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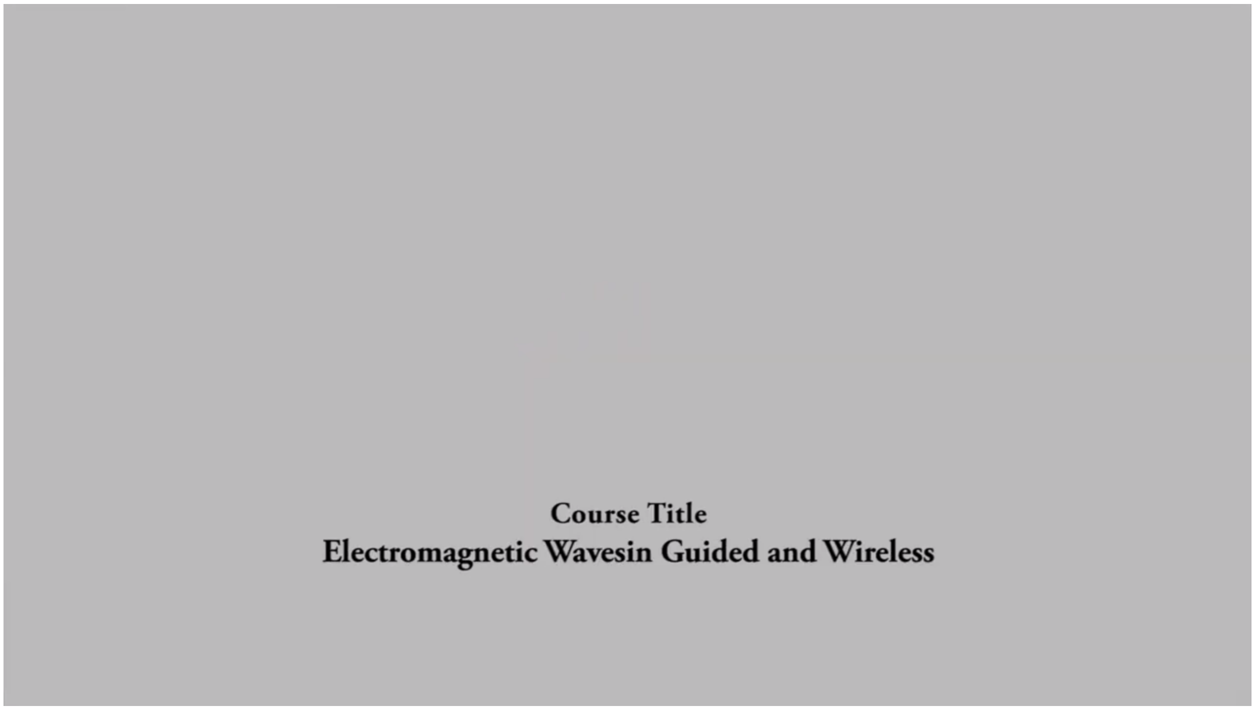
