INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

NPTEL

NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING

COURSE TITLE
ELECTROMAGNETIC WAVESIN GUIDED AND WIRELESS

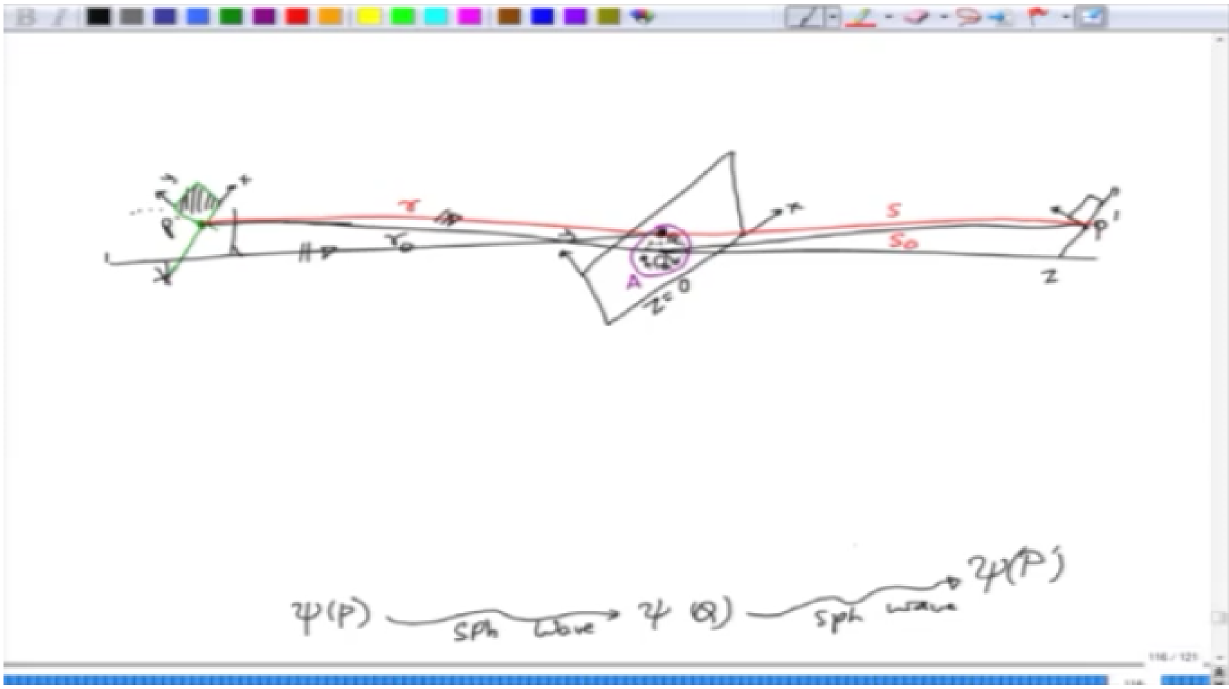
LECTURE-38
Diffraction-II

BY
DR. K PRADEEP KUMAR
DEPARTMENT OF ELECTRICAL ENGINEERING
IIT KANPUR

Hello and welcome to NPTEL MOOC on Electromagnetic Wavesin Guided and Wireless Media. We are looking at the phenomena of diffraction since the last module, because we have discussed the importance of diffraction already, right.

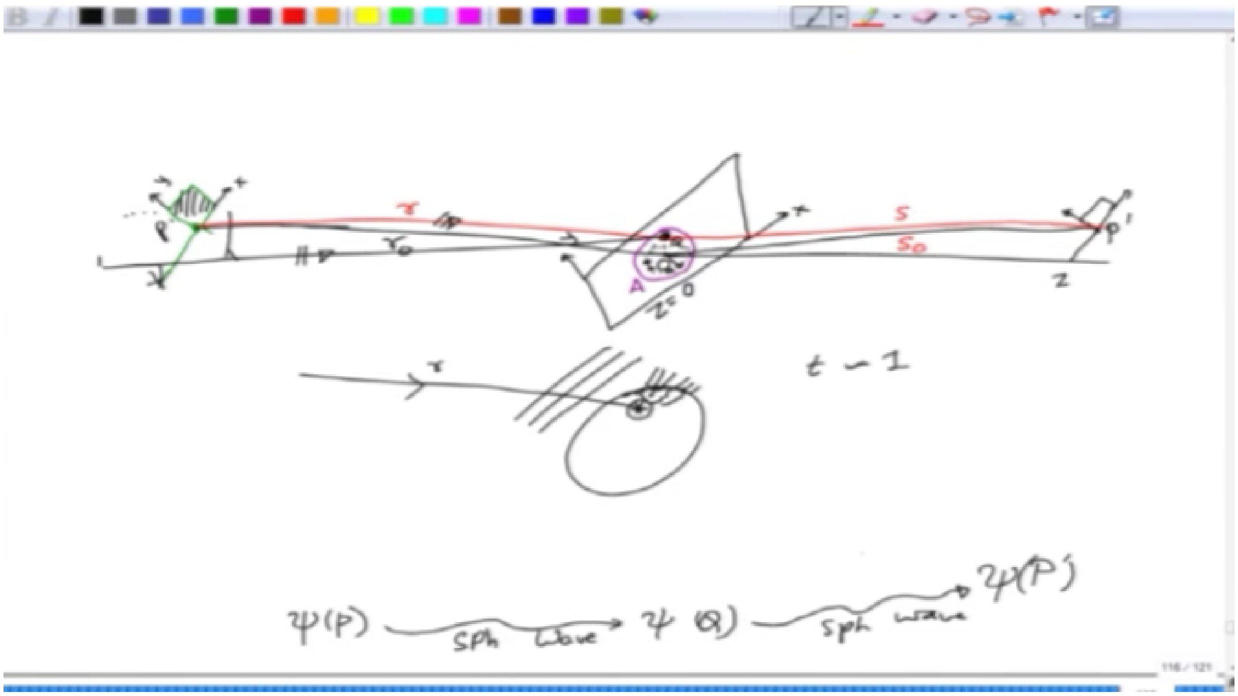
So at the end of the module we were making certain assumptions in order to understand the mathematical expressions which will soon see that there are not that difficult, of course the process of arriving at them is very difficult, so we've not attempted to do that one, but the basic idea is kind of you know intuitively understandable that the field that is you know generated from the spherical wave at the source point, point P or one of the parts of the source would propagate all the way to the point Q on the aperture, and at point Q you would then again have a spherical wave which would propagate to sai of P prime, okay, and the field from P not only goes to Q, it goes to all other points on the aperture, right, so it goes to all other points of aperture and each aperture will have a small area around it and then you will have to look at the field strength at different points on the aperture which would all be propagating at to the point P, and at that point P you have to sum up all the contributions from the different parts of the aperture, okay, that is what Huygens principle is, and that is what mathematically we are going to just write them, write it down.

We have already have made one assumption in the sense that the distances R, R0, S, S0 or very, very far away from the aperture and we call this as the Fraunhofer Diffraction or the Fraunhofer limit of diffraction, okay.
(Refer Slide Time: 02:00)



The second assumption that we make, okay, is that the field that you are going to get on point Q on the aperture because of the point P on the source side would actually be, that would just be that, in the sense that I have an aperture, the aperture could actually modify the field that is coming in, right, so the field is coming far away from this distance R, almost incident in the form of a plane wave and this field could actually be modified here, why would it be modified? Because this aperture is surrounded by the opaque material, so this are this materials and the edge effects may actually transmits some of the light that is falling on the edge on to that and modify the field pattern in the aperture plane itself, okay, this is usually considered by what is called as the transmittance function of the aperture, what we are going to do is that we'll assume the transmittance function which would be dependent on lambda, which would be dependent on the precise location of the aperture point that we are considering that is point Q on the aperture that we are considering, all those things we will assume that this transmission function is actually equal to 1, okay,

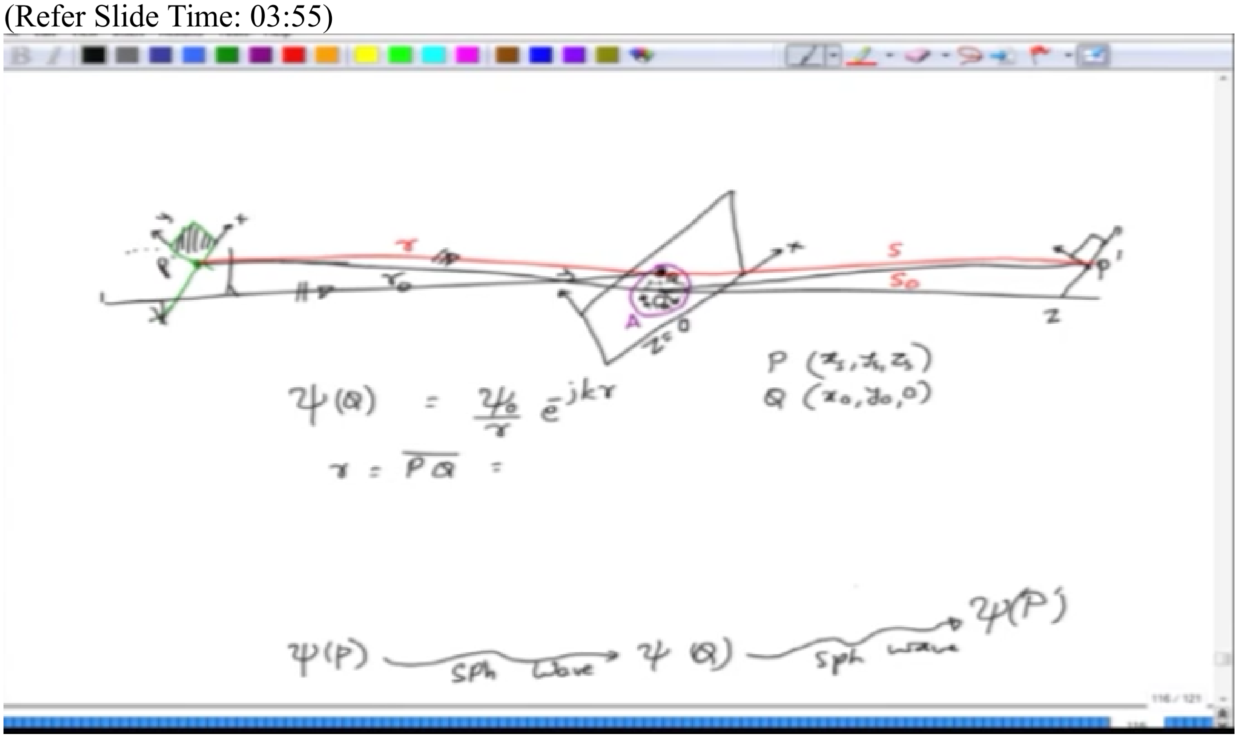
(Refer Slide Time: 03:00)



this is an assumption that we are making but it’s a very good assumption to make, because it simplifies our lives in many ways.

