Fundamentals of Electric Drives

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Lecture – 10

Speed Torque Characteristics of Full Controlled Converter- fed Separately Excited DC Motor, Analysis of Single Phase Half Controlled Converter-fed Separately Excited DC Motor

Hello and welcome to this lecture on the fundamentals of electric drives. In our previous session, we began exploring the continuous current operation of a single-phase fully controlled converter. Today, we will build on that discussion and continue delving deeper into the topic. Let's pick up where we left off.

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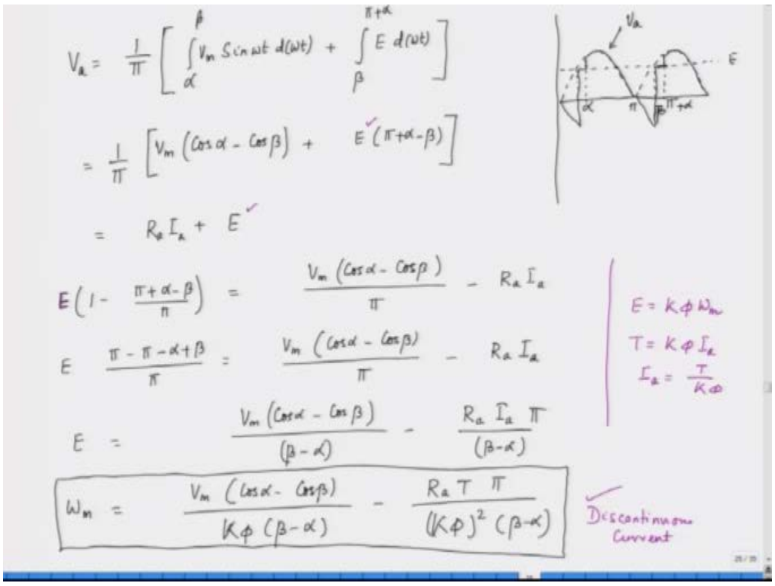

Now, let's consider a single-phase full-controlled converter. Here, the converter is connected to the load, which in this case is the armature of a separately excited DC motor. The current flowing through the armature is continuous, and this is reflected in the current and voltage plots against the angular position ωt.

Our objective is to calculate the average armature voltage. This task is straightforward due to the continuous current operation. To determine the average armature voltage, we integrate the input voltage waveform over one half-cycle. The integration is performed from α to π + α and is given by:

$$\text{Average Armature Voltage} = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin(\omega t)\ d(\omega t)$$

Solving this integral results in:

$$\text{Average Armature Voltage} = \frac{2V_m}{\pi}\cos(\alpha)$$

This represents the average output voltage of the bridge, which is also the armature voltage of the DC motor.

Now, applying this average armature voltage of $\frac{2V_m}{\pi}\cos(\alpha)$ to the motor, we can derive the torque-speed characteristic. Using the fundamental relationship for a DC motor, we have:

$$\frac{2V_m}{\pi}\cos(\alpha) = R_a I_a + E$$

where R_a is the armature resistance, I_a is the armature current, and E is the back EMF. From this equation, we can isolate the speed as:

$$\omega_m = \frac{2V_m}{\pi K\Phi}\cos(\alpha) - \frac{R_a T}{K\Phi^2}$$

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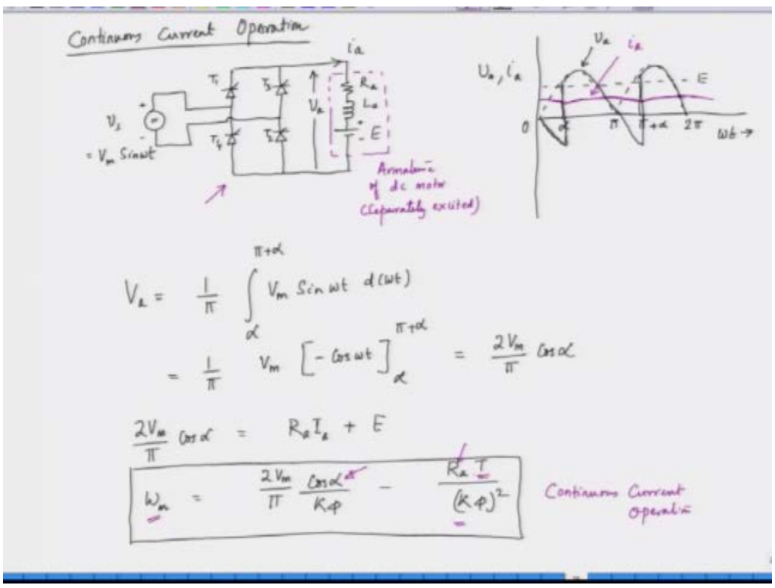

This expression defines the speed-torque characteristic of a separately excited DC motor under continuous current operation. Additionally, we have already derived the speed-torque characteristic for discontinuous current operation.

