

**Fundamentals of Electric Drives**  
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**Lecture No # 14**

**DC Chopper-fed Separately Excited DC Motor for Motoring and Braking**

Hello and welcome to this lecture on the fundamentals of electric drives! In our previous discussions, we explored converter-fed DC drives. Today, we will delve into chopper-fed DC drives.

When we work with a DC source, we can utilize a DC-to-DC converter, commonly referred to as a DC chopper. This device allows us to obtain a variable DC voltage from a fixed DC voltage source. The resulting variable DC voltage can then be applied to the armature of a separately excited DC motor, enabling us to achieve precise speed and torque control.

So, let's dive deeper into the concept of chopper-fed DC drives and understand how they function!

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Chopper-fed DC Drives

Motoring Operation - Separately Excited DC Motor

When T is ON (Powering interval)  
 Diode D is OFF  

$$V_a = V = R_a I_a + L_a \frac{di_a}{dt} + E$$

When T is OFF (Free-wheeling interval)  
 Diode D is ON  

$$V_a = 0 = R_a I_a + L_a \frac{di_a}{dt} + E$$

$\frac{t_{on}}{T} = \delta = \text{Duty Cycle}$

$$V_a = \frac{1}{T} \int_0^{t_{on}} V_a dt = \frac{V t_{on}}{T} = \delta V$$

The diagram shows a DC source V connected to a chopper circuit with a thyristor T and a diode D in anti-parallel. The load is a series combination of an inductor L<sub>a</sub> and a resistor R<sub>a</sub> with a back EMF E. The armature current i<sub>a</sub> is shown as a pulsating current. The graph below shows the armature voltage V<sub>a</sub> and current i<sub>a</sub> over one period T. V<sub>a</sub> is V during the 'on' time t<sub>on</sub> and 0 during the 'off' time t<sub>off</sub>. i<sub>a</sub> rises during the 'on' interval and decays during the 'off' interval. Conditions V<sub>a</sub> > 0 and I<sub>a</sub> > 0 are noted.

Now, we will begin by discussing the motoring operation, followed by the braking operation. Let's

focus on the motoring operation first, specifically in relation to a separately excited DC motor.

To illustrate this, let's draw the circuit diagram of the chopper. We have the armature of the separately excited DC motor connected to a chopper switch, which is represented by a power transistor. Accompanying this is a freewheeling diode and, of course, the DC source providing voltage  $V$ . The chopper transistor, denoted as  $T$ , is designed to conduct current in one direction only, from collector to emitter. This is the direction of current flow through the transistor when it is in the "on" state.

The armature of the DC motor is represented in this circuit with its armature resistance  $R_a$ , armature inductance  $L_a$ , and the back EMF, denoted as  $E$ . The current flowing through the armature is  $I_a$ , and the voltage appearing across the armature is  $V_a$ .

Now, let's explore the various operating modes of this chopper. When the transistor is turned on, current flows from the source through the motor and back to the source in a continuous loop. This indicates that the switch is activated. We can apply the gate drive or base drive between the emitter and the base to turn the transistor on.

When the transistor is activated, the current flows from the source, through the load, and back to the source. In this scenario, what is the armature voltage? The armature voltage  $V_a$  can be expressed as:

$$V_a = V = R_a I_a + L_a \frac{dI_a}{dt} + E$$

This phase is referred to as the "powering interval."

After some time, we switch off the transistor  $T$  while keeping the diode  $D$  in mind. When the transistor  $T$  is on, the diode  $D$  remains off, as it is reverse-biased. However, once we turn off the transistor  $T$ , the current path is interrupted, leading to no current flowing through the transistor.

At this point, while the transistor is off, the current will now find an alternative path through the diode, allowing for freewheeling action. As a result, the current that was previously flowing through the armature will now continue to flow through the diode. This operation occurs when the diode is activated; thus, the diode  $D$  becomes conducting, and this phase is referred to as the

"freewheeling interval."

During the freewheeling period, the armature voltage can be analyzed, given that the current is solely freewheeling through the diode, which we can assume to be ideal, resulting in a diode drop of zero. Hence, we have the equation:

$$R_a I_a + L_a \frac{dI_a}{dt} + E = 0$$

This describes the freewheeling interval where the armature current continues to flow through the diode. Since the armature has sufficient inductance, it stores energy, which then freewheels through the freewheeling diode D. If necessary, additional inductance can be added to the armature circuit to support this operation.

