Fiber-Optic Communication Systems and Techniques

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Lecture – 09
Ray theory of dielectric slab waveguides

Hello and welcome to NPTEL MOOC on fiber optic communication systems and techniques course. So, far in the previous modules we have looked at light as a wave, we have also seen light as rays; we have looked at Snell's law of reflection and refraction.

We have seen how to look at those Snell's laws from, electromagnetic perspective that is starting from Maxwell's equation what should be infer about, what happens when light in incident on a medium and undergoes total internal reflection we have seen that, there is a non zero field outside the medium as well that is the medium into in the medium of the lower refractive index.

So, with all the tools that we have developed in the previous modules, we are now ready to consider the basic problem of communicating information using light from one point to another point ok. You might feel that the simplest possible way of communicating information by light is to somehow get light to change it is properties so, maybe its amplitude can be changed, or maybe its frequency or it is the wavelength can be changed. In response to whatever the information that you want to transmit right so, this information that you want to transmit is itself coming from a non electrical type of a wave form to electrical waveforms.

And these electrical waveforms will be further converted to optical waveforms, that is that process of converting information in one domain to another information, or at least varying one of the properties of the other wave in response to an response to modulating, or information wave is called as modulation right. So, this basic process in which I take information in say electrical form and converted into optical form, by wearing one of the characteristics of light wave would be the modulation. So, once I have this information I modulate the information to the optical carrier ok, which is essentially a light wave.

And then you might think that the simplest way of conveying this light wave would be to just beam it across the communication point. So, for example, you might have played with torch lights, when you were kids or maybe even now, and you would see that you know if you hold up a torch light onto a room which maybe initially dark, then you see that the image of light or rather light actually passes through and illuminates the room.

So, if this light itself would be changing then thus corresponding changes would also be visible across the room right, you might have seen laser pointers. So, instead of using a torch light, if you have used the laser pointer while making a presentation for example, you might actually point out the laser onto the screen and then move it around. So, if you can somehow automate this process in which information would, then change the laser characteristic maybe that it is pointing, or something else then it is perhaps or possible for us to directly convey this information.

In fact, this is a approach of using light wave and communicating light wave directly to the receiver ok, sometimes called as line of sight communicate, is not all together new nor is it not used actually it is quite, gaining popularity in the recent years under the name of visible light communications. And, there are various versions of visible light communications which make use of this fact that, you can modulate the characteristic of light and, then shine this light across directly to the receiver ok.

The problem with this near field I mean the problem with this visible light communication says that it is mostly restricted to near field communication. So, you cannot go long distances for example, the distances that we talk about in optical fiber communication, which is across continents, you cannot really reach that with you know shining a laser pointer, or you know shining a torchlight across over that.

There are lot of technical difficulties and there are a lot of communication channel difficulties as well the channel is very lossy, cannot support high data rates cannot you know allow multiplexing of various users unless, they are specially separated and moreover line of sight communication is also not possible, because of or at least it is very very difficult because of the diffraction nature of light. So, all these technical difficulties that exists both in terms of how to make the technology, necessary for this type of long distance communication, plus overcoming the limitations of the channel itself, because of all this properties and the problem of light having I mean light getting diffracted means that, this type of visible light communication is actually suitable over a distances of a few meters and not more than that.

So, then it is simply not possible for us to do long distance communication with visible light communication, then what is our hope? Our hope in fact, is not new in fact it has been there since 1960s on wards late 60s on wards in the form of what is called as optical fibers. And I have shown you in the in the in the introduction class of this course, how an optical fiber looks like so, you have an optical fiber in the form of a round wire it actually have multiple layers, there are central layer called as core, then surrounding the core you have a cladding, and surrounding the cladding for mechanical you know purposes of giving it a certain strength and durability.

Then you have a coating and after coating you have a jacket. And usually this fibers are not just you know given to you in the form of jacketed fibers, they are usually also for communication purposes, they are actually combined multiple fibers are combined into cables and these are the thick cables that actually are laid underground, mostly underground. Sometimes over ground as well as hanking cables, but that is very rare scenario mostly underground cables and, there have been millions and millions of such fibers that have been laid, which today enables the high speed long distance optical communication systems.

