Fiber – Optic Communication Systems and Techniques
Prof. Pradeep Kumar K
Department of Electrical Engineering
Indian Institute of Technology, Kanpur

Lecture - 56
DSP algorithms for Chromatic dispersion mitigation

Hello and welcome to NPTEL MOOC on Fiber Optic Communication Systems and Techniques. In this module, we will continue the discussion of the DSP operation or the receiver that we were discussing in the previous module on how to mitigate impairments. We have already seen some of the impairments some more impairments we will be seen especially a non-linear phase noise, we have been discuss much about that, but we discuss that I need to first talk about nonlinearity. So, I am going to do that one later on. Let us first look at something that we have already seen and how to mitigate one of the important pairments such as chromatic dispersion, ok.

Let us look at what this chromatic dispersion term. This is the review for you.

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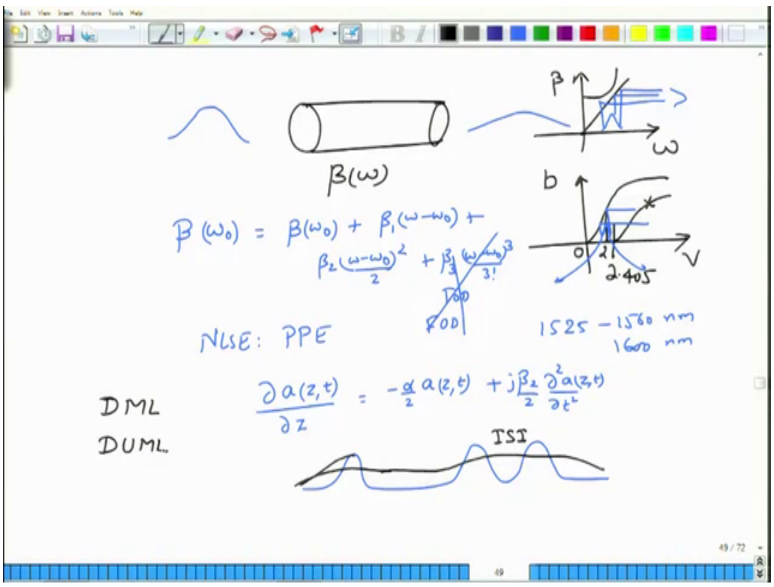

We know that chromatic dispersion essentially if the result of to put it simply, if the result of beta being the non-linear function of omega where beta is the propagation constant, omega is the of course the frequency. If beta of omega were to be perfectly linear that is if you were to plot the relation of omega to beta, if this relationship were to be linear then it could not have been problem for us, because all the different frequency components. So, maybe this is the pulse that you would have transmitted. I am just showing you one approximately Gaussian shape here. It did not be Gaussian depends on what kind of data you have transmitting.

But if all this components arrive with the same velocity essentially experience in the same phase, then the output pulse could not be distorted at all; it could not have broadened, it could not have shrunk but unfortunately, when beta and omega are not linearly related, their actually nonlinearly related. In fact, on optical fiber we have seen those curve in which on the y axis it plot the which is the normalized propagation constant and on the x axis we plot v which is the normalized frequency; for v is kind of f v is kind of b. Then we have seen that even for the fundamental mode right, you are going to see the tap look something like this.

And let me tell you most of this fiber even there single mode actually can support higher order mode which means that when the frequency changes then you will have to I mean they will support higher order modes, but we want really work with these mode. You want to keep the modes much bellow this. So, that necessities such that the v number actually be less than 2.405, but the same time if actually choose very small values of v number, then I would not get appreciable propagation constant at all in the fiber would not have a large propagation constant.

So, be tend to keep this v closer to the edge of stage 2 and there is some possibility that is if the pulse bit actually becomes large, then the fiber can become multi mode, but usually our range of values of v is sufficient first at the fiber can still be treated rather single mode with the reasonably large value of v. So, around this v is what you are going to have channel. They will be multiple channels. I have showing it with a not correct units of there, but if you are actually considered the entire c band which say 15, 25 to about 15, 60 nanometer or the extended c plus L band which course up to 15, 25 to 1600 nanometer. All of them will actually be able to fit into the small region around to ok.