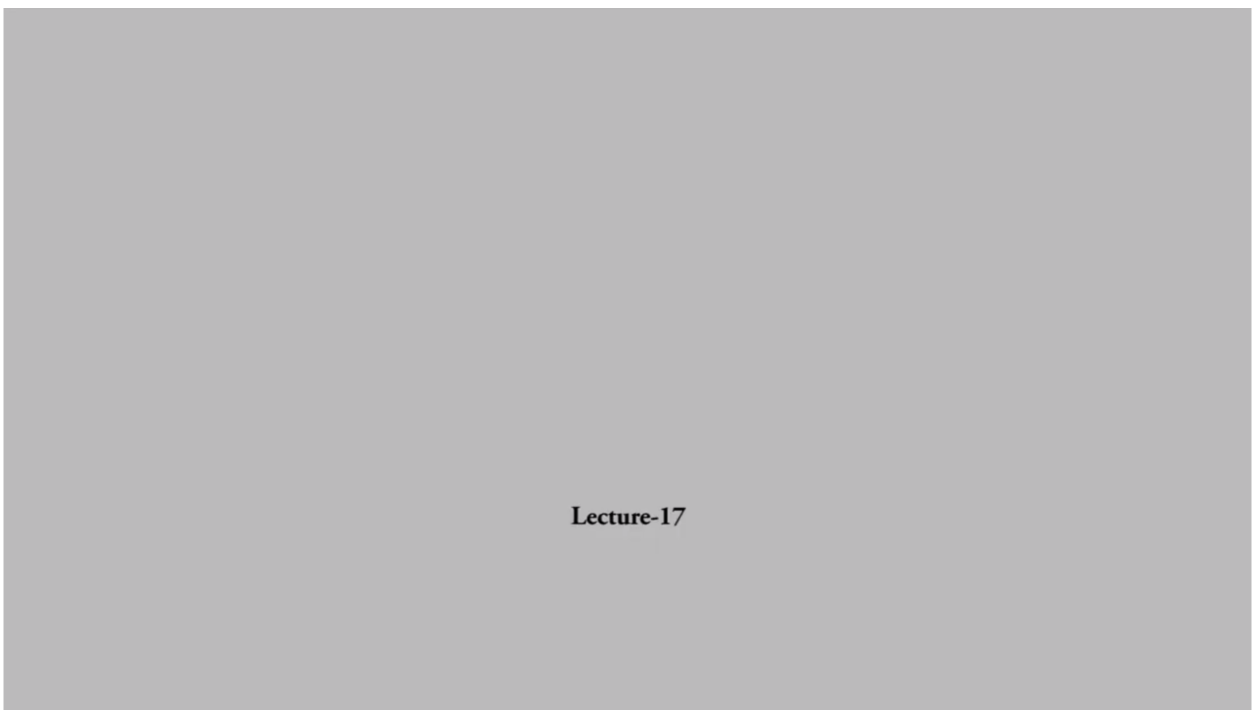
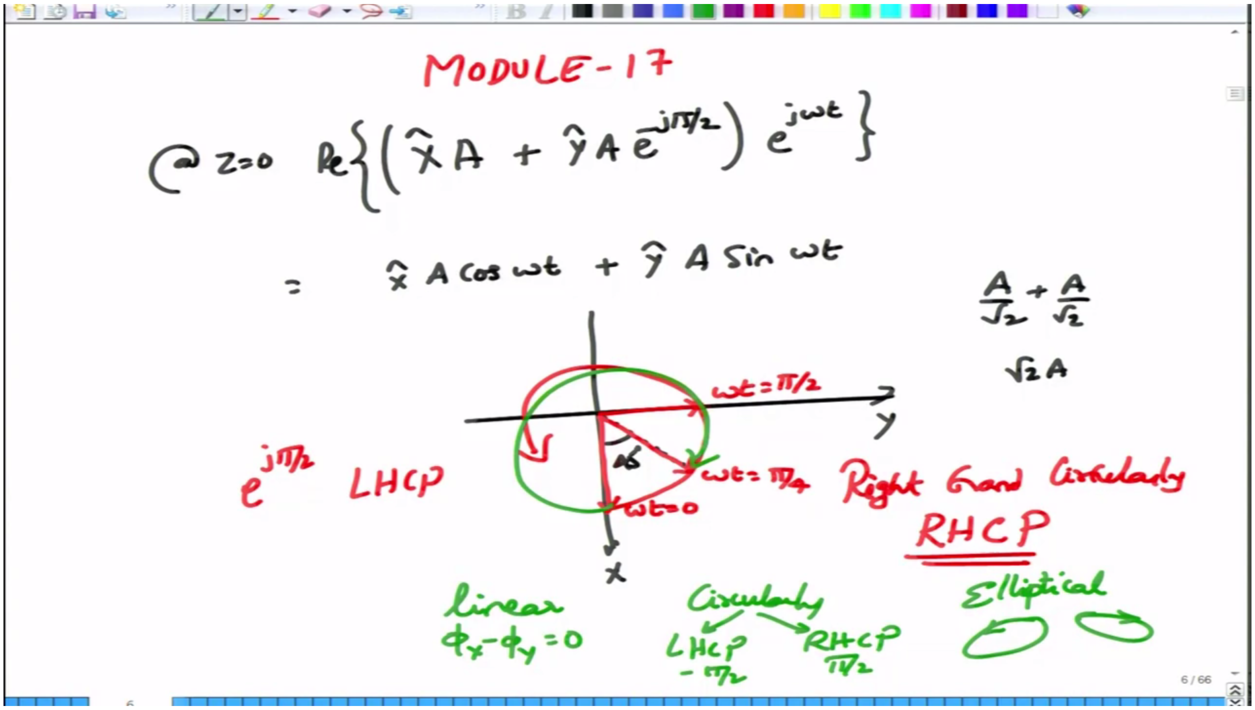




