

**Fiber - Optic Communication Systems and Techniques**  
**Prof. Pradeep Kumar K**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Kanpur**

**Lecture - 53**  
**Higher order modulation & Coherent Receiver**

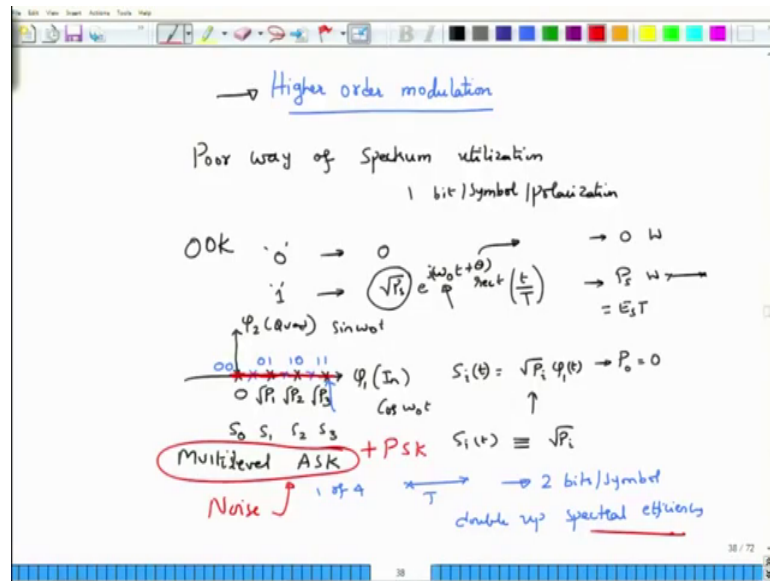
Hello and welcome to NPTEL MOOC on Fiber - Optic communication Systems and Techniques. We will now discuss in this module different way of conveying information using optical fiber; of course, this different way of conveying information is quite well known in the other communication systems especially in the wireless communication system or the older wired copper cable communication system.

But what makes it interesting for optical communication system is that unlike the previous method where we modulated information on the intensity of the optical signal; that is if you wanted to transmit a bit 0, you would transmit a certain power, and then you would transmit another power to send a bit 1. Usually the power that is sent for 0 was very small and the power that is sent for 1 dependent on what is the maximum limit that the channel can actually with stand ok.

And have we saw in the previous modules this type of a simple modulation technique was actually equivalent on off keying system. And at the output of the fiber all you have to do was to place appropriate filters amplifiers and then eventually put in a photo detector follow it up by some simple signal processing in order to retrieve the information that was set ok.

So, the modulator structure was simple and the receiver structure was also simple. And this was actually very nicely done in the sense that the receiver which the front end part of the receiver would essentially consist of a photo detector. And we know that photo detectors are going to measure the intensity of the optical signal, because the current that they generate will actually be directly proportional to the optical power that is incident on it ok.

(Refer Slide Time: 02:09)



However, this type of a modulation is actually very poor way of utilising the spectrum. So, this is very poor way of spectrum utilization which means that the spectral efficiency that is the way in which we are going to use the allotted spectrum is actually not very good. Because, at most you are going to send 1 bit per symbol, right. If even if you were to put in for example: polarization multiplexed system in which you would actually transmit 2 different polarizations and at the output we actually put 2 sets of photo detectors, retrieve the 2 polarization separately and then determine the bits. Your bit per symbol per polarization would always be equal to one bit per symbol.

So, you could even say that even when I use polarization I am essentially not making the full utilization of the spectrum; although yes, considering two polarizations I would be transmitting 2 bits, but then if you actually look for per bit per symbol per polarization that would essentially be just equal to 1 ok. And this is not a new problem. This is an old problem and people have suggested many methods to increase a number of bits per symbol and in this case you can increase a number of bits per symbol per polarization. And what are those methods? Those methods fall generally under the headline of what is called as higher order modulation. In the intensity modulation we sent bit 0 by transmitting 0 here right; over 1 time duration. And for sending bit 1 we transmitted the carrier signal with an amplitude of square root P s or equivalently the baseband signal which has an amplitude of square root P s. And of course, this was a duration of 1 bit

period or symbol period here bit and symbol were actually equal to each other and this is exactly what we transmitted.

