Fiber - Optic Communication Systems and Techniques

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Lecture – 05
Boundary conditions and reflections from a PEC

Hello and welcome to NPTEL MOOC on Fiber - Optic Communication Systems and Techniques course. In this module we discuss Boundary conditions and reflections from perfect electric conductor.

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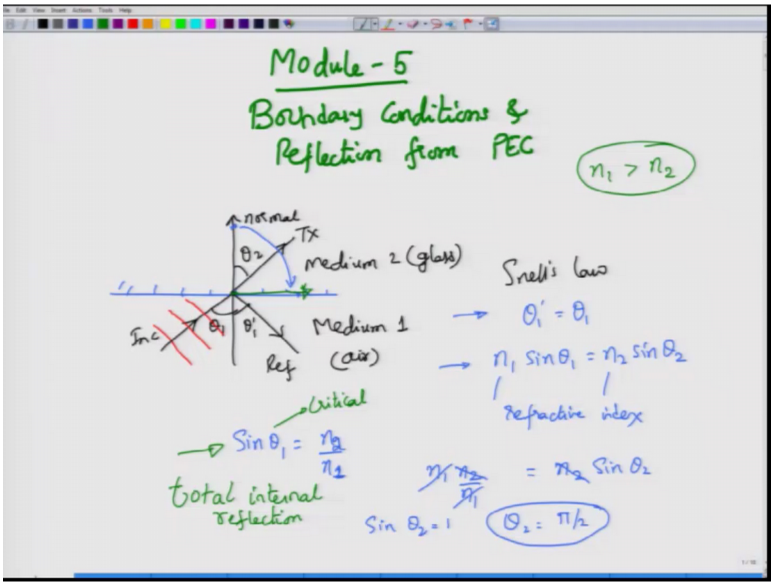

Now, before we discuss boundary conditions and whatever the reflection from a perfect electric conductor, let us go back to our childhood and look at some of the experiment that we would do without really knowing that they were experiments that involve light.

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So, for example, I have a glass of water here and then what I have done is that I have kept this pen at a angle of course, because the cup cannot hold the pen at any different angles. So, just kept it like this and if you focus a little bit on to this, one you will see that the so, if you look at it from the top of course, you can do at home in a matter of minutes you just pick up a glass of water, and then put the water I mean put the pen inside that water and what you see is that this appears the pen all the ways straight and has no discontinuities. It appears that the pen is actually bent.

If you look at it from my perspective what I am holding, you will also see a reflection of this pen coming from the bottom and then it is actually on the top layer of the water. So, you can see double reflection. So, to speak, but the primary thing that we have all observed in our you know childhood is that if you dip a pen here although the pen is supposed to be straight the pin appears to be bent it is. It appears that the pen is actually kind of displaced in the water and disappearing at a different direction.

So, we have seen this phenomena maybe, we have also tinkered with this different angles and then observed how this bent inside the inside the glass would look like. And we have done it without really knowing what causes all this type of a phenomenon. It turns out that this phenomenon that I just showed you and there are lot of interesting thinks that you can do it for yourself.

So, I would suggest that pick up clear glass of water then put up pen and then actually follow what I know goes on to the kind of angles that would that that you would see. And then you are curled up or twist the pen around turn the pen around and then you can see how as you change the angle at which you keep the pen inside the water the corresponding you know the image in the water itself also changes, or other the part of the pen in the water also changes.

So, all these we have seen and this goes by the name of reflection and refraction phenomena. Reflection is when you stand in front of a mirror and you are able to see yourself that is reflection, and when you do experiments like the one that I demonstrate at the beginning of the module is what is called as refraction. Sometimes this is also called as transmission. So, in it seems that whenever light enter from one medium another medium in the example that I showed you, the medium where the light was initially propagating is definitely in the air right. And then the other medium to which we have looked at is the combination water and glass. So, water is the second medium.

So, whenever light goes from one medium to another medium you expect two things to happen. One is partial reflection. So, the entire light would not be reflected unless you specifically create conditions for that to happen. So, they will always be just about partial reflection and there is also partial transmission into the second medium right. These observations are not new of course, these observation have been going on for thousands of years, but it was nilled to first formulated some quantitative loss that connects this reflection and refraction phenomenon right. He specifically considered the following situation.

