Basic Electric Circuits
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Module 5:
First Order and Second Order Circuits
Lecture 21:
First Order RC Circuits

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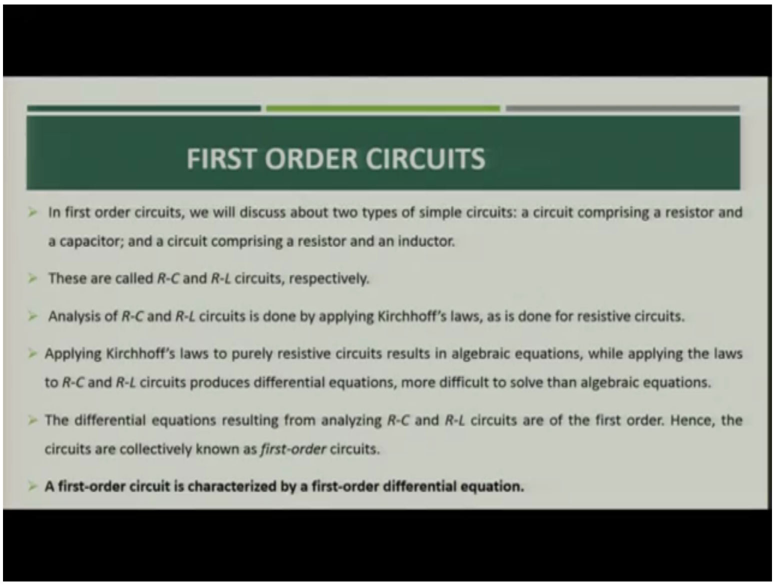

Namashkar. Welcome to the 5^{th} week of our course on basic electrical circuits. In this week basically we will talk about the first order and second order circuits and particularly today we will talk about the first order RC circuits. So, let us see what you mean by first order RC circuits. So, before going into the RC circuit particularly, let us understand what the meaning of first order circuit is. We will discuss about two types of simple circuits which are comprising of a resistor and capacitor and a circuit which is comprising of a resistor and inductor, so these are typically called as RC and RL circuits.

Now, the analysis of this RC and RL circuits, are done by, applying the kirchhoffs law as we did for the resistive circuits, now applying Kirchhoff's law to a purely resistive circuit is simple because it gives an algebraic equation which is easier to solve, now when we apply the same law for RC and RL circuits, it produces a differential equation, it would be little bit difficult to solve when compare with the algebraic equation which we get, when we solve the resistive circuits using kirchhoffs law.

Now, the differential equation resulting from analyzing the RC and RL circuits, are of the first order so that is why we called these type of circuits as first order circuit, so we can say that first order circuit is characterize by a first order differential equation, so wherever you see that there is a first order differential equation coming then, we can say this is our first order circuit.

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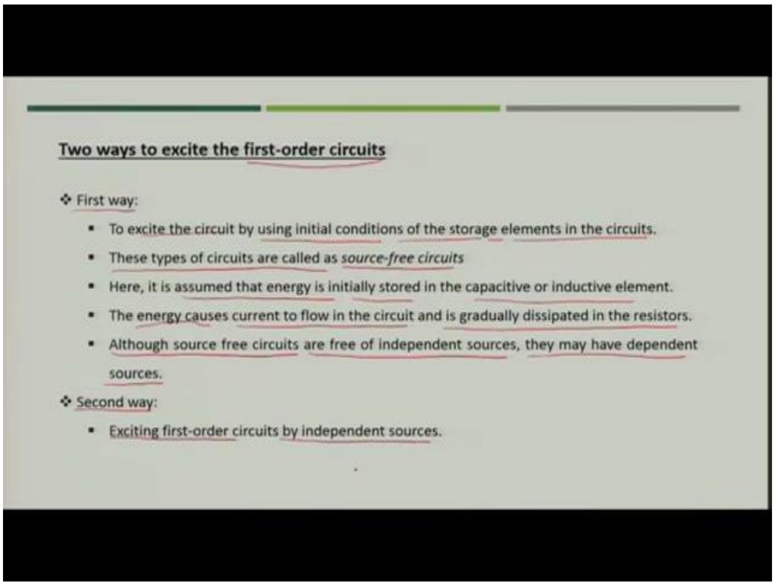

Now we have two ways to excite these first order circuits. The first way is by using initial conditions of the storage elements in the circuit, so that means that initially if you charge either capacitor or inductor and insert that charged capacitor and the inductor into the circuit then, the circuit will have only the source of energy as the total energy stored in either capacitor or inductor.

