Noc19-ee-21

Lecture -14

Uniform Plane Waves-I

Electromagnetic waves in Guided and Wireless

Hello! And welcome to NPTEL MOOC on electromagnetic waves in guided and free space or wireless media. In this module which is module number fourteen, we will look at one-dimensional uniform plane waves; to begin with we recall the following Maxwell's equations, which we have described in the previous module.

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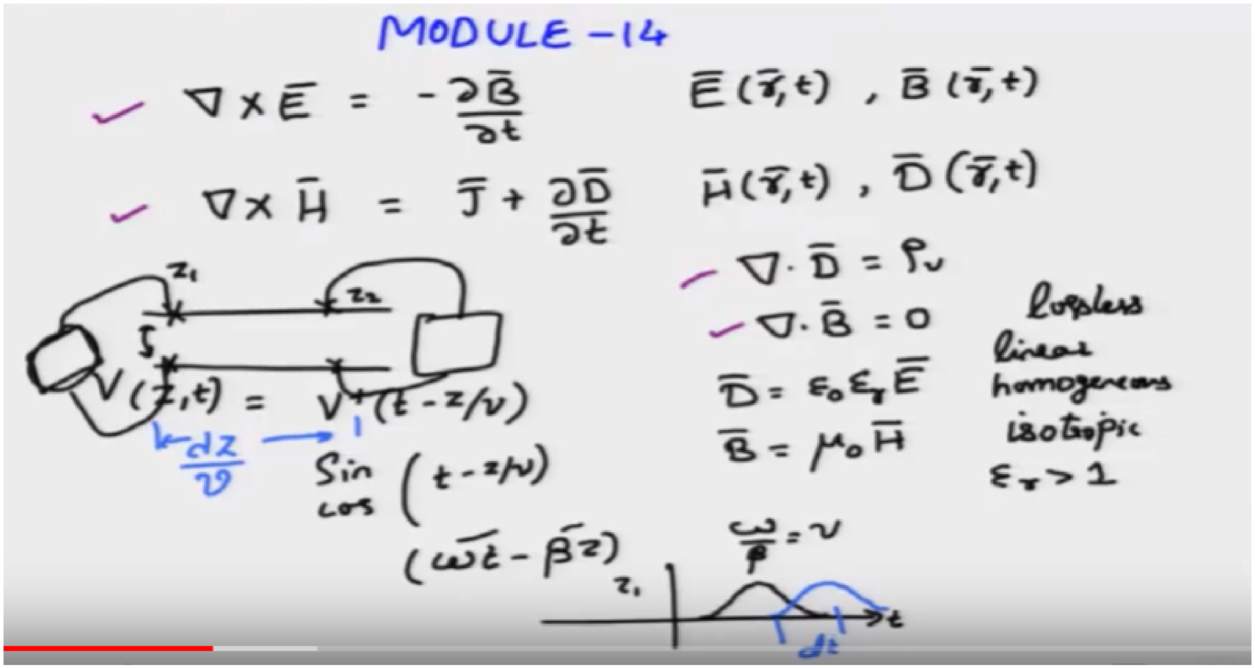

we have del cross E, is equal to minus Del B by Del t, right? Where, we have also already seen that electric field E, is actually a function of both space which is represented by the position vector R, as well as time T, similarly, all the field quantities are typically function and time, of course, if the fields are independent of time or if the fields you know are not varying with respect to time then you get, what is called as electrostatic fields, and in those fields the fields will be functions only of the space coordinates, which is represented by the position vector R. in this module and in our course. We will assume that we are not dealing with electrostatic case; of course that is how you will actually get the waves to be generated and propagated. the other equation that will be of interest for us is del cross H is equal to J, which is the conduction current density, which is the result of the current that is because of the free charge carriers, that are you know moving around in the particular material which we consider, as well as the displacement current density, which is given by del D by Del T, as before the magnetic field H is also a function of space and time, So is the displacement or the flux density electric flux density, which is also a function of space and time. The other two equations that are of interest that we already have seen is Del dot D equal rho V and Del dot B is equal to 0 and we have also seen the relationships between D and B, D is basically, epsilon 0 and in a medium, which is described to be linear homogeneous, okay? And isotropic material of which vacuum or free space or air is an example, Sorry! This is linear homogeneous isotropic medium. Okay? So for such a medium, we can characterize D and E, by a number called as “relative permittivity” epsilon R, Where epsilon zero is the absolute permittivity or the permittivity of the free space. Okay? This epsilon R, is a number at least for the lossless case so we can even add lossless as an additional constraint, in that case epsilon R is a number, which is greater than one usually, So that we can describe D and E, by a simple proportionality relationships. Okay? Similarly B, in this course will always be equal to mu zero times H, in a magnetic material you can also introduce, what is called as magnetic permeability but we are not going to do that because we will consider exclusively only those medium, in which the medium is non-magnetic, Okay. So we have these four equations with us, these are the Maxwell's equations in differential form and using these equations let us, see if it is possible to describe the wave propagation. Okay. Now by what we actually mean, what do we mean by a wave? we have seen waves on a transmission line right, So we had a transmission line, which extended to either infinite length, in the ideal scenario or in a practical case it extended to a finite length, nevertheless, what we found was that, the voltage at any position right the voltage difference between these two conductors at any position is actually or any position and time is in the form of a propagating wave, So V of ZT was any function assuming that, I am only looking at the forward propagating wave was, any function which had this argument T minus Z by V, where V is the velocity of propagation of this voltage. Right?