So, there is no danger of making this fibers become multimode fibers anyway. Because the values of the propagation constant b are not the same for the entire range of the frequencies, we have beta turning out to be a non-linear function. And we have also seen that around an optical carrier we can expand this beta as a Taylor series with beta 1 and beta 2 usually kept as the two terms right. As the two terms that are sufficient for us to describe most of the depression affects here, but in case you also want to discuss amplifiers or other things, then you need to include the higher order terms as well which is the called as a third order dispersion and similarly there is a fourth order dispersion; for our case you are not going to consider this. And we also seen that there is something called as non-linear Schrodinger equation that we talked about all the way we did not specify. This is a non-linear Schrodinger equation.

But we call this as pulse propagation equation; the pulse propagation equation actually allowed us to understand or model the effects of dispersion and we found that the way this pulse changes with respect to the propagation distance is given by minus alpha by 2 times a of z t which represents the attenuation of the pulse and alpha by 2 is because, this is a field attenuation value. And then you have plus j beta 2 by 2 del square a by del t square a of z t and we solved this equation for the case of Gaussian pulses. And then we found that when the dispersion is normal, the pulse actually expands considerably. And on the pulse sequence that has been transmitted shows up as an expanded version. So, may be as it expands of course, it is amplitude also drops out. For then, the pulse would kind of interfere with respect to each other or talk to each other resulting in what is called as inter symbol interference.

Now, in for optical fibers, there are two types of fibers that are currently laid: one is called as dispersion managed link, and the other one is called as dispersion unmanaged links. What you mean by dispersion manage links is this?

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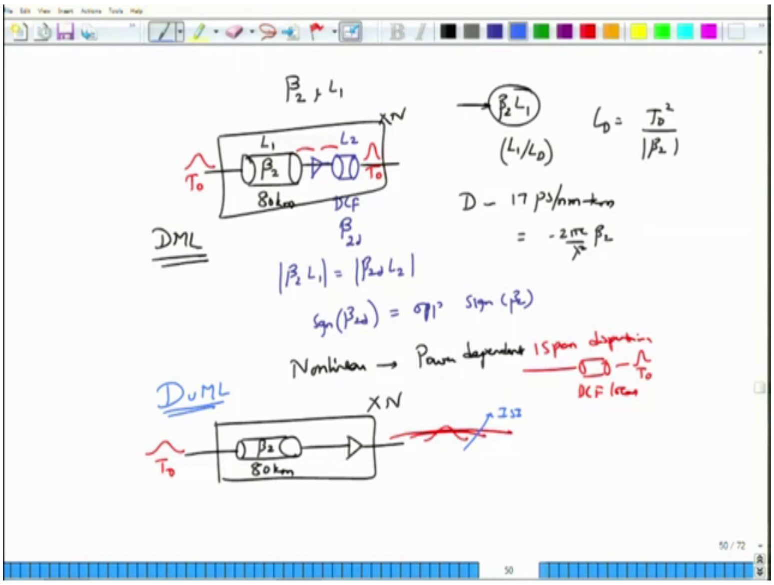

We know that if I consider a pulse of through a fiber with a value of beta 2 around omega 0 and I am considering narrow band pulse to be propagated through the fiber, then if it propagate through a length of L 1, then the pulse broadening will be proportional to beta 2 times L 1 or it is proportional to that the dispersion length that we can talk about and the dispersion length of course, will be proportional to beta 2 itself L D as you know is given by the pulse width beta T 0 square which is the initial pulse width that is large divided by the magnitude of beta 2, right.

So, this is roughly the pulse broadening that you are going to get which is kind of the product of their. Now, what people did in the eighties is to actually do this one. So, they actually had about roughly 80 kilometers of the fiber laid out with that being L 1 and with a value of beta 2 which would represent to a dispersion coefficient of about 17 picosecond per nanometer kilometer or the your relationship between d and beta 2 also we have seen right. D is basically minus 2 pi c by lambda times beta 2 where or lambda square times beta 2 where this relationship shows that when D is positive, beta it is actually negative.