So, there will not be any external voltage or currents source to provide the energy. These circuits are called source free circuits because we do not have any external source available in that circuit. We have the capacitor and inductor which has some initial energy stored so, we assume that energy is initially stored in those elements and this energy will cause current to flow in the circuit, and gradually it will be dissipated in the resistors because when we talk about RC or RL circuit we will have a resistor either in series or parallel in the circuit.

The resistive component would be responsible to gradually dissipating the energy which was previously stored in the capacitor or inductor. Although source free circuits, that means it is free of independent sources, there may be some dependent sources. We will see those cases when we progress in this lecture. The second way to excite the first order circuit is by using independent sources. We have the first case where you do not have any source, so called as source free circuits. In the second is you excite the first order circuit by applying an independent source.

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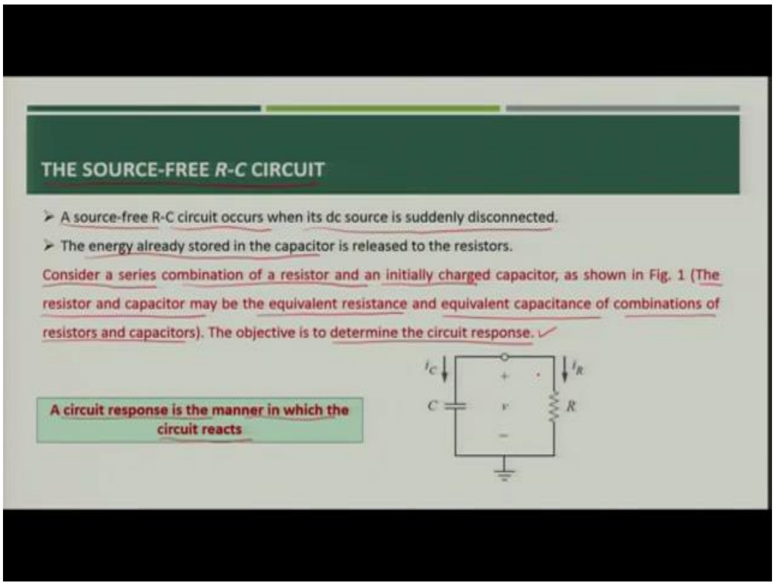

First let us try to understand what you mean by source free RC circuit. Source free RC circuit means we do not have any external voltage or currents source connected to the circuit.

How we can realize a source free RC circuit? It will occur when, a dc source is suddenly disconnected. Suppose you have a voltage source connected through some switch, may be some voltage V. We initially connected this switch and then after some time suddenly we have disconnected this switch. So, in that case when we apply voltage across capacitor, capacitor will get charged and the energy would be stored and when we disconnect the circuit would be like, this as shown in the figure.

So now, let us consider the series combination of resistor and it is initially charged means energy is stored in the capacitor. We will see how the energy stored in the capacitor will be released through the resistor which is there in the circuit. We can see in the figure which is shown in this slide, the resistor and capacitor we have, we are seeing here as a single one, but it can be equivalent resistance or equivalent capacitance of the combination of resistors and capacitor.

So, if you have multiple capacitors multiple resistors in the circuit, you can club all of them and create an equivalent RC circuit, so where C and R can be considered as an equivalent capacitance and equivalent resistance. Now we must find out the circuit response, circuit response means, the manner in which the circuit will react when the energy stored in the elements which is capacitor in this case is released.

