Fundamentals of Electric Drives

Prof. Shyama Prasad Das

Department of Electrical Engineering

Indian Institute of Technology – Kanpur

Module No # 03

Lecture No # 11

Analysis of Half Converter-fed Separately Excited DC Motor

Hello and welcome to this lecture on the fundamentals of electric drives! In our last session, we discussed the half-controlled converter-fed DC motor drive, and today, we will continue our exploration of that topic. To start, let's take a closer look at the circuit diagram.

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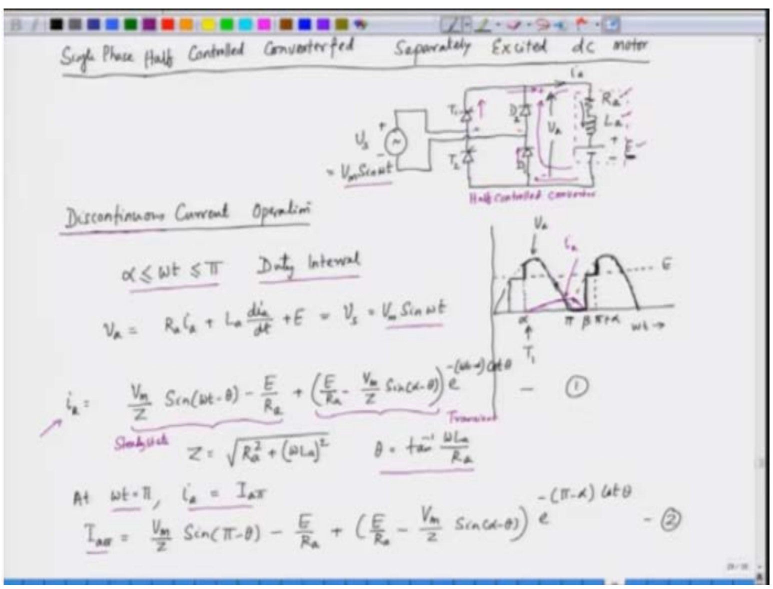

Now, let's take a closer look at the circuit diagram of a half-controlled converter. In this configuration, we have two Silicon Controlled Rectifiers (SCRs) and a diode, which supply power to the armature of a DC machine, specifically a separately excited DC motor. The armature consists of resistance, inductance, and the back electromotive force (EMF), denoted as E.

As we discussed previously, this setup can operate in two different modes: discontinuous current operation and continuous current operation. First, we'll focus on the discontinuous current operation that we covered in the last lecture.

In this mode, we trigger the SCR, say T_1, at an angle α during the positive half-cycle of the AC supply. When we trigger T_1, it becomes forward-biased because the anode is positive, allowing it to conduct. Consequently, the current begins at zero and gradually increases.

When the SCR is in conduction, the side connected to the SCR is positive, while the other side is negative. Both the SCR and the diode are forward-biased, enabling conduction through T_1 and subsequently through the armature, with the current returning via the diode, which we'll refer to as D_1.

During the conduction of SCR T_1 and diode D_1, the input supply voltage is effectively available at the armature, represented as V_a. This constitutes the duty interval where we've triggered at angle α.

The equation governing the armature voltage V_a can be expressed as the sum of the resistance drop, the inductance drop, and the back EMF, which equals the supply voltage. Thus, we have:

$$V_A = R_a I_a + L_a \frac{dI_a}{dt} + E = V_s = V_m \sin(\omega t)$$

Here, V_m represents the peak voltage of the AC supply $V_s = V_m \sin(\omega t)$. This equation is a first-order differential equation, which we are familiar with from our previous discussions.

So, when we solve this equation, we obtain both the transient part and the steady-state part. The transient response is well-known, and it can be distinguished from the steady-state behavior. We must also consider the impedance Z of the armature circuit, which comprises both resistance and reactance. The angle θ represents the impedance angle or the power factor angle.

Initially, the current starts from zero and increases until it reaches a specific value when ω t = π. We plot ω t on the X-axis, and at ω t = π, the current value is denoted as $I_{a\pi}$. If we substitute this into Equation 1, we find that:

$$I_{a\pi} = \frac{\omega V_m}{Z}\sin(\pi - \theta) - \frac{E}{R_a} + \frac{E}{R_a} - \frac{V_m}{Z}\sin(\alpha - \theta) \cdot e^{-(\pi - \alpha)\cot(\theta)}$$

Here, θ is defined as $\tan^{-1}\left(\frac{\omega L}{R_a}\right)$.