Now, if we were to draw the waveforms of voltage and current across the armature, we would observe the following patterns. The x-axis represents time, while the armature voltage  $V_a$  and the armature current  $I_a$  exhibit distinctive behavior. The current will increase and decrease in an exponential manner. In steady state, both the armature voltage  $V_a$  and the armature current will display exponential rise and fall characteristics, clearly illustrated in their respective waveforms.

Now, when the switch is turned on, we denote this duration as  $t_{ON}$ . The switch remains in the "on" position for this time period. Conversely, when the switch is off, we refer to this as  $t_{OFF}$ . Therefore, the total time for one complete cycle of the switch, denoted as  $T$ , can be expressed as:

$$T = t_{ON} + t_{OFF}$$

This is how the chopper operates, and we can define the duty cycle as the ratio of the "on" time to the total time period, represented mathematically as  $\frac{t_{ON}}{T}$ .

In our analysis, we are plotting the armature voltage and armature current. For a DC machine, we are particularly interested in drawing the torque-speed characteristic, which necessitates determining the average armature voltage and average armature current.

To calculate the average armature voltage  $V_a$ , we integrate the instantaneous armature voltage  $v_a$  over the period when it is active. The average armature voltage can be expressed mathematically

as:

$$V_a = \frac{1}{T} \int_0^{t_{ON}} v_a dt$$

Since the voltage is only finite from 0 to  $t_{ON}$ , we focus our integration over that interval. During the period from  $t_{ON}$  to  $t_{OFF}$ , the voltage is zero, so we do not need to include that in our calculations.

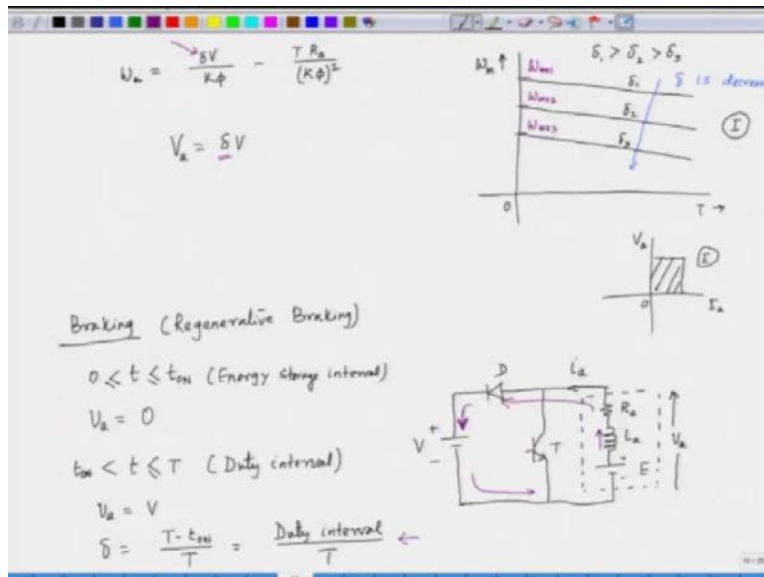
Substituting the known values, where the armature voltage during the "on" time is equal to the source voltage  $V$ , we find that:

$$V_a = \delta \cdot V$$

where  $\delta = \frac{t_{ON}}{T}$  is the duty cycle.

This equation gives us the average armature voltage of a separately excited DC motor. With this information, we can now proceed to plot the torque-speed characteristic, allowing us to analyze the performance of the motor under different operating conditions.

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Now, using this equation, we can plot the torque-speed characteristic of the DC motor fed by a single-quadrant chopper. The armature voltage can be expressed as  $\delta V$ , which leads us to the

relationship:

$$V_a = K\phi - \frac{TR_a}{K\phi}$$

In this equation,  $K$  represents a constant,  $\phi$  is the flux,  $T$  is the torque, and  $R_a$  is the armature resistance. When we plot the speed versus torque, speed is represented on the y-axis and torque on the x-axis.

To control the speed, we need to vary the duty cycle  $\delta$ . As we adjust this parameter, we will observe different plots corresponding to various values of  $\delta$ . For instance, if we decrease  $\delta$ , we will create a new plot that reflects this reduced duty cycle, and so forth.

In this context, we can denote three different duty cycles:  $\delta_1$ ,  $\delta_2$ , and  $\delta_3$ , where  $\delta_1 < \delta_2 < \delta_3$ . As  $\delta$  decreases, we find ourselves with a scenario where the armature voltage  $V_a$  also decreases. Consequently, the no-load speed, which we can refer to as  $\omega_{m0}$ , will also decline.

For example, we can denote the no-load speeds as  $\omega_{m0,1}$ ,  $\omega_{m0,2}$ , and  $\omega_{m0,3}$ . As we reduce  $\delta$ , we can observe a corresponding decrease in these no-load speed values, resulting in parallel lines in our plot.