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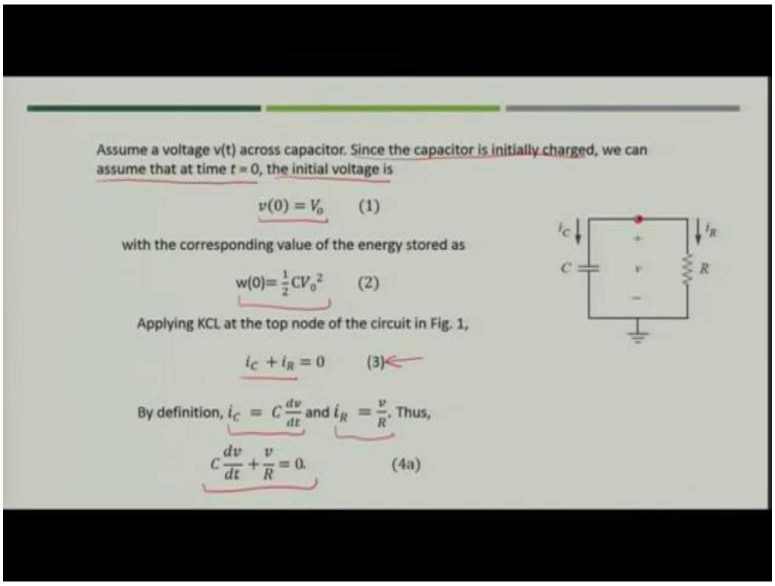

Let see how the performance of the circuit would be when you have the capacitor initially charged and connected to the resistor in parallel. in this case since, capacitor is initially charged so, we can assume that suppose time t is equal to 0, the capacitor was having some initial voltage say, V_0. This initial voltage is the voltage across the capacitor at time t is equal to 0.

So, the value of energy which would be stored in the capacitor would be $1/2CV_0^2$. From the circuit there would be one loop and if you apply Kirchhoff's current law at this particular node you will get, $I_C + I_R = 0$. I_C is the current which is flowing through the capacitor is, $I_C = C\frac{dv}{dt}$. Current through the resistor R is $I_R = \frac{v}{R}$.

So, at any time t we can say the voltage across the resistor or voltage across the capacitor because these two are in parallel, so voltage will remain same across both of the elements, the value of voltage is say v.

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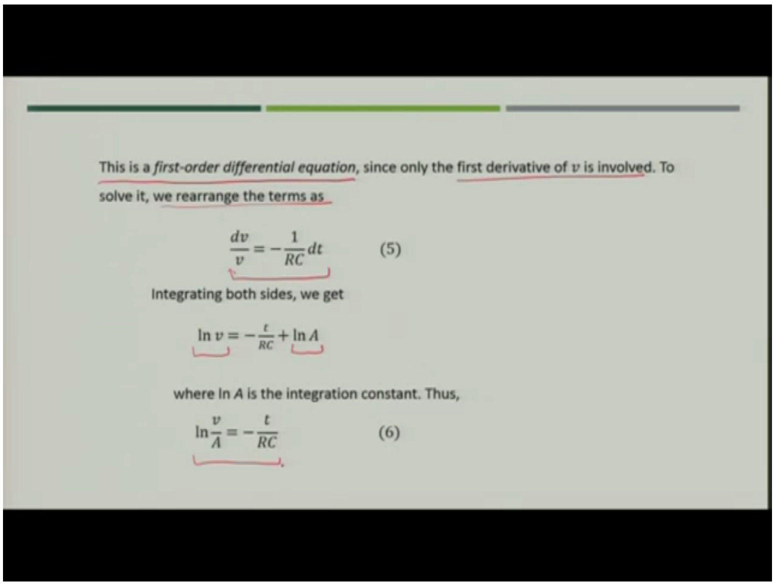

Now, this is our first order differential equation so, why will say first order differential equation because only first derivative of voltage V is involved.

Now, we rearrange the termto get,

$$\frac{dV}{V} = -\frac{1}{RC}dt$$

We integrate both sides to get

$$\ln v = -\frac{1}{RC} + \ln A$$

$$\ln\frac{v}{A} = -\frac{1}{RC}$$

So, in this case for simplicity we will assume that the constant term is ln A where, A is the integration constant.