From π onwards, this equation remains valid for the duty interval spanning from α to π. However, at ω t = π, the voltage attempts to go negative. This negative voltage scenario is prevented by the diodes; we have two diodes present, D_2 and the SCR T_2.

As we move from π to β, the operation enters a freewheeling phase because we still have some inductive current flowing. This inductive current contains stored energy, which will flow through diodes D_1 and D_2.

During this freewheeling interval, the output voltage V_a becomes equal to zero. As a result, the current begins to decrease. The reason for this decrease is that the back EMF, which is equal to E, opposes the current. Consequently, the current experiences an exponential decay due to this opposing back EMF.

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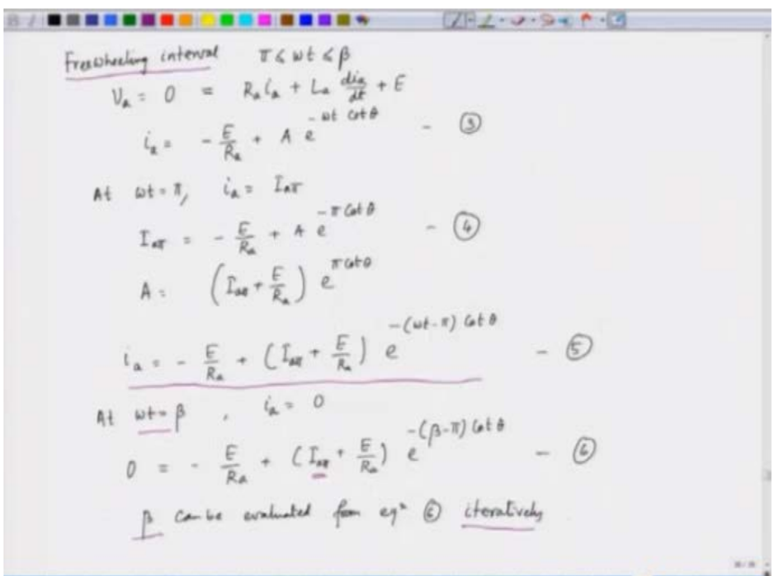

This is how the current decreases during what we refer to as the freewheeling interval. In this interval, we encounter a simplified equation. By substituting the value of the constant A, which we determine when we set ω t = π and $I_a = I_{a\pi}$, we arrive at the final equation for the current during the freewheeling interval, which exhibits this specific behavior.

At ω t = β, the current again reaches zero. This point corresponds to the angle β. By substituting this condition into our equation, we have:

$$I_a = 0.$$

This equation incorporates sine and exponential functions. We already have the expression for $I_{a\pi}$ provided in Equation 2, so we can substitute that value into our current equation. The result is a transcendental equation, specifically Equation 6, which consists of sine and exponential functions. It's important to note that this is not an algebraic equation.

Consequently, we cannot derive a closed-form solution for β. Instead, we need to determine the point at which the current decreases to zero through iterative methods. Therefore, Equation 6 must be solved iteratively to find the value of β.

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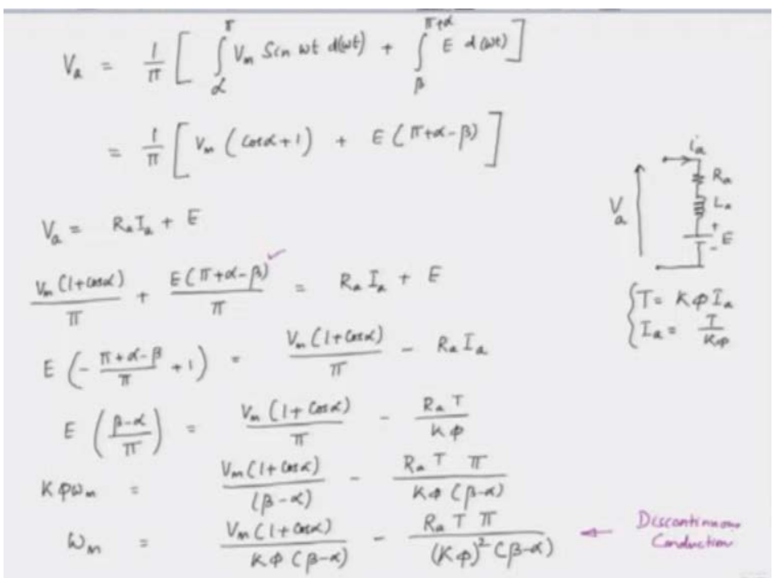

Now, let's examine the average voltage. We have seen in the voltage waveform that the back EMF, denoted as E, is present, while the armature voltage is represented as V_a. Our goal is to determine the average armature voltage, V_a.