Please note that, we may have some phase to the carrier, but we did not exploit this phase nor be exploited the fact that we could actually have modulated  $\omega_0$ . By the way I am showing you the amplitude version of the equations, but if you were to just follow the optical power this would correspond to optical power of 0 watts what and this would correspond to an optical power of  $P_s$  watts over whatever the time duration that you are actually transmitting right.

So, you can also of course, right down this  $P_s$  in terms of energy as well. So, you can say that  $P_s$  is equal to  $E_s$  times  $T$  where  $T$  is the duration of the time for time period of 1 bit. And this is what we actually transmitted in terms of the power. But if you go back and look at it in terms of the amplitude the electric field that would be propagated in the fiber would actually be in this form: exponential to the power  $j\omega_0 t + \theta$  we neither exploited  $\theta$  we neither exploit or we exploited  $\omega_0 t$ , we just change the amplitude. Now you could take this forward: for example, one way of implementing this higher order modulation is to consider four different amplitude levels.

So, let us say that this amplitude level you know is 0, then you have an amplitude level of  $P_1$  then you have  $P_2$  and  $P_3$  with  $P_3$  being the maximum power that the link can withstand and therefore, within the minimum power 0 to maximum power  $P_3$  that the link can be extended we have create a 2 sub levels of actually other powers  $P_1$  and  $P_2$ . Now we want to write in terms of amplitude we do not want write in terms of power. So, therefore, I am going to put a square root of this going to everything. And then if you remember our signal space analysis we said that this any signal which has a certain duration and I know has a certain finite energy could be actually expanded in terms of certain basic functions right. And that is precisely what we have done here.

These  $S_0 S_1 S_2 S_3$  are all different symbols or all different waveforms. And each of those waveforms are actually 1 dimensional signals themselves they have a point up there. So, the  $i$ th signal here can be written as square root of  $P_i$  with  $P_0$  being equal to 0. This is my definition time's  $\phi_i$  of  $t$ . So, I have a one dimensional signal set here. So, all these signals  $S_i$  of  $t$  which are actually square root of  $P_i \phi_i$  of  $t$  where  $\phi_i$  is the

basis for these functions and  $S_1$ ,  $S_0$ ,  $S_2$  and  $S_3$  all are those signals. So, this essentially means that these are the constellation points.

So, any signal  $S_i$  of  $t$  can be put in onto a 1 to 1 correspondence with the appropriate constellation point  $\sqrt{P_i}$  which is defined in the space  $\phi_1$ . Of course, I am showing you 2 dimensional space, but in reality the signals are actually only on one dimension ok. In fact, if you take this  $\phi_1$  to be the in phase component and  $\phi_2$  to be the quadrature component, we actually end up with the even more common in phase quadrature component wise constellation.

So, in phase would essentially be  $\cos(\omega_0 t)$  and quadrature will be  $\sin(\omega_0 t)$ . So, you can use this to even represent all the signals that we are going to consider from now on in terms of the signal space we have already done this for the case of on off keying where we had only 2 levels  $S_0$  and  $S_1$  with appropriate powers of  $P_0$  and  $P_1$  ok. So, this is the way in which you can actually extend the simple concept of on off keying to say multilevel signalling ok. So, this is multilevel signalling or simply called as amplitude shift keying because your amplitude is now going over more than one levels. Now what is the advantage of doing this? Now the advantage here is that since at any time  $T$ . So, since any time  $T$  which is the signalling interval I am about to choose 1 of the 4 messages, I can assign binary symbols to these binary data or binary words to each of this  $f_0$ ,  $f_1$ ,  $f_2$  and  $f_3$ . How I can code this 0 0 or the 0 amplitude wave form or the symbol  $S_0$  as 0 0, then the next amplitude I can consider it to be 0 1 then I have 1 0 and then I have 1 1 ok.

So, these are the 4 different amplitude signals which I am transmitting and each of those signals can be internally binary coded. So, what it has done is to change the number of bits per symbol if you want you can also put per polarization I will leave that on to you. So, I have increased the number of bits per symbol from 1 bit to 2 bits per symbol. So, what I have actually done is to increase the symbol rate while the bandwidth essentially remains the same.