These two key equations allow us to compare the speed-torque characteristics of the motor in both continuous and discontinuous current modes when powered by a single-phase full-controlled converter.

Now, let's examine the characteristic more closely. As we vary α, the no-load speed of the motor changes. This variation in α directly affects the no-load speed. However, despite this change in speed, the slope of the characteristic remains almost constant. The reason for this is that the armature resistance remains unchanged, so while the no-load speed shifts with α, the overall slope of the speed-torque characteristic is not affected.

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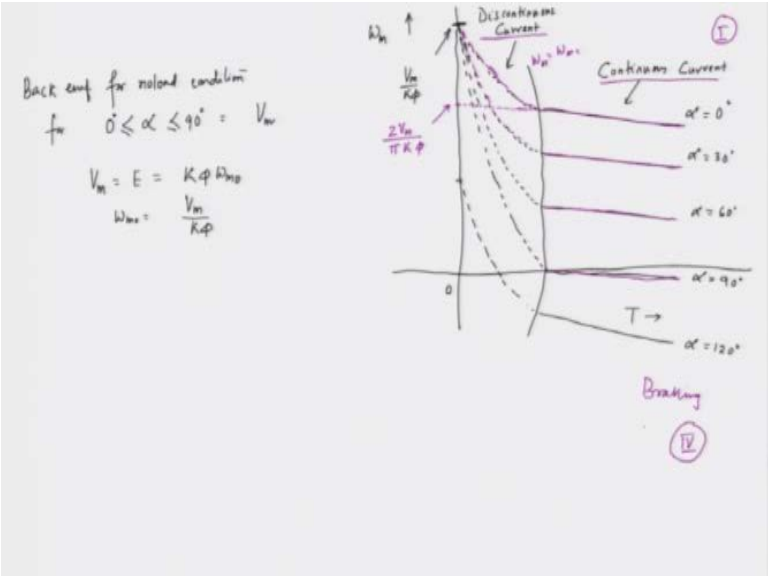

Let's now visualize the torque-speed characteristic by plotting speed on the y-axis and torque on the x-axis for different values of α. We'll consider cases for α = 0°, α = 30°, α = 60°, and α = 90°. When α = 90°, the no-load speed becomes zero, assuming the current remains continuous. However, at lower values of torque, there is a tendency for the current to become discontinuous.

As the torque decreases, especially at low torque values, the current starts to become discontinuous, leading to a shift in the operating region. This behavior is dictated by the earlier derived equations, marking the boundary between continuous and discontinuous operation. For

α = 30°, α = 60°, and α = 90°, we observe that there's a clear boundary, with one region representing continuous current operation and the other discontinuous current operation.

The no-load speed under discontinuous operation is given by $\frac{V_m}{K\Phi}$, as at no load, the back EMF equals the supply voltage, resulting in the current being zero. If we assume perfect continuous current operation, the speed-torque characteristic would intersect the y-axis at $\frac{2V_m}{\pi K\Phi}$.

So, the no-load speed in discontinuous current operation is $\frac{V_m}{K\Phi}$, since, under no-load conditions, the back EMF equals the supply voltage V_m. Hence, when the back EMF equals the peak of the supply voltage, the current becomes zero, and so does the torque. This marks the situation where $V_m = E = K\Phi\omega_{m0}$, giving us the no-load speed $\omega_{m0} = \frac{V_m}{K\Phi}$.

As a result, when the motor enters discontinuous current operation, it typically occurs when the load is reduced for a given α.

For discontinuous current operation, the relevant equation is this one. This equation specifically applies to discontinuous current operation, where it relates the speed with the torque. On the other hand, when the load is higher, the motor operates under continuous current conditions. So, we have a separate equation for continuous current operation, which similarly relates the speed to the torque.

