Fundamentals of Electric Drives

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Module No # 04

Lecture No # 17

Speed Torque Characteristic of Induction Motor, Operation of Induction Motor from Non-sinusoidal Supply

Hello and welcome to this lecture on the fundamentals of electric drives! In our last session, we began our discussion on induction motor drives, and we explored the concept of slip that occurs when the induction motor is in operation.

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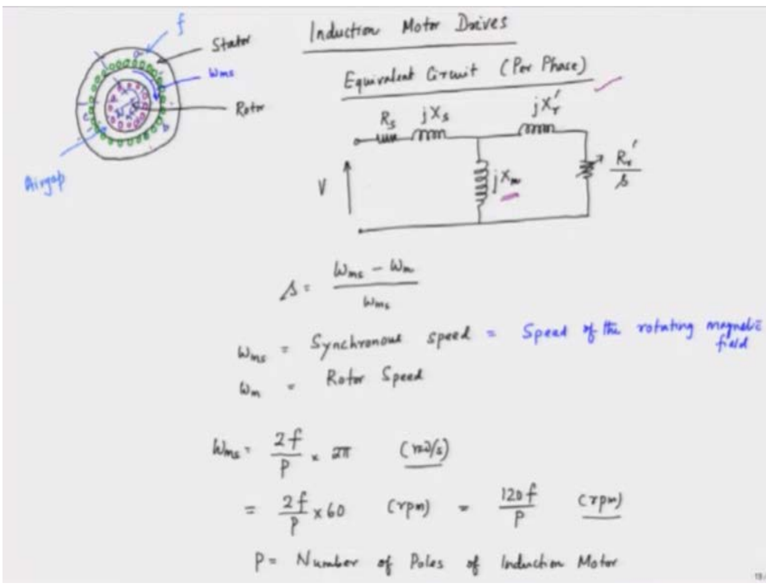

Now, the slip is defined as the ratio of $\frac{\omega_{ms}-\omega_m}{\omega_{ms}}$, where ω_{ms} represents the synchronous speed. To express ω_{ms} in radians per second, we can use the formula $\omega_{ms} = \frac{2f}{P} \cdot 2$. Conversely, if we want to define it in revolutions per minute (RPM), the equation becomes $\omega_{ms} = \frac{2f}{P} \cdot 60$, which simplifies to $\omega_{ms} = \frac{120f}{P}$. Here, the unit for ω_{ms} is RPM, while the previous expression is in radians per second.

In this context, f denotes the frequency of the input supply, which provides voltage to the motor, and P represents the number of poles in the induction motor. This is the speed at which the rotating magnetic field moves. When we apply a three-phase voltage, the magnetic field rotates sequentially from phase A to phase B to phase C. The speed of this rotation can again be expressed as $\omega_{ms} = \frac{2f}{P} \cdot 2\pi,$ in radians per second or $\omega_{ms} = \frac{120f}{P}$ in RPM.

Now, let's redraw the equivalent circuit. We have already observed that this is the equivalent circuit which can be simplified for our calculations. Although we are dealing with three phases, we can represent it in a simplified manner that is sufficient for our analysis. This is the magnetizing reactance, and sometimes this magnetizing reactance can be moved to the input side of the circuit.

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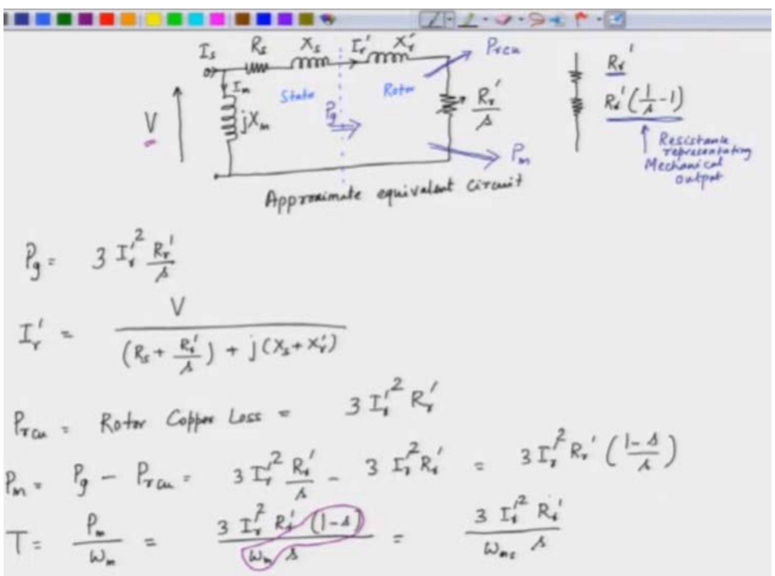