To calculate this, we need to integrate over two specific intervals: from α to π and then from β to π + α. It's important to note that the voltage is zero between π and β due to freewheeling occurring during this interval. Therefore, we don't need to include this section in our integration.

The integration will effectively be performed from α to π and then from β to $\pi + \alpha$. Mathematically, the average output voltage can be expressed as:

$$V_{\text{avg}} = \int_{\alpha}^{\pi} V_m \sin(\omega t)\ d\omega t + \int_{\beta}^{\pi+\alpha} E\ d\omega t.$$

Upon solving this, we find that:

$$V_{\text{avg}} = \frac{1}{\pi}(V_m \cos\alpha + 1) + (\text{back EMF term})(\pi + \alpha - \beta).$$

This gives us the expression for the average armature voltage. We know that the armature circuit comprises resistance, inductance, and back EMF. Specifically, we have R_a, L_a, and E.

With the average armature voltage V_a applied, we can assert that the average inductance drop is zero. Consequently, we can express the armature voltage as:

$$V_A = R_a I_a,$$

where I_a is the instantaneous armature current. Although the instantaneous current is I_a, the average value is denoted as I_a. The back EMF E represents the drop across the inductance, which we have established to be zero on average after the inductance.

Since the inductor is considered an ideal inductor, it has no resistance, which means the average voltage drop across the inductor is zero. Our objective now is to derive the speed-torque characteristic. To achieve this, we can substitute the value of V_a from the previous equation. The expression becomes:

$$V_m \frac{1 + \cos\alpha}{\pi} + E\frac{\pi + \alpha - \beta}{\pi} = R_a I_a + E.$$

Now, we can rearrange this equation to isolate the terms effectively. By moving the terms around, we simplify it to:

$$E\frac{\pi+\alpha-\beta}{\pi}-1=V_m\frac{1+\cos\alpha}{\pi}-R_aI_a.$$

Simplifying further, we find that the E terms lead to a cancellation of π in the numerator, yielding:

$$\frac{\beta-\alpha}{\pi}=V_m\frac{1+\cos\alpha}{\pi}.$$

From our knowledge of torque, we can express the relationship as:

$$T=K\Phi I_a \quad \text{or} \quad I_a=\frac{T}{K\Phi}.$$

Substituting this expression for I_a into our equation, we have:

$$E\frac{\pi+\alpha-\beta}{\pi}=V_m\frac{1+\cos\alpha}{\pi}-R_a\left(\frac{-R_aT}{K\Phi}\right).$$

Additionally, since the back EMF E can be expressed as K Φ ω_m, we can reformulate our equation to find:

$$K\Phi\omega_m=V_m\frac{1+\cos\alpha}{\beta-\alpha}-\frac{R_aT}{K\Phi}\cdot\frac{\beta-\alpha}{\pi}.$$

This leads to the speed expression:

$$\omega_m=\frac{V_m(1+\cos\alpha)}{\beta-\alpha}-\frac{R_aT\pi}{K\Phi^2(\beta-\alpha)}.$$

Thus, this equation represents the torque-speed characteristic of a half-controlled converter fed separately excited DC motor operating under discontinuous current conditions. Now, let’s consider what happens when the current is continuous. In the case of continuous current, the flow is maintained, resulting in a different type of voltage and current waveform.

Now, let's consider the case of continuous current operation. In this scenario, the input voltage gets rectified, and we observe that we are triggering the device at an angle α. This corresponds to π, and subsequently, at π + α, we trigger T_2 during the negative half cycle. The origin is set at zero, with ω t represented on the X-axis, extending up to 2π.