Now, what we observe in continuous current operation is that the torque-speed characteristic forms nearly straight lines, and when plotted for different values of α, the lines are almost parallel to each other. This means the speed regulation in continuous current operation is quite good. However, when the current becomes discontinuous, the characteristic changes significantly, the speed drops quickly, and the regulation becomes poor, with a noticeable drop in speed for small changes in torque.

There is a clear boundary between continuous and discontinuous current operations, determined by the critical speed ω_c. When the motor speed ω_m is less than ω_c, the operation remains continuous. But when the speed exceeds the critical speed, the operation shifts to a discontinuous mode.

Now, let's consider what happens when we increase the value of α. For example, if we set α to 120 degrees, the system enters the braking region. As we further increase α to 150 degrees and beyond, we move deeper into this braking region. In this case, the no-load speed decreases further, and the motor transitions from motoring in the first quadrant to braking in the fourth quadrant.

Thus, in motoring, the operation occurs in quadrant 1, while braking takes place in quadrant 4. Next, we will examine how to achieve braking using a full-control converter.

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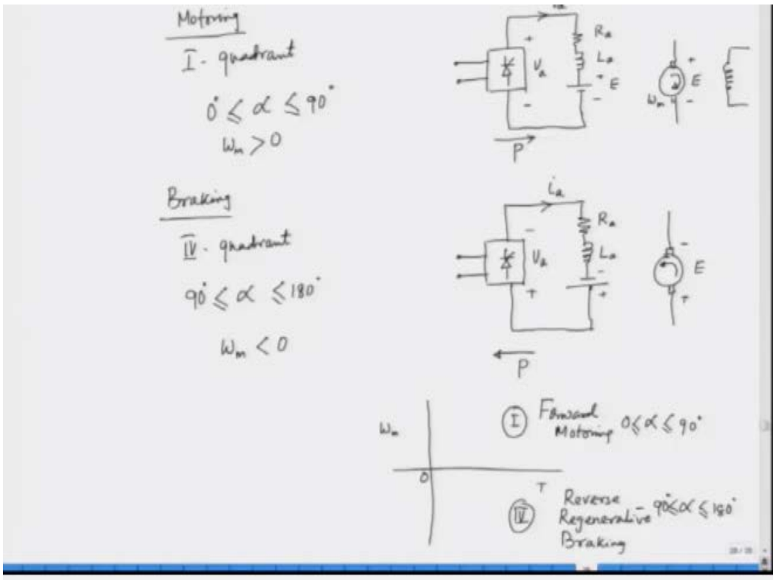

Now, let's examine how a full-control converter operates in both motoring and braking modes. Here, we have a block diagram representing the full-control converter supplying a motor load, which consists of resistance R_a, inductance L_a, and the back EMF E. The terminal voltage V_a is applied across the motor. This diagram reflects the equivalent circuit of the motor, where the actual motor generates a back EMF E, and the field is separately excited. This setup represents a single-phase, fully-controlled bridge converter.

When the motor is in motoring mode, the operation occurs in the first quadrant. In this state, the speed is positive, let's denote it as $\omega_m > 0$, and the torque is also positive. The firing angle α for motoring operation typically ranges between 0° and 90°. During motoring, current I_a flows in only one direction, into the motor, because the thyristor bridge can conduct current in one direction only. So, in this mode, the converter feeds power to the motor, enabling motion in the forward direction.

Now, let's move on to braking. In order to transition to braking, the same full-control converter, still fed by the AC supply, is used. However, braking can only occur if the motor's back EMF reverses. The reversal of the back EMF happens when the motor speed ω_m becomes negative, i.e., the motor is rotating in the reverse direction. This reversed speed results in the back EMF changing polarity, where the positive and negative terminals of the back EMF switch positions, leading to a reversed back EMF.

In braking mode, although the current I_a remains in the same direction (since the thyristors can only allow unidirectional current flow), the back EMF now opposes the current. This reversed EMF effectively means the motor is now supplying energy back into the system, converting mechanical energy into electrical energy, thus achieving braking. The motor acts like a generator, dissipating energy through the resistive and inductive components.

So, during motoring, the speed and torque are positive, and the motor operates in the first quadrant. When braking, the speed becomes negative, the back EMF reverses, and the motor begins to return energy back to the system, achieving braking.