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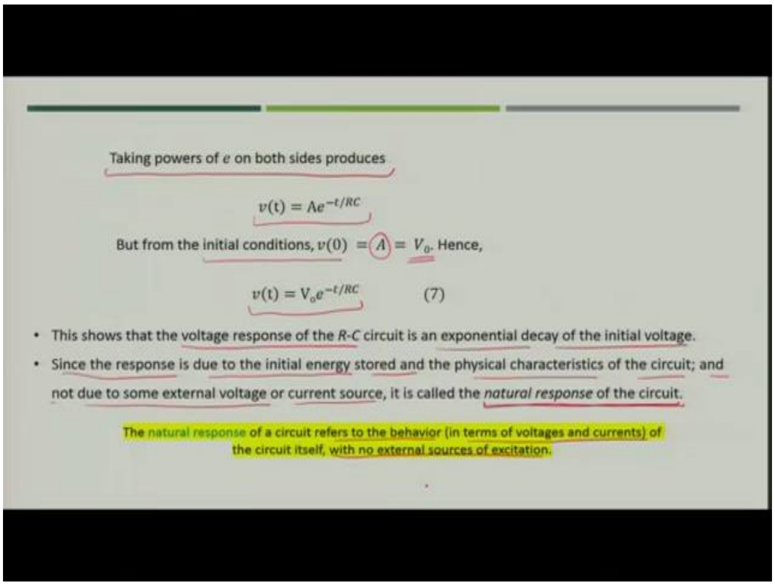

So, next task what we have to do, we have to take powers of e on both sides to get

$$v(t) = Ae^{-\frac{t}{RC}}$$

Now, we assume that as per initial condition $v(0) = A = V_0$. Therefore,

$$v(t) = V_0 e^{-t/RC}$$

You can see that, the voltage across both of the components whether it is capacitor or resistor is are time varying component. So this shows that the voltage response of RC circuit is exponential decay of initial voltage.

Now, since, the response is due to the initial energy stored and physical characteristic of the circuit so, this will not be due to any other external voltage or currents source this is simply, termed as natural response of the circuit. So, when we say that there is no external source connected to the circuit, and the external energy is dissipated the response of the circuit is called natural response of the circuit.

So, we can simply say that natural response of the circuit, refers to the behavior that will be in terms of voltage and current of the circuit itself with no external sources of excitation.

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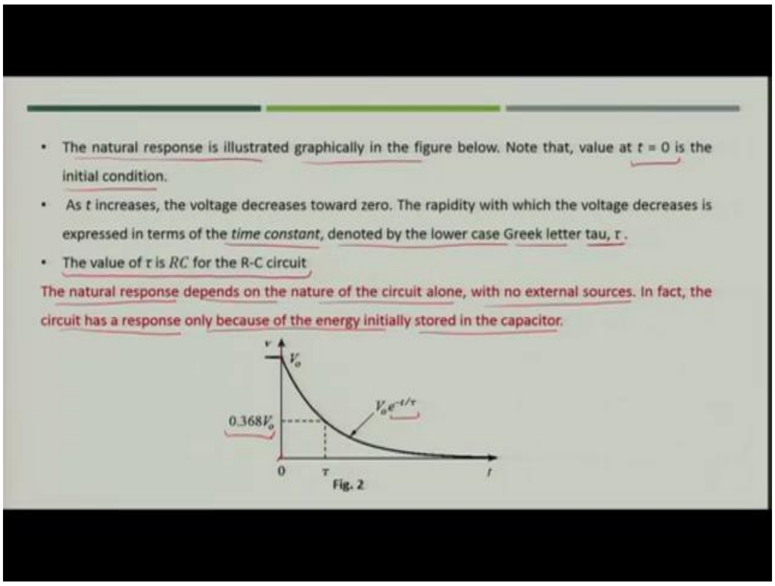

Now, how you will represent it graphically when you represent the natural response graphically it will be, like as shown in the figure, below, so here you will see when you have the component $v(t) = V_0 e^{-t/RC}$. Let us assume that the there is another component called time constant and it is denoted by Greek letter τ. Let us say that this time constant $\tau = RC$. Therefore,

$$v(t) = V_0 e^{-t/\tau}$$

When you plot this function with respect to time what you can see that it is exponentially decaying. So, when we take the time t = 0, we know that the initial value of voltage was V_0 and then after that, because of this factor it is decaying, and eventually all energy would be dissipated in the resistor.

So when we set t is equal to 0, we say it is a initial condition, so the natural response depends on the nature of the circuit alone with no external sources in fact circuit has a response only, because of the energy initially stored in the capacitor, so the natural response will have only the internal energy to be dissipated in the circuit. Now one important thing which we have to know that when time t is equal to τ, that means if you replace t with τ, this would simply become, e^{-1}. So, at that point of time the value $v(t) = 0.368V_0$.