So, clearly the mode of communication is different in the visible light communication you would just take the light shine it across and, then the receiver would capture a portion of it and hopefully whatever the captured portion, that would be sufficient for it to decore the information, or rather de modulate the information. Whereas, in the fiber based kind of a communication you actually have a physical layer, which communicates so, from one point to another point the communication is happening with a help of a physical media which is the optical fiber.

So, we have looked at one type of communication, where distances are limited to about a few meters which is the visible light communication and, we have also looked at or we have at least talked about the communication systems that that are actually very very long range right. So, we have talked about optical fibers as communicating media, or communication channels which covered distances across continents across countries.

There are other situations where it is necessary to guide light from one point to another point ok, where the distances involved are just about a few tense of micrometers to about a few millimeters, or maybe few millimeters about atmosphere about a few centimeters.

What possible communication could happen in this small distances from a few microns, or millimeters to a few centimeters. Such communication actually is necessary in what is called as integrated optical circuits ok.

Integrated optics is essentially like an integrated electronics chip ok. So, you have an optical chip which would take light from its source guide it and, then you know maybe modulated and, then sometimes if it does not modulate it makes simply decide to split the light into two three parts and then send them across. So, in all this light guiding control and manipulation the distances involved are not very large. So, we are going to look at those type of guiding mechanisms, which we call as slab waveguides ok. And these slab waveguides are the basic components of any integrated optical circuit ok.

You might ask why are we talking about slab waveguide when the distances involved are not very large, it turns out that slab wave guides as I have told are very important for integrated optical circuit applications some of those slab waveguides will make an appearance, when we discuss photo detectors. And when we discuss lasers, but for now the reason of introducing slab waveguides is because it provides us with the ideas that go behind, or that are also applicable to an optical fiber. So, to understand light propagation in the optical fiber we consider the simpler case that of the slab waveguide. So, what exactly is slab waveguide, well before we go to slab waveguide let us look at what we have done in the last few modules.

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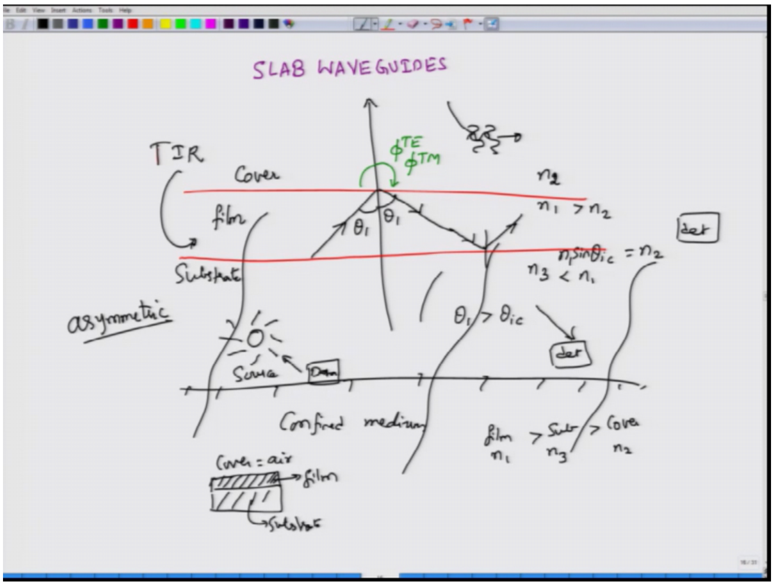

We had an interface which was separating two refractive indices n 2 and n 1 right and, we considered the case of n 1 greater than n 2 so, clearly what we are aiming for is some sort of a total internal reflection right. So, I have an interface normal and, then I send in light at a certain angle theta 1.

Now, as long as the angle theta when its less than the critical angle, there is nothing specific going on about this particular system, or this particular arrangement, but when angle theta 1 becomes greater than the critical angle of the interface, please remember that the critical angle of the interface is obtained by applying Snell's law so, n 1 sin theta i c must be equal to n 2 ok. But of course, from the wave theory point of view, you know that there are actually evanescent waves and this evanescent waves decay perpendicular, or along the normal to the interface, but the essentially propagate along the interface itself right. So, we have seen this kind of a thing when we discussed total internal reflection.