So, this operation takes place in the fourth quadrant. Now, what’s the value of α here? The firing angle α is operated beyond 90°, meaning it ranges from 90° to 180°. In this region, the motor speed ω_m becomes negative, indicating braking. During motoring, ω_m is positive, but for braking, ω_m is negative.

This type of braking is known as regenerative braking because the kinetic energy from the motor is fed back to the supply. The single-phase converter operates with α higher than 90°, causing the output voltage V_a to reverse. This reversed voltage means that if we observe the converter voltage V_a, the power is negative, signifying that power flow is now from the load back to the supply.

In contrast, during motoring, the power flow is positive, with energy being transferred from the supply to the motor and ultimately to the load. However, in regenerative braking, the power flow is reversed, and that’s why this is called regenerative braking.

As this operation occurs in the fourth quadrant, let’s briefly revisit the quadrant system. If we plot speed on the vertical axis and torque on the horizontal axis, quadrant 1 represents motoring, where the motor drives the load forward with positive speed and positive torque. The firing angle α in this case ranges from 0° to 90°.

In quadrant 4, we have regenerative braking, where α exceeds 90° but remains below 180°. This quadrant is called reverse regenerative braking because the speed is negative while the torque remains positive. However, reverse regenerative braking can only occur when the speed is negative, which is possible in overhauling load conditions, such as when dealing with active loads like lifts or elevators that can be braked regeneratively.

That covers the operation of a full-control converter. Now, let's proceed to understand the behavior of a half-controlled converter.

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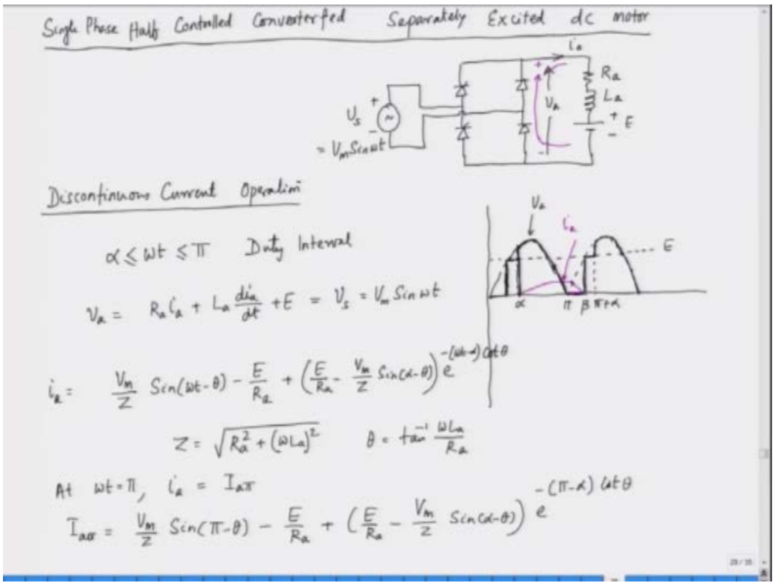

Now, let's discuss the operation of a half-controlled converter driving a separately excited DC motor. This is a single-phase half-controlled converter feeding a separately excited DC motor.

So, what does the converter's structure look like in this case? The converter consists of two Silicon-Controlled Rectifiers (SCRs) and two diodes. It supplies the armature of the DC motor, which has resistance R_a, inductance L_a, and a back EMF E. The input to the converter is an AC supply V_s, and the output is the armature current I_a and armature voltage V_a.

Just like the full-control converter, the half-control converter can also operate in two different modes: continuous current mode and discontinuous current mode. We'll examine these two modes separately, starting with discontinuous current mode.

Now, let's take a closer look at the operation of this converter in discontinuous current mode. The input supply is a sine wave, and the corresponding negative half-cycle of that sine wave is also present. The motor's back EMF is denoted by E, and the converter is triggered at an angle α, with π representing 180°. When α is reached, the current begins to flow from zero, and at π, when the supply voltage attempts to turn negative, the diodes become forward-biased.

At this point, a freewheeling path is formed through the two diodes, preventing the output voltage from becoming negative. Consequently, the output voltage looks like this: after the triggering, it rises, and then at a certain point, it drops to zero. From here, the current in the circuit decreases exponentially, gradually reaching zero.

This extinction of current occurs at the angle known as the extinction angle, β, similar to the case with a full-control converter. Once the current becomes zero, the output voltage also becomes equal to the back EMF of the motor.

