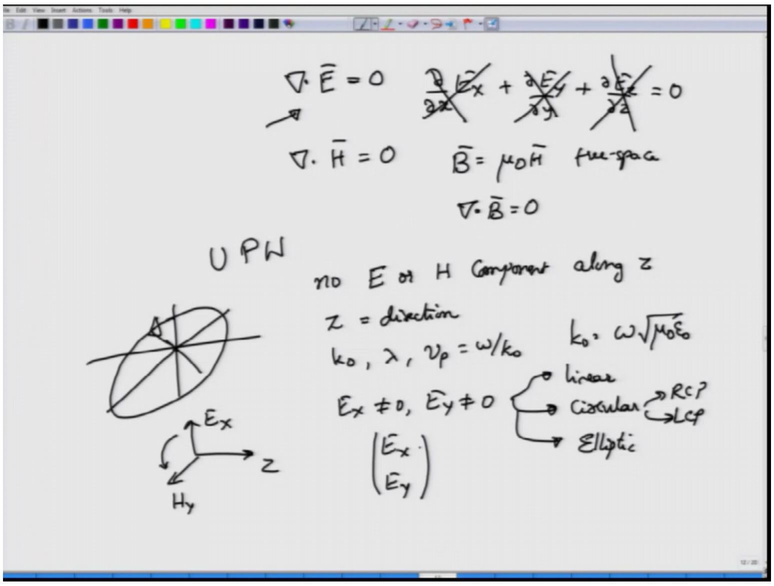

You remember that we talked about some divergence in the last module and we said that del dot E equal to 0, we could actually write this del dot E equal to 0 and we wrote this in and we assume that this is the condition and we expand it out in Cartesian coordinate system. What you have will be del e x by del E x plus del E e y by del y plus del E z by del z equal to 0.

Clearly in the case that we have been considering there is no E z component. Therefore, del E z by del z is equal to 0. Furthermore because this is uniform planewave, E x is not a function of x and E y is not a function of y and this is also equal to 0, ok. So, the conditions that we assumed naturally satisfies the condition for H will be del dot H equal to 0, but this is trivially satisfied because B is equal to mu naught H and del dot B is always equal to 0, and because of this particular case I mean this is always true and this B is equal to mu naught H. In the free space condition, it trivially follows that del dot H is always equal to 0.

So, to summarize we have considered uniform planewaves which are characterized by having no E or H component along z direction, which is the direction of propagation. Z is the direction of propagation. The wave propagates with a wave number k 0 wavelength of lambda phase velocity V p given by omega by k 0 which is all properties of the medium itself. Of course, k 0 is also equal to omega time square root mu naught epsilon naught and the velocity in this particular case for the free space medium is of course equal to c, ok. You can look at all this, right and then, as I said you can have E x non-zero and E y non-zero at the same time and if you know what is the phase difference between x and y components, you can have different type of wave polarizations; one is linear polarized and the other one is circular polarized. In circular you can have RCP and then, you can have LCP standing for right hand circularly polarized and left hand circularly polarized wave.

The other polarization that we did not discuss is Elliptic polarization. In elliptic polarization what happens is that the field vectors rotate in the form of a ellipse, ok. They have a certain minor axis and then, they have a certain major axis, ok. The tilt of this ellipse with respect to the x axis is given by the amplitude difference between x and y or the ratios of the amplitude between x and y, ratio of the amplitude of x and y components and the angle phi which is the phase difference between x and y components. And, for simplicity case or for manipulation case, we introduced Jones vector in which we simply assume that both x and y components are at the same frequency.

Of course, this is a necessary condition and that we can represent E x and E y in terms of their complex amplitudes and call this two-dimensional vector as Jones vector which will tell us whether the wave is polarized in the linear direction or linearly polarized circularly polarized or elliptically polarized. Further the nature of this uniform plane wave was such that as the wave was propagating along z direction, the electric field was if you assume it to be along x direction, then the magnetic field will be along y direction because then, E cross H would point along z. Incidentally I would not derive it here.