So, this statement I am not going to prove here you can look at your undergraduate textbooks for that, but essentially what we have done is to double up the spectral efficiency. So, I have doubled the spectral efficiency what is the problem with this approach? When the problem with this approach is that one you have to switch the laser

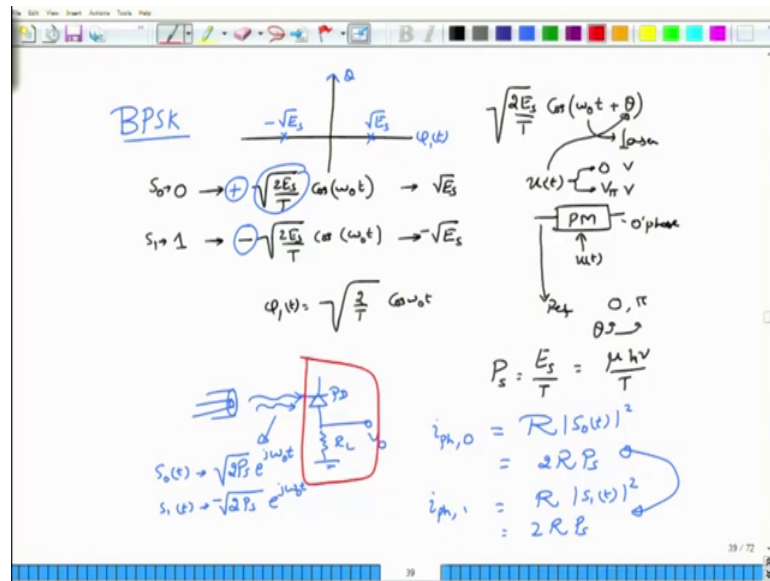
amplitude into 4 different levels that requires you to generate multi level signals in the electrical domain because you know we can control the output power of the laser that is coming out of the modulator, by adjusting the modulating signal  $u$  of  $t$ . So, I have to have multi level electrical signal which is not easy to generate and moreover I cannot continue this approach ok. Because the maximum amplitude or the maximum power that the link can with stand before link gets degraded and I will tell you why it is gets degraded later on, is actually usually fixed.

So this is the maximum amplitude. So, if I want to increase the spectral efficiency further I will have to start making smaller and smaller intervals between the different symbols right. But what I have so, far shown you is only the signal points. But in a real system when you take this constellation point and actually look at what the received constellation will be; the received constellation is going to be looking something like this and this looks because all this red indicates that even when you transmit just say P 3 or corresponding to S 3, you will still have noise which will be distributed. So, the received constellation points will not exactly follow on P 3, but they will be distributed around this one of course, this distribution will actually be 2 dimensional, but I am showing you only 1 dimensional to bring out that this is a 1 dimensional set.

So, as I start making the interval between the symbol smaller and smaller that is increase a number of amplitudes, I may increase the spectral efficiency, but I am actually going to be very very prone to the noises that are present. So, noise actually kills this multilevel ASK system. It was tried ASK systems with 4 levels for tried earlier, but this did not give the required performance that people were looking for. So, today we do not really use multilevel ASK alone we actually combined ASK with what is called as PSK.

So, what is PSK? Now that is your, we are going to get very interesting thing out there.

(Refer Slide Time: 11:27)



Now, you have your carrier signal; in general when you take a carrier signal you can either specify a carrier signal by its amplitude. So, you will have signals which are actually transmitted in this form right. So, this is the same thing whether you are looking at a wireless system or an optical system; here  $\omega_0$  comes from the laser. So, that's all the differences in the wireless or rf communication this comes from an RF source or a crystal whereas, in this lay in optical communication this comes from a laser. And then I have this plus theta which is the phase that this laser would you know emit.

Now I have my signal  $u$  of  $t$  and as we have seen earlier, one of the natural ways of converting an electrical signal into an optical signal is to go and modulate the phase of the optical carrier right. So, I can go and modify the phase of the optical carrier and in order to do that I will be using the phase modulator. What if you choose  $u$  of  $t$  to be that it will either produce a 0 volt or it would produce a  $V \pi$  volt when you send  $u$  of  $t$  with 0 volt the output and input electric fields are in phase with respect to each other.