So, this is how the current and voltage waveforms behave in a half-controlled converter operating in discontinuous current mode. The current starts from zero, rises, and then decays to zero exponentially, while the output voltage initially follows the input voltage, drops to zero, and eventually becomes the motor's back EMF.

At this point, the output voltage builds up and eventually becomes equal to the motor's back EMF. The nature of the output voltage waveform looks like this, and if the operation is periodic, the voltage waveform will follow a similar pattern continuously. Now, we can break down the discontinuous current operation into several distinct modes.

The first mode is called the duty interval. This occurs when ω t is greater than or equal to α and less than or equal to π. During this interval, the supply voltage is applied to the output, meaning that between α and π, the output receives the supply voltage. Essentially, we are working with an RLE circuit where the armature voltage V_a is defined by:

$$V_a = R_a I_a + L_a \frac{dI_a}{dt} + E$$

Here, V_a is the voltage across the armature, R_a is the armature resistance, L_a is the armature inductance, E is the back EMF, and $V_s = V_m \sin \omega\, t$ represents the supply voltage. So, during the duty interval, the supply voltage is a sine wave, expressed as $V_m \sin \omega\, t$.

As before, this equation has both a steady-state component and a transient component. The current equation during the duty interval can be written as:

$$I_a = \frac{V_m}{Z}\sin(\omega t - \theta) - \left(\frac{E}{R_a}\right) + \left(\frac{E}{R_a} - \frac{V_m}{Z}\sin(\alpha - \theta)\right)\exp(-(\omega t - \alpha)\cot\theta)$$

Here, Z is the impedance of the circuit, given by:

$$Z = \sqrt{R_a^2 + (\omega L_a)^2}$$

And θ is the phase angle, calculated as:

$$\theta = \tan^{-1}\left(\frac{\omega L_a}{R_a}\right)$$

This is the current equation during the duty interval.

Now, let's move on to the freewheeling interval. At ω t = π, the current I_a reaches a value $I_a(\pi)$. The expression for $I_a(\pi)$ can be derived by substituting ω t = π into the current equation:

$$I_a(\pi) = \frac{V_m}{Z}\sin(\pi - \theta) - \frac{E}{R_a} + \left(\frac{E}{R_a} - \frac{V_m}{Z}\sin(\alpha - \theta)\right)\exp(-(\pi - \alpha)\cot\theta)$$

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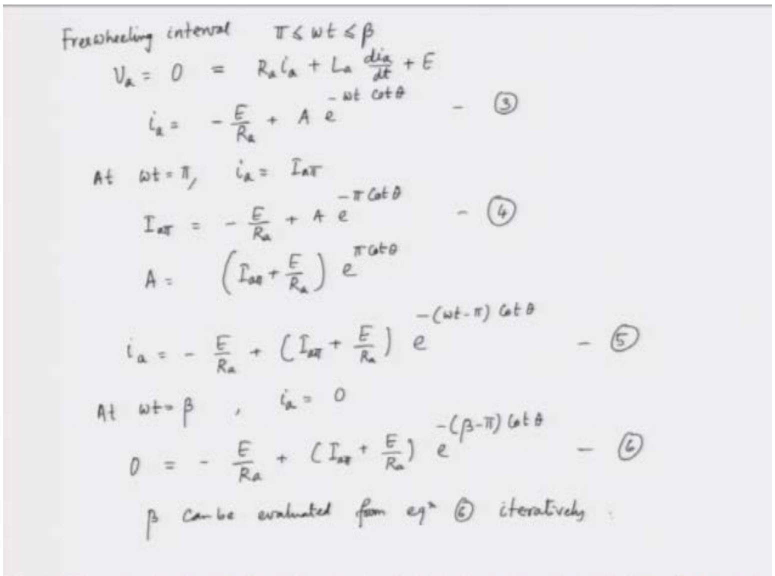

After the duty interval ends, we enter the freewheeling interval. During this period, the voltage across the load cannot become negative because the freewheeling diodes provide a path for the current. As a result, the current freewheels through the diodes, keeping the voltage at zero. The current decays exponentially during this interval, while the voltage remains at zero due to the freewheeling action of the diodes.