Dr. K. Pradeep Kumar: Hello and welcome to NPTEL MOOC on electromagnetic waves in guided and wireless medium. So this I module 17 of the course and we have already seen uniform lain waves, the wave equation, and we have seen different type of polarization. In the last module, I'll give you an exercise. Let me first solve that exercise to show you that we don't only have linear polarization, but we have polarizations of different nature as well.



So we took the electric field to be of the form x̂ A at z=0 plain of course. We took this as (x̂ A + ŷ A $e^{-j\pi/2}$). Now I will first convert this phaser at z=0 plain into the real time dependent expression. So I will now have x̂ A cosωt + ŷ A cos (ωt-π/2). Now cos (ωt-π/2) would actually be equal to sin π/2. So I will have, instead of writing this as cos (ωt-π/2), I can write this as sin π/2. So then I write that what I will have as A -- sorry I wrote cos again -- it's actually sin ωt, so cos (ωt-π/2) is sine ωt. So now this is what we have in terms of the time dependent electric field. As before, we will draw the axes, mark x and y, and now look at what happens at ωt=0. So at ωt=0 cos0 is 1, sin0 is 0, so the electric filed have to lie entirely along the x axis, with am amplitude of A. So this is a situation at ωt=0.

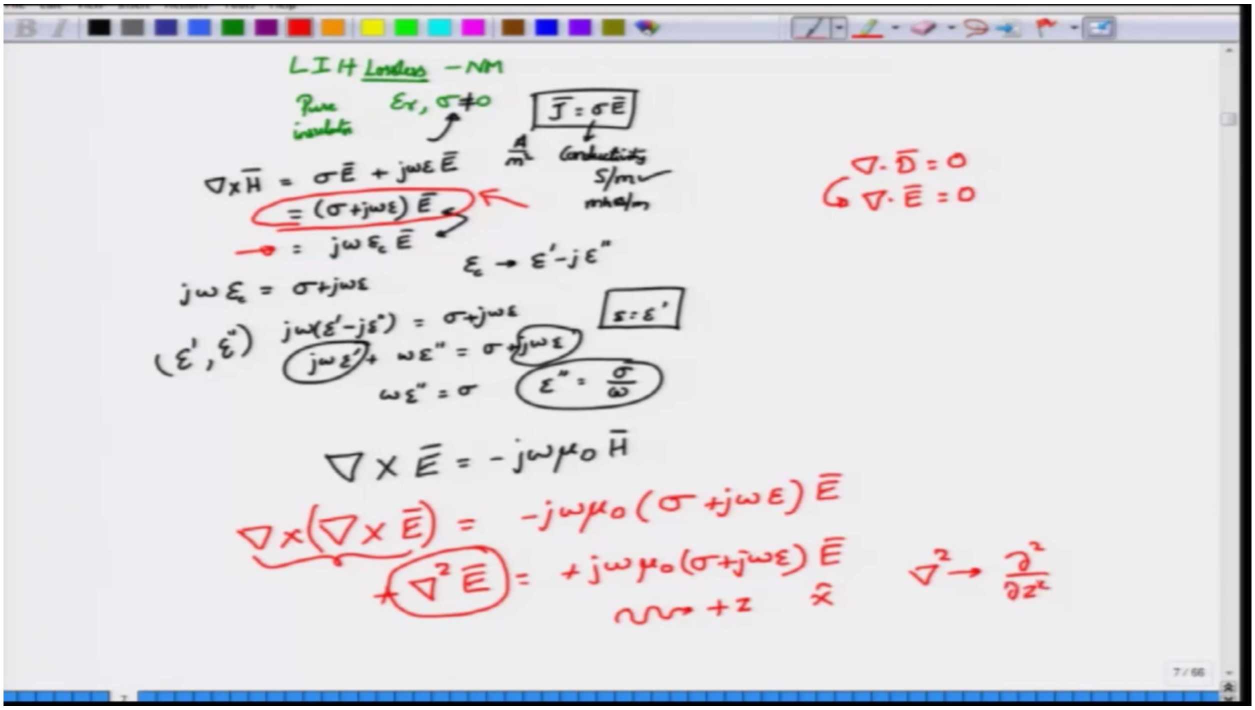
Now what would be the situation at ωt = π/4? At ωt = π/4, both cosine and sin have equal amplitudes and because you have x̂ A + ŷ A divided √2 on both sides, the angle with which the resultant vector will be at 45 degrees, okay, so that would be 45-degree angle. However, the amplitude of this one would be at A/√2 + A/√2. That would be square√2 A and you can plot that. So you have A here and then you have A here and then A/√2, so what you'll actually be able to find is to find the magnitude of this one and then write the vector at ωt = π/4, okay.

Now what is the situation of $\omega t = \pi/2$? In which case $\cos \pi/2$ will be 0. So the x component will contribute to 0 length vector whereas the y component will contribute to the full A magnitude length vector, okay. So you actually have A here at $\omega t = \pi/2$. Now you can see a pattern emerging, right. So as I start increasing ωt further, I will note that the field vector seemed to be going along a particular circle and they would be moving in this particular direction, which is basically counterclockwise. So because you move in the counterclockwise direction, so you can actually start with x and y, and then when you move in the counterclockwise direction, your thumb will be pointing along the z axis.