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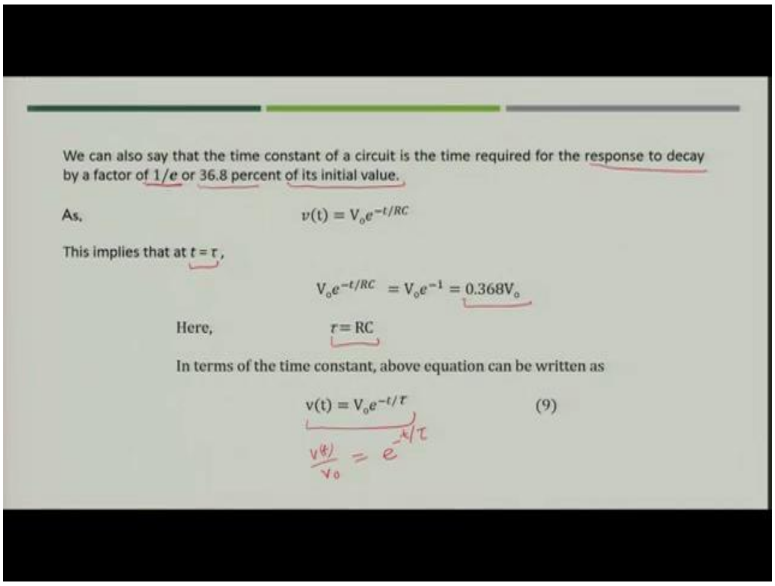

Now, what does it mean let see, so when we say that the time t is equal to time constant the response will decay by a factor of 1/e that is e to the power minus 1. It means that it will decay by 36.8 percent of its initial value. When we simplify by replacing RC as t is equal to τ, we get this value, so we say that when we represent τ in the circuit,

$$\tau = RC$$

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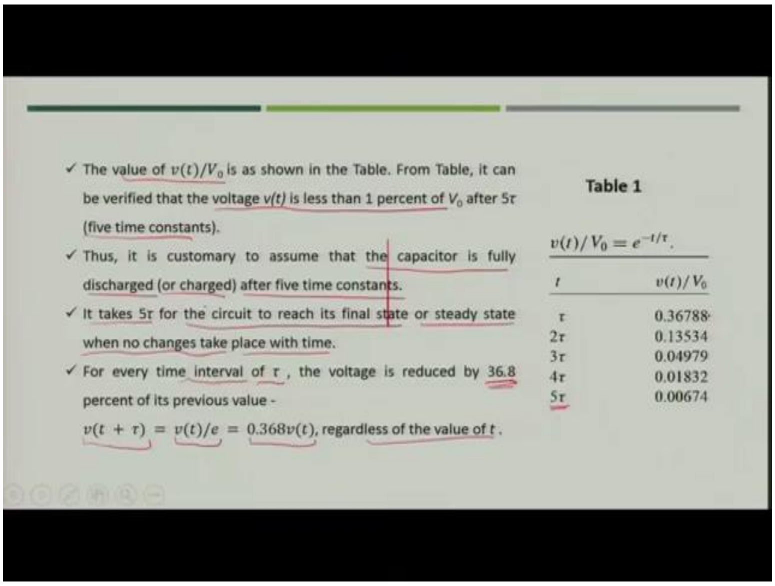

So, if you plot the value of $v(t)/V_0$, that is, if you take V_0 in denominator so, this will become $v(t)/V_0 = e^{-t/\tau}$. We will plot this value with respect to time t and let us take time t as a multiple of τ, that is the time constant so, if you compare you will come to know that after 5 τ, that means after 5 times constants the value of voltage *v(t)* is less that 1 percent.

So, what we can say, that, the capacitor is fully discharged or alternatively you can say is fully charged after 5 times constants so, means it takes 5 τ for the circuit to reach its final state or steady state when, no changes take place with time, so every time interval of τ the voltage is reduced by 36.8 percent of its previous value.

