Fiber - Optic Communication Systems and Techniques
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Lecture - 06
Obliquely incident waves-I (TE and TM waves, Snell's laws)

Hello and welcome to NPTEL MOOC on Fiber - Optic Communication Systems and Techniques. In this module we continue the discussion of reflection, but not from perfect electric conductor, but from in medium, where the interface or the second medium is in perfect dielectric. So, first medium is also perfect dielectric. So, the situation is shown here. I have now considering, what is called as obliquely incident waves.

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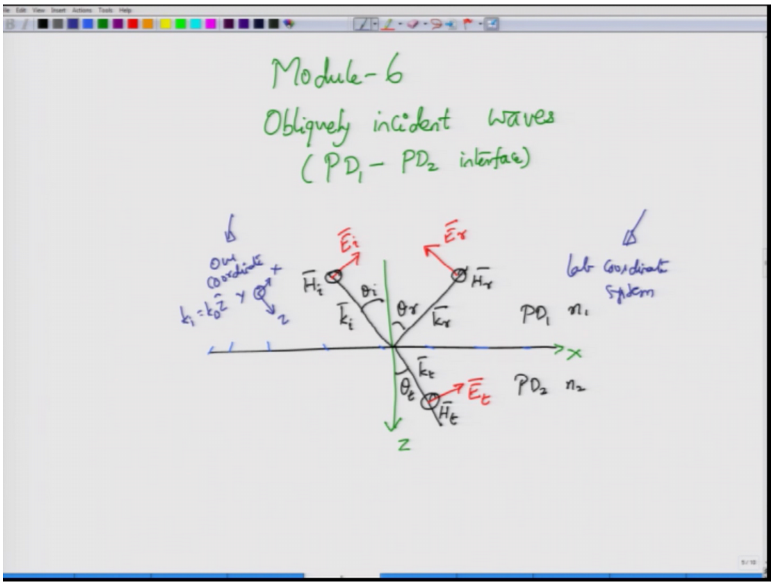

Obliquely incident means that the incident light wave, which we are assuming to be a uniform plane wave, arrives at the second medium at an angle theta i. What is this significance of angle theta i and the diagram that you shown here, cannot be just make an our own coordinate system by perhaps writing the incident direction, as the Z axis denoting the direction of the electric field as the X axis. And whatever the direction, the other one that is remaining, which essentially is parallel to both the shown system as well as to our own coordinate system as the Y axis.

So, can we not make this right. So, this is our coordinate system right whereas, this is the lab coordinate system. So, when I say lab coordinate system, it is the coordinate system that is chosen before we actually had this light coming in. So, if you go to look at our coordinate system, in our coordinate system we just have a transverse electromagnetic wave, which is the plane wave propagating along the Z direction. Whereas, if you look at the lab coordinate system, the wave is propagating, not only along the Z axis right, purely along the Z axis, but it also has a component along the X axis. Why? Because the incident k vector, which is denoted by k i here, has both a component along Z as well as the component along X.

So, this seems to be needlessly complicating our problem, because if we could go to our coordinate system then k i in our coordinate system will be along, you know Z axis whereas, in this case, it is slightly complicated right. So, can you not do this? Unfortunately you, if you work with our coordinate system then the interface will have to be expressed in the coordinate system that we have chosen our coordinate system. So, you are transferring the problem of obliqueness from the wave to the interface. So, earlier what we have is the interface is in, coordinate system which is lined up perfectly along the X Y Z axis, whereas the wave is obliquely incident where as, if we change the coordinate system that is our coordinate system, such that we make the wave lined up X Y Z axis then the interface will appear not exactly normal, but it would appeared at an angle.

So, we are transferring the problem of obliqueness from the coordinates that we have chosen to make the electric field, magnetic field and the direction of propagation coincident with X Y Z axis to the problem in the lab coordinate system. So, it is matter of perspective, whether we choose the lab coordinate system and then let the wave be oblique or we choose our coordinate system in which the wave is in a proper oriented access, whereas the obliqueness is now, on the interface right.

Because it is kind of convenient to work, both methods of course, give you the same results, but this method of denoting the coordinate system first and making it line up with X Y and Z and clearly the interface is actually at Z equal to 0 plane right. So, the plane of interface is the Z equal to 0 plane, which itself is described by X and Y axis right. So, the interface plane is described by X and Y axis.