So you start off with x axis, which is where at $\omega t=0$ the electric field vector was, and then you move it, move your fingers along or curl it along the direction of the increased chaining electric field vector, and you will see that your thumb is pointing along the z axis, and you've actually used your right hand in order to get to the z axis, and this is therefor called as right circularly polarized wave, okay, or sometimes called as right handed polarization, right hand circularly polarized. Why it is called circularly polarized? Because these electric field vectors are moving along a particular circle of radius A here, correct. So this is right hand circularly polarized wave or sometimes simply called as RHCP vector.

You can show I will leave this as an exercise that if you change the phase from $-\pi/2$ to $\pi/2$, you will end up with a left hand circularly polarized vector, which would actually move in this direction. So it would move in the direction that is opposite to that of the right hand circularly polarized wave. So you do have different types of polarization, linear polarization is one where the phase difference $\varphi_x - \varphi_y = 0$ and you have, let's say, the circularly polarized version. You have two types of circularly polarized cases. One you have the left hand circularly polarized phase and then you have the right hand circularly polarized wave, depending on whether the phase $\varphi_x - \varphi_y$ is $\pi/2$ or $-\pi/2$, and then the most general case is when you have what is called elliptical polarization. In the elliptical polarization, you can still have it along -- I mean the movement of the electrical field vectors can be either left or to the right.

However, they would not move along the circle, but they would actually move along the ellipse that you can see here. So they would actually be moving along the ellipse, both kinds of ellipses are possible, left handed ellipse and the right handed ellipse. So these are the different types of polarization of a uniform plain wave.



Now let us go back to the type of media that we were considering. We considered a linear medium, which is fine, we will consider the same medium. We consider isotropic medium, meaning that your direction of the propagation would not matter. The properties of the matter would remain the same whether you were propagating along the z direction, oriented along x or oriented along y.

Then we have a homogenous medium, meaning that the material properties, say, the permittivity and permeability would remain constant and it would remain the same at different points in the space, and finally, we had considered the lossless materials. In the lossless material scenario, what we had was the material made out of pure dielectric, meaning that εr was present and any conductivity of the medium that could normally be present in a typical medium was actually taken to be equal to 0. So the material was pure insulator or pure dielectric. Of course, we have non-magnetic medium which we will continue to use.

Now what I want to do is to relax this assumption of losslessness. I mean we want to consider after all propagation of waves in real media, and most real media will actually be lossy. So if you take a glass lab for example, and then send light in, and then you measure the power of the light that is coming out and compare it to the power of the light that was put into the glass, then you will see that there is some difference. That difference is actually being observed or is the result of absorption of electromagnetic wave, some part of electromagnetic wave by the material itself.

What it does is to slightly change the temperature of the material but we will not worry about that one. For us, whatever that has gone in is not the same as that is coming out of he medium, then it means that we are considering a lossy medium. Of course, we could also have the output of the particular medium being having the power which is greater than the input power. In that case, we would be considering what is called as active medium or gain medium. We will not consider the gain medium in this course. We will stick with lossy medium.

Now I've already given you a hint as to how to approach modeling of a lossy medium. Lossy medium can be modeled by letting σ be non-zero and having some finite value of that. Now the notable change that would happen in Maxwell's equation for such a lossy medium is this curl expression of the Maxwell-Faraday expression wherein the previously neglected term, which is j is now brought back into it, so which is given by σe plus assuming still that we are working with phasers, I can write the phaser expression as jωε E.

This comes from another observation timing, perhaps a different course that conduction current density is actually proportional to the electric field and this proportional to constant is called as the conductivity of the medium, and this conductivity carries units of segment per meter. The old notation was most per meter. Now this is the preferred SI notation of siemens per meter. And of course, it makes sense, because j is current density, which would be measured in ampere per meter square, and this fellow is siemens per meter and there's voltage, v per meter, so v into s is like voltage times conductivity, which is basically current ampere, so ampere per meter square is what you're going to get.