So, therefore, you can think of this thing as 0 phases being transmitted ok. So, this is 0 phase being transmitted whereas, when  $u$  of  $t$  becomes equal to  $V \pi$ , then the input and output are 180 degree out of phase with respect to each other and taking this input as the reference your either being in phase with the output is either in phase with the input or it is  $\pi$  phase shifted with the output. And these 2 can be you know mathematically captured by allowing theta to go from 0 or  $\pi$ .

So, when you want to transmit a bit 0, you consider transmitting square root of  $2 E_s$  by  $T \cos \omega_c t$  plus 0 phase right. So, that is you set  $\theta$  equal to 0 and when you want to transmit 1, you transmit this way. So, you have square root of  $2 E_s$  by  $T \cos \omega_c t$  plus  $\pi$  which of course, can be simplified because  $\cos a \pm b$  formula can be applied  $\cos a \pm b$  is  $\cos a \cos b \mp \sin a \sin b$  or plus  $\sin a$ ,  $\sin b$  I think, but  $\sin$  of  $\pi$  will anyway  $b$  equal to 0 and  $\cos$  of  $\pi$  will be minus.

So, what you would be transmitting is minus square root of  $2 E_s$  by  $\cos \omega_c t$  and because  $\cos \omega_c t$  plus 0 is the essentially  $\cos \omega_c t$  these are the 2 carrier wave forms that you are going to transmit. But you can express this carrier wave forms by first defining say is square root of 2 by  $T \cos \omega_c t$  as this basis function  $\phi_1$  of  $t$  and in terms of this basis function, 0 symbol which let us says  $S_0$  and  $S_1$  ok.  $S_0$  equals 0,  $S_1$  equals 1. These are the wave forms that correspond to the symbols this would actually have a constellation point which is given by square root  $E_s$  this will will have a constellation point given by square root, sorry minus square root  $E_s$ .

Of course, in the way that we have written I could I have easily used the power here, but I am sticking with energy because this is kind of quite common in digital communication terminology. So, it might be better for us to get used to seeing this energies here and if I were to ask you what is the energy of a laser over the time period of course, it produces whatever the laser output power, but energy is basically the average photon number that you have times the energy of each photon which is about  $h \nu$  and this is what is the relationship between energy power and time ok. So, you can clearly see that it is possible for us to actually relate energy and power and energy and the actual energy of the photon pitch. So, in terms of the photon pitch also we can specify the energy. So, but you notice here something. Right?

So, I have square root  $E_s$  and I have minus square root  $E_s$  in terms of the carrier of course, I have square root of  $E_s$  times  $\phi_1$  of  $t$  and minus square root of  $E_s$  times  $\phi_1$  of  $t$ . In terms of the constellation, I have two points here, 1 corresponds to square root  $E_s$  and the other corresponds to minus square root  $E_s$  on  $\phi_1$  of  $t$ . This is the (Refer Time: 15:48) component anyway this is the quadrature component and I am transmitting this constellation simple ok.

So far so good we have transmitted binary phase shift keying signal. This is called as binary PSK or binary phase shift keying and we have transmitted this one. Now, how do I receive? If you for a moment, don't give it a deeper thought you might think that I can simply put in a photo detector, have a load register  $R_L$  and then take out the voltage across that load register and I should be all right. Alright, that is what you would think.

So, this is at the receiver after we do it has come out from the fibre right. So, after it has come out from the fibre the light is incident on the photo detector and then we hope that the photo detector current  $i_{ph}$  or the photo current would essentially allow us to distinguish between 0 and 1 ok. What is a complex electric field that you are receiving here? The complex electric field that you receive here will be square root of  $2 P_s$ . I am now writing  $P_s$  equals  $E_s$  by  $t$  e to the power  $j \omega_0 t$ . When you transmit  $S_0$ , that is, for the symbol 0 you are transmitting this one. For the symbol  $S_1$  the electric field will be minus 2 square root of or minus under root of  $2 P_s$  e power  $j \omega_0 t$  right, And what would be the photo current When you transmit Bit0?