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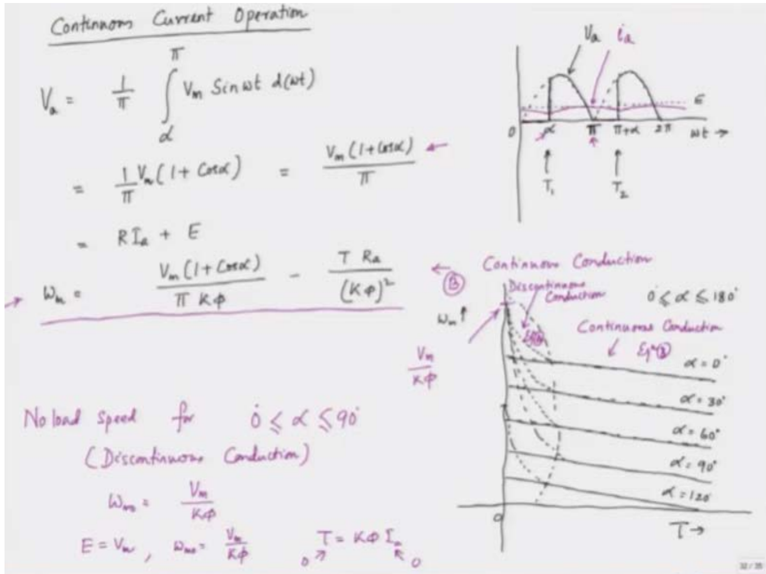

In continuous conduction mode, there is typically some back EMF, denoted as E. Here, we note that the speed is relatively low, resulting in a lower back EMF. The current waveform in this mode does not drop to zero; instead, it begins from a certain value and maintains a continuous flow throughout its cycle. This characteristic shape illustrates that the current never reaches zero.

Now, examining the voltage waveform, we trigger at an angle α. The input voltage waveform exhibits free-wheeling behavior from α to π, marking the beginning of the free-wheeling interval, which lasts from π to π + α. After this point, we trigger again, and the voltage waveform takes on a specific shape.

In steady-state operation, we see that the voltage waveform is periodic. The armature voltage and armature current are both captured within this periodic behavior. To evaluate the average output voltage V_a, we can approach it straightforwardly since there is no back EMF to consider and no intervals of zero current. We only need to integrate from α to π, because between π and π + α, the voltage is zero, and the same applies from 0 to π.

Therefore, we can express the integration as follows:

$$V_A = \int_{\alpha}^{\pi} V_m \sin(\omega t)\ d(\omega t).$$

Upon completing this integration, we find:

$$V_A = \frac{1}{\pi}(1 + \cos\alpha)V_m,$$

which represents the average output voltage for continuous conduction. This scenario typically occurs at lower speeds.

Now, if we were to draw the torque-speed characteristic for continuous conduction, we can use a simpler equation. The relationship can be expressed as:

$$I_a R_a + E = \omega_m = \frac{V_m(1 + \cos\alpha)}{\pi} K\Phi - \frac{TR_a}{K\Phi^2}.$$

This equation elegantly captures the dynamics of the torque-speed characteristic for a system operating under continuous conduction conditions.

This equation we just discussed is valid for continuous conduction. Now, let's take a moment to compare it with the previous equation we derived for the speed-torque relationship under discontinuous current conduction. We have two distinct equations: one for discontinuous conduction and another for continuous conduction. This allows us to effectively draw the torque-speed characteristic.

One important observation to note is the range of the firing angle α. The range for α extends from 0 to 180 degrees. Within this full range, the average voltage remains positive during continuous conduction. This means that, in the case of a half-controlled converter, the average armature voltage can never become negative.

Consequently, this is classified as a one-quadrant converter, which operates exclusively within a single quadrant since the average voltage cannot be negative. As a result, there is a free-wheeling path where the voltage can drop to zero.

When we draw the torque-speed characteristic, we will utilize the continuous conduction case represented by this equation. Each line on the graph corresponds to different values of α. For instance, let's label these straight lines, starting with the first one. We can denote subsequent lines for various angles: α = 0°, α = 30°, α = 60°, α = 90°, α = 120°, and so forth.

On the graph, the speed is plotted on the Y-axis while torque is plotted on the X-axis. For low torque values, the speed equation will be governed by a different equation related to discontinuous conduction, indicated by dotted lines. This illustrates the boundary between continuous and discontinuous conduction.

Even for α = 120°, we still observe regions of discontinuous conduction, leading to a no-load speed. Thus, we can clearly delineate the regions: one for continuous conduction and another for discontinuous conduction.

If we designate the equation for discontinuous conduction as Equation A and the equation for continuous conduction as Equation B, we can better analyze the differences in their respective torque-speed characteristics.

This situation is governed by Equation B, while the previous scenario is described by Equation A. Now, let's discuss the no-load speed for discontinuous conduction. For α values ranging from 0 to 90 degrees, the no-load speed can be expressed as:

$$\omega_{M0} = \frac{V_m}{K\Phi}$$

This indicates that when α is greater than 0 but less than 90, and the conduction is discontinuous, the no-load speed is indeed given by the above equation.