Now, what I have is I sending light at an angle theta 1 and I at this point I do not concern myself with whether I am dealing with a transverse electric polarized wave, or transverse magnetic polarized wave. So, I just have light you can think of this as in the form of a ray, or in the form of a wave ok. For now we think of this in the form of a ray and do not worry particularly about whether it is TE or TM, what happens when theta 1 is greater than theta i c you know very clearly that the light incident would actually get reflected back right.

So, this is the phenomenon of total internal reflection of course, from the wave theory point of view, you know that there will be an additional phase shift that this particular reflected light will you know experience. And this phase shift depends whether we are dealing with the TE, or TM. That is the reason why I have not yet introduced the picture of this phase information over here, and simply dealing with the ray kind of an approximation over there, over in this case. So, let us actually look at the importance of this scenario, by total internal reflection ok.

What we have done is to actually confine light entirely into this medium. So, if this medium goes of all the way to infinity, then light is confined in this medium right. So, light has not penetrated or does not really exist in this lower refractive index medium of course, forget about the evanescent waves at this point, because in the ray picture, there are no in evanescent waves the ray of light completely reflects back.

But this really does not solve my problem, for example let us say this is where I have kept my optical source the only way I can communicate now is that if I keep a detector exactly at this particular angle. So, that the ray of light gets reflected and whatever the information that you have that you want to transfer is now received, because information is riding on the light source. Unfortunately if my receiver stands somewhere at this point, there is just no way, or at least it is not possible with this kind of a mechanism I have to adjust the angle theta 1 to send light across this and may sometimes I might not even be able to do so right.

You might of course, say that why do not I move the source up and then make it parallel to the detector, or you know like in the line of sight of detector, but this is simply the problem of VLC that we have been talking about which is what we do not want to perform right. So, what is the way out right? And of course, there is another technological problems here I mean I cannot really have a medium of infinite refractive index right. So, I have to have a finite medium so, that solution for our problems is to actually put another interface ok.

And this interface in integrated optics is normally called as a cover, this is called as film or sometimes called as a guide, and this one is called as a substrate ok. Here we have seen that the cover has a lower refractive index than the film layer and, in this particular substrate if I were to have a refractive index which is greater than n 1 that is of the film. Then light when it is incident at this interface, would then have partial reflection and partial transmission.

And after a few such back and forth reflections then the amplitude of light would actually go down to 0, because at every reflection it will start losing out light ok. To avoid that the refractive index n three of the substrate is also chosen to be less than n 1 usually the substrate is intermediate between film and the cover, that is film refractive index which is n 1 is greater than the substrate refractive index which is n 3 which is greater than the cover refractive index which is n 2.

Such slabs, the substrate need not be infinitely long, it may you know b terminated at this point and this is essentially like a slab that you are carrying around. So, you have you know if you want you can imagine this to be your duster ok, where you have a lower this one and then there is a sponge on top of it which is used to erase like I mean erase the board right.

So, this is what we would call as the film the cover in this case would be just air and, this would essentially be the substrate. The substrate should be sufficiently thick so, that any evanescent rays remember I, if I have the condition that n three is less than n 1 and the angle of incidence here is greater than the critical angle, then light will be totally internally reflected back into the film medium leaving only the evanescent wave here.

So, if your substrate is sufficiently thick such that the evanescent amplitude is almost gone to 0, then that essentially would have allowed you to confine light entirely into the film region. So, this is what is normally done so, a few micron thickness of substrate, which is usually larger than the film thickness is sufficient to confine light entirely into the film region. And because we considered three different layers each of them having different refractive index, this is called as asymmetric slab waveguide ok. This is called as asymmetric slab waveguide.