So this you can see from this particular table also that, the every increment of time equal to τ the value of, the voltage would reduce by 36.8 percent of its previous value because it is simply Vt by e, so if you keep on changing the value of t this will always being equal to 36.8 percent of Vt. So, this would be regardless any value of t, so if you put the value of t, as τ, 2 τ or 3 τ you will see that, whatever the value you get, is 36.8 percent of its previous value.

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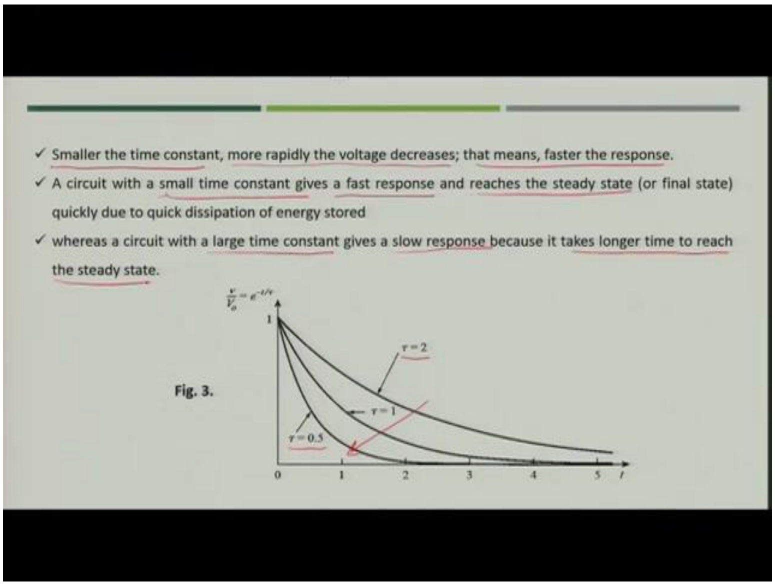

Now, smaller the time constant more rapidly the voltage will decrease so, that means that you have faster time response of the RC circuit. If you see this particular figure when time constant is small that τ that is nothing but RC the product of R and C, if it is small the slope of the curve shows that the voltage is decaying very fast as compare to other so, more you decrease the value of τ more speedily would be the decay of voltage in the circuit and the you have τ lager then you will see slow response of the circuit, so we can say that is small time constant will give us a fast response and it will reach the steady state quickly whereas when we have large time constant it will give slower response because it takes longer time to reach to its steady state value.

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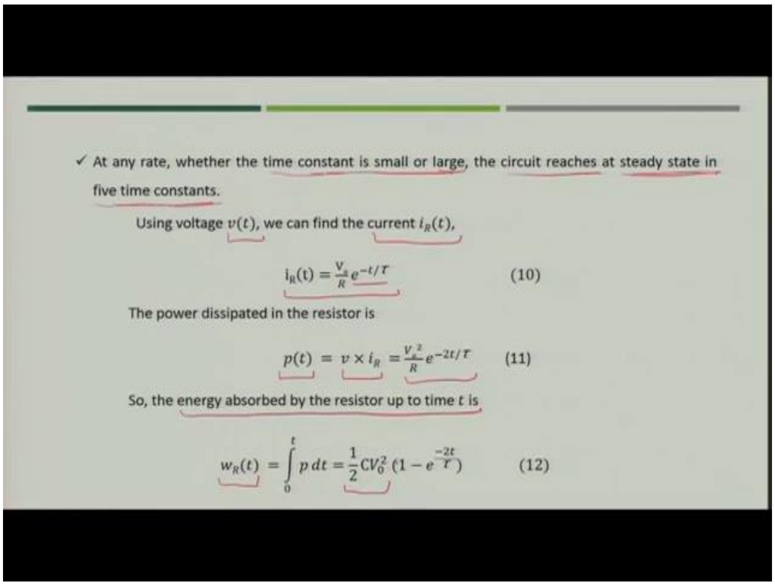

Now at any rate, whether the time constant is small or large, circuit will reaches at its steady state in 5 time constant. Because in 5 time constants the circuit voltage is there, then voltage value will decrease to less than 1 percent of its initial value so, the value of time constant is small or large will not impact, the steady state time, it will always be 5 times of the time constant. When time constant is small, means 5 times τ would be small in that case, and the circuit will reach to steady state value quickly.