So, most of the fiber that you considered for long haul communications are actually anomalous is dispersed fibers. Anyway, what we are concerned with this only the product beta 2 L 1 and if I know put in what is called as a dispersion compensated fiber after amplifying another things, I am going to put up a Dispersion Compensating fiber whose length is L 2 and whose value beta 2 let us call this as beta 2 D is chosen in such a way that beta 2 L 1 in magnitude should be equal to beta to D times L 2 in magnitude of course, the sign of beta 2 d right will be opposite of the sign of beta 2 itself. And these links when you cascade them one after the other and then you have an n such cascaded links are called as dispersion managed links and what is the key point of this dispersion management links.

At this point when you launch of optical pulse, then the optical pulse would broadened here it would be amplified, but still broadened here, but because I am using the D C F the amplifier would of course, consider the losses of both L 1 and L 2 fibers. And therefore, provide you again with that much. But then after you have amplifier it and the losses in the link or this span has been eliminated, the pulse width is now back to the original pulse width. This is very important. Here, you stirred of with T is 0, here again you end up with T 0.

So, which means that the pulse when it comes to the receiver has only undergone one span dispersion right; you can even put a local D C F here. So, I will call this as D C F local to make the pulse go back to T 0 itself. Now, this mechanism of course, compensate for any dispersion loss in the system, but there is a big problem with this system because you have the pulse width going back, the pulse is as strong at the input side as it is at every span itself ok. Why this is important, because fibers are unlike traditional channels are non-linear and nonlinearity is a power dependent quantity in fibers.

Which means, if you take this power could as the power reduces because of attenuation, but then you put the power back up to the original value, then the nonlinearity can build up in this span. Next time, as the power goes down the nonlinearity goes down in the fiber, but again at the end of the second span you have put the amplifier back. So, the power goes back and more importantly, unbroadened the pulses the pulses broadened you now squeeze the pulse. So, the pulse again goes back to the same amplitude or same power and therefore, they will be additional nonlinearities affected.

So, dispersion managed links are very good for dispersion management, but they are very poor for nonlinearity mitigation or at least they are very poor in terms of nonlinearity because they are going add nonlinearity in every span. And therefore degrade the system performance at higher launch powers and this is a very very important parameter that limits the data rate of higher order modulations and it also limits the reach over which this optical communication systems can be deployed.

Then, there are other systems which are called as dispersion unmanaged system. Here, discussion unmanaged system what we do is we remove this extra fibers while the fibers were good they are not so good for non-linearity. So, we actually remove this fibers and we simply putting an amplifier and if you want something other that can be done, but you are not going to put a dispersion compensating fiber in this span and you are going to take this multiple spans in this way. So, the pulse that would start off with some initial pulse width T 0, after the first span would be broadened. After the second span, would be further broadened and after many many such span would be almost flat which means the inter symbol interference keeps on increasing.

So, chromatic dispersion will be very bad, but nonlinearity can be kept reasonably small. It would still be there, it would still be present and it can be kept reasonably small using this scheme. So, these schemes are called as dispersion unmanaged link. So, no dispersion management is done at the span level. So, the full effect of dispersion is going to be present at the receiver. Then, how may going to overcome that one? Well, there are optical signal processing techniques based on non-linear fiber optic itself which can mitigate this dispersion problem, but we are interested at this point with using D S P techniques right. Using D S P technique, we have many such techniques available.