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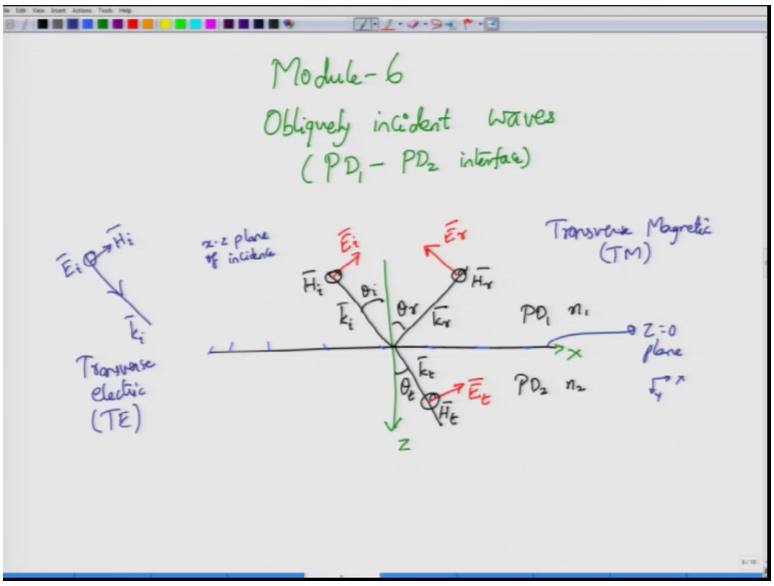

The plane of incidence right which is the essentially the X Z plane, this is the plane of incidence and it also turns out, that the plane of polarization can be defined, which would include both the electric and magnetic field. The electric field will be in the direction which is given by X and Z and the magnetic field will be along the Y direction. So, I have lined up this problem in such a way that the interface is at Z equal to 0 plane and the Z equal to 0 plane can be described any point on the Z equal to 0 plane can be described by X and Y axis. And, the plane of incidence in which the k vector lies in all the k vector incidence reflected and transmitted vectors. They all lie in the same plane of incidence, which is the x and z plane of incidence.

So, k i k r and k t incident reflected and transmitted right; this is the reflected or the transmitted wave vector, they all lie in the x and z plane and the magnetic field H does not change sign in this representation, the magnetic field is always oriented along the Y axis and if you look at this the magnetic field is actually perpendicular to the plane of incidence as well as to the I mean. So, it is perpendicular to the plane of incidence.

So, this scenario, in which the magnetic field is perpendicular to the plane of incidence is called as transverse magnetic polarization. There are some additional waves in which these polarizations are denoted, but I feel that having to spell out, transverse magnetic makes it better for our understanding, because this clearly indicates that the magnetic field will not be having any X and Z component or rather the magnetic field will have only the Y component, which happens to be transverse to this plane of incidence ok. These are for short called as T M, this is of course, not the only wave in which we have, you know we can orient r field vectors. We can also orient r field vectors in a wave bad corresponding to the incident k vector. It is the magnetic field which may lie in the plane of incidence, whereas the electric field may lie in the perpendicular plane. So, this is the case where we have transverse electric conditions.

So, we have for short, we write this as T E waves. So, any wave of course, can be split up into T M and T E that is if I have a wave, which has both, some components in the plane of incidence as well as a component along the Y axis. Similarly, for the magnetic field, which has some component along the plane of incidence as well as a component along the Y axis, we can split this total electric field and magnetic fields into T E and T M waves. You can pair them of independently.

So, that you are dealing with T E waves, you are then dealing with the T M wave and then the results can be super imposed right, you can do a super position, because Maxwell's equations are all linear and this problem that we have considered actually linear problem. So, we can considered separately, what happens to T E waves upon reflection and transmission T M waves upon reflection and transmission and then combine the two results ok. This is perfectly valid way of approaching this problem. And for simplicity, we will look at transverse magnetic case, not for simplicity for as an example, we will look at transverse magnetic case.

I will leave the corresponding derivations of transverse electric to you right. So, I want considered further the transverse electric case, I, I only consider the transverse magnetic case of course, the equations are all available and we will discuss both T E and T M case. What happens in a quantitative manner in the next module. So, this module is devoted to come up to the point that we have transfers magnetic wave and then how do we describe the various reflected and transmitted components to do. So, of course, I need to write down the expressions for E i E r E t H i H r and H t right. Before I can do that, let me write down the expressions for k i k r and k t k i.