For the simple case of a sinusoid, this V plus, was a function, which like sine or cosine and the argument of this one was t minus Z by v, instead of t minus z by V we also wrote the argument as Omega t minus beta Z and then were related, Omega and beta to the velocity of propagation we write, So Omega by beta was actually equal to V and what we meant by wave? So, if you actually hook up an oscilloscope here, okay? So, this is an oscilloscope and then you hook up another oscilloscope at a certain distance, which is greater than which is at a distance farther away from the initial position, So you can call this as plane Z one and another this plane has Z two and then you looked at what kind of a wave form would you, would you, would be displayed? Assuming a general arbitrary V Plus kind of a function, if this was the wave that you saw, at z1, the corresponding wave that you would have seen or the corresponding voltage waveform that you would have seen, would be displaced by a distance that is actually proportional to these two or in terms of time, it would be whatever the distance that has been that is the difference between these two planes, divided by the velocity that you had would be the amount of delay which we will call as say DT, by which this particular pulse would be delayed. Right?

So, it see it's conceivable that if you actually take a you know very very long transmission line and then start hooking up imaginary oscilloscopes at every point. Okay? We have considered lossless transmission line and then you start noting down the voltages, a pattern begins to emerge. What is the pattern? That whatever the functions that may be there you know V plus of T minus Z by V, that would be progressively delayed, as you keep moving along the transmission line. So this is in fact the behavior of a propagating wave. Okay? They have also seen a different kind of a wave, that is called as a standing wave, in which you would have a forward propagating voltage, hitting upon some discontinuity for example, that could be a load whose characteristic impedance sorry! Load whose impedance would be different from the characteristic impedance of the transmission line and it would generate a reflected wave, right? So, these two waves would you know combined together there and it would not be moving so much but at any particular position if you were to stand, you would actually see the amplitude to be changing. Right? But these are essentially what is called as standing waves? which we will not concern consider now, what we are interested is the progressive waves or the waves which are propagating I arbitrarily assume that, the propagation is along Z direction, for the transmission line case, we will continue to make that assumption although, there is nothing in space that would tell that my Z Direction should coincide with your Z Direction, but, what is important is that if you pick a particular direction, in that direction this should be a propagation type of a behavior. Right? So you imagine putting up a oscilloscope and somehow, being able to look at electric fields or magnetic fields, these electric and magnetic fields, should exhibit a behavior, which is given mathematically by this function, the plus of t minus Z by V, So that is essentially what we mean by a wave more ordinarily you may have done lot of experiment, you know you take a string tie it up onto one end and then you actually start doing this, you know moving behavior of the other end the free end of the string and then you would actually visualize that the string is actually going up and down and there is some sort of a waviness into that string, right? So there are these different types of this for examples, seismic waves are the waves that are generated because of the plate movement, you know I'm not an expert, but those are essentially also type of avails because they would also move. Right? So any of this phenomenon, which has this function of t minus Z by V, kind of a behavior would qualify for a wave. Please remember that it's not only some function of t minus Z by V, it could be, t plus Z by V, it could be t minus X by V or it could be any general direction that the wave could be propagating. Okay? However, coming back to these electromagnetic waves, what makes it very different from the other kinds of waves is that? These electromagnetic waves can travel in vacuum as well, because there are no material charges pushing this wave, right? so in in contrast to a sound wave, which requires the particles to be pushed and pulled, sort of a elastic motion that you would actually see, there is nothing like that that is required for an electromagnetic wave, a moving electric field would generate a moving magnetic field or rather time varying electric field would generate a time varying magnetic field which in turn would generate a time varying electric field and magnetic field and these couplings can go on in create a propagating wave as we will shortly see.

So we have to understand that these waves when they propagate in free space they actually have a different velocity whereas, if the waves propagate in a material for example, light waves moving in a small slab of glass would have a different velocity, right? In most cases, that velocity is governed by epsilon R, in some rather very specific cases, the velocity is also governed by other characteristics, when epsilon R, itself becomes complex, Okay? But that story's for something later.