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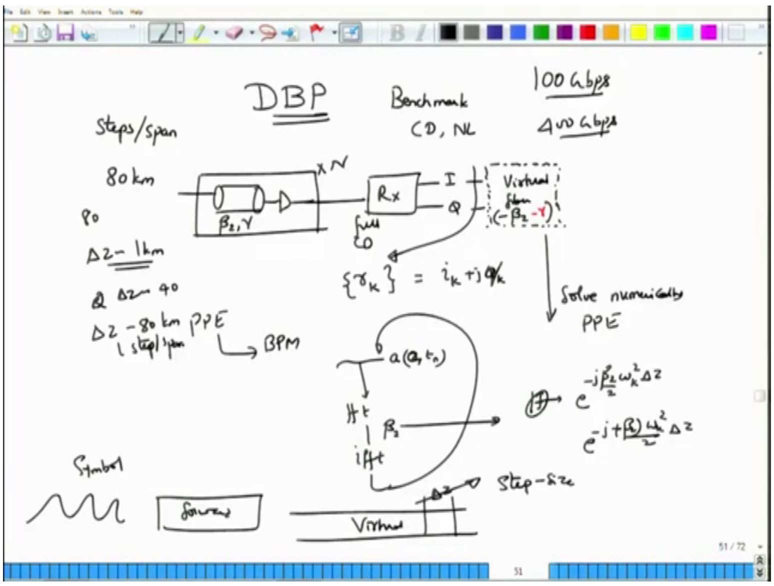

One of the most popular technique and this is the benchmark again to which we normally compare both C D composition as well as the nonlinearity compensation, I tell you not every nonlinearity can be compensated by this.

But deterministic nonlinearities can be compensated by this algorithm. In fact, this is algorithm it is currently used in the 100 GBPS standard and it is also expected to be used for 400 GBPS, but there are also now good other algorithm that have come up, but d b p has been benchmark for most of the last decade in dealing with C D composition. In method is actually quite simple the idea is the same as D C F but except that you actually putting inside of a real fiber, you put it a virtual fiber. So, you have your no signals propagating, you have your spans. So, do not do anything in the middle to amplified that is all to compensate for the losses, we come to know cascade about n sections of which and then you now have a full C D affected signal arriving at the receiver.

Now, you put in the receiver extract I x, not going to write x here because now I know that I am actually dealing with only this one. Right, I am only dealing with x polarized receivers. So, I am simply going to write this as I and Q. So, these are the samples I might even write down the complex received sample as r k; have this complex receive sample without doing anything further what I am going do is to put in a virtual fiber. A virtual fiber is simply an algorithm that we are going to implement on a computer or on a real time S P G, a kind of a device right. So, do have dedicated processer for that one or we can put it on your offline processing conclude on your computer, take the sample sequence that you are going to receive this r k will be equal to I k plus j q k where I and q are the in phase and quadrature components are the k th sample time ok.

So, now I have this r k sequence, what I am going to do is to put this through virtual fiber whose beta 2 will be negative of the beta 2 of this one and that is all essentially. So, attenuation you do not need to worry about it I have already taken care of it. The only thing that we need to worry at for dispersion compensation is to put in minus beta 2, but fiber also has a non-linearity. So, you can actually put in appropriate value of gamma. It tells out that gamma in that case. We will define gamma later on when we talk about nonlinearities. For now, we do not need to worry about that. So, this is the virtual fiber, all that we are saying with a virtual fiber is if the forward transmission fiber has beta 2 and gamma ok, you simply put in sorry you should not be in gamma 2, you should be just gamma.

So, you just put in minus beta 2 and minus gamma and then you are done right. Of course, when we say you are done you are not exactly done, what you have to do is you solve numerically that is on a computer you solve the pulse propagation equation. Remember, we solve the pulse propagation equation by a method which we called as beam propagation method in which we took a of z t and then discretized it you have 0 to n discretized it, then we took the effective of this one with appropriate frequency resolution depending on how we have sample the signal and then after f f t we have propagated it and then you take this I f f t and then you repeat this operation many many times, right.

So, this is our model of solving the pulse propagation equation. Of course, on a fiber it is simply happen in does not really depend on how we are solving the equation, but when it comes to numerically solving the pulse propagation equation, you assume that the samples are obtained by fiber after and then model the fiber as a pulse propagation equation kind of a system. Then in the virtual fiber, what you do is you take I f f t your account for dispersion. Here, you would have accounted for dispersion by multiplying the transfer function right. So, you had this e to the power j beta 2 by 2 or other minus j beta 2 by 2 omega k square times delta z. That would be the way in which you could account for dispersion at this particular step right and you do the same thing. Except now, instead of beta 2 you take this e power minus j and minus beta 2 omega k square divided by 2 delta z.