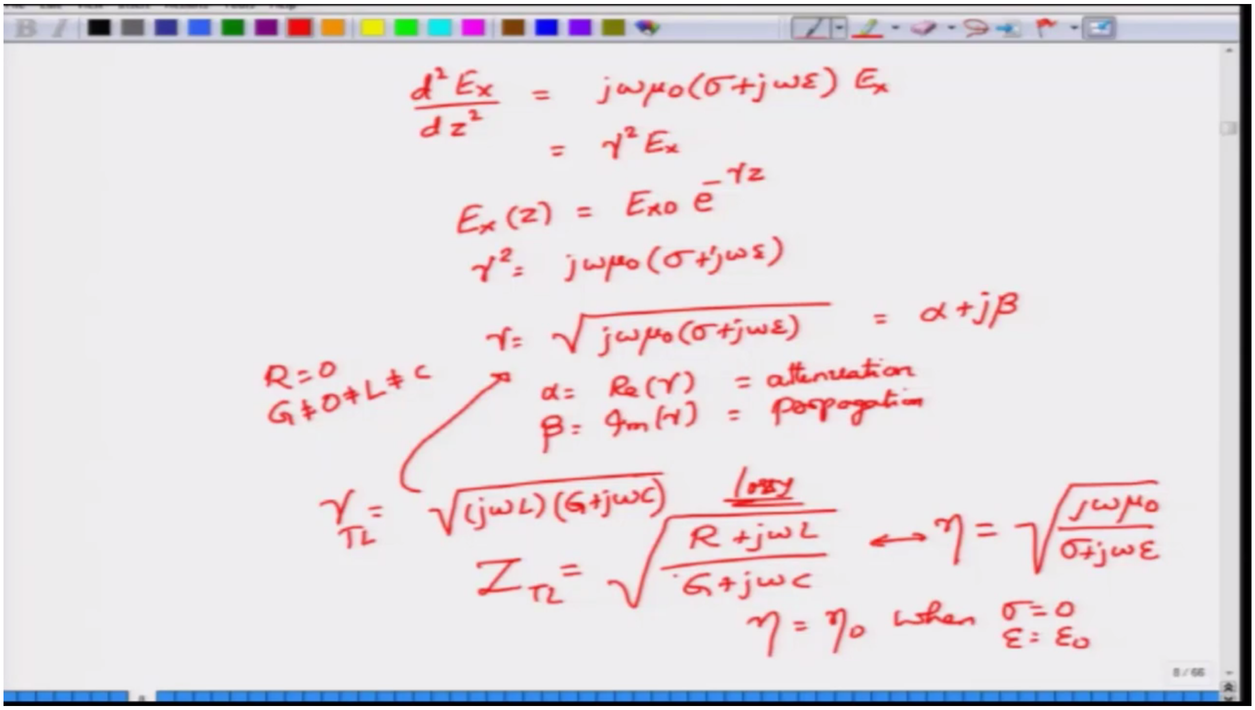
So this is the only chain that would happen in your Maxwell's equation. The other equations are as they are. So you can even simplify this expression that I think this σ plus jωε E and I can write this as $j\omega\varepsilon_c$ E, where you can see that ε_c is a parameter that has simply introduced and this parameter should actually be equal to some sort of -- that would actually be complex and that should be related to σ and ε. How should it be written. So we know that $j\omega\varepsilon_c$ should be equal to σ plus jωε, these two equations has to remain the same, and then I know that ε_c is a complex permittivity that has introduced, which I can write as ε' -jε". So I now have jωε' - jε" should be equal to σ + jωε, where ε is your original ε that we had considered not the complex permittivity that we have considered. So I can write this as jωε + jω -- sorry j is not there, because that has gone now -- so ωε". This should be equal to σ + jωε. So the complex part can be equated to this one saying that ε, which we had taken, is actually the real part of the complex permittivity ε' and then whatever that is left out, ωε' is equal to σ. Alternatively, ε" is equal to σ/ω.

So I can write the equation without changing anything, by simply introducing this complex permittivity ε_c and working with the entire thing, identifying that the original permittivity of the lossless media can be taken as the real part of the complex permittivity. So this is something that I can do. In fact, you would see ε' and ε' being used in many electromagnetic materials that we considered.

So what we have seen is this ε" can be written as σ/ω and this can be included to talk about the lossy medium. Let us now see what happens to the wave function when we include this lossy materials or lossy medium. So I have Δ x E = -jωμ_0 H as before, so there's no change in this one, and now I will take the curl of this electric field again. So I will get curl of electric field. Sorry, I should take the curl here. So this would be curl of the electric field that should be -jωμ_0, and then I will end up with Δ x H which as I have seen can be written as σ + jωε E or equivalently I can write this as jωε_c E.

I'll go with the former, because I want to bring in the transmission line analogy as well. So I am going to use the full expression for complex permittivity. So I will write this as σ + jωε only, and in this case ε will be a real quantity. Σ will of course also be real. The left hand side we have already seen can be reduced down to -Δ^2 E. the presence of conduction current density does not alter this equation, Δ.D = 0. This equation will still remain the same and consequently Δ.E will also be equal to 0, and in this expression of the vector Laplacian, you can see that when you expand this out, that Δ(Δ.E) term will go to 0. So I am having on the left hand side a -Δ2.E, which of course would be onto the right hand side given by -jωμ_0 (σ + jωε) E.