When we refer to a no-load condition, it implies that the torque is zero. Now, how can the torque be zero? The torque will be zero when the current is also zero. Since we are dealing with a separately excited DC motor, we know that torque is a function of both flux and current. Therefore, if we say that the torque is zero, it follows that the current must also be zero.

In this case, when the current is zero, the back EMF reaches its maximum possible value, which is equal to V_m. Hence, when the back EMF E is equal to V_m, the no-load speed can be calculated as:

$$\omega_{M0} = \frac{V_m}{K\Phi}$$

This represents the no-load speed for α values between 0 and 90 degrees. Now, if α exceeds 90 degrees, we must use a different equation to determine the no-load speed in that scenario.

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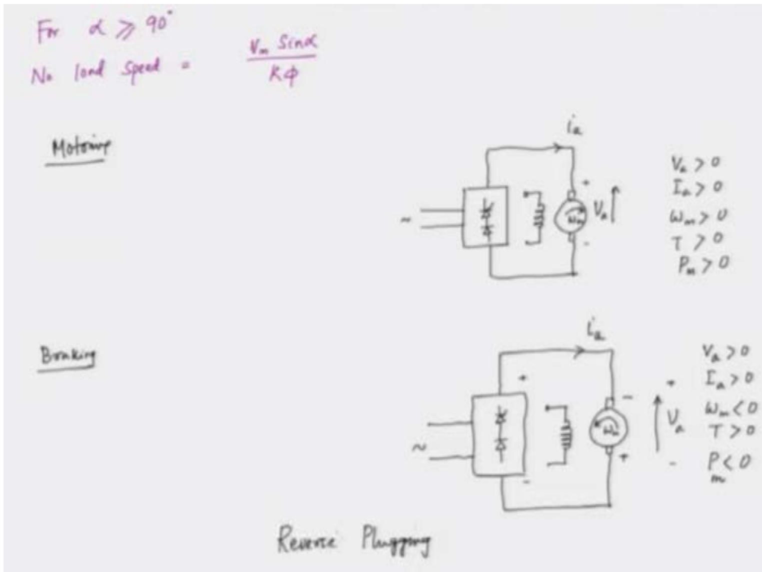

In this scenario, the no-load speed is expressed as:

$$\omega_{M0} = \frac{V_m \sin \alpha}{K\Phi}$$

This is because $V_m \sin \alpha$ represents the maximum voltage achievable when α exceeds 90 degrees. To ensure that the armature current I_a equals zero, the back EMF must equal this maximum possible armature voltage, which is $V_m \sin \alpha$.

Now, let's consider the motoring case, which is quite straightforward. In the context of motoring, we have a half-controlled bridge configuration, consisting of thyristors and diodes, supplying the armature circuit. The field is partially excited, and we are dealing with a single-phase AC input.

In this motoring scenario, the armature voltage V_a is positive, the current is positive, and consequently, the speed is also positive. However, when we shift our focus to braking, the process is not as direct. To initiate braking, we must reverse the back EMF. This reversal is crucial because, in this case, we are still using the same half-controlled converter circuit.

It's important to note that regenerative braking cannot be achieved here. In order to reverse the back EMF, we can either reverse the speed or the field. Suppose we initially have a motor speed of $\omega_m > 0$. If the speed ω_m is reversed, the back EMF will also reverse, provided that the field is separately excited and remains constant. In this configuration, the armature continues to be supplied by the AC voltage V_a.

In this scenario, the armature voltage V_a is supplied by the converter. Importantly, the current remains in the same direction because the converter circuit only permits current to flow in one direction. Consequently, we find that V_a is positive, the current is positive, and, in contrast, the speed is negative. This results in the power, P, also being negative.

This situation describes the braking phase; however, it's crucial to note that the power generated in this context cannot be fed back into the supply. Here, the mechanical power P_m is negative because, despite the torque being positive, the speed is negative. Previously, we had both positive torque and positive speed, but in this case, we have positive torque alongside negative speed.

As a result, the mechanical power is negative, in contrast to our earlier scenario where it was positive. This form of braking is referred to as "plugging," and more specifically, it is termed "reverse plugging" because the speed has reversed direction.

In the context of a half-controlled converter, regenerative braking is not achievable. This limitation arises because the converter cannot reverse either the voltage or the current. Thus, during electrical braking, the energy dissipated is wasted in the armature resistance.

In summary, we see that while the half-controlled converter allows for forward motoring, it only facilitates reverse plugging during braking. We will continue our discussion in the next lecture.