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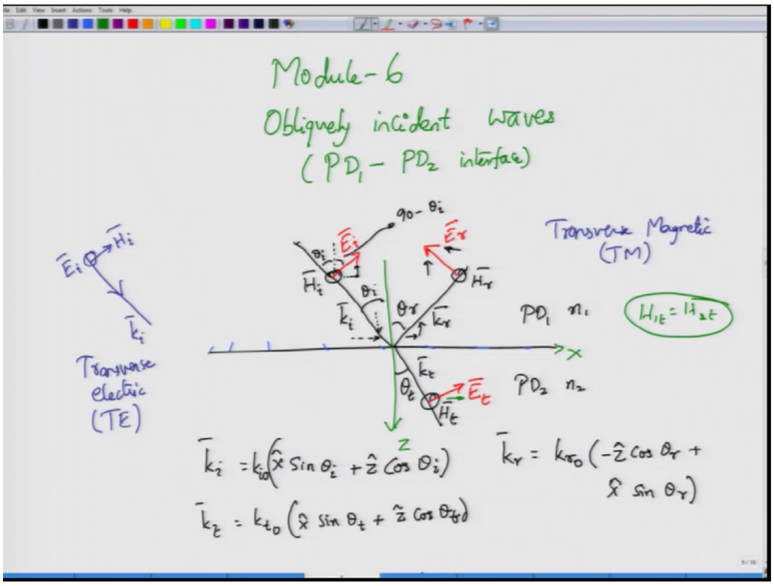

If you look at the way, this line is going around. So, it will have one component along X and then there will be another component along Z right. So, this will have X hat. So, let us say, I have k i 0 where k i 0 corresponds to the magnitude of the incident wave vector. So, it is k i 0 X hat and what is the component along the X axis? It is the sin theta i component correct. So, you can break this up as sin theta i plus Z hat cos theta i. Please, convince yourself that this k vector expression that we have written is correct, it has a sin theta i component along the X and cosine theta i component along the positive Z axis and it is this reason that we actually say that the wave is along Z axis. It is positive Z axis, because it has a positive Z component for the k vector.

How about the k r vector; k r vector should move away from the interface, should move away from the interface. It must travel in the negative Z direction, if it simply travels in the negative X direction, but it goes in the same direction as the Z, then it is not propagating in the or this is not a reflected wave right. So, for a reflected wave means that the wave should move away from the interface for which Z direction should become negative or the Z component should be negative for the reflected wave vector whereas the X component can be positive.

So, if you go again and decompose this k r vector into it is corresponding components, you would see that it has a negative Z component, whereas a positive X component and how much is the positive X component? This will be k r 0, which is again the magnitude of the reflected wave vector. So, k r 0 minus Z hat cos theta r theta r being the reflection angle, cosine of the reflection angle plus X hat sin theta r, k t of course, follows very similar to k i, because they are essentially in the same direction. So, k t will be k t 0 X hat sin theta t plus Z hat cos theta t ok. So, these are the expressions for the k vector. Now, we come to the expression for the E vector, to obtain the expression for the E vector, I am going to follow maybe this, not really required you, you will know, you will have much easier ways of doing this problem or finding out the component, but this is what I follow, makes my life a little bit simpler and avoids certain mistakes.

So, I know that this line, which is black line, is actually making an angle of theta i right and I also know that the black line is perpendicular to the red line, the red line being the direction in which the incident electric field is polarized. Therefore, clearly the angle with which the electric field make with respect to the normal right will be 90 minus theta i. Now, it is very easy for me to write down the corresponding components. So, I have one component of electric field E i directed along minus Z direction right. So, that would be this one and then I have an electric field, which is directed along the plus x direction, which is along this one, with of course, the sin and cosine things considered, appropriately right.