So, you can again think of the pulse propagation equation has the same as a forward propagation for the forward link now connected with a virtual fiber whose properties are all mirror images of the forward fiber and if you are lucky and if things are alright by changing the value of delta z, you can almost reconstruct what has been transmitter and you can do it for the entire sequence. It does not have to be symbol by symbol, you can actually take the entire transmitted symbol and then run it through this virtual fiber.

Alternatively, you can do it block by block you take some symbols, run it through some symbols it can do keep doing it right. You can do a sliding window or you can do an entire batch in a single processing and you can do it for the entire sequence in case you have lot of time in the offline processing case. And then this parameter delta z which is called as the step size is an important parameter and in specifying D B P, we normally talk about number of steps per span; one span is usually 80 kilometer. So, if you say 80 steps then your delta z is roughly one kilometer; doing so we will improve the numerical accuracy.

But it will also you know mean that if you are simulating over say 2000 kilometers, the simulation time will be prohibitively large. So, you want to do not do that one. You can do say two steps per span. In that case, delta z will be roughly 40 kilometers. The kind of minimum required would be to do delta z of 80 kilometer. This is called as one step per span so. In fact, by increasing the number steps, you can kind of get closer but as I told you simulation time will be longer.

So, this is how you use D B P to compensate for dispersion. There are other methods.

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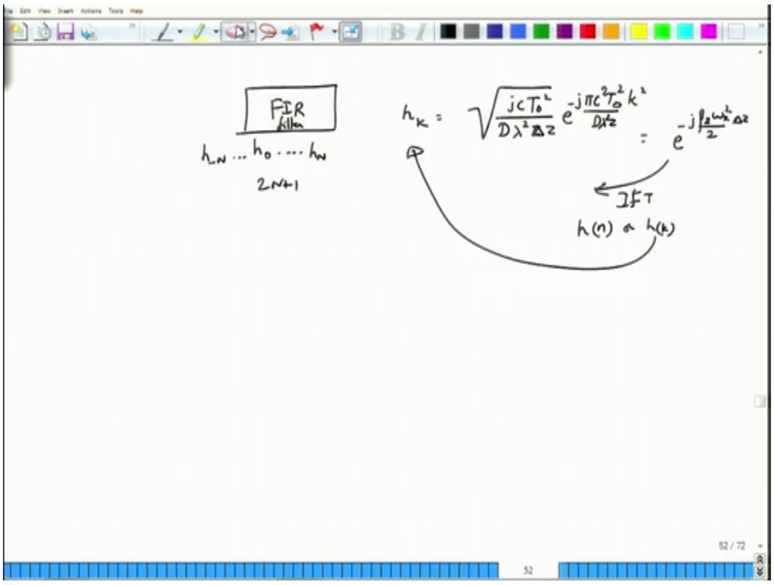

One method is based on putting up an F I R filter whose coefficients let say h of minus n to h 0 to h of n. There are about 2 n plus 1 coefficients that they have chosen this coefficients are actually obtained by sampling the impulse response at all those places.

So, remember that transmission or the h of omega of a fiber is given by e to the power minus j beta 2 omega k square divided by 2 delta z. You can actually take the inverse Fourier transform of this one to obtain h of n and or h of k and these are the h of k s that we are actually writing here and of course, when you take this inverse transformation you have to keep in mind that this is Gaussian. So, appropriate amplitude also should be multiplied. It turns out that this is what you are going to get delta z maybe this is the amplitude and then have e power minus j pi c square t 0 square k square divided by d lambda square z.

So, instead of writing this one in terms of beta 2, we have written it in terms of e d which is more accessible to you. So, this is t 0 square times k square and what are the values of k, this is a finite impulse response filter.