The photo current will be  $R$ . Where  $R$  is the responsivity times the power and power is basically  $S_0 t$  magnitude square. Therefore, this is equal to  $2 R P_s$ . The factor of 2 is not very important you can just ignore that one. I just did not want to change the equations here and therefore, I have included here, but normally what we do is when we have a cosine  $\omega_0 t$  They will also be half associated with that when you express this in terms of  $e$  power  $j \omega_0 t$

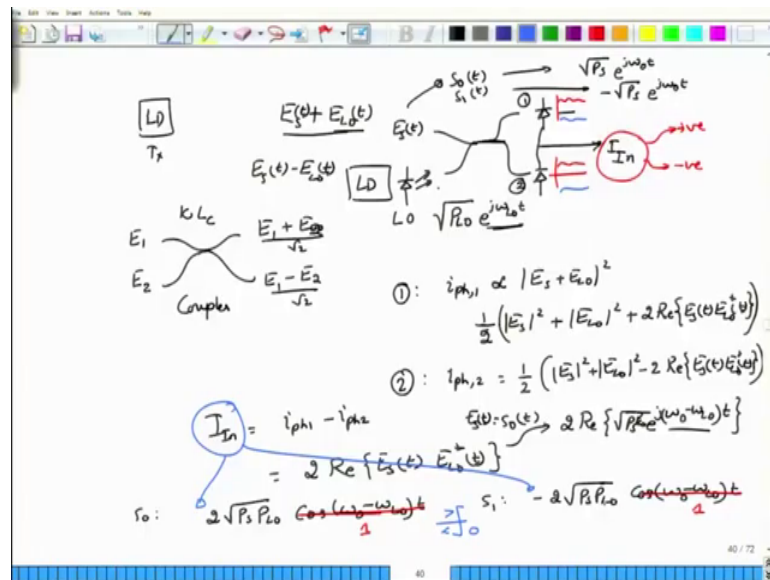
Therefore, they should have been written as square root of  $P_s$  by 2, but our goal is not to really look at this multiplicative constants they are just going to change the photo current levels here and there in terms of our mathematical model. but the basic idea here is that, when you transmit  $S_0$  of  $t$  then you would have received a power which is proportional to  $P_s$ .

Now, the photo current; when you transmit bit 1 will be  $|R$  magnitude of  $S_1$  of  $t$  square which would again be equal to  $2 R P_s$  in this case because  $S_1$  of  $t$  magnitude square is the same of  $S_0$  of  $t$  magnitude square. Now, we are you know fix. We get this photo current, but then both photo current values are the same, but my information is not in this intensity. My information is actually in the says whether I am transmitting a plus carrier or a minus carrier that is I am transmitting phase 0 or  $\pi$  it is what determines whether I



have received a big 0 or 1, but if by putting a simple photo detector I am not able to distinguish between the 0 phase signal and the pi phase signal then this entire detection system is useless right. Is there a way around for us to overcome this problem? Yes there is a way around to overcome this problem and that is a very clever idea ok.

(Refer Slide Time: 19:05)



Consider for example, what would happen you know when you have say some  $E_s$  plus  $E_{LO}$ . Now I have to apologize for using these symbols this in these symbols they actually stand for electric fields not the energies ok.

So, let us say I have two lasers one laser is at the transmitter. So, I have the transmit laser diode and then at the receiver I have another laser diode which I will call as the local oscillator. So, I have a local oscillator and what I do is I sum the two signals I sum the 2 signals and then send each of those signals to a photo detector right. So, on this branch what I have here is the received signal  $E_s$  of  $t$  which of course, could be either  $S_0$  of  $t$  or  $S_1$  of  $t$  depending on what you have transmitted, but here what I have is the local oscillator signal which I have referred to as  $E_{LO}$  of  $t$  this is the electric field.

So, in terms of your powers you can write this as say square root  $P_{LO}$  e power  $j \omega_{LO} t$  ok. Now  $E_s$  of  $t$  also we will rewrite this as square root of  $P_s$  e power  $j \omega_0 t$  or for  $e_1$ . We will write it as minus square root  $P_s$  e power  $j \omega_0 t$  ok. Now we are ready to look at what exactly is happening with this kind of a receiver. Now instead of using one photo detector I am putting in 2 photo detectors and for some reason I want to

actually generate the some signal which is going to go in this branch and then on the other branch, I want to generate the difference signal ok.