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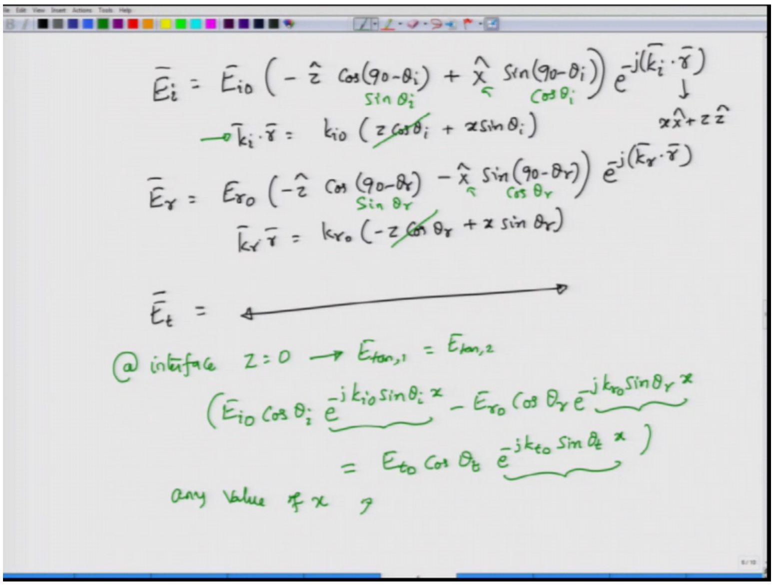

So, I will write down the expression for electric field incident, electric field. Please, ensure that you understand this equation and in case I have made a mistake, please let me know if I have not made a mistake then the expression for incident electric field will be minus Z hat cosine 90 minus theta I, that is the angle between this red line, which is the electric field incident, electric field and the black line, which is parallel to the normal of the interface. So, minus Z hat cosine 90 minus theta i plus X hat sin of 90 minus theta i.

This is just the electric field expression, you need to know multiply this one with the face fact k i dot r e to the power minus j k i dot r, where k i dot r will be equal to k i 0 z cos theta i plus x sin theta i why? Because you can go back to k i expression here and then take a dot product with the position vector r and in this case the position vector r is given by x x hat plus z z hat. This is the plane of medium that we are actually interested in. So, this becomes x x hat plus z z hat and when you look at k i dot r this is what we are actually going to obtain.

Similarly, you can write down what would be E r and you can go back to this expression and then follow up either the similar procedure. You will notice that there will be one component along minus z direction and then component along x is also minus now. So, because you can have to decompose this one in such a way that this would be this way and this would be in this way. So, clearly the x component will be along minus direction, the z component is also minus direction and you can write down E r as E r 0 minus Z hat cos of 90 minus theta r minus X hat sin 90 minus theta r.

This we have done for the electric field component itself, but you need to have e power minus j k r dot r. I am not putting e power plus j k r dot r, because the reflected wave is only negative for the z component not for the x component. So, this is e power minus j k r dot r. We know: what is k r dot r, because we have already seen: what is k r from the previous expression. So, we can calculate that. That would be k r 0, the magnitude times minus z cos theta r and plus x sin theta r.

So, this expression goes into the exponential. Shall I leave the transmitted electric field as an exercise for you. So, I will leave this one as an exercise and you can fill out this expression and you can verify whether you have done it correctly, when I write the next set to obtain certain other relationships. Well I also know that cosine of 90 minus theta i can be simplified, because this is sin theta i whereas, sin of 90 minus theta i is basically cos theta i cos of 90 minus theta r is sin theta r sin of 90 minus theta r is cos theta r.

Now, we have all the expressions in our and except for the magnetic fields for which will come to it in a moment ok so, now what? What we now, want is to look at the interface? So, at the interface, I need to apply the boundary condition, what boundary condition should I apply the interface? Of course, is described by z equal to 0. So, I have to go and apply z equal to 0, in all these expressions, which I will do, but what am I looking at this expression, I am actually applying the tangential electric field condition. So, I have the tangential electric field in medium 1, being equal to tangential electric field in medium 2, at the interface.

So, if you look at medium 1 here, I have the tangential electric field, which is the incident electric field along the plus x direction and the reflected field component along the minus x direction, the component along z are normal. So, they do not really, I mean do not really need to worry about that, for the second medium the transmitted electric field will have a plus x component and that would also be the tangential component.