So following this basic idea, first what would be the field at point Q due to point P, that is source at point P, this would be some sai 0 divided by R, E to the power –JKR where R is the distance between source point P and the aperture point Q, correct, this is the spherical wave which we already have seen earlier.

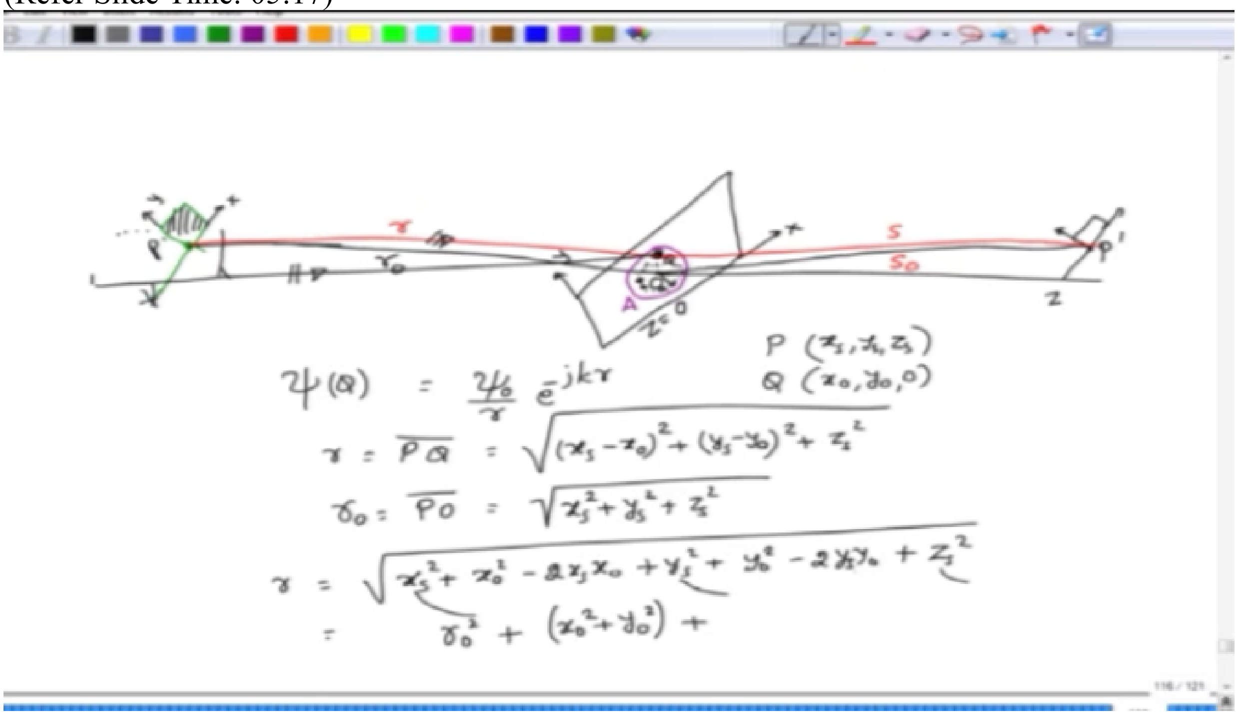
(Refer Slide Time: 03:55)



Now what is R? We know that R is basically the distance PQ and we know that the coordinates of P and Q are given by, P coordinate is basically XS, YS and ZS, right, and the Q coordinates are X0, Y0 and 0 because we have assume that this is located at Z = 0 plane, then these are the coordinates that we actually have, right, so what would be the distance PQ, remember I'm only interested in the distance, right, this would be XS – X0 whole square + YS – Y0 whole square + ZS square under root, correct.

Now what is the distance R0 itself? R0 is a distance from the point P to the origin, origin of course is given by 0, 0, 0, so the distance from P to 0 is given by square root of XS square, YS square + ZS square, right, now we can expand this distance PQ, okay, we can expand this distance PQ or this value R, and then write this as XS square + you know you will have X0 square – 2XS, X naught + YS square + Y0 square – 2YS Y naught + ZS square under root, clearly utilizing the fact that R0 is given by this particular expression, what will I have? XS square I can you know take this, I can take this YS square and ZS square and call this entire thing as R0 square, then I have X0 square + Y0 square, then I have you know X0 square + Y0 square is kind of given by the radius, so if it was a completely spherical aperture then X0 square + Y0 square

(Refer Slide Time: 05:17)

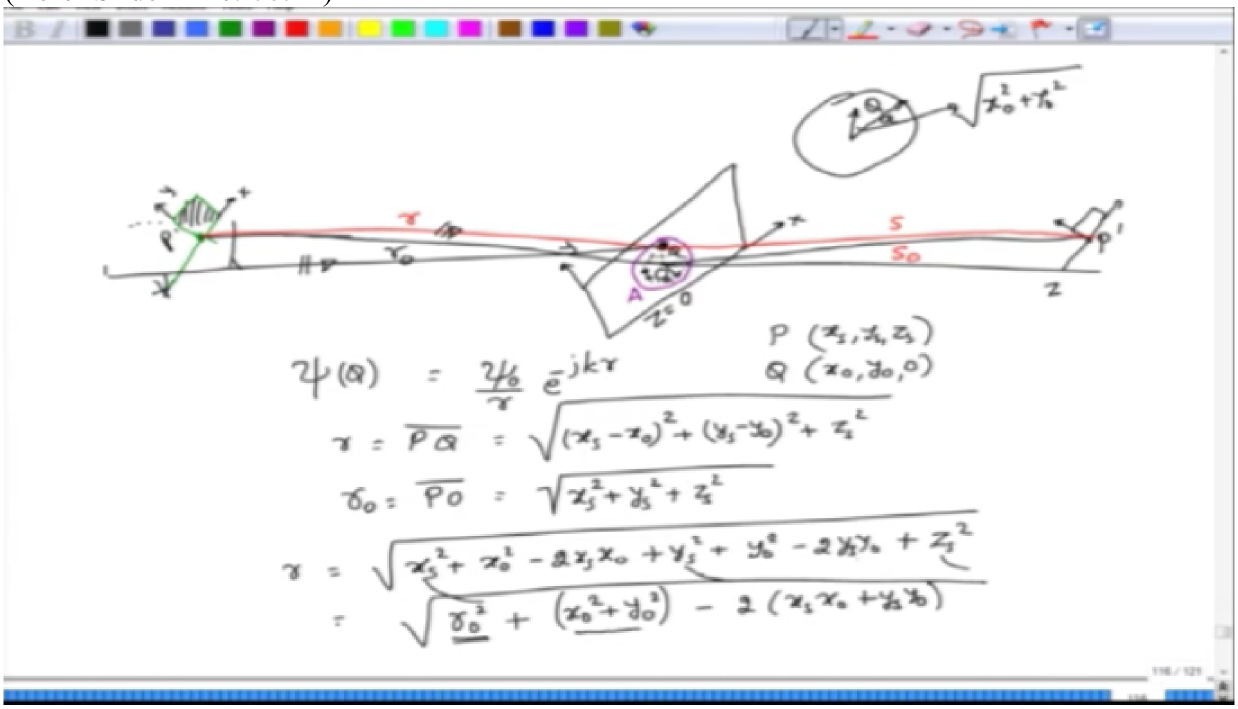


would essentially be telling us the area of a circle, right, so that put our, the radius of the circle, and as we change X0, Y0 go to other points on the aperture eventually this is going to cover the aperture area, right, so this is X0 + Y0 square.

In other words we can also think of the point Q and from the origin to the point P, and this distance can be given as X0 square + Y0 square under root, because Q is given by the coordinates X0, Y0 and 0, so that's all that this particular part is, and of course the length of this arrow can become equal to the radius of the aperture assuming the spherical aperture when the point Q lies on the edge of the aperture, right, so that's all this particular thing is.

Then you have inner product of this XS and X0, so you have -2, XS X naught + YS Y naught, okay, so this is what you have entire thing under root. Now we are going to make some assumptions, we are going to assume that this R0 square is much, much larger than any of these two terms,