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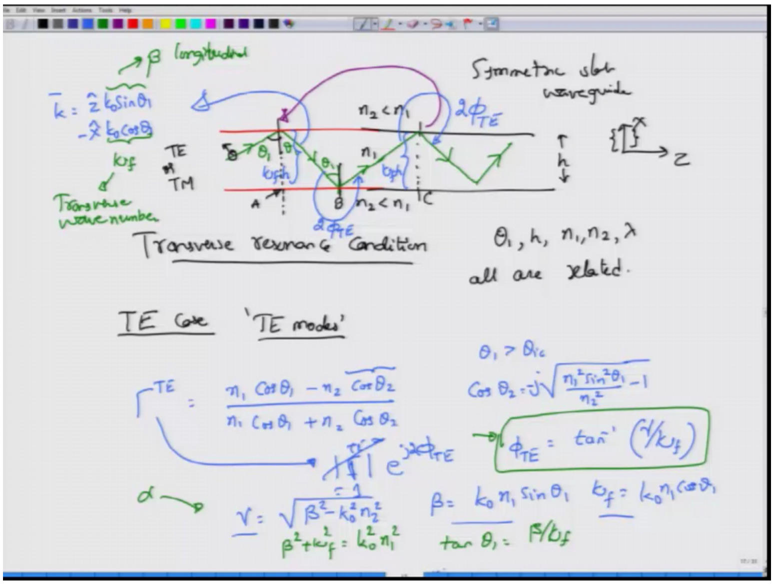

Transverse resonance Condition

$\theta_1, h, n_1, n_2, \lambda$ all are related.

TE Case 'TE modes'

$\theta_1 > \theta_{1c}$

$\cos\theta_2 = -j\sqrt{\frac{n_1^2 \sin^2\theta_1}{n_2^2} - 1}$

$\Gamma^{TE} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$

$|\Gamma^{TE}| e^{-j2\phi_{TE}}$, $|\Gamma^{TE}| = 1$

$\boxed{\phi_{TE} = \tan^{-1}(\gamma/k_f)}$

$\alpha \rightarrow \gamma = \sqrt{\beta^2 - k_0^2 n_2^2}$

$\beta^2 + k_f^2 = k_0^2 n_1^2$

$\beta = k_0 n_1 \sin\theta_1$ $k_f = k_0 n_1 \cos\theta_1$

$\tan\theta_1 = \beta/k_f$

But, what we will do is to actually consider the symmetric slab waveguide, in which I have the film of refractive index n 1 and the cover and substrate both with of same material ok. So, this is called as symmetric slab waveguide, this is the reason I am doing this is just to kind of get some reasonable expressions.

The analysis is not really difficult if you were to replace this symmetric slab waveguide with a asymmetric slab waveguide ok. So, what am I looking at here so, I know that I need to incident light right, somehow I need to couple light into this film in such a way that the angle that it would make theta 1 has to be greater than the critical angle. And once I ensure that, then I know that the light that gets reflected of would also be making the same angle theta 1, and it would also make the same angle theta 1 with the lower interface as well right. So, this would also be theta 1 and therefore, light would be reflected back from this.

So, the net effect is that after a few zigzag you know reflections right, the light would then arrive at the receiver, please remember that although I have shown this symmetric waveguide to be this particular long you know, a few centimeters long. In an actual integrated optical circuit, this will be very very short moreover it would not be just straight like this sometimes it might also be curved ok. But those are topics of integrated optics which we are not dealing with in this particular course.

The idea that we have done is or discuss is that it is possible by use of total internal reflection, to confine light entirely into the film region. And then transmit information from one point that is at one end of the waveguide to another end of the wave guide, simply by allowing light to have multiple refractions right. So, multiple reflections total internal reflections.

Now, having looked at this that is not really telling me you know more information for example, what is the relationship between the slab thickness, and what are the allowed values of theta 1, what happens if the slab thickness is made smaller, what happens if the refractive index difference between n 1 and n 2 is made larger or smaller, is it any useful to have a control over the refractive index profile.