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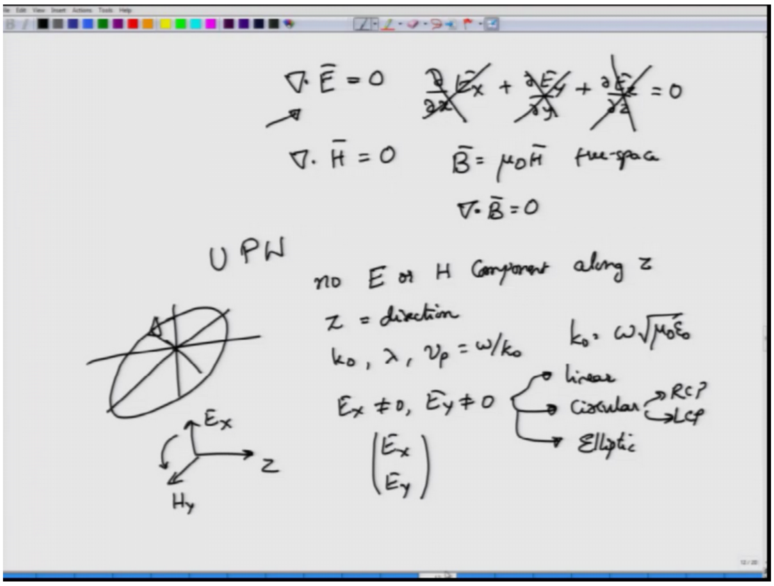

We can define what is called as a Poynting Vector Density or sometimes called as Poynting Vector itself or sometimes also called as Power Density Vector which is given by S equals E cross H, ok. If you use the full electric field in the real form, magnetic field in the real form, then this S corresponds to the power density of the wave that is being propagated along z axis.

So, you can use full electric field E in the real and H form, but if you were to use the complex form that we discussed, or the time harmonic form as we call it where the time dependency is always E power j omega t, then the equation that you can write in that particular case will be slightly different. You can actually write the poynting vector as half real part of E cross H. I mean half of E cross H star, this half is just for you know normalization purposes and this S is the poynting vector which tells you how much is the power density being carried away, right.

Of course, the power itself is obtained by integrating over the area and in this case the area will be along x and y direction. So, this would be S dot ds, where ds is the surface element given by z hat dx dy, right. So, I hope that you remember this surface density or the surface integral concept and whatever the surface that you consider which has certain bounds along say x equal to 0 to x equal to y equal to 0 to y equal to b over this surface, if you know what s vector is, right which is the poynting vector, then you can integrate over that limit and obtain what is the time average power that is being carried by this particular wave. So, this is the time average power. If you simply write E cross H that is called as the Instantaneous Poynting Vector, but most of the times we are not interested in the instantaneous power because these are oscillating at terahertz and I really do not have a detector which can follow those instantaneous variations.

So, I am mostly interested in the average powers and in that particular case, the time average power is given by half E cross H star, where E and H are represented by their complex forms, that is we do not have E power j omega t in that? That kind of a thing is already assumed implicit, ok. So, the time harmonic form or the complex form or the phasor form, they all are essentially the same and just the names are different. You know the fundamental idea is the same, the time dependence is dropped out assuming E power j omega t.

So, this was all about poynting vector and previously we have seen Jones vector, then to describe polarization we have another vector called as Stokes vector which I will not describe it now, but we will postpone the discussion of Stokes vector when we want to discuss polarization of the wave that is propagating inside the fiber, ok. So, we will postpone that discussion later on and for now just recap that planewaves that we considered were characterized by wave propagating along z direction, no E and H component along z direction as well we had defined a few of the parameters such as wave vector or the wavenumber wavelength and we described the phase velocity.