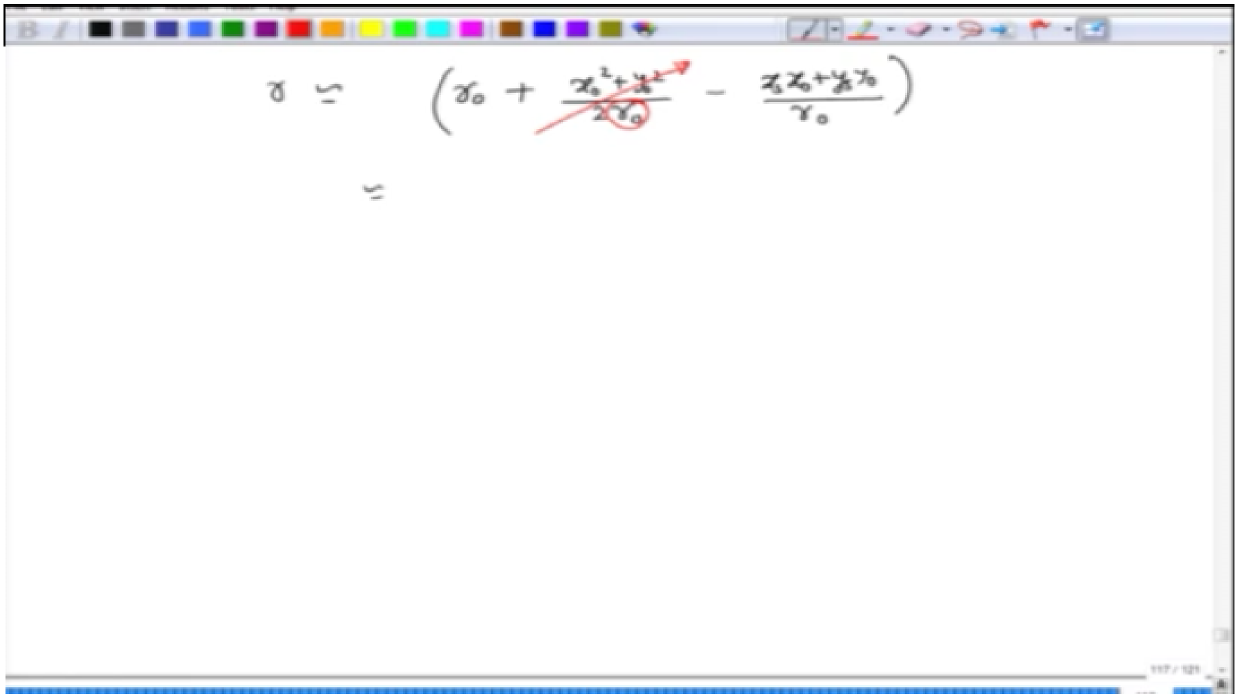
(Refer Slide Time: 06:21)



we are justified in this assumption, we have already said this, we are working in the Fraunhofer Diffraction limit, in the Fraunhofer Diffraction limit the distance from the source to the aperture and from aperture to this one itself is very, very large compared to the dimensions of the aperture, okay, so because of that I can take this R0 out of this square root and then I will get, this two terms to be divided by R0 and then I'm going to you know just write this out as approximately R0 into 1 + X0 square + Y0 square divided by 2R0 square – XS X naught + YS Y naught divided by, so the 2 will cancel so you can show this as a simple exercise, you are going to get this as R0 square or taking R0 into inside the integral or sorry inside the brackets I can remove one R0 down here, and then what I will get is this term, okay, so I will have R0 + X0 square + Y0 square by 2R0 and this quantity.

So further in the Fraunhofer limit I can neglect this, okay, why could I neglect this? Well, this is the Fraunhofer limit and R0 which is the distance PO will be very, very large compare to X0 square + Y0 square,

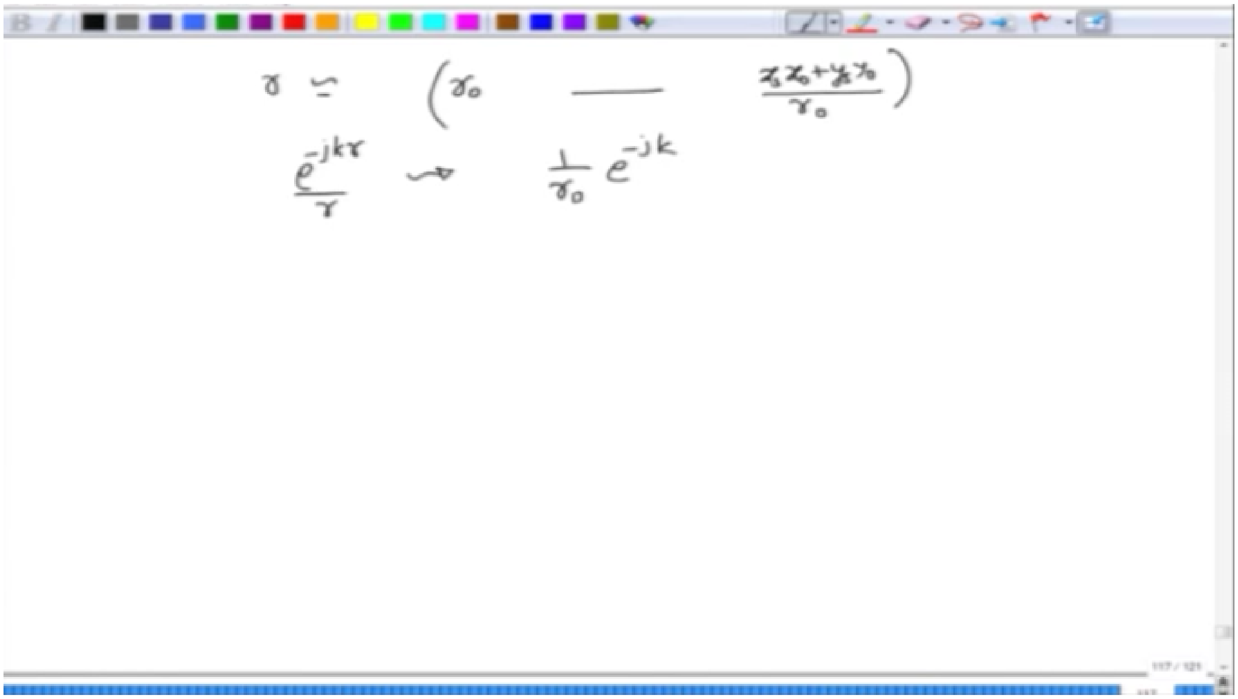
(Refer Slide Time: 07:41)



so now with that I can remove this term, but we will see that this term needs to be included later on at that point we will include, okay, for now we are going to neglect this because we are working in the Fraunhofer Diffraction limit, okay, so this is what we are going to get in terms of R, so in the denominator for E power –JKR/R what I can do is to further approximate the denominator as 1 over R naught, because you see you know again making this similar approximation that we had made in antenna analysis as well, right, there also we had an E power –JKR, and in the denominator we had an R, there we made this kind of an assumption, okay, this assumption is sometimes called as paraxial assumption, so especially in the context of diffraction and that is what we are actually making now.

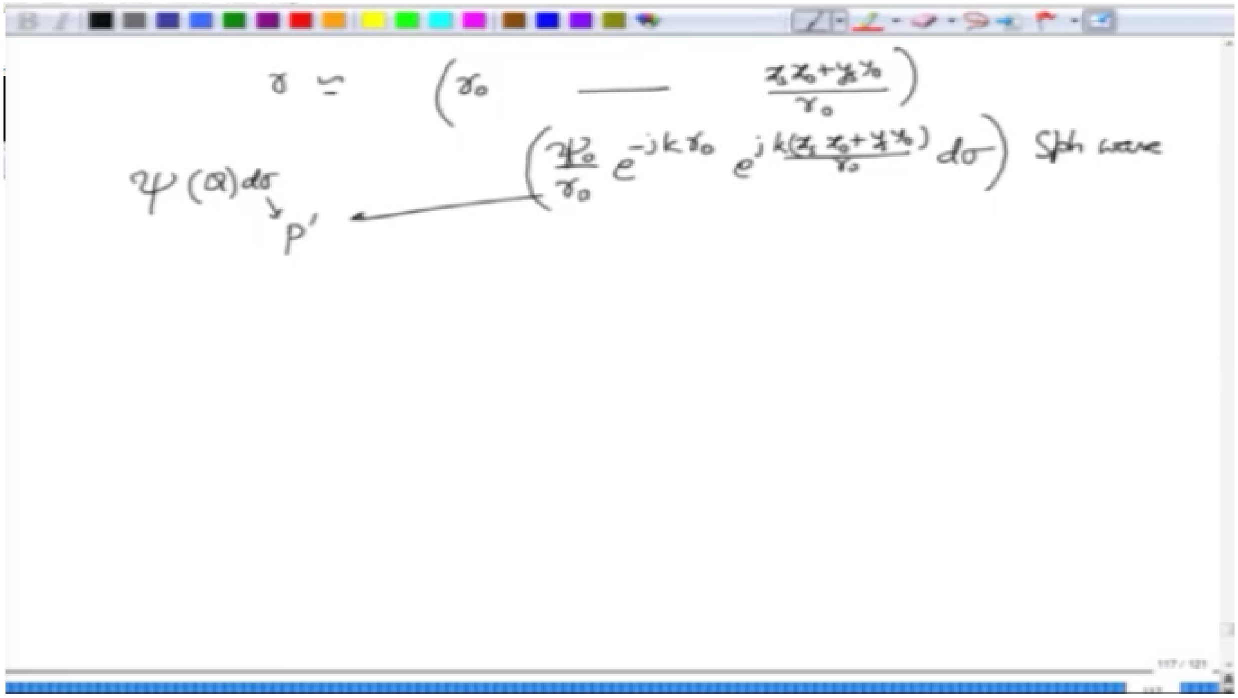
We assume that this is 1/R naught in the denominator, and in the numerator we will write down this approximation, okay, so I can't make in a numerator the approximation to be equal to R = R naught,

(Refer Slide Time: 08:35)



I can make that approximation in the denominator because of small you know deviation about R0 would not be too much change in the amplitude of this spherical wave, okay, so now in the numerator I will have E power –JKR0, then I will have, this is a minus sign, okay, so I will have E power JK, XS, X0 + YS Y0 divided by R0, okay, so please note this particular expression, this is if you multiply this entire thing by sai 0, this expression would be the you know the field light that is incident at, from the point P and now incident at point Q, okay, and when you multiply this one by D sigma, this would correspond to the contribution of the wave or the aperture point Q with the cross section D sigma on to the observation plane P or P prime, okay, so please note that, so sai of Q times D sigma is my contribution on to the field point P prime, okay, so that is what this particular thing is.