So, retaining only the tangential components here, which are the x components from all the field quantity, what we obtained is an equation, which says E i 0. So, this would be E i 0 cos theta i from the incident expression times e power minus j k i 0 sin theta i times x. Remember, in this expression k i dot r, I am going to put z equal to 0, which causes this term to go to 0. Similarly, in k r dot r, this term will go to 0. So, I am only left with this terms, with theta i and theta r with respect to x. So, this is one expression, which is coming from or this is one component, just coming from the incident field, then there is a component, which comes from the reflected field, which will be directed along the minus x direction.

So, this would be E r 0 cos theta r e to the power minus j k r 0 sin theta r times x, which must be equal to e t 0 cos theta t e to the power minus j k t 0 then I have sine theta t x. Notice this expression, in this expression which you can contrast with the reflection from the perfect electric conductor, where we considered the normal case, there we did not have any face term, which was dependent on x. Here, you have all three terms depending on x as well. So, this E i 0 cos theta i e r 0 cos theta r e t 0 cos theta t can be consider to be constants given theta i, theta r and theta t, but these are functions of x and the only way that this can be equal know this condition can be satisfied for any value of x.

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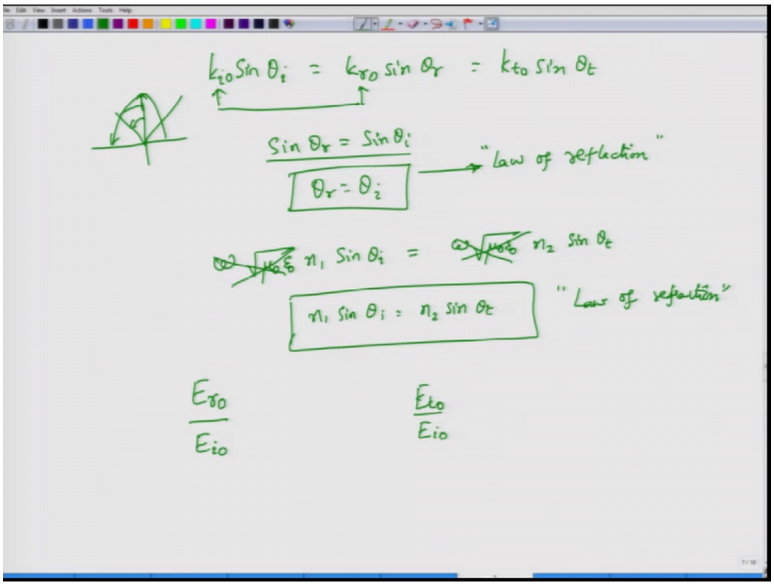

For any value of x, if you want to satisfy this condition, the only way it is possible is that is satisfy the face condition, that is k i 0 sin theta i equals k r 0 sin theta r, which is equal to k t 0 sin theta t very interestingly k i 0 magnitude will be equal to k r 0 magnitude. Why refer to the previous module right is not go back to this figure and realise that incident and reflected fields are in the same medium? Therefore, they k vectors will depend on the medium property.

So, which essentially means that there magnitudes are going to be equal ok. So, if this magnitudes are equal then the condition here is that sin theta r equal sin theta i. Now, theta r and theta i, theta i is this one, write this angle that we are measuring and this angle can be from 0 to pi by 2 and a sin of 0 to pi by 2, if it is equal to, for the reflected angle in that first quadrant, if sin is wearing in this manner from 0 to pi by 2 sin theta i, will go from 0 to 1 and correspondingly that is equal to sin of theta r, the only condition for the only conclusion that we can draw from this is that, theta r is equal to theta i and what is theta r equal to theta i from previous module? We know that this is Snell's law of reflection.

So, what Snell found out from experiments and from other considerations was an law right, which we call is law of reflection. Now is simply a condition on the face of the wave, give us this law without really calling that I has a law of models right. So, within experimentally find out, but we do know that mathematically, this must be true and fortunately this is been verified experimentally also. So, Maxwell's equations are very nice in that way that we can actually obtain this called Snell's law. Adjust face conditions, know we have not put lot of effort into it, a simple face matching condition is given as theta r equal theta i.