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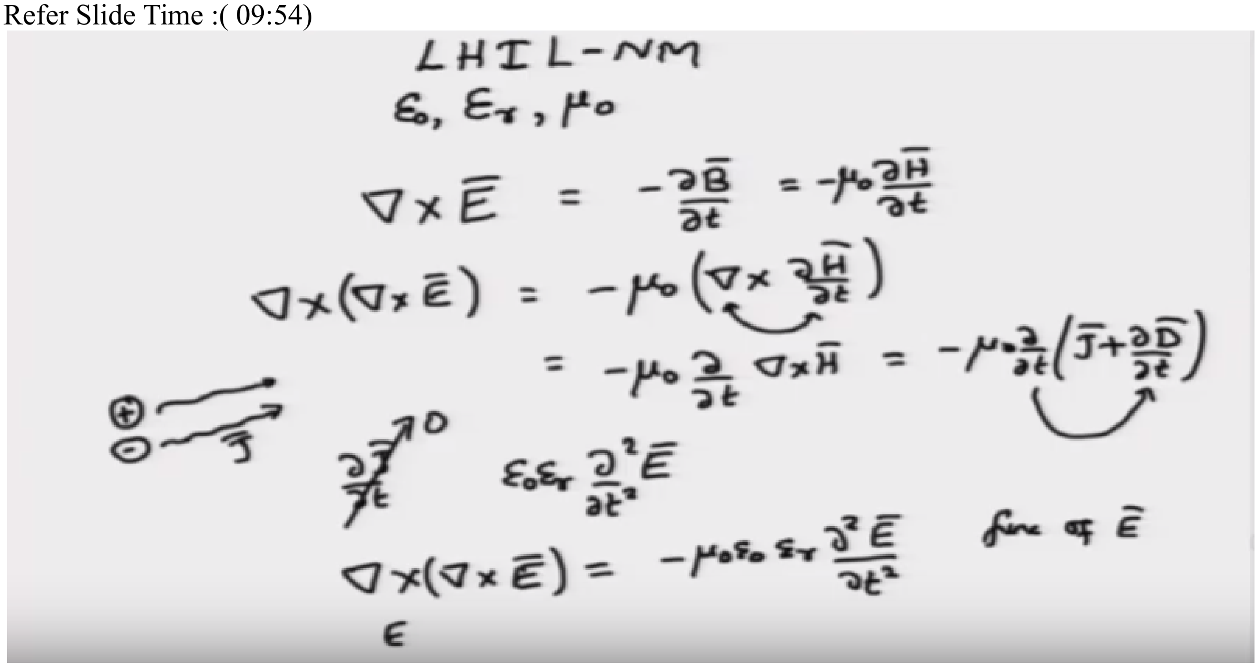

So we will concentrate on the simple scenario, where we are going to consider a medium to be linear, homogeneous, isotropic, as well as lossless, okay? and for this medium the characteristic of the media is given by specifying epsilon R, as well as, mu naught, epsilon naught anyway is already defined. Okay?

this is decidedly non-magnetic medium, so you can even attach a minus or a hyphen and say NM, where NM, would stand for a noun magnetic medium. Okay? Our starting point at least from the math perspective, would be to go back to this equation Del cross E, is equal to minus Del B, by Del T, Okay? and then replace B with H, that I can do because I already know what is the relationship between B and H and I also know that Mu naught, is a constant, so I can pull that out of the differentiation and therefore, partial derivative and then I have minus mu naught Del H by Del T.

What I do now is, to take the curl of the equation again, Okay? So I'm going to take the curl of this first equation which is Faraday's law and what do I get? I will get minus mu naught, Del cross Del H by Del T, Okay? Right now without going into lot of mathematical justification I, I will simply interchange this operation of curl with partial derivative. Okay? I am allowed to do this under no certain special conditions which you can read about in any math textbook, but when I do that what do I have? I have minus mu naught del by del T del cross H, okay? But I know what is Del cross H? There is another equation which tells me that Del cross H is equal to, so you can fill up that equation which would be J plus Del D by Del T, okay? let us, consider the first term, I have del J by Del T, J is because of the conduction current, so which requires that I have no free charges plus or minus whatever the type of charges that are possible and these charges have to physically move in order to constitute that J field or the conduction current density field, Okay? but my medium is a complete insulator there is no free charges anywhere the medium also extends all the way to infinity everywhere that you can think of the medium extends all the way to infinity and it is only specified by the parameters epsilon 0, mu 0 and epsilon R, Ok? And epsilon R is also real quantity so clearly there is no sight of any free charges and therefore there should not be any conduction current density, right? So this Del J by Del T term readily goes to 0 and you can eliminate it and you can now consider the second term which is Del square by Del T Square, why did it become second partial derivative? Because there is a Del by Del T here which goes on to another Del by Del T, therefore, this becomes the second partial derivative with respect to D ok or rather of the quantity D, right? but we already know that D can be written as epsilon 0, epsilon R, both of which are assumed to be constants so you can pull them out of the partial derivative and then write here as del square E by del T Square. Ok?

Now, let's complete the left hand and the right hand side equations after this simplification, so del cross, del cross E, which is still unknown. we don't know what exactly to make out of this quantity, Okay? but the right hands is at least is now simplified you will get minus mu naught, epsilon naught, epsilon R, del square E by Del T square, at least this is some sort of an okay, thing right? On the right hand side, I have a function of E alone right and on the left hand side presumably, I have a function of E alone right? Because this is a curl operation on E that depends only on the electric field components taking the curl of E would also depend only on the electric field component, so on both sides. I have an electric field component and an electric field component or rather functions of electric field and function of electric field. Okay?