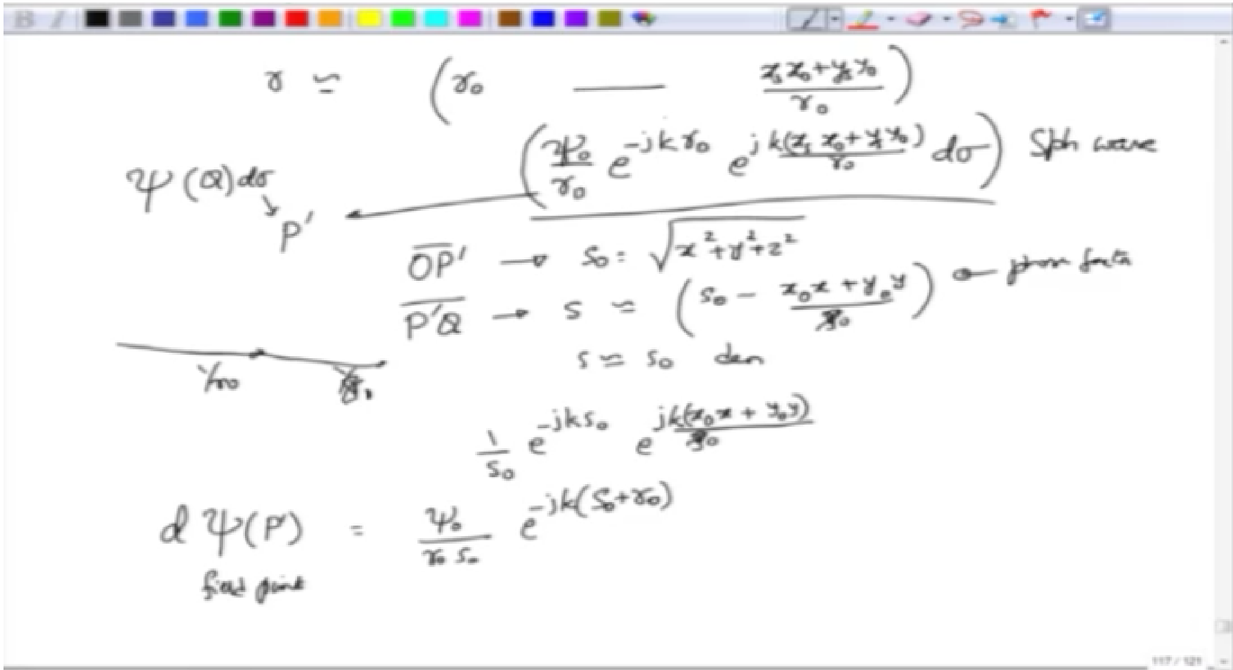
However this is the amplitude of the field that is being incident, this further needs to be multiplied by another spherical wave, right, because this amplitude would now be the input amplitude kind of a thing which would then begin to propagate as a spherical wave, all the way to the field point at P prime,
(Refer Slide Time: 10:05)



P prime is located at the coordinates X, Y and Z.

So similarly to the assumption that we have made you can show that the distance OP prime can be written as in the denominator as it can be written as 1/S0 or rather in fact OP prime is not that one, OP prime is basically S0 which is given by X square + Y square + Z square under root, and utilizing this X square + Y square under Z square defined as S0, you can then show that P prime to Q or rather Q prime to P distance which we have denoted as S can be approximated as S naught -, okay, this would be X0X + Y0Y divided by, so I think yeah so this is divided by R0, sorry not R0 this would be S0, right, because I'm actually looking at this one, so I can make this approximation, I can do this based on whatever we have already discussed in the denominator we are going to make the approximation that S will be approximately S naught in the denominator and in the numerator or in the phase factor expression we are going to retain this approximation, okay, this same paraxial approximation, so with this as the amplitude the next part of the spherical wave would be given by E power –JKS naught, right, I'm just following the same ideas that I have already written down, so there will be 1/S0 E power –JKS naught you can observe that this terms are essentially the same as we have written earlier, times there will be this additional phase factor which would be E power JKX0X/S0 and + Y0Y, this entire thing divided by S naught, right, so this would be present, so the field differential field that you are going to get, differential field that you are going to get at the point P prime which is the field point, okay, would be given by sai 0 divided by R0 S0 which of course would tell you the overall amplitude clearly because you had propagated a distance up to 1 over R0, and then another distance of 1 over S0, yeah you would expect that this would actually be like 1/R0 + S0 , but it turns out that because of this contributions that you are going to get, slightly different you will have to start the spherical wave reach Q and from Q you have to count the next part of the spherical wave, right, so that's why you have sai 0/R0 into S0, not R0 + S0, okay, so this part is anyway understood.

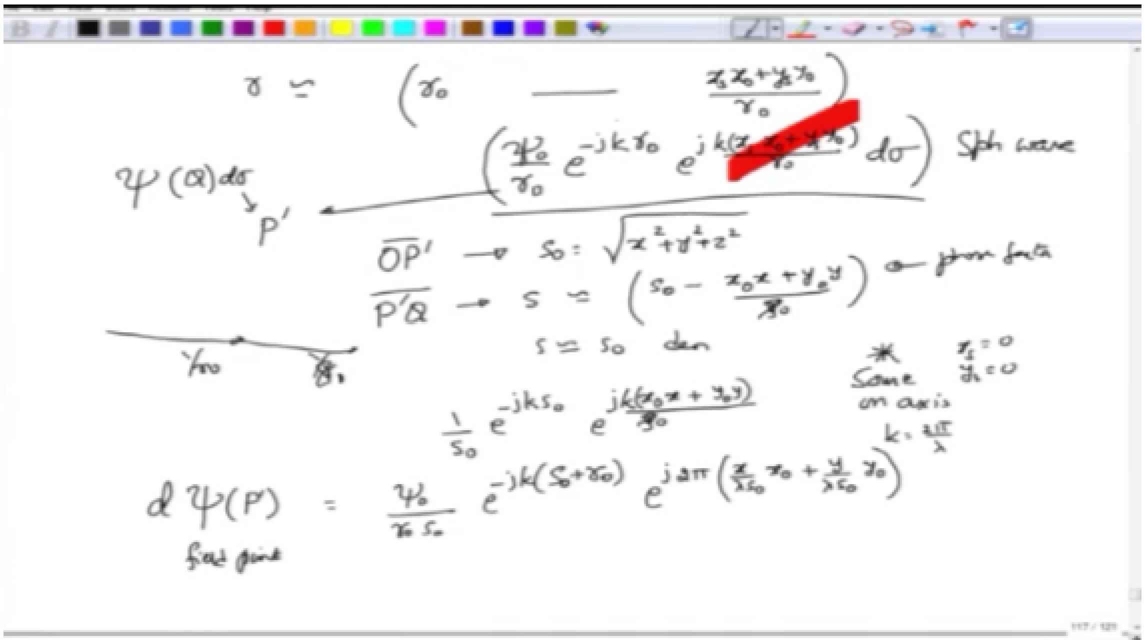
(Refer Slide Time: 12:55)



The overall phase that you are going to see would be E power –JK S0+R0, this is now understandable, because as you move a distance of R0 to begin with you would have change the phase by E power –JK0 or rather KR naught, and then moving an additional distance of S0 would have changed the phase to KS0 and therefore the sum of the phase would be the one that would actually determine the overall phase, so this is E power –JK S0 + S0 and then you have these two phase factors to go with.

Now you can return to the case that we considered earlier that is we will assume that the source is located on the axis, okay, so for a non-axis source XS will be equal to 0, similarly YS = 0, because the source is now located at the point ZS, right, so on the axis, so when you make this assumption, this point source assumption further then you can drop the phase term that is appearing here, so I can drop this phase term and whatever the phase term that I would be left, with only be this and knowing that K is given by 2 pi/lambda, I'm going to write down the phase term slightly differently, I'm going to write this as 2 pi X/lambda S0 times X naught + Y/lambda S0/Y naught, please note the variables that I have grouped here,

(Refer Slide Time: 14:08)



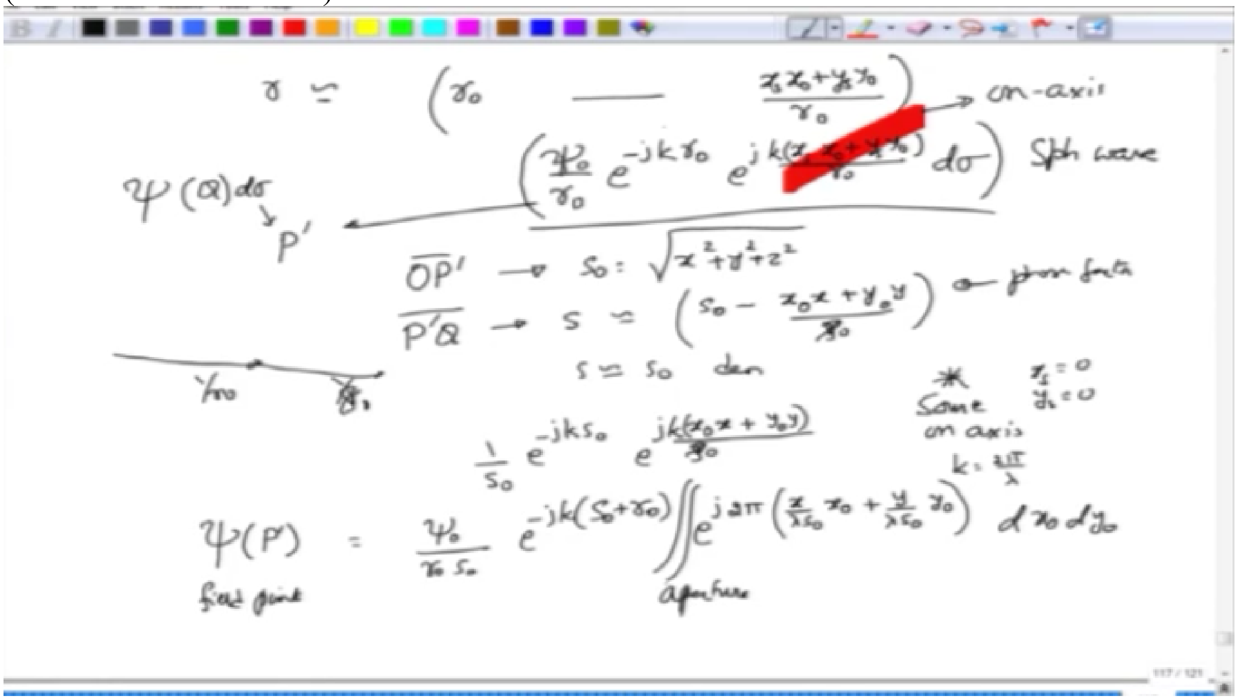
I’ve now fixed my field point to point P prime, of course sorry this is the part that we have written right, yeah, so this is the differential amount, I need to also multiply this one by the cross sectional area D sigma that we were considering at the aperture point, so this is the differential field that I’m going to get, and what I have now seen is that there is an overall phase factor acceptable, there is an overall attenuation factor which is 1/R0 S0 which is also acceptable, and the initial amplitude sai 0 and there is an extra phase shift that we have obtained which I have put in here, this other phase shift that I have neglected will actually be, we need to include that phase factor when we go to what is called as Fresnel Diffraction Limit, okay.