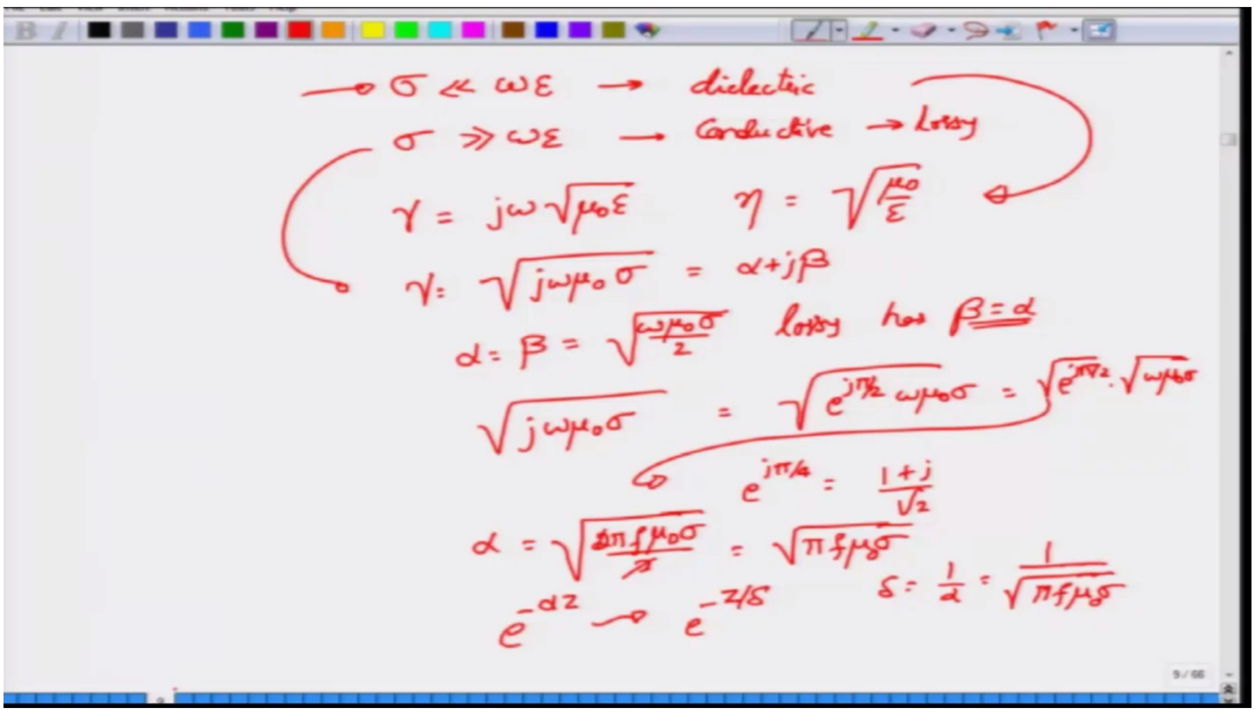
Minus sign can be cancelled off and also considering the fact that we're not looking at waves in arbitrary direction, we're looking at waves which are propagating along the z direction, and x polarized waves is what we have been considered, x polarized linear waves that we have considered. I can replace this Δ^2 simply by $\Delta^2/\Delta z^2$, which is what I am actually interested in.



When I do that, I'll have Δ2 Ex/Δz2 to be equal to $j\omega\mu_0$ (σ + jωε) Ex. So this would be the expression that I would have. Of course, instead of Δ2/Δz2, because we are dealing with a phaser. I can simply write it as d2 Ex/dz2. I can now write this entire thing as say γ2 Ex and my solutions with respect to z, the phaser solutions with respect to z will be some amplitude, which we will call as some Ex0, and because we are considering z direction wave, so we can write this as e-γz, where γ2 is equal to $j\omega\mu_0$ (σ + jωε), or γ itself I equal to √$j\omega\mu_0$ σ + jωε, which can be further written as α + jβ, and α is the real part of γ, which is called as attenuation constant or attenuation coefficient, and β is the imaginary part of γ, which is called as the propagation constant or the propagation coefficient.

This has given by √$j\omega\mu_0$ (σ + jωε) that γ. Now if you recall the transmission line where we have taken, say, let's assume that we'll take R=0, we'll take g not equal to 0, we will take l not equal to 0 and not equal to 0, c also not equal to 0, then this γ expression in the transmission line case would actually be given by (jωL) (G+jωC). Is that correct? Yes, because I've taken only R=0, these, I am neglecting the series resistance of the wires. So I am assuming that the wires actually have some amount of or the material that I have considered is lossy material, which is modeled by having this conductance G there. So this is what your γ in the transmission line expression would look like, and now you can see that it would essentially be similar to the expression that we have written in a lossy medium.