How do I generate these 2 sum and difference signals? Well this is what the device that we are going to use this device are we have seen is a coupler. Of course, I should actually have the aptitude split ratio of one by root 2 with this I am just not writing those 1 by root 2 terms here, but you understand the point right. So, I can send in electric field one in this port another electric field in this port there is 2 different light signals at the 2 different ports can the phase can be adjusted such that I am going to get  $E_1 + E_{LO}$  in 1 branch of course, I should get by root to here and then I get  $E_1 - E_2$ ,  $E_1 + E_2$ ,  $E_1 - E_2$  by root 2. I can adjust this phase instead of being ninety degrees I can make this phase to be equal to 180 degrees ok.

So, that we have already seen with by choosing appropriately the value of  $\kappa$  and  $L$  C the length of the coupler I can get whatever the phase I want and this is what I have. Now what is the use of these 2 sum and difference signals? Consider this as the first photo detector and these are the second photo detector. What is the output of the first photo detector? We will call this as  $i_{ph1}$  this would be equal to or this would be proportional to  $E_s + E_{LO}$  magnitude square remember there will be a one by root 2 terms.

So, you can also put that half term here, but if you expand this  $E_s + E_{LO}$  what you get is  $E_s$  magnitude square plus  $E_{LO}$  magnitude square. this is a simple this one plus you will get 2 times real part of  $E_s$  of  $t E_{LO}$  conjugate of  $t$  ok. If you want you can also put in this half later on it might be useful for you, then the output of the second one yeah as you can easily guess will be actually equal to half  $E_s$  magnitude square plus  $E_{LO}$  magnitude square minus 2 real part of  $E_s$  of  $t E_{LO}$  conjugate of  $t$ .

So, except for the minus sign here these 2 expressions are essentially same. Now got I am going to do is to actually connect this 2 and take the current which I will call this as  $I$  with a subscript  $I$  you will soon see I have put a  $I$  subscript of  $i$  or if you want I can put in  $I_{in}$  ok. So, I will tell you what  $I_{in}$  is  $I_{in}$  is actually equal to  $i_{ph1} - i_{ph2}$  now when you take the difference between these 2 the constant terms are at least the magnitude terms which are common to both photo currents will cancel with each other

and then you have  $2 \cos(\omega_0 t - \omega_{LO} t)$  will become  $4 \cos(\omega_0 t - \omega_{LO} t)$ , but there is a half here. Therefore, that becomes  $2 \cos(\omega_0 t - \omega_{LO} t)$  again.

So, what you get is  $2 \cos(\omega_0 t - \omega_{LO} t)$  times real part of  $E_s$  of  $t$   $E_{LO}$  conjugate of  $t$  ok. So, I will write this as real part of now I am going to assume that I have transmitted some bit 0 or 1 and then look at the output of this one for the 2 different symbols. So, initially suppose I am have transmitted  $S_0$  of  $t$  which means that  $E_s$  of  $t$  will be equal to  $S_0$  of  $t$ .

So, the output of the photo detector would be or the not the photo detector the output of this I or the output photo current  $I_{In}$  will be actually equal to  $2 \cos(\omega_0 t - \omega_{LO} t)$  times real part of square root  $P_s e^{j(\omega_0 t - \omega_{LO} t)}$  is that correct, because  $E_{LO}$  we have already written it here and conjugating oh sorry I should also multiply this one by  $P_{LO}$ . So, let me rewrite it in this case.

So,  $S_0$  will produce an output in which I can take  $2 \cos(\omega_0 t - \omega_{LO} t)$  pull this  $P_s$  and  $P_{LO}$  out because they are essentially constants at least I am not considered them to be time varying at this point and then I have real part of  $e^{j(\omega_0 t - \omega_{LO} t)}$ . Now what is the real part of exponential  $e^{j(\omega_0 t - \omega_{LO} t)}$ ? That is precisely  $\cos(\omega_0 t - \omega_{LO} t)$ . You can show that when you transmit  $S_1$  right then there will be a minus sign up there and then what you get is  $-2 \cos(\omega_0 t - \omega_{LO} t)$  and then you get  $\cos(\omega_0 t - \omega_{LO} t)$  times  $t$ .