So far it’s not necessary that we need to include, not this term the quadratic term, this term which we have neglected now is because of the on axis field, so if your field is not on axis then you have to include this particular term, okay, and as I have said we have nowhere indicated the orientations, so we have assume that all the obliqueness is actually not there, everything is kind of collinear which of course we realize is not true, but this assumption makes our life little easier in terms of getting the you know expressions, I’m going to use this, of course this is just the differential point at P prime, the contribution from point Q, what if I start moving Q around the aperture, every point should contribute to the field that P prime, right, so when you do that, you know that instead of considering the field at that point as a differential point, you simply have to integrate over the aperture distribution, okay, please note the aperture is actually described according to the coordinates X0 and Y0, okay, so not X and Y, X and Y are not your variables, they are kind of fixed here, X0 Y0 is the variable that is moving and you have to simply integrate this one over D sigma, D sigma itself is given by DX0, DY0, okay, this expression that we have, what you can observe that is that this S0 and R0 are kind of independent of X0 and Y0 right, the distances that P prime Q as well as the distance PQ are independent of DX0, DY0.

Similarly if you assume that sai 0 is a constant and this R0, X0 is also independent, then you can actually moves these outside the integrals, okay, so you can move this outside the integrals and then integrate this factor over the aperture distribution, okay.

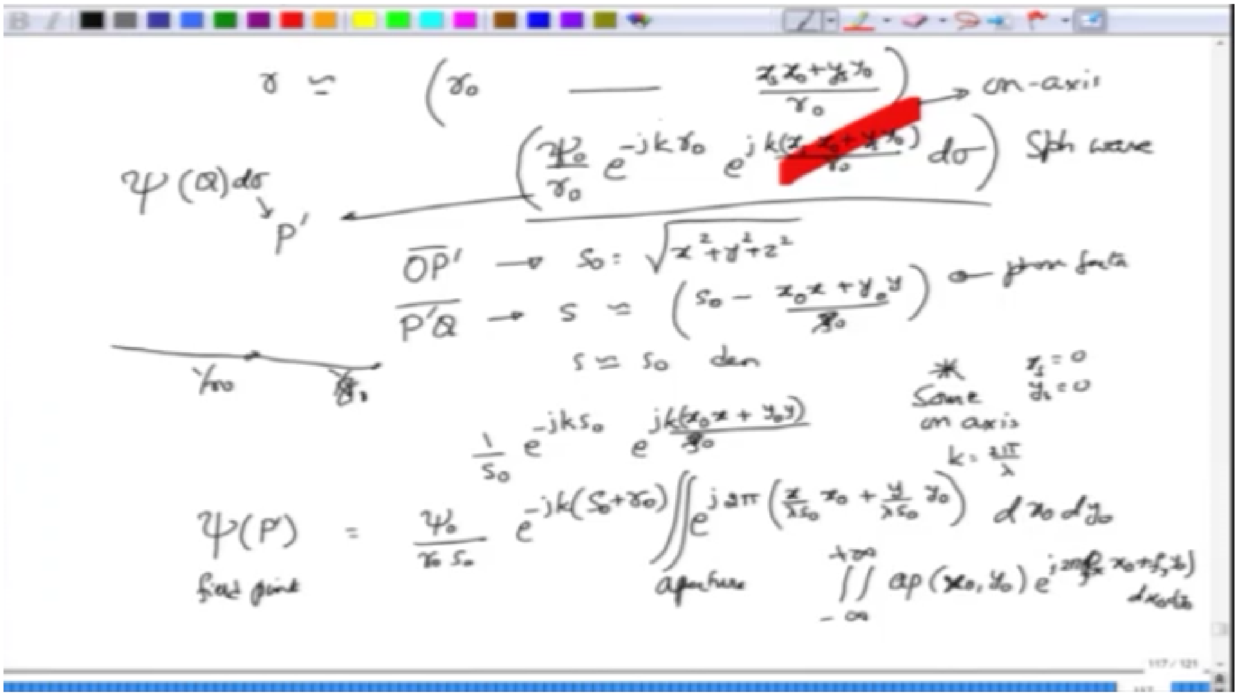
So whatever the aperture shape that you have is being integrated, if you are not happy with this integration over the aperture,

(Refer Slide Time: 16:40)



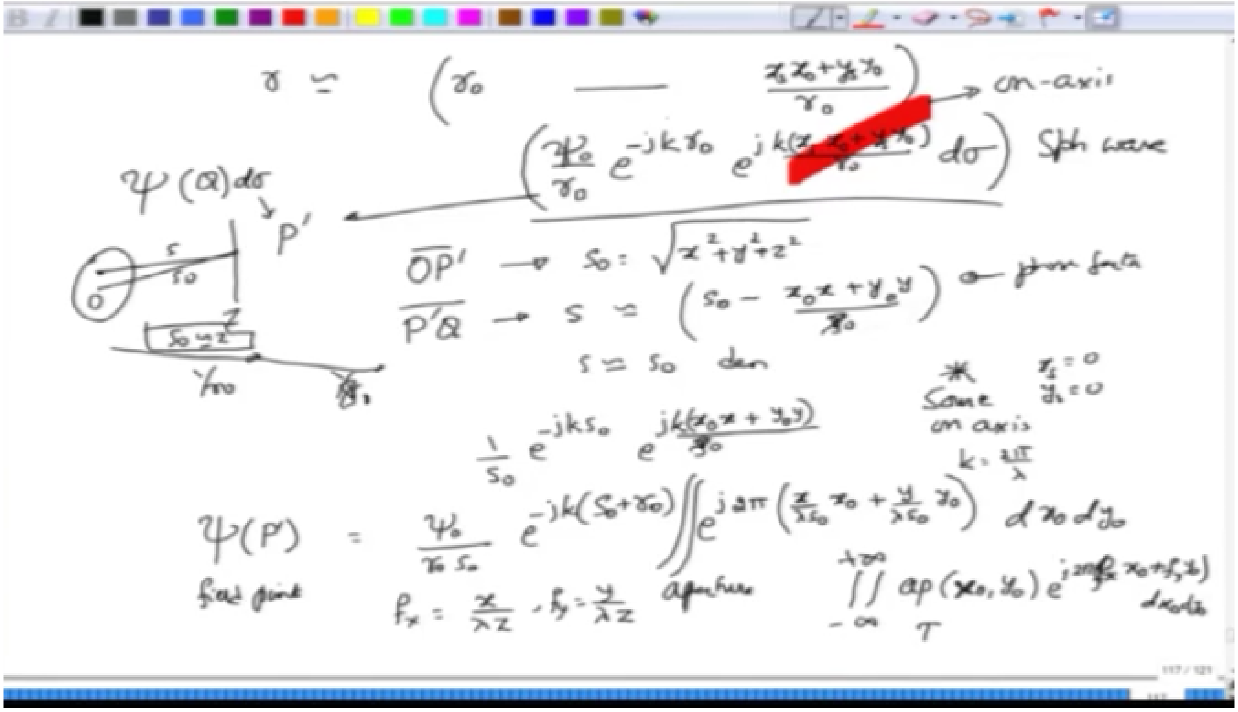
you can make the integration go to –infinity to +infinity, and then you call this as aperture function of X0 and Y0, right, so if you call this as aperture function of X0 Y0 that would tell you how the aperture itself is distributed and the integration could be –infinity to +infinity which is also okay, I mean we can also do this and then you have this E power J2 pi, I'm going to write as KX or rather FX, X naught + FY Y naught, okay, and then I have DX naught and DY naught,

(Refer Slide Time: 17:17)



for those who have seen Fourier series and Fourier transforms you can clearly see that this is the Fourier synthesis equation, right, given the aperture as a function of X0 and Y0, if you take the Fourier transform of this aperture function, what you are going to get would be the complex field that is the field that you are going to get at that particular point P, okay, and FX and FY are clearly given by, FX is given by X/lambda S naught, and FY is given by Y/lambda S naught, one final assumption that we are going to make is that this distance on the aperture that you had, right, so this was the origin of the aperture, this was S naught, and this was S, and this aperture was located in the plane Z, right, so what we are going to assume is that this aperture itself is so small that S0 can be approximated to the value of Z itself, okay, so I can remove this S naught which would give me the distance from the origin, but rather I can just put it in terms of the plane distance itself which would be X/lambda Z and Y/lambda Z, so clearly this is a wave, I mean this is similar to a frequency except that this is called as a wave number, okay, so this is the Fourier transform or