So, you have this blue line here indicating the interface between 2 medium. So, as a it showed you one medium is air the other medium is glass. For example, or it could be water I have written glass here, but the same kind of idea supply of the air right. So, we had light represented in the form of a ray, but because you have looked at or you have listened to the last module, you perfectly know that light is not just composed of a single line that we would call it as a ray, but also this red lines which are all the constant amplitude friends along which you will have electric field, and perpendicular to both these electric field as well as a direction propagation would be the magnetic field. So, light is actually a wave.

But for our application or for our consideration in this first few minutes it is sufficient for us to consider light as a ray and then we know that right. So, when light is incident to the second medium at an angle theta 1 then there is a partial transmission into the second medium where the light makes an angle of theta 2. Just like pen dipped in the water you would seem to have an angel theta 2 with respect to normal to the interface right. So, with respect to normal to the interface there is an angel theta 2 and there is a partial reflection as well and the refraction angle would be denote in this case I have denoted it as theta 1 prime. What is Snell observed was that the angle of reflection which is theta 1 prime is exactly equal to the angle of incidence.

So, this is called as the first Snell's law of reflection or first Snell's law. And this pertains into reflection. And the second law that he formulated by lot of careful experiments was that the amount of angle theta 2 that the ray into the second medium makes with respect to the normal actually depends on several parameters. It depends on the refractive index of the medium not only of the second medium, but also of the first medium so. In fact, the law goes like this n 1 sin theta 1 will be equal to n 2 sin theta 2, where n are the refractive index of those particular media.

So, for medium 1 it is n 1 and from medium 2 it is n 2. So, you can use this equation to calculate what would be the angle theta 2. You can use this equation to know: what is the angle of reflection and as you change the angle of incidence theta 1 this is theta 1 is the angle of incidence the corresponding angle theta 2 also changes right. There is also a certain specific case where you might you know end up having a same following scenario. So, for example, if I somehow make sin theta 2 equals n 1 by n 2, right then what will happen if I make sin theta 2 equals n 1 by n 2 then Snell's law tells me that n 1 sin theta 1 will be equal to n 2 or rather I am sorry; so, if I try and make sin theta 1 not theta 2.

So, if make sin theta 1 which is the angle of sin of angle of incidence into something like n 2 by n 1 right. Then what I will have is n 1 times since it is sin theta one and we have just made sin theta 1 equals n 2 by n 1. So, this becomes n 2 by n 1 and then you have n 2 sin theta 2 and n 2 cancels on both sides and then what you have is sin theta 2 equals 1. Because theta 2 can be at most from 0 to pi by 2 correct because 0 corresponds to propagation, along the normal and pi by 2 corresponds to propagation at 90 degrees to the normal.

So, what this equation it is telling me is that theta 2 can be equal to pi by 2 which means that the ray instead of being transmitted into the second medium actually starts to travel parallel to the interface. And this phenomenon where the angle of incidence is chosen in certain manner which is given by this expression such that the corresponding transmitted light propagates parallel to the interface is called as total internal reflection.

So, this phenomenon is called as total internal reflection. And this angle at which the total internal reflection just barely is possible is called as the critical angle. Now, observe certain thing in this sin theta 1 equals n 2 by n 1 expression, is that possible for n 2 to be greater than n. It is not possible for n 2 to be greater than n 1, because in that case this right hand side will be greater than 1. And we know that real solutions of sin theta can never be greater than 1. So, if you plot sin theta you would see that the limit on sin theta is always plus 1 on the positive side and minus 1 on the negative side.

So, it cannot go beyond that. So, clearly total internal reflection cannot happen when the incident medium has a refractive index which is less than the medium to which we are trying to or which we have incident the light. But the other situation is possible. So, I can always have a situation where n 1 is greater than n 2. In this is case theta 1 will have a real solution which means that one of those angels becomes the critical angle for which the ray of light actually propagates parallel to the interface. So, total internal reflection phenomenon happens only when n 1 which is the medium of refractive index of the medium or the incident medium is greater than the second medium ok.

So, this is a fundamental condition for total internal reflection. We have talked about all these angles reflection refraction phenomena, but if I were to ask you how much power is actually reflected when light is incident and is that the only thing that happens, is it also possible for us to look at the face of the light upon reflection or face of the light after transmission. Is it possible for me to have light totally internally reflected, and if that is happened if if that happens then what will happen to the fields on the second region in or the medium 2. These kind of questions cannot be answered by the simple picture that we have. So, for considered the simple picture being that of light rays right.