More of this will be talked about in the next modules, but let me ask you one thing. So, is it possible for the wave to propagate in any other direction apart from z? It is possible. For example, if the wave is propagating in x and y plane, right or in x and z plane making an angle of certain theta with respect to x axis, then the direction of the propagation will not be purely along z hat or along x hat. It would be in the direction of the vector, the unit vector along whatever the direction that the wave is propagating.

In that case, electric field will not be purely in x and y directions because remember if we are considering transverse electromagnetic waves, then the electric field has to be perpendicular to x. I mean perpendicular to the direction in which the wave is propagating. So, if the wave is propagating along z axis, electric field could be along x axis and magnetic field could be along y axis right, but if we turn around and then say that the direction of the wave propagating in free space is at a certain angle with respect to the z axis, then clearly electric field will also be now having two components. It will have a component along x and it will have a component along z, right.

Similarly, magnetic field will also have a component along x or rather y and the corresponding z components. So, this type of a wave is called as Obliquely Travelling Wave. Oblique in the sense that it is not directed along z axis, but also remember our x y z coordinate systems need not be the same. Coordinate system for the wave also for example is possible for us to define z axis as the direction of the wave propagation, then x axis as the electric field and y axis as the magnetic field. But, if we fix the coordinate system first and then, we find that the electric are the wave is propagating at a different angle, then we should be ready to consider the components of that direction vector. In that case instead of wave number, we talk about wave vector that is k itself will become a wave vector, ok.

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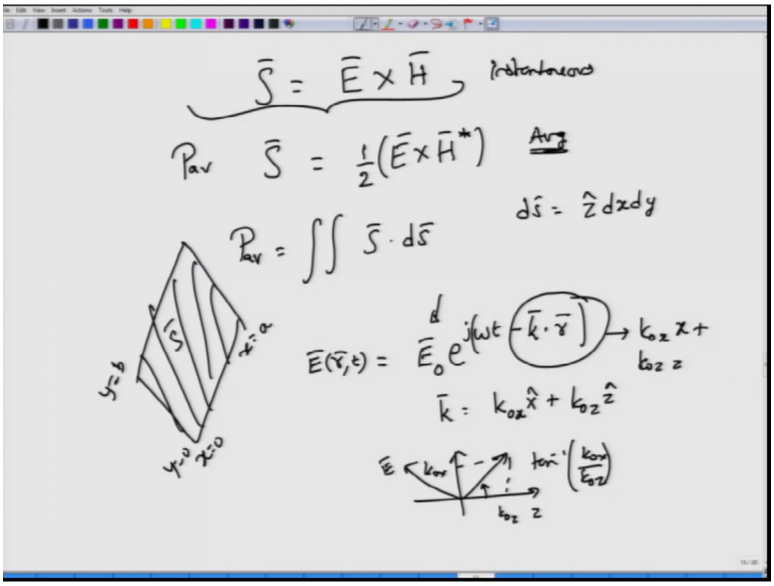

In that case, the direction of propagation of the wave will be given by the dot product or rather the direction of the wave propagation will be denoted by k and this k itself will be a vector and the corresponding special part will be given by k dot r, and the electric field direction will also be determined by what the direction of this k vector is.

So, what it means is that k can be k 0 x x hat plus say k 0 z z hat. In the previous case, we have to consider only that the wave k will be directed along z axis, right. In this case, I am considering k vector to be along both, I mean in the direction which is at a certain angle with respect to z axis having values, along x and having a value of along z. So, in this case, it is not purely propagating along z axis, but it is propagating at a certain angle which is given by tan inverse of k 0 x by k 0 z, and the electric field itself will be given by the electric field which is now a function of r and t is given by its vector which is dependent on this direction. So, the electric field could be this way directed. This is the electric field direction and then, in terms of the phase, it would be k dot r. Of course, in this case r is equal to x x hat plus y y hat and you will see that k dot r will turn out to be k 0 x x plus k 0 z z indicating that wave is propagating in x and z planes.

So, we stop here in this module and in the next module, consider what happens when a way which is propagating in free space goes and hits a medium, right.

Thank you very much.