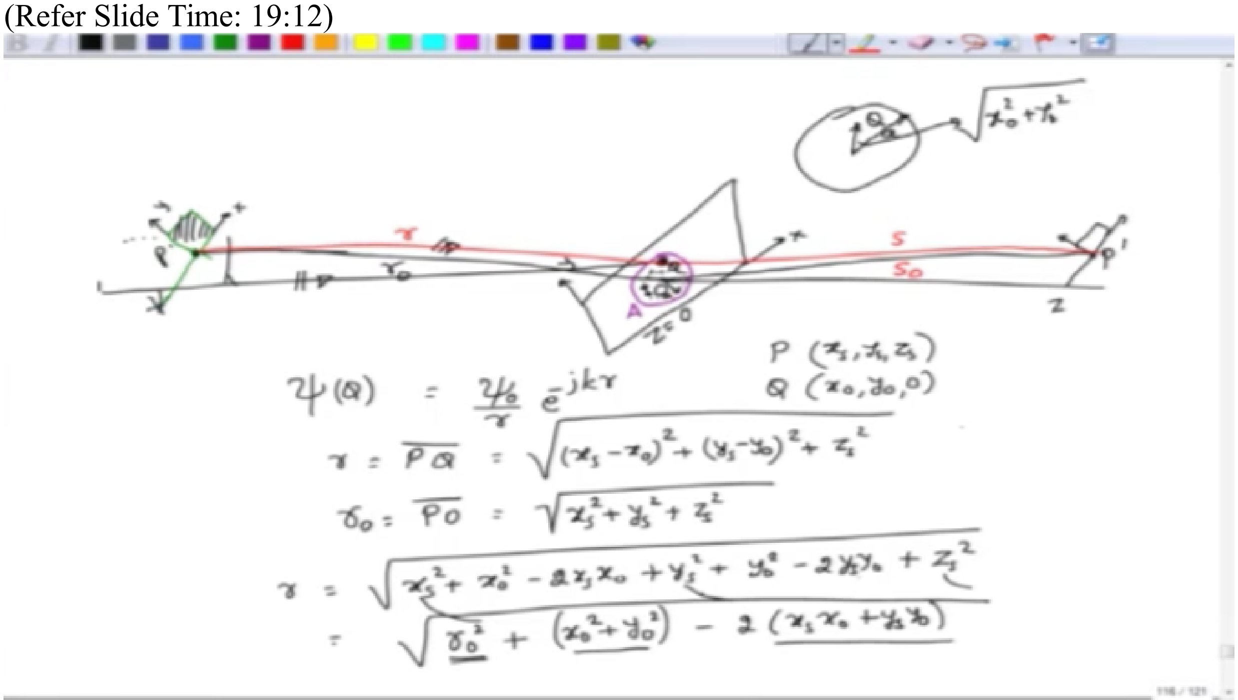
(Refer Slide Time: 18:34)



rather if you are that you know not happy with the sine then this is the inverse Fourier transform of the aperture X0, Y0, so in order to calculate the field at the point P.

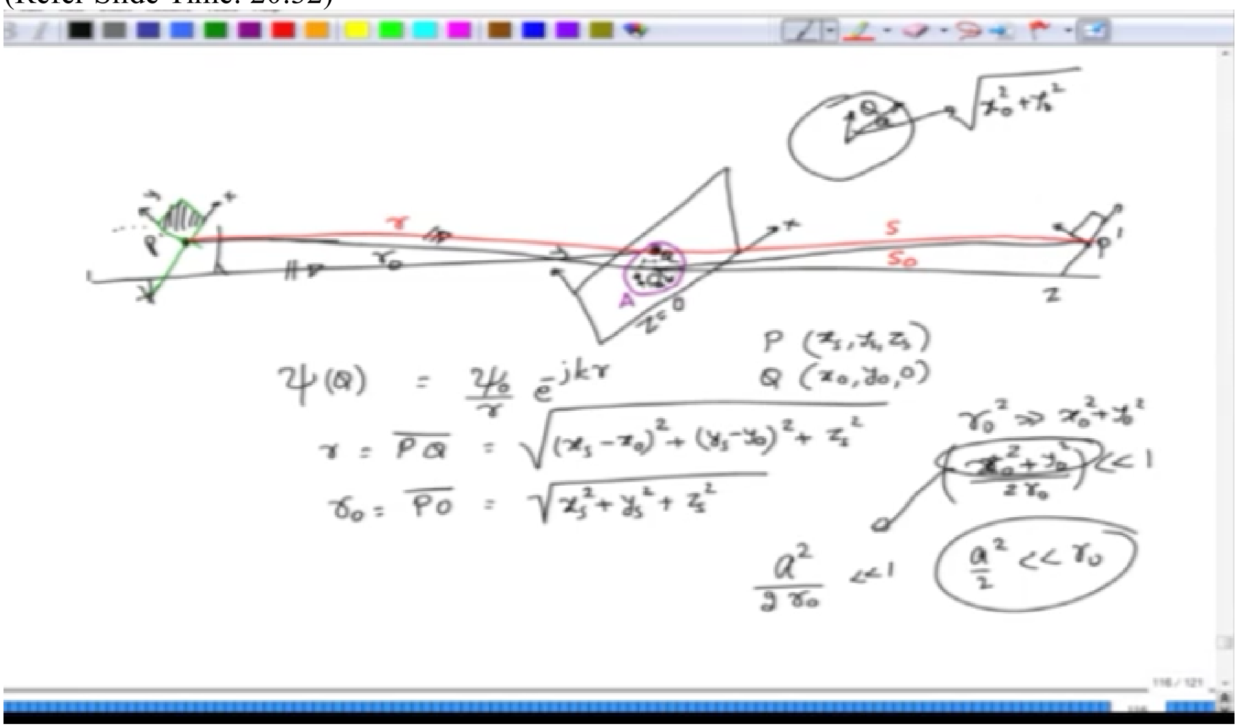
if you neglect all this constant phase factors and if you also assume that this thing is very well known, this 1/R0 S0, then what you have is the inverse Fourier transform of the aperture, okay, this gives us a nice starting point for understanding diffraction from different apertures, before we go to that we can slightly more rigorously consider, what have been neglected and what is the assumption that we have made? We have made this assumption, right,

(Refer Slide Time: 19:12)



so we said R0 square is much larger than X0 square + Y0 square, actually when you take the binomial theorem then what we would have said is that, so the term that you would have obtained and neglected is X0 square + Y0 square divided by 2R0 square, right, so this was neglected in comparison with R0 itself which we had taken out, so or you know this would be neglected with respect to 1, because this is the one that is going to give us a phase, right, so we neglected this component, we neglected this after multiplying by R, so we said this is 2R0, but then after multiplying by R0 which was actually coming in from that other side, we had to take out this R0 on both sides, so this is the part that we said, and this one was you know consider to be very, very small compared to 1, equivalently what we have here is that if you think of this X0 square + phi 0 square, what's the maximum value of this numerator term? The maximum value of this numerator term would be the distance that you are going to have, right, so that would be some A square, where A is the radius of the aperture, right, so what you have is A square divided by 2R0 to be much less than 1, so A square/2 is much less than R0, so this is the condition that we actually had, okay.

(Refer Slide Time: 20:32)



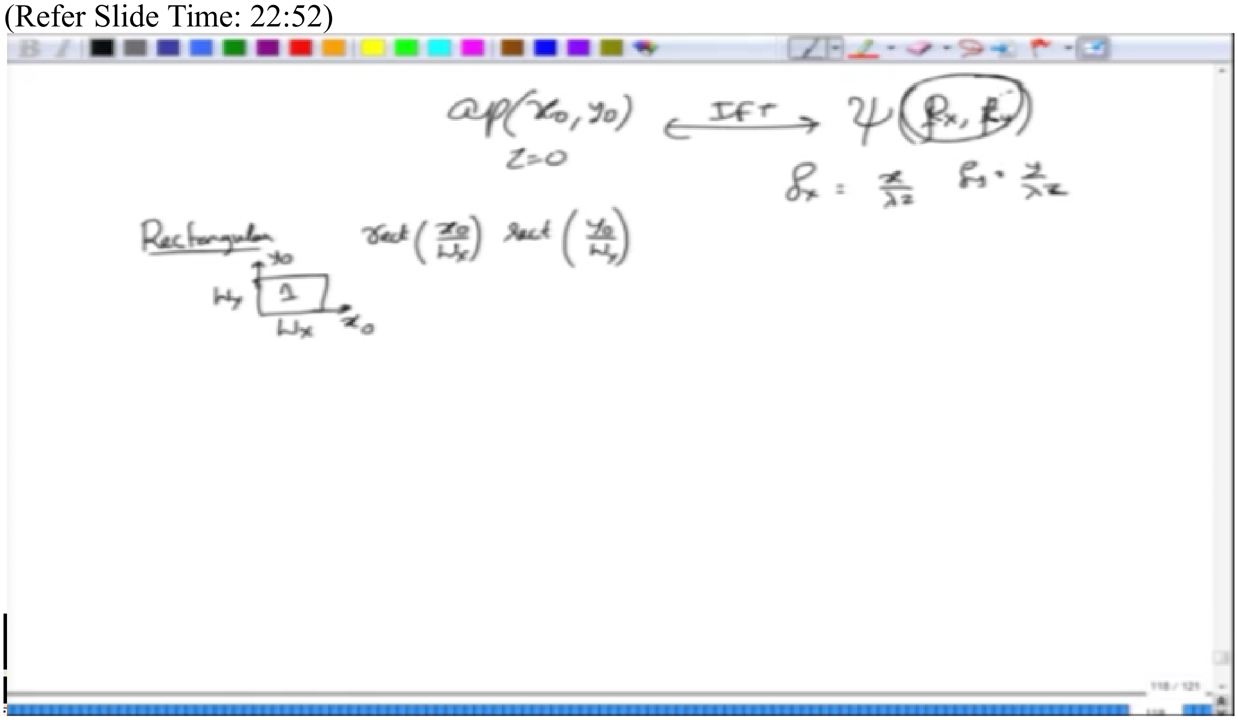
Of course there is a fact of lambda involved, what I will do is I will give you the correct expression, the correct expression is pi A square/lambda is consider to be much smaller than the, so instead of R0 we have to of course consider the S0, because R0 is usually consider to be so far away that we assume this is a plane wave, right, so this is pi A square/lambda should be much less than S0, as you can actually obtained from the expression for S0, okay, so I will develop this one more clearly in the exercise, and then I'll also tell you what the implications of not having to satisfy this equation.

For now we know that if I've given aperture function in the form of X, aperture at X0, Y0, located at the plane Z = 0, the aperture is located there, if you take the inverse Fourier transform then what you are going to get will be a function phi KX and KY, of course or rather phi FX and FY, of course FX = X/lambda Z, FY = Y/lambda Z, so you are actually getting this in terms of the X and Y coordinates itself, okay, so this is the inverse Fourier transform expression.

Now let us look at two important cases, okay, first I'm going to consider a rectangular aperture, okay, so I will assume that the aperture has a width of WX, along the X direction, and WY along the Y direction, so this is Y, this is X, so this is like you know taking a cardboard and cutting the rectangle out of it, the rectangle is defined by the dimensions WX and WY along the X and Y directions, and we want to understand what is the inverse Fourier transform of this.