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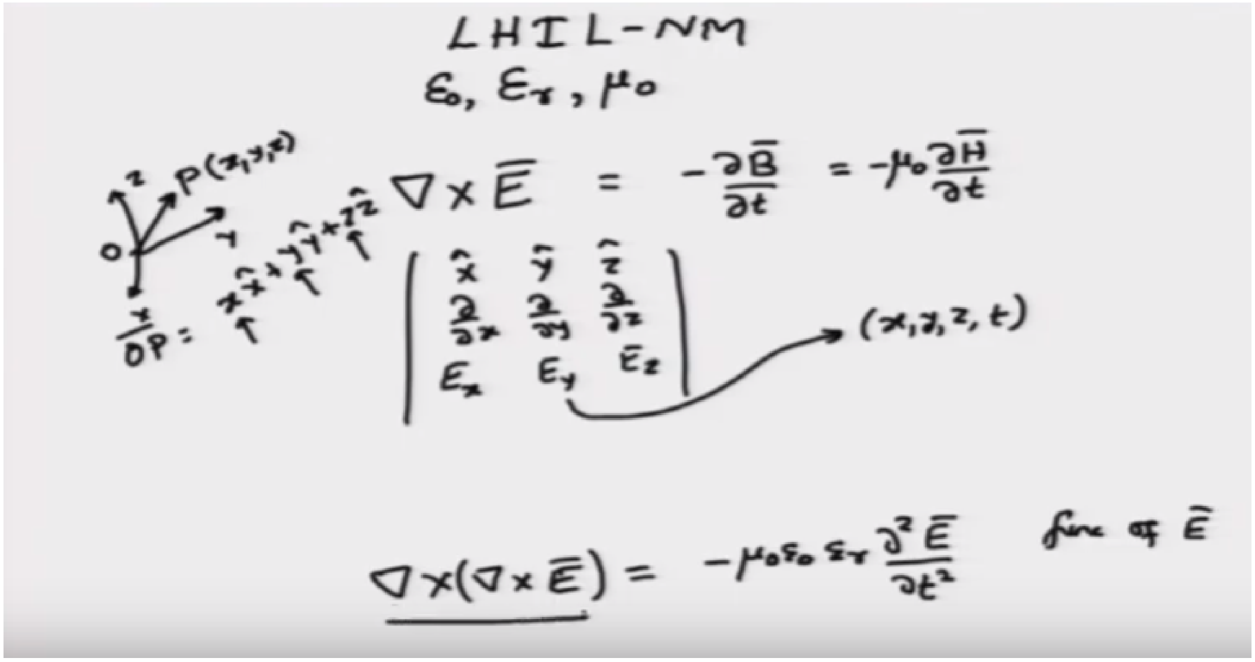

Now what do we do about the left hand side okay, I'm going to erase the equation right here because I want to preserve this equation that we have already written and discuss the meaning of the left hand side term okay? what is the term you have a curl of, curl of E, right now one can actually go to specific coordinate system that you are talking about for example, in the Cartesian coordinate system, this del cross E could be written as X hat,Y hat, Z hat, these are the unit vectors along the coordinate system, which is given by X,Y and Z. X, Y & Z, are three mutually perpendicular lines or axis and any point on this one can be specified by giving the three points X,Y and Z are the corresponding vector OP are the position vector P, can be given by this particular quantity, X, X hat, Y, y hat, plus Z hat. This is something that you already know. So curl of E, in this coordinate system would be X hat, Y hat, Z hat, del by Del X, del by Del Y, del by Del Z and please remember that electric field E is also vector, so it will be Ex, Ey and Ez and each of this Ex, Ey and Ez themselves are functions of the position vector and in the coordinate system that we considered the position vector can be specified by giving the components of the position vector, which are XY and Z right? So you have X Y and Z, T that would be, that is each Ex, Ey and Ez, will be a function, how and what we still don't know but it would be a function of X, Y, Z and T, right? So this is, this is, what you would have for the curl of E and after you have evaluated this curl of E you will actually have the curl expressions, you can put them back into this expression of this curl and evaluate the complete left-hand side, Okay?

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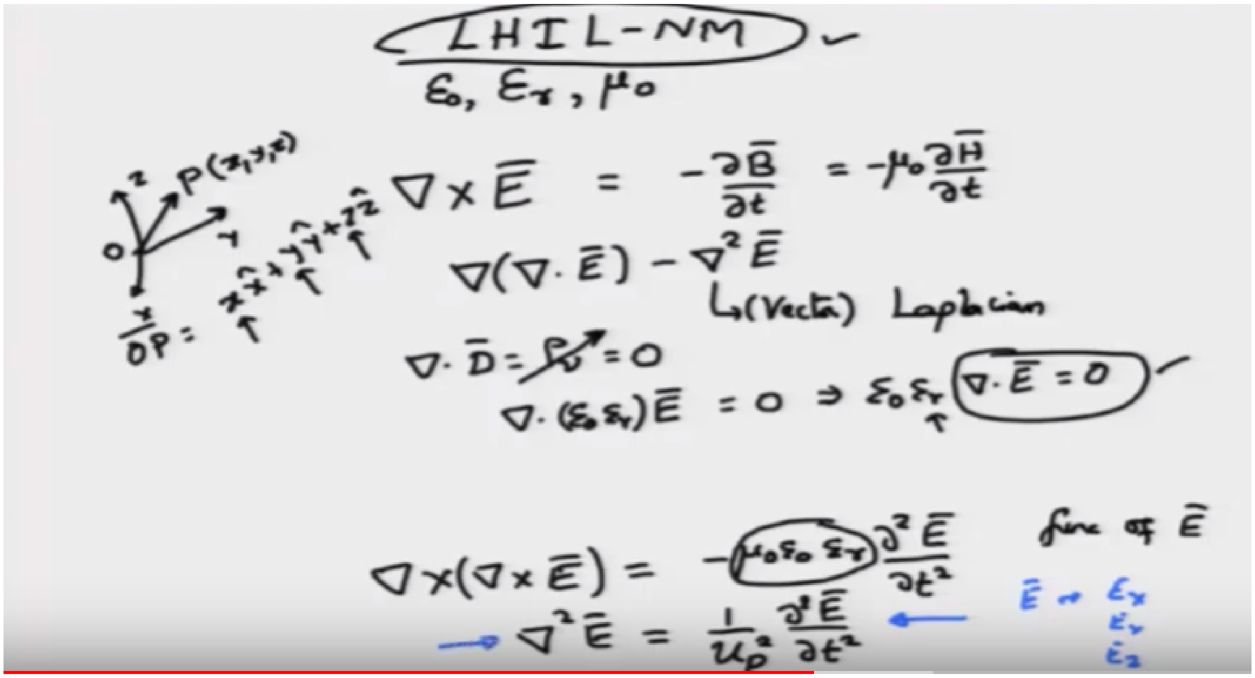