It's essential to note that this analysis pertains specifically to the motoring operation, and this particular converter only operates within the first quadrant of the torque-speed characteristic. Hence, we classify this as a one-quadrant converter, focusing solely on the first quadrant of operation.

In this scenario, when we plot the armature voltage  $V_a$  and the armature current  $I_a$ , both are positive, which indicates that the operation is confined to the first quadrant. Specifically, this chopper or DC-to-DC converter operates exclusively in quadrant number one.

Now, let's shift our focus to the braking operation, specifically regenerative braking. In this mode, we have a different circuit configuration. Here, we include the armature resistance, inductance, and back EMF. Notably, the switch is positioned right after the armature, and the diode's position is now interchanged with that of the switch. The DC supply remains, represented by voltage  $V$ . We can denote the switch as  $T$  and the diode as  $D$ .

In this setup, the armature of the DC motor is the same as before, and we define the armature current  $I_a$  in the same direction as before. However, it's crucial to note that the current direction has reversed: during motoring, the current flows into the armature, while in braking operation, the current flows out of the motor. This change is evident in the circuit diagram.

The chopper circuit now appears as follows: the switch is in series with the voltage supply, while the diode, which was previously in parallel with the load, is now positioned in series with the source.

When the switch is turned on, we can denote this time as  $t$  where  $t > 0$  and  $t \leq t_{ON}$ . Once the switch is activated, we observe that the back EMF generates a positive voltage. This positive back EMF facilitates the circulation of current through the switch. This phase is referred to as the freewheeling interval or energy storage interval, where the energy is temporarily stored before it can be utilized for regenerative braking.

The inductance of the armature plays a crucial role in this process, allowing us to add additional inductance to the circuit. The kinetic energy stored within the inductance of the armature circuit is significant. This back EMF, representing mechanical energy, stores energy in the form of magnetic fields within the armature inductance. This phase is referred to as the energy storage interval.

Now, during this energy storage interval, what happens to the armature voltage? The armature voltage is effectively zero during this time, meaning it is not connected to the supply; in essence, the armature is short-circuited. When we say the armature is short-circuited, it indicates that the voltage across it is equal to zero.

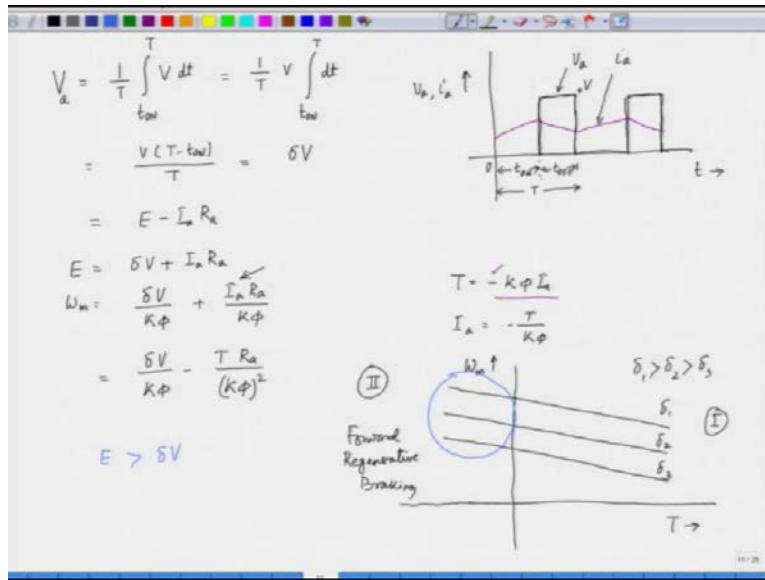
Once we turn on the switch, we can denote this time as  $t$  where  $t$  is greater than  $t_{ON}$  and less than or equal to  $T$ . This phase is referred to as the duty interval.

Now, what occurs when the switch is turned off? At this point, the current path is interrupted, breaking the circuit. With the switch off, the current stored in the inductor, the inductive energy, flows back to the source as the diode turns on. This action effectively charges the source, feeding energy back into it, and this constitutes the duty interval.

So, what about the armature voltage during this process? The armature voltage now equals the

source voltage, denoted as  $V$ . In this context, we can redefine the duty cycle based on the duty interval, which ranges from  $t_{ON}$  to  $T$ . Therefore, the duty cycle can be expressed as  $\frac{T-t_{ON}}{T}$ , representing the proportion of time the switch is off within one complete cycle.