So, what happens if I change the refractive index profile itself, all these information is not available to us if you were to simply follow this so called re approach. So, what we do is that we do a mixed approach ok we will first look at what is called as transverse resonance condition which is absolutely critical that tells us what values of theta 1, or allowed for light to be guided from one point to another point. So, it is important enough that I actually mention this; this is called as transverse right resonance condition I will come to this in a moment ok. What this transverse resonance condition does is to actually, show that not all values of theta 1 are allowed and in fact, if I were to fix the waveguide the film thickness.

So, let us let me pick the film thickness to be h if I fix the film thickness h, then theta 1 h refractive index n 1 and n 2, and the light wavelength lambda all these quantities are related to each other ok. So, this are all quantities which are related to each other as we will now see ok.

Go back to our case and of course, we before we go to the transverse resonance condition this one additional thing that we need to assume here or consider here, that we need to consider whether we are dealing with TE or TM. For simplicity I am going to consider TE case, I am assuming that the light propagating inside the film is that of transverse electric polarized is a wave which is transverse electric polarized. And therefore, the allowed values of theta 1 which are called as modes, or the allowed a guided modes are called as TE modes of propagation, a precise definition of modes and the concept of modes will appear in the you know not the next model, but the module after that. So, for now we simply think of mode as allowed values of theta 1.

So, it is important not every value of theta 1 greater than theta critical angle will be allowed to propagate; there is a certain condition that you need satisfy. And that condition is the transverse resonance condition ok, what is transverse resonance condition? Now, imagine that you started off at some point here ok. So, this is the point which I have been marking let me call that point as A right, I know that this is the refract you know this is the k vector right this is the electric field, or the magnetic field in this particular case this would be the magnetic field and of course, the electric field would be perpendicular to this interface and pointing along the y direction right.

That is why we have called we have considered the case of TE in this one. So, and the TE light is incident and get is reflected you may now think of this case so, as the light leaves this interface at you know at the plane z equal to A. And then starts to propagate along the green arrowed direction that I have shown, reaches point B. And then goes back again to point you know again to the plane which is at z equal to C right.

What it has done is it has done one complete round trip propagation right. So, you started off with the plane at a so, you followed from A to B and from B to C plane when you came back, you have actually traversed one round trip. What we now say is that, in this traversal if the overall phase shift. In the direction perpendicular to the film that is along the x direction for example, so, this is conventionally taken to be the z direction this is conventionally taken to be the x direction for the slab wave guide.

So, if the total phase shift along the x direction happens to be an integral multiple of 2 pi ok. Then the field at this point would be in phase with the field that is field at point C would be in phase with the field at point A and, then this wave can then further undergo reflection further reflections and sustain itself, or be guided from one point to another point ok.

So, this transfers condition is transfers resonant condition is the condition on the total phase shift along the x direction, where is that coming from well let us look at this one right. So, we considered that this is a TE wave and, we know that for the TE wave the refractive index can be written as or the reflection coefficient I am sorry, gamma TE can be written as n 1 cos theta 1 minus n 2. I think this is cos theta 2 divided by n 1 cos theta 1 plus n 2 cos theta 2, please remember that theta 1 is already greater than critical angle which means that cos theta 2 is going to be imaginary component. We have seen this one in fact, cos theta 2 will be equal to square root of n 1 square sin square theta 1 divided by n 2 square minus 1 with the plus j right with a plus with a minus j, because of the appropriate evanescent wave condition that we have described ok.

So, what we have seen from earlier TE case or TM case discussion not really TE case, but TM case discussion is that, when there is total internal reflection the reflection coefficient can be expressed in terms of its magnitude and the phase ok, which we will call as 2 phi TE ok. Because there is a complex number in the numerator, there will be a complex number in the denominator. And the magnitude of this complex numbers are equal, but their angles are opposite and the magnitude of course, turns out to be unity of whatever gamma TE that we have this is actually equal to 1.

So, that is the phase shift that I was actually talking about so, as the ray of light leaves from the top you know at the plane z equal to A arrives at B, and then gets reflected back on to the film itself, there is a phase shift that it will undergo and this phase shift is 2 phi TE. So, I had given this as an exercise, for you to find out what is phi TE and in fact, this phi TE is given by tan inverse of gamma divided by kappa f ok.