So, in order to answer these questions it is necessary for us to look at the electromagnetic nature of light. And that is what we are going to do. So, what we actually have is the following scenario.

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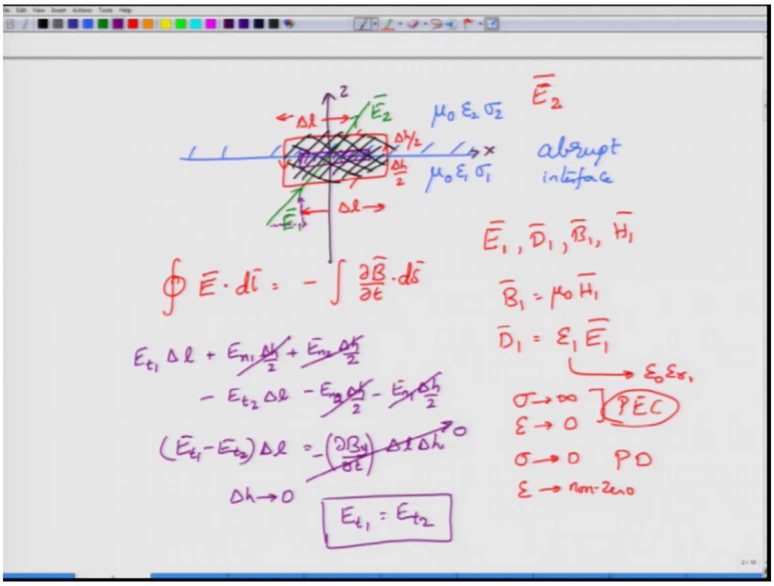

So, I have a certain you know interface. So, this is how I just represent an interface and this is a very specific kind of an interface this is called as an abrupt interface, meaning that this is a idealization of an actual interface that would happen I mean practice you would not have a scenario, where the medium properties let us say mu one epsilon 1 and sigma 1 and the second medium has a property of mu 2 epsilon 2 and sigma 2. And for simplicity I have considered what is called as linear isotropic homogenous media both in the first medium as well as in the second medium none of these quantities are complex, there is no loss none of these quantities are mattresses which means anisotropy ok.

So, we have considered a simple medium move for most of our discussions in this course this type of media is perfectly ok. And we even have certain simplification of this media. So, we do not consider non magnetic media; so, which means that I can put the value of permeability as the same for both medium. In fact, it is many cases we do not even have sigma 2 and sigma 1, but there are a few cases where you would take a for example, you would take a fiber and then coat it with a metal for some sensing applications.

So, in that case you will have an interface between one dielectric, which is the glass material and coating which is a metallic material or a conductor. And of course, using a conductor also gives us couple of ideas about how to talk about modes and that is primarily the interest in which I am introducing this conductor to the discussion ok. So, anyway those things come later on.

So, I have an interface up here and this interface has a normal. What is normal? Normal simply means a line which is perpendicular to the plane of the interface. So, if I have an interface in this particular form these correspond to the normal. So, the direction of the pen here would correspond to the normal. Now, look at it from your side. So, maybe I will turn it around like this. So, you look at it from this side if you are looking at this plane right. So, I have one I mean I have a set of fingers going this way which I can call as the x axis, the direction of this pen we would call as the z axis. And then there is another line you know the another finger which is a thumb that is pointed towards you this would be the y axis.

So, I have set up a Cartesian coordinate system in which the interface itself is in the plane which is described by x and y, the normal is along the direction z. And the plane of the incidence will be the plane x and z because, light ray would actually come at this particular angle. So, this line or this finger is the one that is depicting uniform plane wave coming in at an angle theta 1 as measured with respect to this pen this pen is the normal direction. So, this is the setup that we have considered.

And when I considered such a set up I am actually interested in knowing: how would the following fields change as we go from one medium to another medium. So, let us say we put in all are field quantities with a subscript of 1 to indicate that these are medium 1 values and then we put the corresponding subscript 2 for second medium. Of course, I know that electric field magnetic field and all the other field quantities are all vector quantities. So, I do have a vector sign on this.