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Now, let's take a closer look at the waveforms of the voltage and current for the circuit we just discussed. During the energy storage interval, which lasts from 0 to  $t_{ON}$ , the voltage across the armature is zero. Therefore, when we plot the armature voltage,  $V_a$ , on the y-axis against time on the x-axis, we observe that the armature voltage remains at zero while the current builds up.

This current buildup occurs because the armature is effectively short-circuited at this stage. At  $t = t_{ON}$ , when the switch is turned off, the diode comes into play, feeding the stored energy back to the source. As a result, the current begins to decrease because the supply voltage  $V$  is generally greater than the back EMF of the armature. Thus, the current has to flow against this voltage, leading to an exponential decrease in its value.

The cycle continues to repeat periodically after  $t_{ON}$ . So, if we visualize the armature current,  $I_a$ , it displays a characteristic pattern, and alongside it, we have the corresponding armature voltage waveform.

Now, let's focus on the average armature voltage. The waveform we have can be described as a

rectangular voltage waveform, which includes both a zero interval and a positive voltage interval, varying from +V to 0.

To determine the average armature voltage, we consider the time interval of  $t_{OFF}$  for the switch, with one complete cycle defined as the period T. To find the average armature voltage, we need to integrate the voltage from  $t_{ON}$  to T, during which the armature voltage is at V. This integration will give us the average armature voltage, allowing us to better understand the behavior of the circuit over time.

Let's analyze this step by step. When we integrate the average armature voltage, V, which remains constant, we can factor it out of the integration sign. This leads us to the following expression:

$$V \int_{t_{ON}}^T dt = V(T - t_{ON})$$

Now, if we divide by the time period T, we have:

$$\frac{V(T - t_{ON})}{T}$$

This expression represents the armature voltage, and we recognize that  $T - t_{ON}/T$  is the duty ratio, which we previously defined. Thus, we can rewrite this as:

$$V_a = \delta \cdot V$$

where  $\delta$  is the duty cycle. This means the armature voltage can be varied by controlling the duty cycle or duty ratio.

Next, we can formulate the complete equation relating to our circuit. The average voltage across the armature can be expressed as:

$$E - I_a R_a = V$$

Here, E is the back EMF,  $I_a$  is the armature current, and  $R_a$  is the armature resistance. If we rearrange this to find E:



$$E = V + I_a R_a$$

Now, if we analyze the speed, we can express it in terms of the voltage:

$$\omega_m = \frac{\delta V}{K\phi} + \frac{I_a R_a}{K\phi}$$

where  $K$  is a constant related to the motor's characteristics and  $\phi$  is the field flux.

Moving on to torque, the expression for torque is given by:

$$T = -K\phi I_a$$

The negative sign here indicates that we are assuming the direction of the current is opposite to the conventional direction (where current normally enters the armature). Thus, we can rewrite the current as:

$$I_a = -\frac{T}{K\phi}$$

Now, substituting this expression for  $I_a$  back into our equation gives us:

$$E = \delta V + \left(-\frac{T}{K\phi}\right) R_a$$

This simplifies to:

$$E = \delta V - \frac{T R_a}{K\phi}$$

Now, this leads us to the torque-speed characteristic of this motor. If we plot the speed on the y-axis and torque on the x-axis, we observe that the torque is negative. This means we are operating in the second quadrant of the torque-speed plane. So, we can conclude that in this configuration, the motor operates effectively in a regenerative braking mode.

In this analysis, when we plot the equation, we observe straight lines, much like before. These lines correspond to various duty cycle values, specifically  $\delta_1$ ,  $\delta_2$ , and  $\delta_3$ , indicating a decrease in

the duty cycle. We can establish that  $\delta_1 > \delta_2 > \delta_3$ .

Now, let's focus on our operational quadrant. In this scenario, we are functioning within the second quadrant of the speed-torque characteristic because the torque is negative. To clarify, the first quadrant is shown here, while the second quadrant is designated for forward regenerative braking.

Despite the equations remaining unchanged, it is crucial to emphasize that we are indeed operating in the second quadrant of the torque-speed characteristic. In this context, the torque is negative due to the current flowing out of the armature, while the speed remains positive.

An important observation to make is that the back EMF ( $E$ ) is greater than the armature voltage ( $\delta V$ ). By adjusting the duty cycle, we effectively ensure that the armature voltage is lower than the back EMF. This situation allows the motor to operate as a generator, thereby confirming that  $E$  is indeed greater than  $\delta V$ . This condition is what facilitates the regenerative braking effect.

In conclusion, we will pause our discussion here for today's lecture. So we stop here for today's lecture we will continue the chopper-fed DC motor control in the next lecture.