But there is a small shortcut for us, which makes use of the vector identities. Okay? What is the shortcut? That means the shortcut is actually the rule which says that Del cross, Del cross E, is actually gradient of divergence of E minus Del square E, okay? And this Del square is called as the "vector" or simply sometimes called as the "Laplacian", okay? And of course you have Del dot E and gradient of this one, does it make sense you know in terms of the vectors? Yes, Carlos electric field will result in a vector taking the curl of a vector will result in a vector, which is fine, this Del square is a scalar operation but because it operates on a vector, this is sometimes called as vector "laplacian" and this Del square of e results in a vector, good! del dot E will result in a scalar but taking the gradient of a scalar will get back the vector, so this equation makes sense at least, so now with that, let us also look at another equation we had del dot D equal to rho v right. Now we said that the medium is infinite and all that there are no free charges, if there are no free charges no conduction current density J, there is also no you know free charges Rho V itself right, so del dot d equal to Rho V simply becomes equal to 0, because there are no free volume charges or free charges.

Now D is related to epsilon 0epsilon R and D, or rather related to Ey, R epsilon 0 and epsilon R, which when you put them here and realize that this epsilon 0, epsilon R, is a constant, okay? which can be pulled outside the divergence operation you will see that epsilon 0, epsilon R, del dot E equal to 0, the only way you can this equation to be valid is when, either epsilon R equal to zero, but we have ruled out that possibility or this del dot E itself equal to zero, which we will readily accept. Okay? So, I have in the vector laplacian Del of del dot E, minus Del square E, Del dot E, equal to zero under this particular medium, Okay? For a different medium this may not be true in fact, as we will see for waveguides this equation is not true in general okay? But luckily for us we are dealing with this kind of a medium which is, LHIL - NM medium and for which this left-hand side can simply be written as, minus del square E, now the right hand side also has a minus sign so I am going to remove the minus sign from both left as well as the right hand side terms and then re arrange this mu naught epsilon 0, epsilon R and call it as, 1 by UP Square and del square E, by del T Square, okay? Please note that, this equation is true for E, which is you know itself consists of Ex, Ey and Ez components, okay? And in the rectangular Cartesian coordinate system, you can take this equation, which we have written separately into three scalar equations. Okay?

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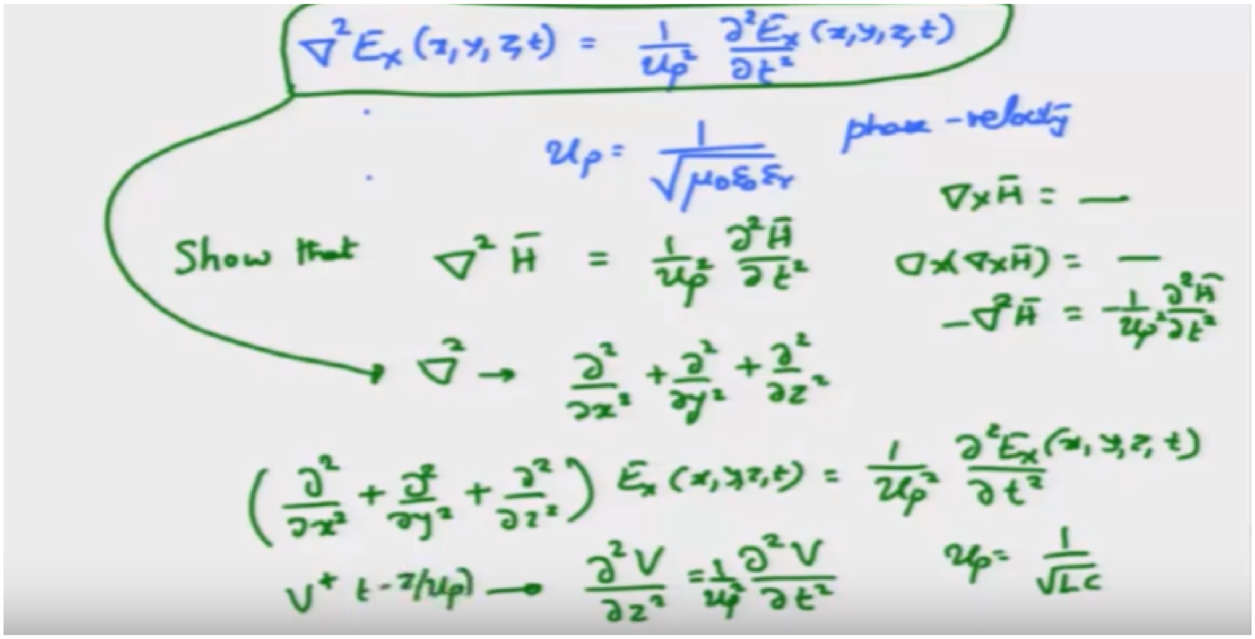