So, on the first medium I have E 1 D 1 B 1 and H 1, but if you really look at this; since we have considered only non magnetic material B 1 is equal to mu naught H 1. So, apart from the small scalar value there is nothing really happening with. Or there is nothing really different from B 1 and H 1. So, we could use one of them to discuss about the. So, called boundary condition where as D 1 and E 1 are not normally. So, simple.

So, if the medium is perfect dielectric, then this D 1 can be written as epsilon 1 E 1 where my notation epsilon 1 is actually epsilon 0 epsilon r 1. So, instead of writing all the time epsilon 0 I have combined that epsilon 0 with the relative permittivity and called it the total permittivity epsilon 1, but if the medium happens to be a perfect conductor then for the perfect conductor sigma would go off to infinity whereas, epsilon would go off to 0. So, this corresponds to a perfect electric conductor with which has no losses. On the other hand, I have a perfect dielectric for which sigma is 0 whereas, epsilon goes to whatever the non 0 value it cannot go to infinity or it does not go to infinity normally.

So, a perfect dielectric is 1 which has a epsilon which is non 0, but the condition for dielectric is that there are no I mean there is no non 0 sigma value. So, these are some of the definitions that we have and for today's module we will be focusing mostly on the perfect electric conductor ok. So, what I am I interested in; so, after having set up the coordinate system. So, maybe this is the coordinate system. So, this is the z axis this is my x axis, y axis would of course, be coming out are going into the page does not really matter to us and what I have is a case where there are 2 fields. So, there is an incident electric field. So, please note that this is not ray of flight this is actually the field vector itself that we have drawn.

There is a reason for this it will become very clear. Once we go to the next part of the course you are next part of the module here. So, for now just imagine that there is an electric field E 1 here and there is a corresponding electric field E 2 in the second medium. Now, imagine that I am you know putting up this looped ok. I will also label the direction of this loop the sense of direction around which I have to pass through the loop.

So, this loop let us say has a length delta l. So, one half of the loop is laying in the medium 1 the other half of the medium I mean loop is laying medium 2. The one that is lying in medium 1 has a height of delta h by 2 here, and the same height in the other medium as well. So, we have to imagine that I have this interface plane and then I have. So, I have this interface plane and then I have this line which is delta l long. And then it plunges onto the second medium the total height of this one is delta h goes down at delta l again and then comes back up to complete the loop.

So, we have looked at it carefully the direction of the sense is that this is along x direction whereas, in the other loop it is a long minus x direction. So, it does not really matter which way you take, but you have to be consistent with whatever you take ok. So, you cannot take both delta l to be positive it does not really go that way.

Now, what am I going to do with these equations, with this picture? Well I know that integral closed loop integral of E is actually given by minus integral of del B del t dot ds ok. What is the ds that we are considering that is the surface element or the surface area which is actually created by this loop which is enclose by this loop. And of course, this surface area has to be open whereas, the loop here on which we are integrating the electric field has to be closed.

So, writing this left hand side with whatever the electric field that I have I first noticed that, or I first note that this electric field can be broken up into 2 parts. One part will be along the delta l axis or the x axis or the tangential component as we would call it. And the other component will be perpendicular to it would be along the z axis right.

And in this day d l expression I am only interested in the horizontal component there is the reason why that will be true I will show you in a minute because if I write down the left hand side what I have is E t 1 where t stands for the tangential component and one of course, stands for the medium 1 I have E t 1 delta l which is along the x axis here. Then I have plus E n 1 delta h by 2 then I have E n 2 delta h by 2 correct I have E n 2 delta h by 2 minus E t 2 delta l why is it minus because the delta l sensors are different in the medium 1 and medium 2 correct.

So, this is what I have and then I have minus E n 1 or rather E n 2 delta h by 2 minus E n 1 delta h by 2 clearly these cancel out. And leaving the left hand side simply as E t 1 minus E t 2 times delta l. What about the right hand side? Well for the right hand side we have the surface area on the surface area that we are considering actually has a total surface area of delta l times delta h.

And then you have a certain magnetic field changes that are happening in this particular loop right. So, if I have the magnetic field then I need to know what the corresponding magnetic field. The sense of direction of ds will be along the y axis in this case. So, I need to know del B y by del t in the 2 regions of course, right. So, I need to know with the minus sign as well.

So, this is from the right hand side of this expression and what I do now is that I take the limit of delta h going to 0. So, which means that I have this loop and I am basically going to make the loop hug this x axis or the plane of interface very tightly ok. So, that delta h goes up to 0; as delta h goes to 0 and we also assume that the B fields are going to be finite everywhere.