Here also the transmission line was lossy because of the presence of this G, and clearly because G is kind of similar to σ, this corresponding medium is now lossy as a result of this. So the expressions that we have written for γ is essentially similar to the expression that we have written or used in the transmission line case as well. Now you may also find out that the impedance that we are going to write, earlier the characteristic impedance of a transmission line when it was lossy was given as R + jωL/G + jωc under √. So by taking this analogy between the transmission line and the lossy medium that we are considering the characteristic impedance or the wave impedance or the intrinsic impedance of the medium when the medium is lossy would be correspondingly given by $j\omega\mu_0/\sigma + j\omega\varepsilon$. This expression equal η_0, that is eta = η_0 when σ = 0 and ε is actually equal to ε_0. So this expression that we have written, the medium impedance or the wave impedance or the intrinsic impedance, this would be equal to the -- it would be similar to the expression for a lossless or lossy transmission line. So this is the propagation of wave in a general loss media.



There are some situations where the σ is very, very small compared to ωε. In this case, the medium is mostly a dielectric medium. However, at different frequencies it may not remain a dielectric material. When σ is much greater than ωε, then you have a pure conductive medium which is very, very lossy. That is the reason why most metals actually are very lossy at very high frequencies. So basically their σ value becomes very high ωε. Of course, you can bring them to be kind of a dielectric, but you have to actually increase the value of ω to very large extent there.

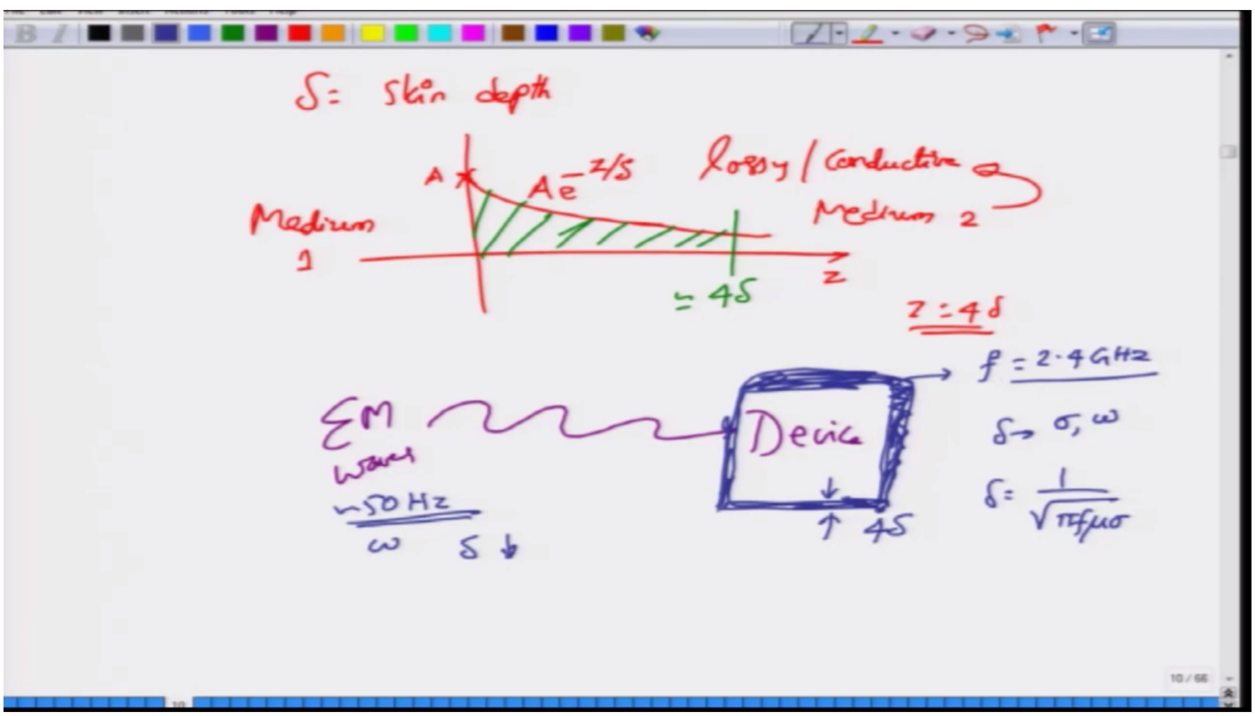
So it's also interesting to look at what would be the expression for γ when it is pure dielectric. When it dielectric, we know that γ is given by pure jω $\sqrt{\mu_0\varepsilon}$,

$\varepsilon = \varepsilon_0$ will give you the propagation constant in free space, ε - $\varepsilon_0 \varepsilon_r$ will give propagation in a dielectric medium where this condition holds. So this σ is so small that I can neglect that one. The corresponding impedance is also going to be quite real. It would be $\sqrt{\mu_0/\varepsilon}$. This is for the dielectric scenario.