what we mean by that is, I can have this equation decomposed into XY and Z terms as del square VX, which is still a function of XYZ and T, to be equal to 1 by UP square, del square E, Ex by Del T Square X of course as a function of X Y Z and T and of course, you can now write two similar equations for Ey and Ez ok? and what is this UP, UP of course is now given by 1 by square root of, mu naught, epsilon naught, epsilon R, actually corresponds to the phase velocity, the meaning of phase will come back later on we'll come to that later on, or but it is essentially the velocity with, which the wave is actually moving. Okay? Of course we haven't established that, this is exactly the wave solution, we will do so shortly but in anticipation of the fact that we are dealing with waves, I just call this UP to be the face velocity. Okay? As I've told, you can write down a similar equation for Ey and use it and everything Ex, even and easy we'll all be functions of X Y Z and T and it will satisfy a similar equation. I will give you a short exercise to show that not only the electric field satisfies this equation, even the magnetic field; you know H would also satisfy the same equation ok It would be given by 1 by UP square, Del square H by, Del T square, the development of this equation is very simple. You start off with Del cross H given by the right-hand side, which you now can fill up take the curl of curl of H, ok? and you will get the right-hand side and show that the left-hand side reduces to only minus del square H and on the right hand side you will have minus 1 by UP square, del square H by Del T square, cancel off the minus and then you will get this equation so, I encourage you to do this exercise just to get home know a kind of a mathematical, you know hold on the mathematical identities that we have used in deriving this equation. Now we consider, first only this equation, ok, to talk about the further development, what I have here on the left hand side is this laplacian. Now I have already made my choice of coordinate system to be rectangular Cartesian coordinate system, in that coordinate system this Del square can be written as Del square by Del X square plus, Del square by Del Y square, plus Del square by Del Z square. Okay?

So let's write down del square by Del X square, plus del square by Del Y square, plus del square by Del Z square, this entire thing you know acting on Ex, which itself is a function of all these four variables being equal to 1 by UP square, del square Ex, which is a function again of these four coordinate so rather four variables x del T square. Okay? what you observe is that the left-hand side, is a function which is only of space changing like the derivatives of Z X and Y, tell you how the space derivatives of the electric field component Ex would be there and on the right hand side you have a time derivative, second time derivative and this type of an equation, where on the left hand side you would have seen the space derivative and on the right hand side view you would have seen the time derivative, is something that you would have seen in the transmission and so if you recall the transmission line equations, for the voltage there we had, del square V by Del Z square equals del square V by Del T square, of course you still had this 1 by UP square kind of an equation there. Okay? Except that there UP was actually equal to 1 by square root LC and this was the case for the transmission line, which was loss less right and it had a uniform cross section. So you have seen this equation? So this equation you know is going to give you a wave right, so when you choose mathematically the function V then the equation can be started with the general equation that would satisfy would be either V Plus of t minus Z by u P or it would be V sorry, V Plus of t minus it by u P or it would be V minus of T plus Z by u P, you have already seen that, this is this equation is going to give you 1 dimensional wave, which is basically to say that it is a wave which is propagating either along the plus Z direction or along the minus Z direction, but it is going to be a wave.
Now if you compare this equation with the previous equation for the electric field that we have written Ex component that we have written you will see that in addition to this del square by Del Z square terms, you also have two additional terms one is del square by Del X square and then you have del square by Del Y square, if you can remove these two terms right then the equation would be identical to whatever, the voltage equation on a transmission line is, right? let us mathematically take it. Okay? We'll worry about how to generate this later on in some other you know module. Okay? However nothing compels us to stop taking this Del square by Del X square and Del square by Del Y square turns to be zero, meaning that what I am assuming