The right hand side will completely B equal to 0 and then living the left hand side where delta l is non 0 the only condition that can be satisfied here are the only conclusion that is possible is that E t 1 equals E t 2, which is tangential electric field equals tangential electric field on the second medium as well. So, this is regardless of whether this medium is dielectric that medium is conductor.

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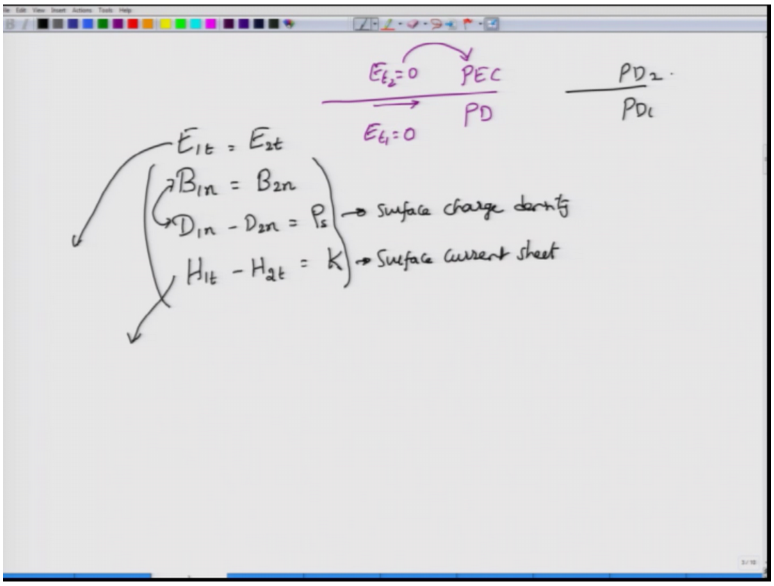

For example if I have perfect dielectric medium and the perfect electric conductor this condition of E t 1 equals E t 2 simply means on the interface plane E t 1, itself will be equal to 0. Why? Because E t 2 is already equal to 0 by definition of a perfect electric conductor; so, perfect electric conductor cannot sustain electromagnetic fields inside it. So, clearly in that case E t 2 will be 0 which also means that, on the surface E t 1 will also be equal to 0. I will leave the discussion of other boundary conditions ok, but I will just mention those boundary conditions. B 1 n equals B 2 n indicating that in medium 1 the normal component is the same as medium 2 normal component.

For D it is likely different D 1 n minus D2 n will be equal to rho s again do not worry about the signs here, whether it is minus or plus there actually determined by the kind of problem that you are looking at the signs depend on whether the charge is positive or negative, but for our consideration as of now D 1 n minus D2 n corresponds to the difference in the normal d components. And that difference in the normal d components should be exactly equal to the surface charge that is contained in that particular surface.

So, you can do this or you can obtain this equation by applying Gauss's law to the 2 medium situation that we have discussed I have not done. So, there is the tangential magnetic field condition, which says that the difference in the tangential field components from medium 1 and medium 2 should be equal to the surface current sheet ok. So, this is the surface current sheet and this is the surface charge density. So, the discussion of these boundary conditions is nicely summarised in the textbooks that I have suggested.

So, please take a look at them I am not going to derive it, but to just complete whatever that we have been writing, we have even t equals E 2 t all the time no problem whatever the medium are B 1 n is equal to B 2 n all the time, but the d field and h field are discontinuous by the exact amount of surface charge density that can be present and surface current sheet that is possible.

In turns out that in most of the course I mean in most of the modules on the course we do not really bother with the normal conditions ok. So, we are only interested in the tangential conditions and even here we are most of the times fortunate, because we will not be considering the perfect electric conductor interfacing with a perfect dielectric most of the times we are interesting p D 1 and p D2 where p D 1 is perfect dielectric in medium 1 p D2 is perfect dielectric in medium 2. So, these are the boundary conditions that are necessary for us to investigate the phenomenon that we have described at the beginning of the module.