Now, using voltage v(t) we can find the current as

$$i_R(t) = \frac{V_0}{R} e^{-t/\tau}$$

If power dissipated in the resistor is P at particular time t then the instantaneous power would be

$$p(t) = v * i_R = \frac{V_0^{\ 2}}{R} e^{-2t/\tau}$$

The energy absorbed by the resistor till time t is,

$$w_R(t) = \int_0^t p dt = \frac{1}{2} C V_0^{\ 2} (1 - e^{-2t/\tau})$$

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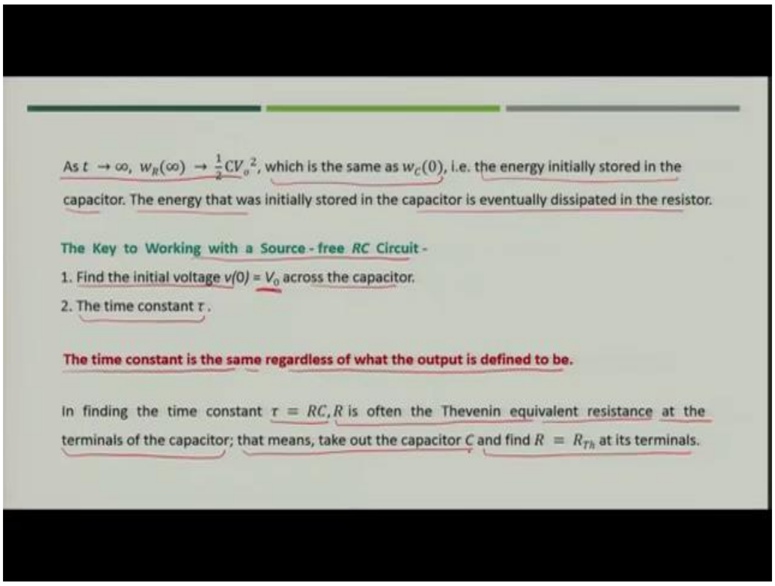

Now if you see this equation at time t is equal to 0, the energy will be 0. When say that at time t is equal to infinity the term in the bracket will become 0, so.

$$w_R(\infty) = 1/2CV_0^2$$

This energy is nothing but the initial energy which was stored in the capacitor.

So we will say that the total energy which was there in the capacitor, is discharged in the resister in time that is infinity, so it will take infinitely longer time to finally dissipated energy completely from which is available in the capacitor. Now, there are two important point to remember, when we work on the source free RC circuit. One is that, the initial voltage and the other is the value of time constant τ.

When we get these two values, we can simply use the equation which we derived and find the various parameters like voltage or current in the circuit. So, the time constant is the same regardless of what output is defined to be, so in finding the time constant τ that is equal to RC, R is often the Thevenin equivalent resistance. So when you have given the larger circuit and you want to find out the circuit response you can simply use the Thevenin theorem, and R would be simply a Thevenin equivalent resistance, at the terminal of capacitor.

So, that means that if you take out the capacitor and find the value of Thevenin resistance, that would be sufficient to find the time response of the circuit.

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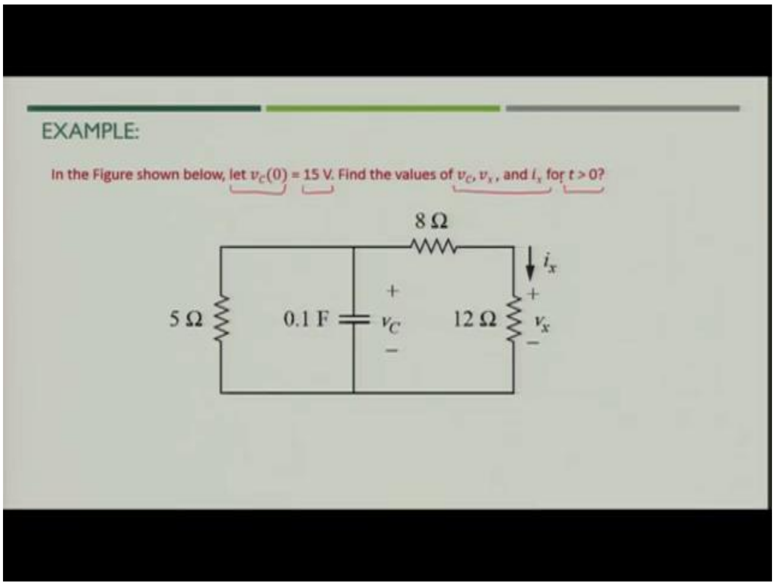

So, let us now, understand the concept with the help of one example so that you can understand the concept clearly, if you see this particular figure in this figure let the initial voltage across capacitor is, 15 volts so, what we have to do, we have to find the value of v_C, v_x, i_x.