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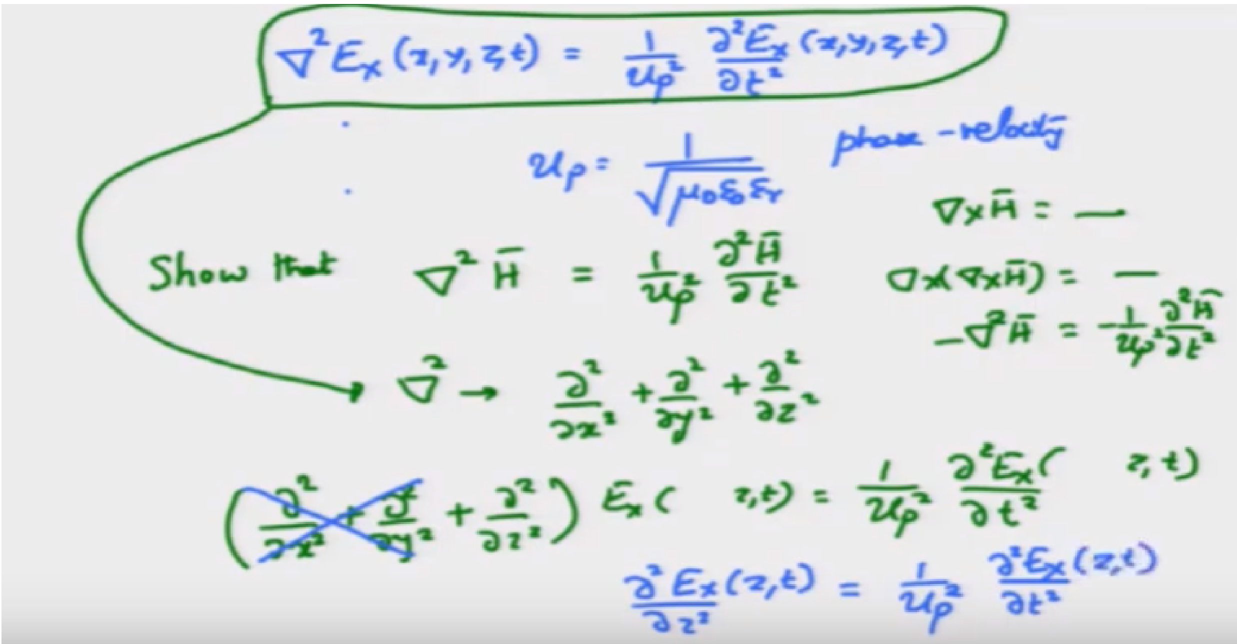

Is that Ex is not a function of X, it is not a function of Y, It is simply a function only of Z and T. Okay? As I have told you, I still have not, you know, exactly told you right that how we are going to make this x and y dependence go away, but take it mathematically that you can always do this right? and when you do this you know, assume that X is going to be just a function of Z and T, then you will land up in a very interesting scenario saying that, you have del square Ex by Del Z square, where Ex is now a function of Z and T, to be equal to 1 by UP square del square Ex by Del T Square, D X is a function of Z and T.

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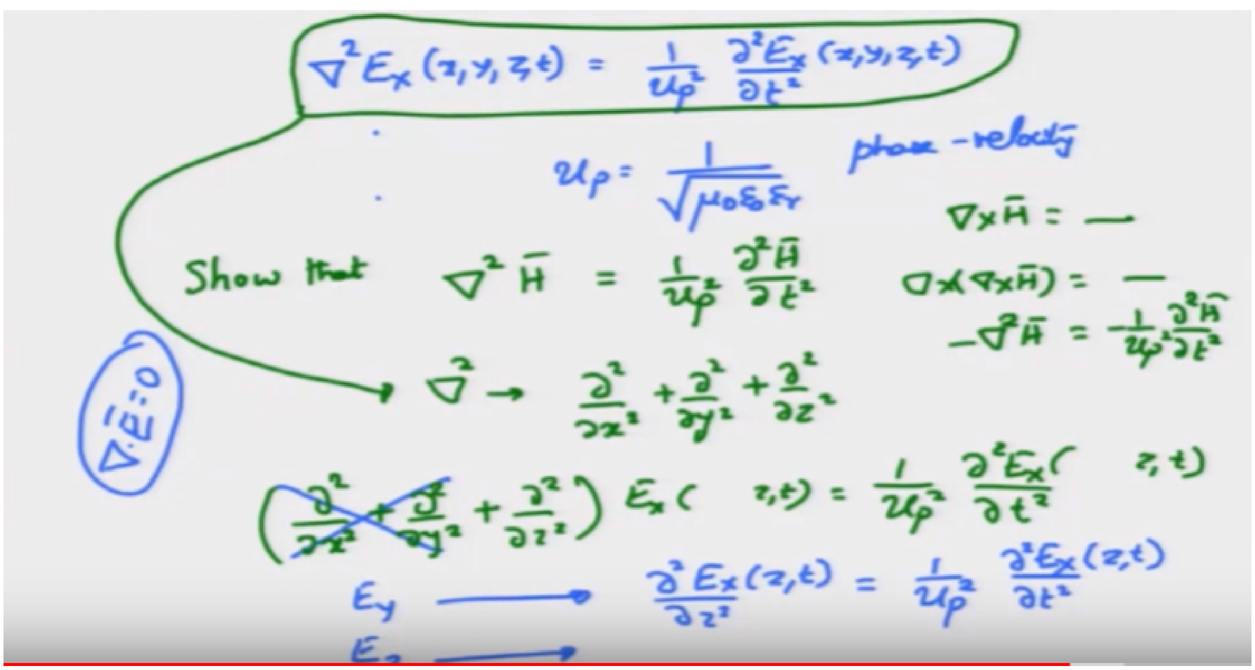

And any such solution on Ex, which would be in the form of Ex of Z and T, which would be in the form of some E plus of t minus Z by UP or E minus, T plus Z by UP, are potentially the solutions for this equation. Ok? The story is not complete yet, because by following the same logic I can show that even Ey will satisfy the same equation and then EZ will also satisfy the same equation, ok? but we will later on see that, I cannot have all three of them satisfying this type of an equation, because there is another constraint called del dot E equal to 0, which forces something else to happen and We are going to consider that one in the next, sorry! That we are going to consider that one shortly.