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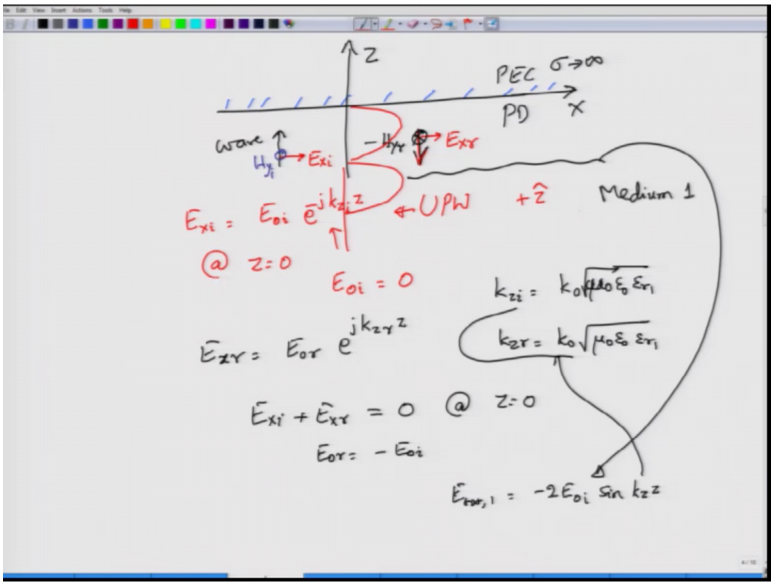

So, let us specialize to a very interesting case. This case happens to be an interface between a perfect dielectric and the perfect electric conductor ok. And then I have a normal here which of course, will be along the z axis and this one will be along the x axis this is my interface. Let me just indicate that this is a interface that I have considered. And now what I consider is not electric field I actually considered a wave propagating along the direction of z in medium 1 ok. So, let us say the medium 1 has a certain refractive index or equivalently the relative permittivity. P E c of course, simply means that sigma is equal to infinity or sigma is set to infinity.

So, I have a wave here. So, this is wave whose electric field is oriented along the you know x direction. So, I have an electric field along x direction. Of course, I have a magnetic field in the y direction and the subscript of I indicates that I am considering the incident medium. So, clearly if you look at the interface E x i happens to be that of a tangential component if I write down the expression for E x i E x i will be a function of z of course. So, we have some incident amplitude. And then I have E power minus j k z i times z right. This is the uniform plane wave that we have considered which is propagating along the plus z axis and this is there in the medium 1 right. So, this wave keeps on propagating approaches and approaches the interface.

Now, what happens at the interface clearly there is no electric field in the second medium. So, whatever that must happen must happen at the interface plane z equal to 0 living me, or living us with the condition that E o i will be equal to 0 correct. If I go back to this expression substitute z equal to 0 the exponential term will be 1 and then you are requiring that the total tangential electric field heating the perfect dielectric be actually equal to 0.

If that condition is imposed, then it also turns out that E o I itself is 0 that is incident amplitude itself is 0 which means fields are 0 everywhere. This is not a very helpful situation. So, the only wave we can reconcile with non-zero wave propagating rights and non 0, wave propagating and heating the perfect electric conductor is that there is a reflected wave.

So, there is a wave which is reflected back and the reflected wave propagates in the direction which is opposite to the incident wave. I have assume I am going to assume that the reflected electric field will be in the same direction where as the magnetic field will be in the opposite direction. So, I will have minus h y r. Because E x r is given by E o r which is the amplitude of the reflected field, but this is propagating in the other direction right. So, it is propagating in the minus z direction.

So, I have k z r times z where k z i is equal to the free space k 0 times square root of mu naught epsilon naught epsilon r 1. And similarly k z r will also be equal to k 0 times mu naught epsilon naught epsilon r 1 because, these 2 are actually in the same medium. So, k depends only on the medium properties not the direction of propagation. So now, the total condition that I actually have is that E x i plus E x r must be equal to 0, at the interface z equal to 0 which clearly gives me E o r equals minus E o i. So, if you are at some particular distance and then look at the total electric field, the total electric field in this case in the medium one will be equal to minus 2 E o i let us say, and then sin of k z times z where k z is a simple notation for representing both k z i and k z r.

Now, this condition make sense because a z equal to 0 the electric field actually goes to 0 and after that the electric field would actually you know increase go to 0 increase go to 0 and so on. So, this is how the electric field gets reflected from a perfect electric conductor. We will postpone the discussion of perfect electric conductor to some other module where it is really required when we talk about the modes.

For now, thank you very much.