What about the second condition, well for the second condition will first write down, what is k i 0? k i 0 is given by k 0, which is the free space wave number times square root of mu naught epsilon naught n 1, where n 1 is the refractive index of the medium 1 times sin theta i is equal to k 0 square root mu naught epsilon naught. This is not k 0, this is omega 0 right. This is omega 0 or rather omega naught even 0, made a mistake here. So, this is omega times square root mu naught epsilon naught, but this is n 2 sin theta t right. Clearly, this will go away, all the wave will have the same frequency and this component, this of course, the way. So, now, what we are left with is the second Snell's law, which tells us that n 1 sin of angle of incidence must be equal to n 2 sin of angle of reflection or refraction right.

So, this is your second Snell's law, which is called as law of refraction ok. So, both Snell's law turns out to the simple face matching conditions and you can use this equations to predict what would be the angle theta t, you know, understand the phenomena of total internal reflection, all that thing you can do of course, we will do all that in the next module.

But we also are interested in knowing couple of other quantities. I want to know, what is the reflection for the amount of reflected lined and what is the amount of transmitted light. Since, talking about reflected and transmitted light without telling about incident light, makes know sense. What we are actually looking for is the ratio of reflected field amplitude to the incident field amplitude and ratio of transmitted field amplitude to the incident field amplitude to do that I need to go back to this expression and now, that I have realised that all the space terms are essentially going to be the same. I can remove the space terms from this expression.

So, I can remove this e power minus j terms from the face expressions, I have removed it here ok. So, I have gone ahead and removed it and I have obtained the reasonably simple equation. So, this equation is the result of boundary condition for the tangential electric field. Now, what about the magnetic field? Well we go back to our condition or our situation here, we have a perfect dielectric. We have another perfect dielectric, there is no chance that we have actually have a surface current distributed.

So, I have not placed any current, there are no current or free charges available, which can constitute current therefore, the relevant boundary condition for the tangential component of the magnetic field is that the total tangential component in medium 1 be equal to total tangential component in medium 2 for the magnetic field and luckily for us. Magnetic field is tangential everywhere to the interface plane right.

So, because h is along y axis y axis is parallel to the x and y plane and therefore, all of the incident reflected and transmitted. Magnetic fields are oriented along the y axis and therefore, they are tangential therefore, the second condition that we will obtain in terms of tangential magnetic fields will look something like this right.

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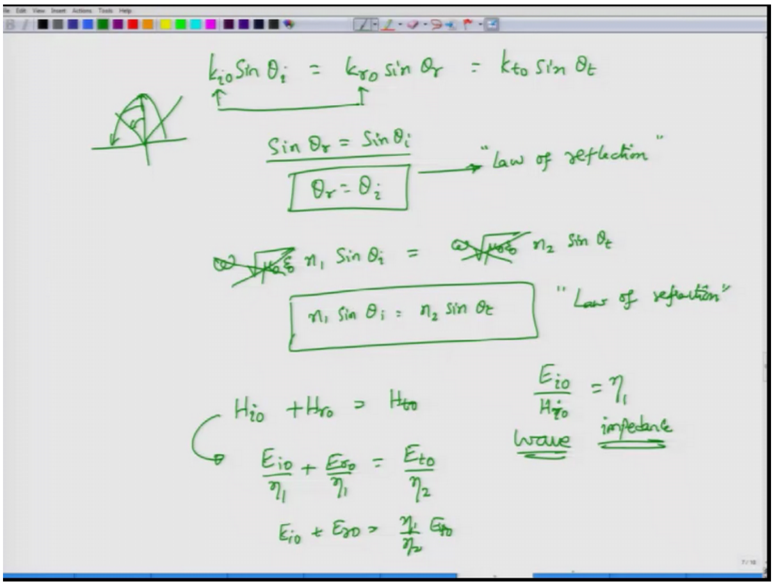

I will have H i 0 plus H r 0 equals H t 0. Now, what this H i 0 H r 0 H t 0, is it possible for me to relate these quantities to electric field? Yes, it is possible. It turns out that the there is something called as a wave impedance, where in E i 0 over H r 0 sorry, H i 0 is given by eta of that particular medium. So, since is a medium 1, this eta 1. So, this eta is called as the impedance of the medium ok.

So, this is the impedance of the medium and this is called as the wave impedance, because this the ratio of the wave components that is field components, electric and magnetic fields. So, with that and which is the same condition for the reflected and transmitted magnetic fields, the condition on magnetic field can be rewritten in the form where E is H i 0 can be written as E i 0 by eta 1 plus E r 0 by eta 1.