Now you might not have seen the, you know two dimensional Fourier transforms, but in this particular case you can actually split the two dimensional Fourier transforms into two products of two single sided Fourier transforms, one acting along X0, the other one acting along Y0, okay, or rather X and Y, so this aperture function that we have, which we have defined to have an amplitude of 1 here and 0 everywhere else can be expressed mathematically as rectangular function of X0/WX multiplied by rectangular function of Y0/WY and only in this partition coordinate system in this particular example you can split the two dimensional Fourier transform into product of two, one dimensional Fourier transforms,

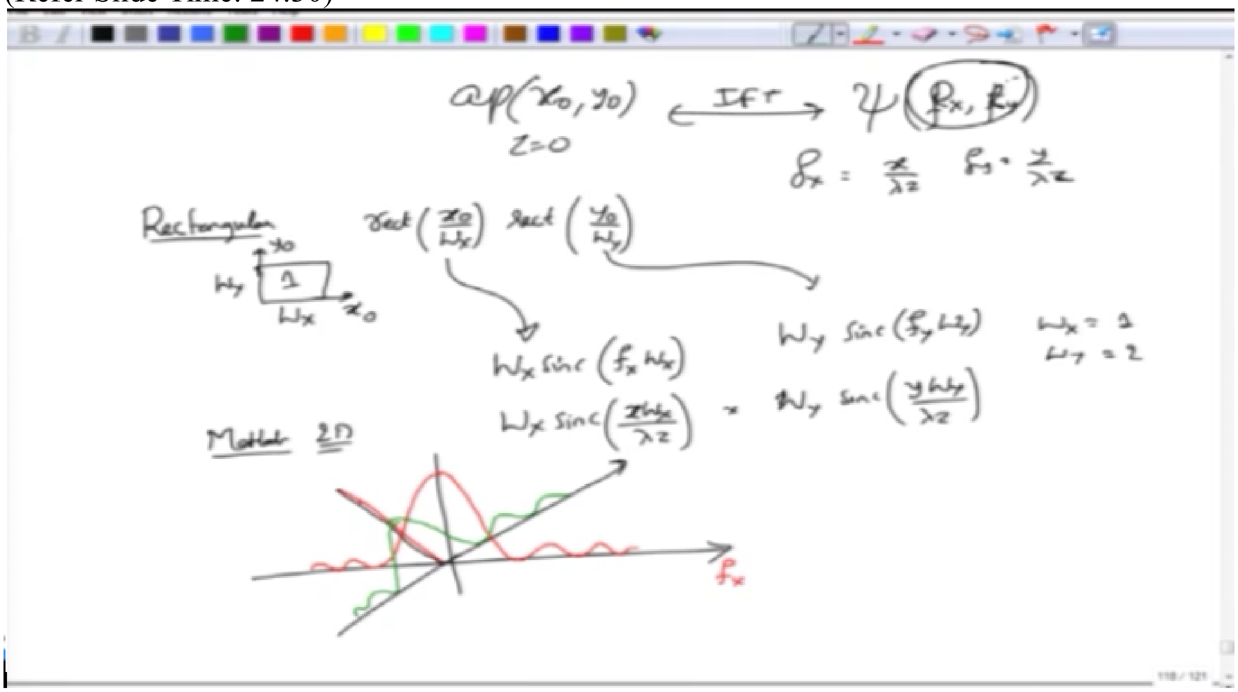
(Refer Slide Time: 22:52)



and knowing very well from your classes on no signal systems or Fourier transforms, this rectangular function in one of the variables will actually turn out to be a sinc pulse in the complementary variable, right, so the expression would be WX sinc of FX WX, right, and you know what is FX, FX is X/lambda this thing, so therefore for the X part this would be WX sinc of X WX/lambda Z, okay, so this is the expression that you are going to get.

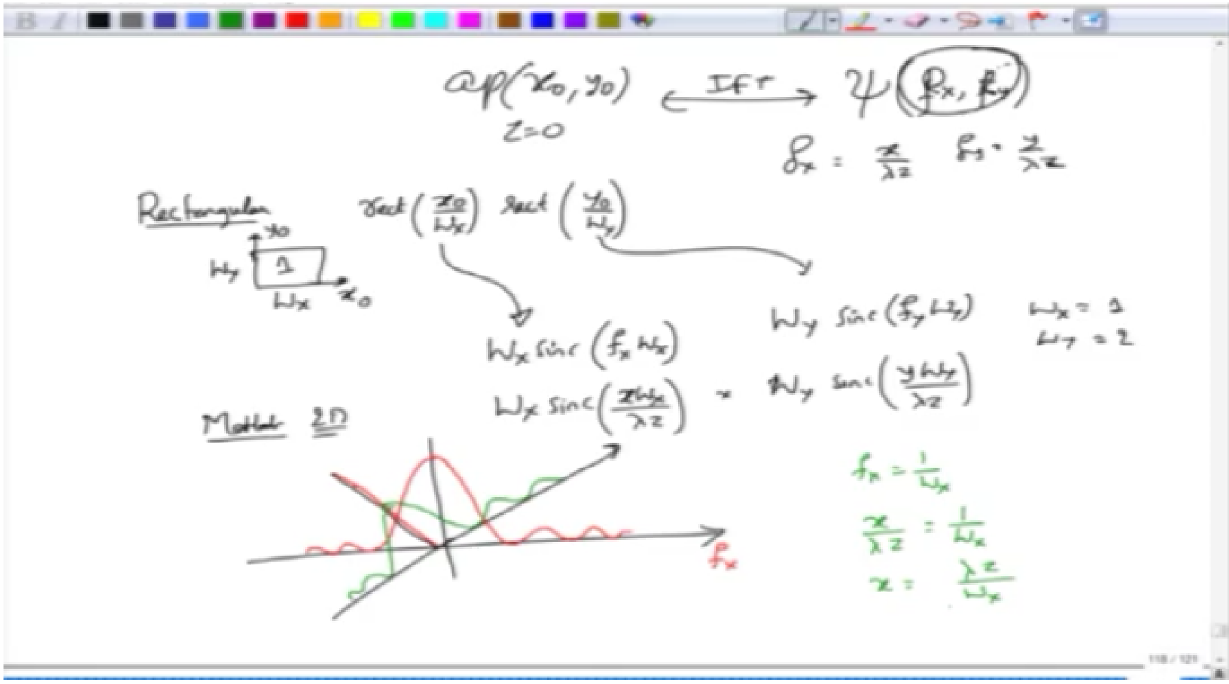
Similarly the rectangular function along the Y direction would give you WY sinc of FY WY, okay, and this way it can be written as WY sinc of YWY divided by lambda Z, okay, so this is all in terms of FX and FY being replaced in terms of X and Y, so overall will be the product of these two, okay, and if you sketch what you are going to get in terms of X and Y, actually you can use Matlab which would, to give you a 2D graph for this, so you just define, so imagine that WX is given by 1, WY is given by 2, and then you can write down the simple Matlab script in order to see what the two dimensional distribution would be, of course one dimensional distribution is very well known to you, right, so this is a sinc pulse distribution, so if you take the argument of the sinc pulse as FX and of course you will also have you know another sinc pulse, okay, which would be along the Y direction, so I'm going to write this one, so it would actually be a 2 dimensional sinc pulse that I have written here.

(Refer Slide Time: 24:30)



For clarity I have shown it in two different access, and where does the minima occur? Remember what we are looking is the sinc pulse and sinc pulse will actually have a minima at FX = 1/WX, right, but FX we have already written it as here, so it would actually have unity at say X/lambda Z = 1/WX or at the spatial location on the field point X = lambda Z/WX, so if you fix the value of Z, you know what distance that you are looking at,

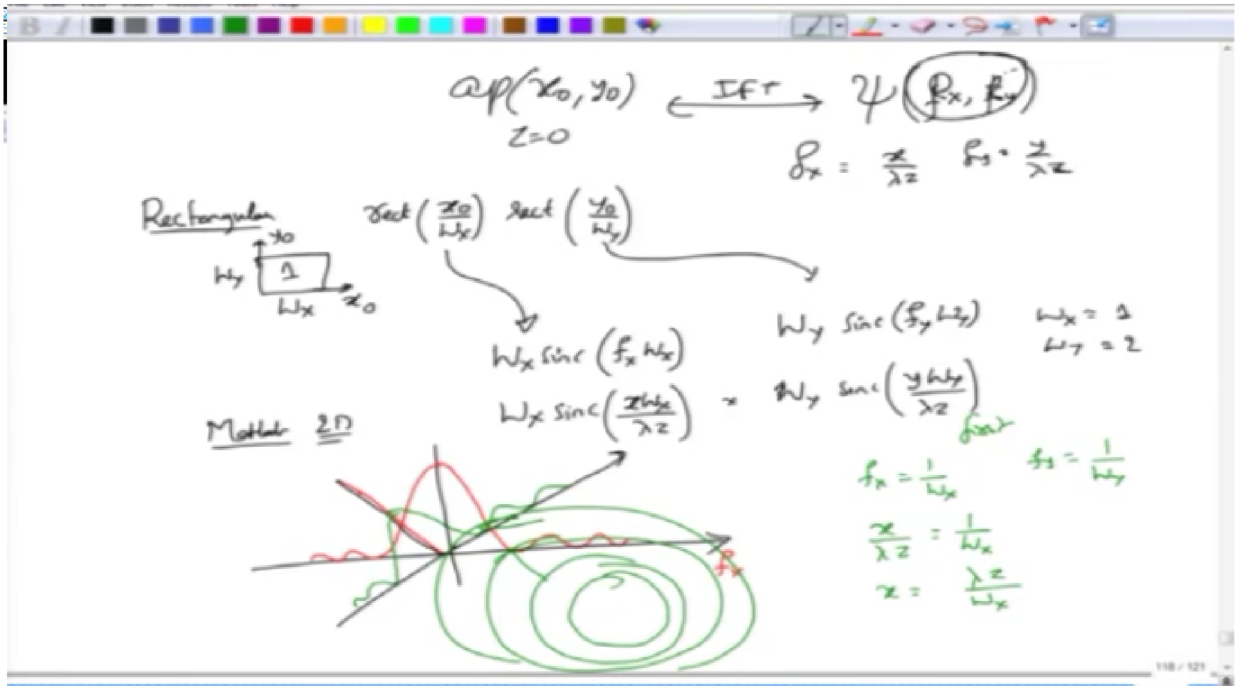
(Refer Slide Time: 24:59)



then that width WX will move the 0 on the X axis, right, so on the X axis where the 0 occurs will be determined by what the width WX is.