Where gamma is equal to square root of beta square minus k 0 square n 2 square, beta itself is equal to k 0 n 1 sin theta 1 and kappa f is equal to k 0 n 1 cos theta 1. So, where are these beta kappa and gamma coming from, well you look at the film case so, this is the k vector right.

So, this k vector can be written as k vector will be this is directed along minus x direction, and plus z direction right. So, along the z direction the value of this wave vector will be z hat k 0 sin theta 1. And along x this is minus x hat k 0 cos theta 1 and, we have simply relay I mean labeled this k 0 cos theta 1 as kappa f and we call this kappa f as the transverse wave number ok.

And we call this k 0 sin theta 1 as the propagation vector beta, or sometimes called as longitudinal propagation vector ok. So, this is what I have and I have use this definition and also written instead of alpha in the previous module, I have written this as gamma ok, gamma being the decay constant of the evanescent mode, or evanescent wave outside the interface. And of course, you can also show that tan of theta 1 will be equal to the ratio of beta over kappa f, and you can also show that beta square plus kappa f square must be equal to k 0 square n 1 square.

So, we know all this if you do not know this one then please know this one. So, these are some simplified notations that I am using to denote these expressions ok, what is important for you to understand at this point is that as the ray of light leaves, the plane z equal to A arrives at B it under goes a certain phase shift. And this phase shift intern depends on the refractive index as well as the angle of incidence and of course, what wavelength light is travelling ok, and that is what is given by this expression ok.

So, this total phase shift upon reflection is two times, the value that have shown here two times the value here and, when this ray now leaves B and arrives at the plane C it undergoes another reflection and, refraction means it picks up an additional phase which would be 2 phi TE right, and what is that transfers resonance condition? We have had two phase shifts one upon reflection here at the lower interface, one upon reflection at the top interface, but there is also a phase shift that you would have seen along the x direction right. The corresponding component you know along the x direction would be kappa f and, because the distance or the height of the film is h.

The resonance or sorry the phase shift here would be kappa f times h, and this phase shift here would be additional kappa f times h. And it turns out that if you add up all the phase shifts ok. The phase shift up on reflection actually you know, because kappa is h and 2 pi TE are in opposite signs when it goes to the expressions right.

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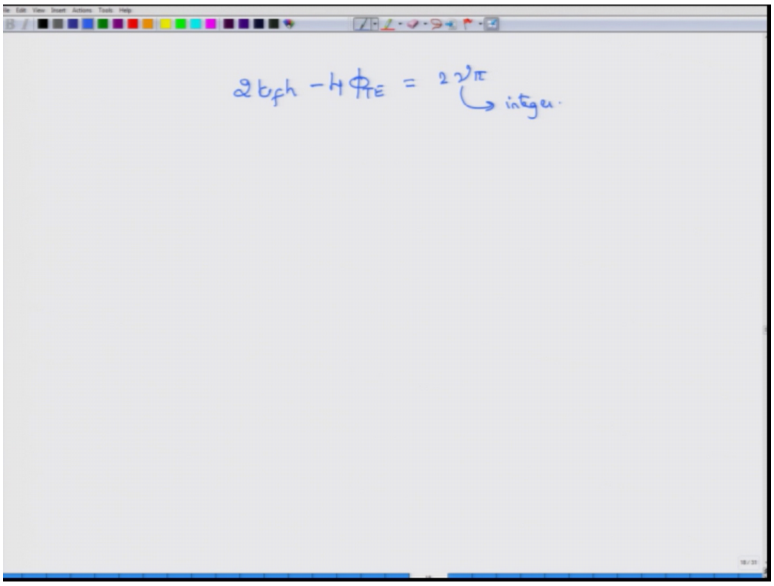

$$2\kappa_f h - 4\phi_{TE} = 2\nu\pi$$

↳ integer.

So, the transfers resonant condition will be 2 kappa f h minus 4 total phase shift was pi TE is equal to 2 nu pi nu being an integer. So, this is a transfers resonance condition, which we will discuss in the next module the importance of this. As well as how this can be used to understand guided light, I mean guided propagation inside the slab waveguides.

Thank you very much.