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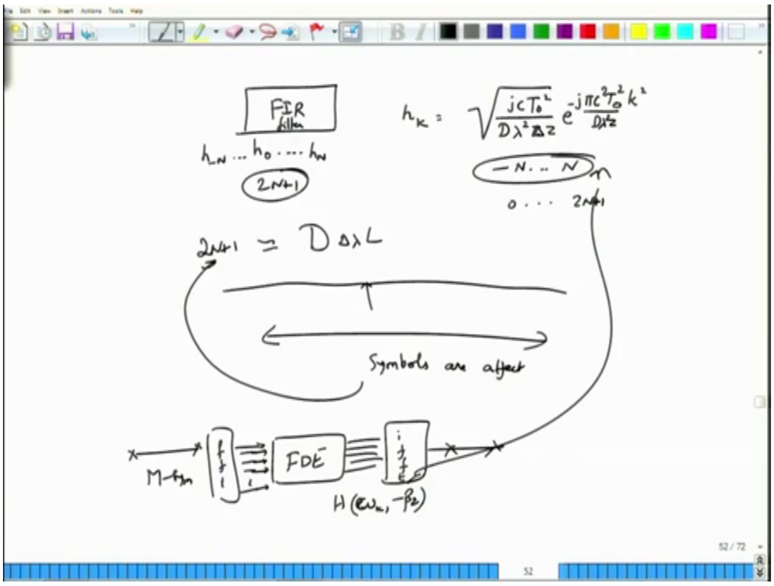

So, your k can go from minus n 2 plus n. You can choose any values you want, but then choosing minus and plus indicate that you are actually dealing with non causal filters, but because in F I R, you can always shift every coefficient such that it starts from 0 and goes all the way up to 2 n plus 1.

So, this is just the way you write down the notation and writing it with a minus indicate simply that your centre is at t equal to 0 ok, but these are stable filters and these are F I R filters. The way to decide the number of tabs n or 2 n plus 1 which we will call as L as a number of tab is that if you are fiber actually has a length of or it is not call this as L and just call at as 2 n plus 1. If the fiber actually had a dispersion coefficient of D and it had delta lambda as the line width of the bandwidth of the data and it may propagated over the entire length L, this is the amount of pulse broadening that you are going to get.

So, 2 n plus 1 should roughly account for this d delta lambda L because from this point the amount of trading is going to characterize how many symbols are affected right and there will be n plus one symbol 2 n plus 1 symbols with you because n symbol from the left, n symbol from the right get affected because you are assuming that the symbol is sitting at the centre. Now, this for way of putting up an F I R filter and then passing the incoming sequence the complex sequence r k, if you pass them and then get the output, you would have compensated for the dispersion and this is a time domain method of analysis; you could of course perform what is called as frequency domain equalization in which you take n symbols that are coming in or n symbols that are come again you take the Fourier transform via the F F T.

So, you have all those n symbols or the samples at the output and multiply each of those by the inverse this 1. So, h of omega k with a minus beta 2, you multiply them. So, each omega k component will be multiplied by the transfer function that we had indicate some points which is the Fourier transform of this fellow actually and you after multiplying that is what the F D E part is you then take the inverse F F T so that you recover the block again. Now, by varying the number of symbols in a block or wearing the block size and by wearing the appropriate frequency resolution because number of block determine the frequency resolution

So, by varying this properties or varying this value for parameter, you can then almost achieve complete dispersion you know equalization. So, this is called as dispersion compensation or dispersion equalization, you can do all of this.