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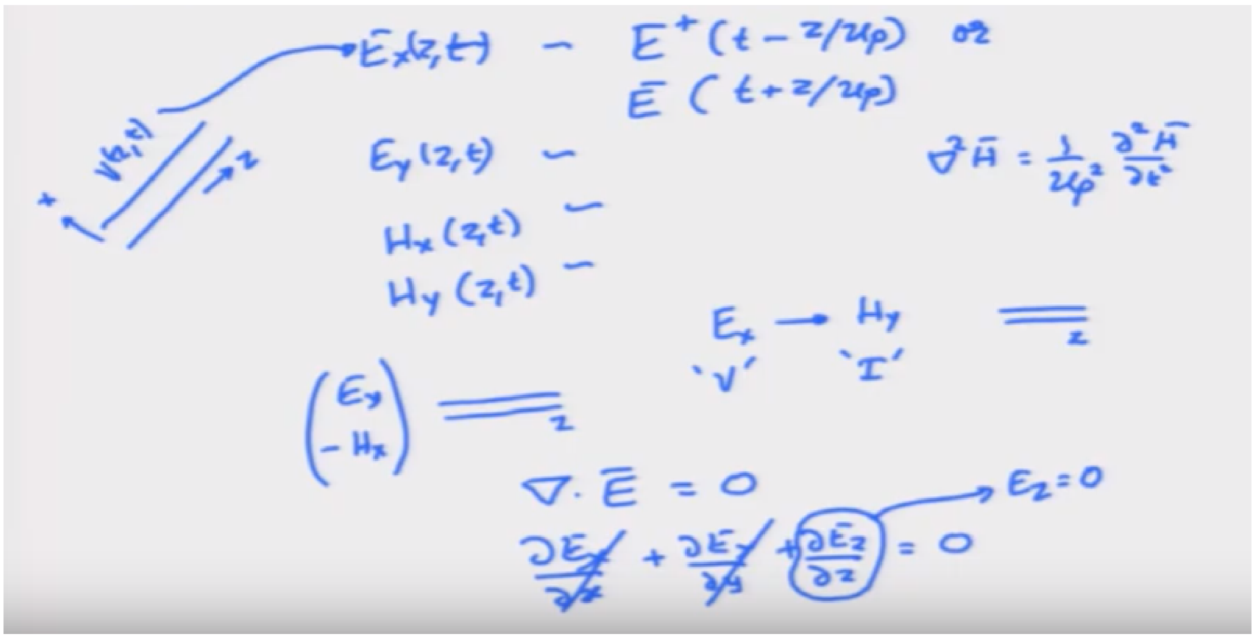

But for now, we know that Ex satisfies this equation and you can also put down Ey which satisfies the same equation. Now let us come back to this H case, the equation we had was del square H, equal to 1 by UP square, del square, you know H by Del T Square, is what we had similarly you will have HX and Hy okay? You will have all these equations satisfying very similar wave equation and therefore, have the forms to be similar. now if you go back to the transmission line analogy and then think of a certain transmission line, which is propagating along Z or rather transmission line, which is lying along Z and then choose one of the axis to be the x axis right and then say the potential difference, is going to be because of the Ex component then the voltage V of Z T on the transmission line is analogous to the voltage or rather the component Ex, which is propagating as a function of Z and T as well. So in fact, mentally you can imagine that there is a transmission line which is associated with the potential difference V, which is analogous to Ex component. Okay? So Ex propagating along Z, can be associated with the transmission line, a uniform lossless transmission line, but now you may ask well the transmission line not only has the voltage it also has the current, so what? Shall we do about the current component or the corresponding component of the current? There well, we can show that by writing down this Del cross equations that there is a very natural pairing of Ex and Hy okay? You can treat this VX as the voltage V and H X as the current I on the transmission line. so this one pair EX plus Hy both propagating along Z you know with a given velocity UP can be thought of as having a transmission line or analogous to a transmission line along the z axis. Now this is not the whole story because you can find similarly EY and minus HX. Okay? I just put minus HX for a reason that will come out later on in the other module, but I now go ahead with that one, Ey and minus Hx, will also be associated with the same transmission line Z. Okay? So these pairings Ex, Hy, both traveling along the I mean both being the components of the waves, which is propagating along the z axis, as well as this Ey and minus Hx, both can be associated mentally and formally one can show that it is true that they can associated with the uniform lossless transmission line. Okay?

Now, what about the easy component? Well we have this condition that Del dot e equal to zero and, if you go back to the Cartesian coordinate system, this translates to Del Ex by Del X, plus Del EY by Del y, plus del Z by Del Z, Okay? This would be equal to zero. Okay? However, what we are going to assume but what we have already assumed is that Ex is not a function of X, EY is not a function of Y, so that leaves us only with Ez del EZ, by Del Z equal to zero, which you know the simplest solution for this case would be to make Z itself equal to zero, ok? So now our wave for the electric field would have only components Ex and Ey, which both will be functions of Z and T, similarly you will have Hx and hy, Okay? Because del dot H is also equal to zero and these four nonzero components together constitute a wave which is propagating along the z direction, but because we have paired them like Ex and Hy, Ey and minus HX, you can treat these two as, two sub components, okay and you know think of this as one type of a wave, and the other one has another type of a wave. So one can be thought of as the X polarized wave, the other can be thought of as the Y polarized wave, both polarized waves propagating along the z axis. We will continue a discussion in the next module.
Thank you! Very much!