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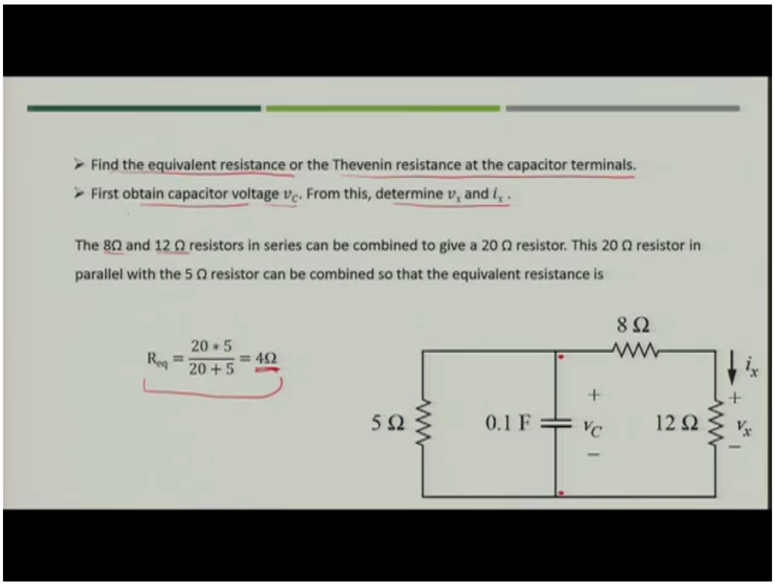

So, what you will do, first we will find the equivalent resistance or you can say the Thevenin resistance at the capacitor terminal, and then we will obtain the capacitor voltage and then, from v_C we will determine the value of v_x and i_x. Now, 8 ohm and 12 ohm resistors are in series and this combination is in parallel with 5 ohm resistance. From the calculations in the above slide it can be seen that R equivalent is 4 ohms.

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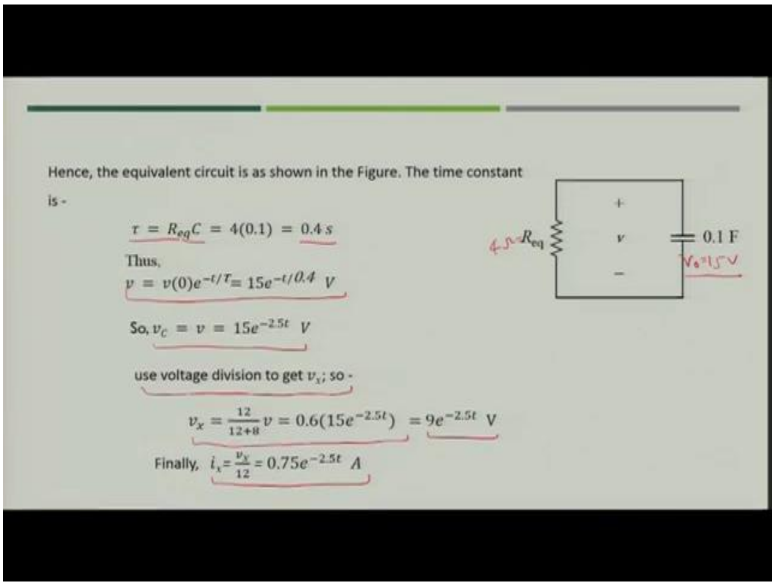

So now, finally the R equivalent that is 4 ohms, is connected in parallel with 0.1 Fared capacitor who is having initial voltage as 15 volts. From the above slide, the calculations can be seen to find the unknown parameters.

So with this we close our today's session, in this session we discuss about the natural response of RC circuit and in next session we will discuss about the natural response of RL circuit. Thank you.