Now, let's focus on the freewheeling interval, which occurs when ω t is greater than π and less than β. During this freewheeling interval, the armature voltage V_a ideally equals zero because the output is effectively short-circuited by the diodes. If we assume the diodes are ideal, we can express the output voltage V_a as:

$$V_a = R_a I_a + L_a \frac{dI_a}{dt} + E$$

In this scenario, since V_a is zero, we arrive at a first-order differential equation. We can solve this equation similarly to how we approached the previous cases, addressing both the steady-state and transient responses.

The solution to this differential equation reveals that the steady-state component of the current I_a is given by:

$$I_a = -\frac{E}{R_a}$$

The transient part takes the form of a constant A multiplied by an exponential decay, leading to:

$$I_a = -\frac{E}{R_a} + Ae^{-\omega t} \cos \theta,$$

Now, let's evaluate the initial conditions at $\omega t = \pi$. At this point, the current I_a is denoted as $I_a(\pi)$, and we can express it as:

$$I_a(\pi) = -\frac{E}{R_a} + Ae^{-\pi} \cos \theta,$$

From this equation, we can solve for the constant A:

$$A = I_a(\pi) + \frac{E}{R_a} e^{\pi} \cos \theta,$$

We can now substitute this expression for A back into our differential equation, which we will refer to as equation (1). This updated equation will be labeled as equation (2), and we will consider the equation we derived previously as equation (3). Let's now denote our new equation with A substituted as equation (4).

Thus, we have:

$$I_a = -\frac{E}{R_a} + \left(I_a(\pi) + \frac{E}{R_a} e^{\pi} \cos\theta\right) e^{-\omega t} \cos\theta,$$

This equation represents the current during the freewheeling interval.

As the freewheeling interval concludes, the current approaches zero. We can set $I_a = 0$ to determine the extinction angle β. Substituting this condition into our earlier equation yields:

$$0 = -\frac{E}{R_a} + I_a(\pi) + \frac{E}{R_a} e^{-\beta} \cot\theta,$$

This equation, which we will refer to as equation (6), contains the value of β and can be solved iteratively.

Finally, if we wish to plot or calculate the average voltage value during the discontinuous current operation, we will need to perform an integration based on the established relationships and equations.

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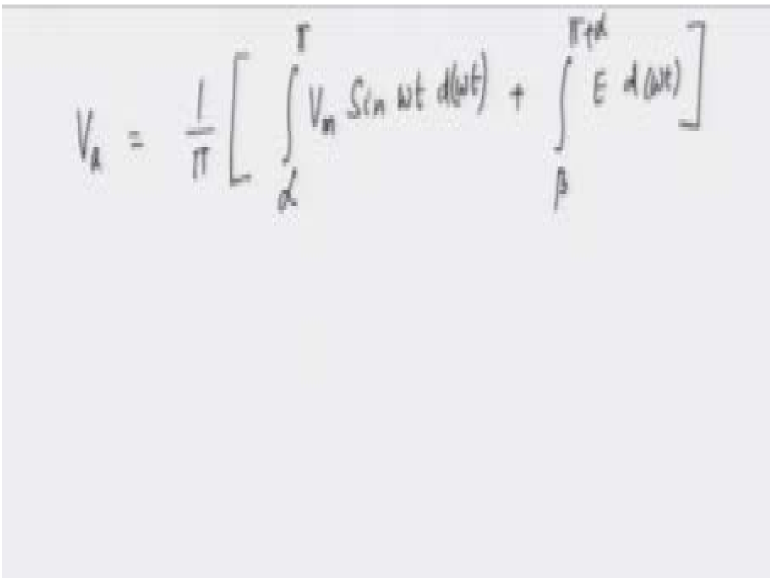

Let's discuss how we can perform the integration to find the average output voltage V_a over one half-cycle. The integration is executed as follows: we integrate from α to π, where the voltage can be expressed as $V_m \sin(\omega t) \; d\omega t$. Then, we need to integrate from β to π + α, where the back electromotive force (emf) E is present.

To visualize the voltage waveforms, we observe that from α to π, the voltage is represented by V_a, followed by a zero voltage condition. After this, from β to $\pi + \alpha$, the voltage takes on a finite value, which is E.

By solving this integration, we will derive the average value of the output voltage for the discontinuous current operation of a half-controlled DC converter. Furthermore, this analysis will also enable us to determine the torque-speed characteristic for a separately excited DC motor fed by a half-controlled DC converter. We will discuss the analysis of the torque-speed characteristics in the next lecture.