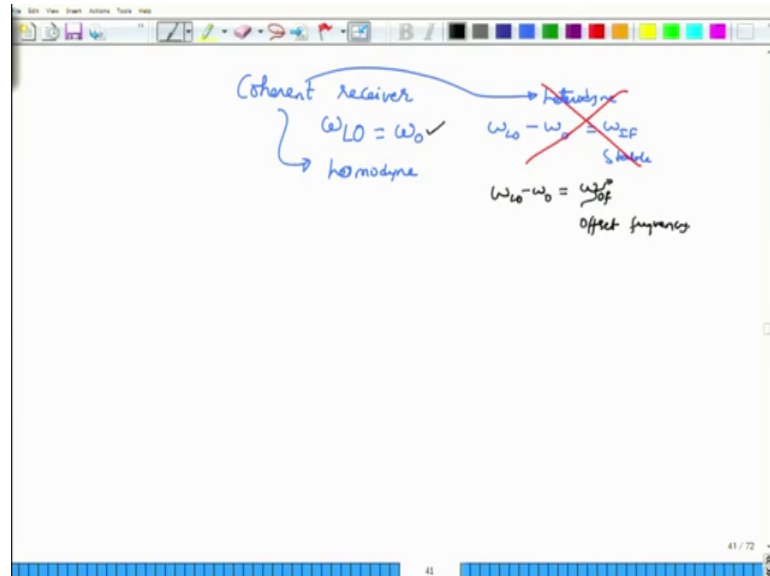
Now the magic is that if somehow I take  $\omega_0$  equals  $\omega_{LO}$  then this will be  $\cos(0 t)$  which would actually be equal to 1. Similarly this will also be equal to 1 and now you look at this the current  $I_{In}$  can be positive or negative please note that current can be positive or negative because we are actually looking at these differences.

So, whatever the dc values that are there. So, this is the actual received current here let us say this is the actual received current here and then the difference is what I am looking at and this difference could be positive or negative depending on whether you have transmitted bit 0 or bit 1 ok.

So, those possible combinations are there, but what is the magic here is that the current  $I_{In}$  can actually switch between it signs to positive and negative now if you were to put just a threshold of 0 here I am not considered noises here therefore, I am putting as threshold 0 here if my current happens to be greater than 0 then I know that I have

transmitted bit 0 if it is less than bit 0, then I know that I have transmitted bit 1 or I have received bit 1.

(Refer Slide Time: 26:21)



So, in this way write what I have done is to actually received the signals in the form of a coherent receiver or a coherent detection in which I have also assumed that  $\omega_{LO}$  is equal to  $\omega_0$  or rather  $\omega_{LO}$  is equal to  $\omega_0$  if  $\omega_{LO}$  is not equal to  $\omega_0$ , but it is stable right  $\omega_{IF}$  which is stable than this is called as heterodyne communication system. This is called as homodyne communication system.

So, what you have is coherent homodyne or coherent heterodyne usually we do not use this coherent heterodyne system not really useful for us or at least not really useful in this today's scenario. But this is what we trying to achieve, but if the difference  $\omega_{LO} - \omega_0$  is not equal to 0, but it is actually  $\omega_{off}$  and it is not very stable than this  $\omega_{off}$  is called as the offset frequency or frequency offset and this will actually impact our communication system as we will see in the later modules.

So, the lesson here is that a simple photo detector can never receive or directly measure the phase what you have to do to measure the phase is to do a coherent receiver. It is also possible to use the incoming signal itself as a reference, but that is again that is not something that is quite popular today. The popular thing today is the coherent receiver in which two different electric fields are going to bit against each other if the conditions are right.

Then the output could be there positive or negative and by looking at the signs you can decide whether you transmitted bit 0 or bit 1. So, in the next module we will bring out the two systems on off keying and the coherent BPSK system and then show that BPSK systems are actually going to perform 3 d b better than on off keying systems for the same operating conditions.

Thank you very much.