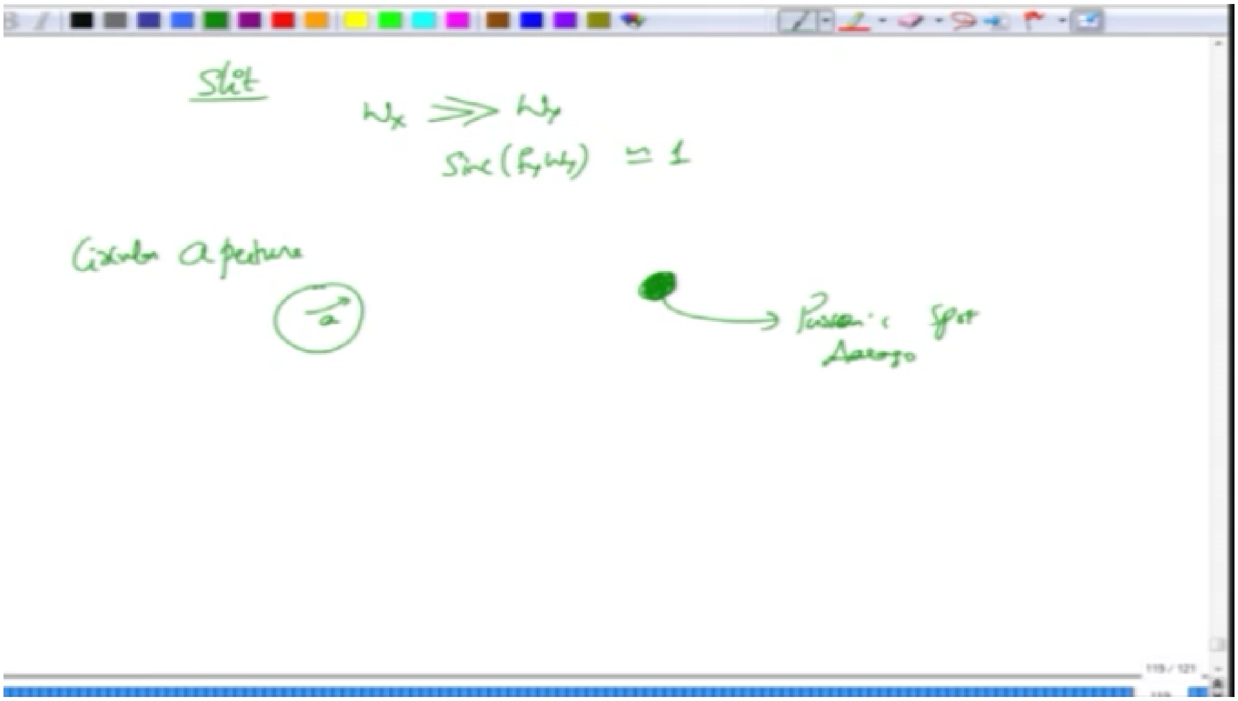
Similarly you will get when FY = 1/WY, this will be the first side lopes, right, so the first points where you are going to get the zeros, of course the integral multiple of this will actually give you higher zeros as well, but as you can clearly see where at point your optical field is going to zero can be controlled by the aperture WX and W0, right, so in the two dimensional thing you will actually see a central bright spot, then there will be these rings, you know, around this, this is what you are going to see in a 2 dimensional plane, if you actually do it nicely.

(Refer Slide Time: 25:46)



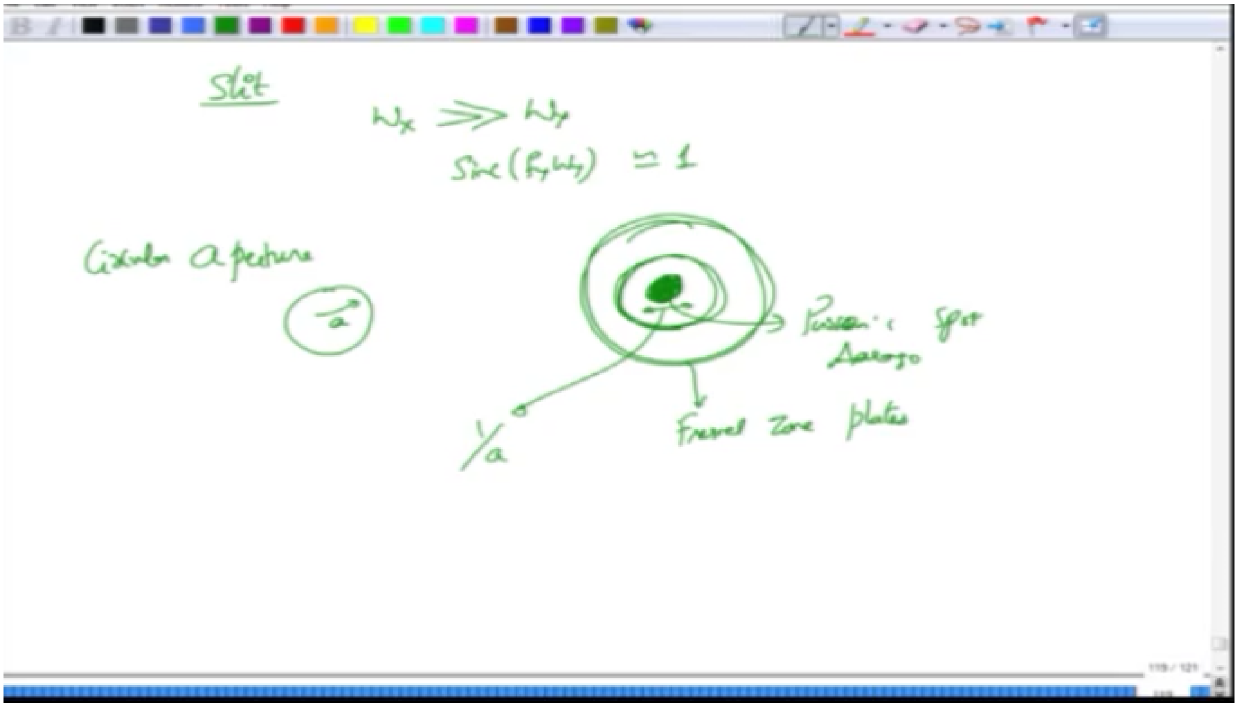
Another thing that you can do is to consider diffraction by a slit, okay, this slit is the one that would correspond to the small aperture thing that I actually talked about, what you can now assume is that if the slit is along X axis then you can take WX to be much larger than WY, okay, so when you do this one, what you're essentially making is the sinc of FY, WY with WY tending almost to 0 can be approximately equal to 1, so then you don't really get any variations in the Y plane, will get variations in the X plane, so if you put up a screen along X you will see minima, slight maxima minima, maxima minima, there is a main loop maxima and then it goes on in that way, right, so this is the sinc pulse, please keep the expression of slit in mind because we are going to use this for another phenomena called as interference, in fact we can describe interference using diffraction of course that's not how historically it was obtained to begin with, historical what was obtained was interference and then diffraction to show the wave nature, but we will look at it as a interesting and amusing you know exercise in the next module, okay.

Finally we mention the circular aperture, and in this circular aperture which would be like you know aperture of size A or the radius A, unfortunately the corresponding you know expressions are not so straightforward, in fact you can show that there will be a central bright spot, this is sometimes called as Poisson spot or sometime, and first discovered or first realized experimentally by a person called as Arago, okay,
(Refer Slide Time: 27:30)



so sometime it is also called as Arago spot, but this also called as not Poisson ratio I'm sorry, this is called as Poisson's spot, okay.

And there is an interesting bit of history behind this which we will not be able to tell you, but you can look up Wikipedia for this, so there will be central bright spot followed by this many rings, okay, in fact this rings are known as Fresnel zone plates, we will meet this Fresnel zone plates later on, and what you can see is that the diameter of this disk, okay, which we will call as airy disk sometimes, so this diameter is roughly, I mean it's inversely proportional to the aperture area, and an exact expression for circular aperture, the field will be Bessel functions,
(Refer Slide Time: 28:00)



and unlike the rectangular aperture we can't separate the 2 dimensional Fourier transforms, I'm taking 2 dimensional Fourier transforms, I'm taking 2 dimensional Fourier transforms in the circular or the fanatical coordinate or the spherical coordinate is kind of difficult, so I'm not going to cover that, but you have to understand that if a point source is present, what you get at the field or the screen point is not just a point, but actually a disk of certain aperture which is about 1.22 WA divided by something like that, okay, so this expressions I will look at, I mean I'll give you in the exercises for you to work on, and you can use Matlab in fact to a great extent to understand this diffraction problems.

Our study of diffraction is not complete yet, because we are going to next consider Fresnel diffraction which will be much more important for us to understand the wave bending nature and what is the effect of that on the wireless channel you know attenuation. Thank you very much.

Acknowledgement

Ministry of Human Resource & Development

Prof. Satyaki Roy

Co-coordinator, IIT Kanpur

NPTEL Team

Sanjay Pal
Bharat Lal

Ashish Singh
Badal Pradhan
Tapobrata Das
K. K. Mishra
Ashutosh Gairola
Puneet Kumar Bajpai
Bhadro Rao
Shikha Gupta
Aradhana Singh Rajawat
Sweta
Nipa Sikdar
Anupam Mishra
Ram Chandra
Manoj Shrivastava
Dilip Tripathi
Padam Shukla
Sharwan K Verma
Sanjay Mishra
Shubham Rawat
Santosh Nayak
Praduyman Singh Chauhan
Mahendra Singh Rawat
Tushar Srivastava
Uzair Siddiqui
Lalty Dutta
Murali Krishnan
Ganesh Rana
Ajay Kanaujia
Ashwani Srivastava
M.L. Benerjee

an IIT Kanpur Production