On the other hand, for the conductive scenario, γ will be equal to $\sqrt{j\omega\mu_0\sigma}$, because $j\omega\varepsilon$ is very small compared to this one. So γ will be equal to $\sqrt{j\omega\mu_0\sigma}$, which of course will be equal to $\alpha + j\beta$, correct. Any complex number is equal to real and imaginary party. So what is α here? In fact, the magnitude of α and β both are equal, and they are given by $\sqrt{\omega\mu_0\sigma/2}$. Why is this? Because I can write this $\sqrt{j\omega\mu_0\sigma}$ as $\sqrt{e^{-j\pi/2}}\ \omega\mu_0\sigma$. This can be written as $\sqrt{e^{-j\pi/2}}\ \sqrt{\omega\mu_0\sigma}$, and the $\sqrt{e^{-j\pi/2}}$ is basically $e^{-i\pi/4}$, which is actually $1 + j/\sqrt{2}$. So now you can see that α and β both should be equal to $\sqrt{\omega\mu_0\sigma/2}$. So this actually means that the medium is lossy, but also has a propagation constant, lossy, but has a propagation constant β whose magnitude is actually equal to α. So this is a very interesting, this one.

In fact, this α which is given by $\sqrt{\omega\mu_0\sigma}$ and writing ω as $2\pi f$ and substituting ω as $2\pi f$ and canceling this 2 will give $\sqrt{\pi f\mu_0\sigma}$.

The expression that you have $e^{-\alpha z}$ as the z propagating wave or the attenuation along z can be re-written as $e^{-z/\Delta}$ where Δ is equal to $1/\alpha$, which is equal to $1/\sqrt{\pi f\mu\sigma}$ or $\mu_0\sigma$ for non-magnetic medium.



This Δ is a very important parameter called skin depth. This is called a skin depth meaning that if we launch an electromagnetic wave into a completely lossy or conductive medium, with an initial amplitude of A, then the amplitude decays exponentially as A $e^{-z/\Delta}$, this is a z axis, so this is the medium, let's say, which is medium 1 have sent in the wave medium 2. We will have to say more about what happens when medium 1 and medium 2 interact, but for now, we will simply assume that the wave that has been launched into the second medium, which is basically lossy and conductive actually starts off with an amplitude to A at z=0, but as you have seen here, the amplitude decays exponentially and over phase z equal to some four Δ or so, the amplitude would have actually decayed almost to 0. The amplitude would be just about 1% of what it started out with.

So that is why this Δ measure the depth over which the wave can actually, the typical width over which there is important distance of the depth over which the wave can go into the lossy medium can penetrate the lossy medium is about 4Δ. So Δ is called as skin depth, and it actually is important not only to know what is the depth, but actually is important in practical scenario when you're designing shields.

Suppose I have a source of electromagnetic waves and I want to prevent. So I have some device let's say, so let's say, this device is motherboard and these EM waves are being generated externally for whatever reasons, maybe lightning or whatever other reasons, or it could be that EM waves are generated by the motherboard and this device is, let's say, television or some other kind of a device that I am looking at, and I want to shield the electromagnetic waves between these two. The EM waves which are going to impinge on the device, if the device is not shielded would actually simply go and fall onto or incident onto the device.

However, if I build a metal, I build a metal shazi or I kind of build a metal ground around this one of thickness which is approximately 4 Δ, the thickness of this metal would be approximately 4 Δ, and please remember this Δ is now dependent on σ as well as the frequency. So if the EM waves are coming in at 50 hertz, then frequency ω is very small, Δ is actually going to be very small as well. However -- sorry, Δ is inversely proportional to this. So Δ we wrote it as $1/\sqrt{\pi f \mu \sigma}$ and at low frequencies the skin depth will be larger, meaning that the waves can penetrate more into the metal or the lossy material, whereas when frequency increases, the skin depth reduces.

So if you know that the worst case electromagnetic waves that would be incident on your device would be at 50 hertz, then if you build a metal house or you house this device inside a metal with a depth of, say, 4 Δ as determined by the material conductivity, then that metal or that house is actually sufficient, or the thickness of the device is sufficient to withstand, even when your f is, say, 2.4 gigahertz, which would come from your cell phone.

So this idea of what should be the thickness which is determined by the material conductivity and the frequency of operation, the worst case operation and best case operation. It's not so straightforward as I have written, but this still gives you a basic idea of what to expect when you want to shield certain devices from electromagnetic interference. The concept of skin depth of a material is very important in this case in order to determine the requisite thickness of the material.

So with this, we stop our discussion on uniform lain waves in loss materials. We will have to say more about the interfaces of these waves. So as we have seen here that we start off with a wave in one medium, and then we somehow said that the medium will carry the wave, but exactly what happens at the interface and whether this -- I can just take the electromagnetic wave from one medium and then convert or push the electromagnetic wave into the second medium without any changes at the boundary is something that we have to explore, and we're going to do that in the next module. Thank you very much.

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