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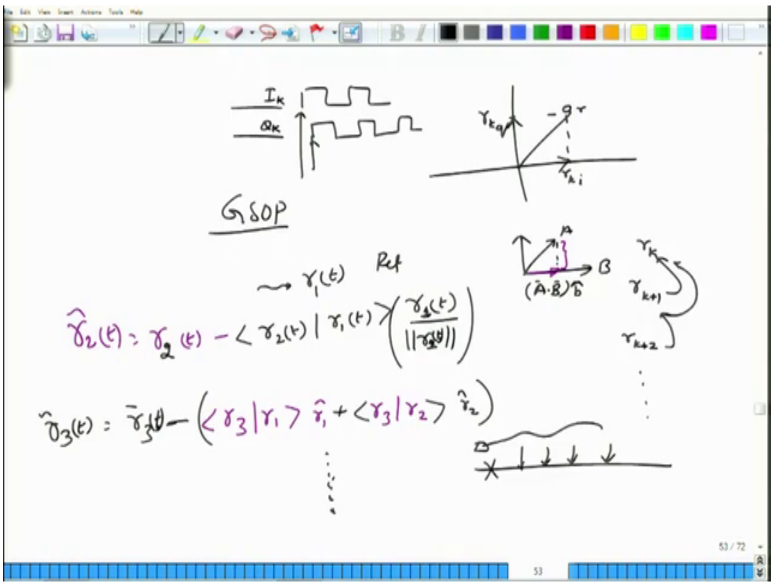

I also told that once you sample the signals the I and the Q samples are usually not going to be aligned together, right. So, if this is the clock, then the clocks are usually not going to be aligned. So, truly speaking I and Q are orthogonal to each other correct because if you take any received signal here, then this is the I component and this is the quadrature component and clearly these components are going to be orthogonal with respect to each other.

But if they are not because of this clock problem, you can make them orthogonal by this process called as G S O P. What is G S O P does is, it takes the first received symbol or the received vector or received signal let us say ok. I am going to work with the continuous time portions of this to make the point clear. So, let us say the first symbol that you are going to get all the first waveform that you are going to observe will be taken as a reference waveform and about that reference waveform, the next signals will be classified into orthogonalized. How we get the next symbol or next waveform let us say r 2 of t, what I do is I take the inner product of r 1 of t with r 2 of t ok.

And then, when I multiply this one with r 2 of t right, then I would have obtained a component of r 2 that is along r 1. So, if this is the 2 vectors, or let us say this is a 2 vectors A and B, the inner product of these two will be A dot B correct and then multiplying this one by sorry not the entire vector, but let us say the B vector which is the unit vector. So, to obtain the unit vector, I simply have to take the vector or the signal r 2 obtain then divide that one by it is norm ok. So, when I do that one, I am going to get a unit vector in this form.

So, after taking the inner product, multiplying it by the unit vector I actually obtain this vector. Once, I obtained that vector this is the error vector that I have or there is a vector that I have which is actually orthogonal to this vector correct. So now, if I take r 1 of t, subtract it out, I am going to get the new value of vector r 2 of t which would actually be perpendicular to r one of t. And once I have r 2 of t done, then for r 3 I am going to take the inner product with respect to r 1 multiply that one with the unit vector along r 1 and r 3 r 2. Sorry this is actually supposed to be with r 1 not with r 2. I take the inner product of r 3 with r 2 multiplied by r 2 and then form the sum, subtract it out. Sorry this also has to be r 2 of t, I am sorry I am making small mistakes here.

So, this is r 3 of t subtracted out with the inner product components out there that will tell you the new r 3 of t which will be orthogonal to both r 1 of t and r 2 of t. So, in this manner you can actually build up this equation. So, if you go back to this I and Q to start of with one sample r k, then take the next sample that would come in say r k plus 1 to make this one orthogonal to r k. You take r k plus 2, you make that an orthogonal to both these components and then you continue to do this way or continue to go in this way and eventually obtain at the output with the first one has a reference, all the other components that you are going to get are going to be orthogonal to each other.

So, you can actually derived orthogonal samples from non orthogonal samples that you would have received at the end of the sampling process. So, we have looked at dispersion compensation, we have looked at this other you know this skewing problem that you are going to get. This is sometimes also called as I Q imbalance. So, all these things have to be taken care by themselves.

So, we are now left with phase estimation and frequency offset. I am not going to consider frequency offset because those methods are you know something beyond our course. We will look at phase off set, but then we have to first also talk about the nonlinearity in the fiber which we are going to talk about it in the next module.

Thank you very much.