Why would I write E r 0 by eta 1, because they are in the same medium right. So, both incident and reflected media are in the same reflected fields are in the same medium. So, this must be equal to E t 0 divided by eta 2. Now, I have two simultaneous equations. So, I can either put them into a matrix form.

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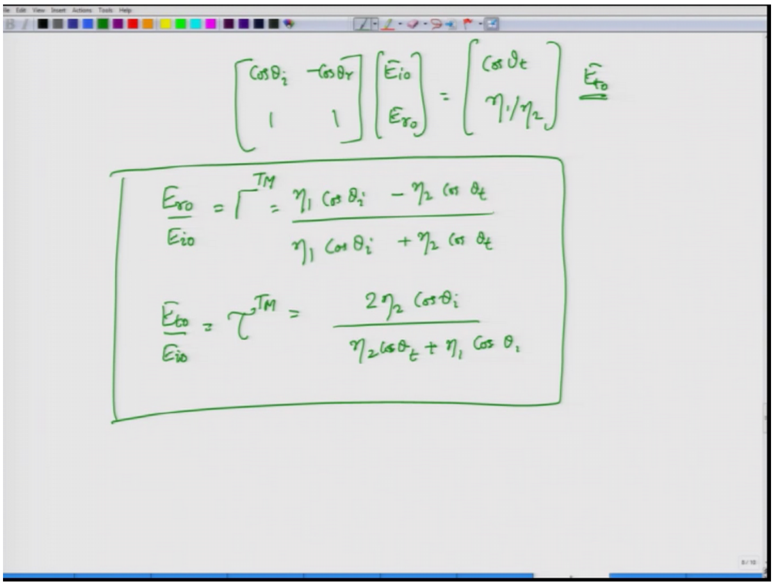

So, if I put them in the matrix form, the first condition would be that cos theta i and then I have minus cos theta r E i 0 E r 0 to be equal to cos theta t and then what is the other condition that I have from the magnetic field. So, I can write this as E i 0 plus E r 0 equals eta 1 by 2 times E t 0. So, I can put this in the matrix forms in 1 and 1, this must be equal to eta 1 by eta 2 for E t 0. Now, you can simply these equations with whatever way you know.

So, either you just take them add them subtract them do that or you can use matrix, find out the determinant and do that all simplification to eventually end up with the ratio of the reflected field, to the incident field given by eta 1 and denoted by gamma. Gamma denotes the reflection coefficient and since this is the T M case, we have denoted gamma with a T M superscript right.

This is for the transverse magnetic case and this will be equal to eta 1 cos theta i minus eta 2 cos theta t divided by eta 1 cos theta i plus eta 2 cos theta t and correspondingly E t 0 to E i 0, which is the ratio of the transmitted electric field amplitude to the incident electric field amplitude, which will call as some tau T M will be given by 2 eta 2 cos theta i divided by eta 2 cos theta t plus eta 1 cos theta i may be little confusing here. So, let me rewrite this.

So, it is eta 2 cos theta t plus theta 1 cos theta i. So, these are the formulas that we are looking for and these formulas are sometimes called as fersnel formulas and what this formula represent is that given the medium properties of medium 1 and medium 2. We are now able to find out the radiation or the ratio of the reflected electric field, to the incident electric field and transmitted electric field, to the incident electric field of course, we could also derive the same thing is for magnetic field cases as well, but we choose not to do that and these are dependent on the angle of incidence and material properties.

So, if I change the material properties, if I change the angle of incidence, what will happened to this gamma T M, what will happen to this tau T M, we have said eta as a wave impedance, which is the ratio of the electric field component to the magnetic field component. Magnitudes of course, but can it be related to the other quantity such as say, the refractive index, which is quite commonly used in the optical frequencies rights, in the optical frequencies you do not normally, talk about impedances, but you normally talk about the refractive index.

It turns out that this eta, which is the impedance is inversely proportional to the refractive index itself. So, you can simplify these equations in terms of refractive index as well and we have; so, far looked at only the amplitude values. We can also use poyntings vector, you know concept of poynting vector to calculate what would be the power density of the wave that is being reflected and the power density of the transmitted wave. And hence, find out how much power is getting reflected, when incidence light on a medium and how much power is transmitted into the second medium, when incidence light on it.

Thank you very much.