So, we derive another circuit representation that includes the resistance, reactance, rotor reactance, rotor resistance, and the magnetizing reactance, which is now moved to the input side. This is represented as jX_m, and it includes the stator resistance and the leakage reactance of the stator. Additionally, we have the rotor reactance referred from the primary side and the rotor resistance referred from the primary side, divided by the slip. In this circuit, we apply the power phase voltage, which is an alternating current (AC) voltage.

The input current is denoted as I_s, while the current flowing through the rotor is referred to as I_r', and the current flowing into the magnetizing branch is labeled I_m. This configuration represents the approximate equivalent circuit. This approximate equivalent circuit is quite accurate for calculating the torque and speed of the motor.

Now, let's explore how to determine the torque and speed. Here, we have the air gap, with one part being the stator and the other being the rotor. The induction motor can be conceptualized as a rotating transformer. The power flowing from the stator to the rotor is referred to as the air gap power, denoted as P_g.

In the rotor circuit, we observe both reactance and resistance. The active power consumed by the rotor resistance, represented as $\frac{R_r'}{S}$, contributes to the overall power dynamics. Therefore, we can express the air gap power as:

$$P_g = I_r'^2 \cdot \frac{R_r'}{S}$$

Since we are dealing with a single-phase circuit, for a three-phase system, we multiply this expression by 3.

Now, what exactly is I_r'? I_r' represents the rotor current referred from the primary side. To evaluate I_r', we consider the applied voltage V, which is the AC voltage, and we also take into account the impedance present in the circuit. Thus, we can express I_r' as:

$$I_r' = \frac{V}{R_s + R_r' + j(X_s + X_r)}$$

Here, R_s represents the stator resistance, while R_r' indicates the rotor resistance. Additionally, X_s and X_r denote the leakage reactances of the stator and rotor, respectively.

Let's delve into the rotor current, denoted as I_r'. Now, what about the copper loss? The rotor copper loss, represented as $P_{r\,cu}$, is essentially equal to the product of the rotor current and the actual rotor resistance. This resistance can be divided into two components: one is R_r', the physical rotor resistance, and the other is a fictitious resistance given by $R_r' \cdot \left(\frac{1}{S} - 1\right)$.

In the context of an electric circuit, when we aim to derive the mechanical power, we need to incorporate the concept of slip, which is a mechanical variable. This aspect represents the mechanical output. Therefore, we can consider the resistance that represents the mechanical output.

The rotor copper loss can be expressed as $I_r^2 R_r$, and since we are dealing with a three-phase circuit, we multiply this value by 3. Consequently, we have the air gap power, with a portion allocated to the rotor copper loss, $P_{r\,cu}$. Thus, the mechanical power, $P_{\text{mechanical}}$, is defined as the air gap power crossing the air gap minus the rotor copper loss:

$$P_{mechanical} = P_g - P_{r\,cu} = 3I_r^2 R_r \cdot \left(\frac{1}{S}\right) - 3I_r^2 R_r'.$$

This can be simplified to:

$$P_{mechanical} = 3I_r^2 R_r' \cdot \left(\frac{1-S}{S}\right).$$

Now, let’s discuss how to determine the torque, because ultimately, our goal is to find the motor torque. The motor torque T can be calculated as:

$$T = \frac{P_{mechanical}}{\omega_m}.$$

Substituting in the expression for mechanical power, we have:

$$T = \frac{3I_r^2 R_r' \cdot (1-S)}{\omega_m \cdot S}.$$

By replacing the term for mechanical speed, we recall that:

$$\omega_m = (1-S) \cdot \omega_{ms}.$$

When we substitute this into our equation, the (1 - S) term cancels out, yielding:

$$T = \frac{3I_r^2 R_r' \cdot \omega_{ms}}{S}.$$

This expression represents the torque of the induction motor. Now, let’s explore how we can write a more detailed expression for the torque.

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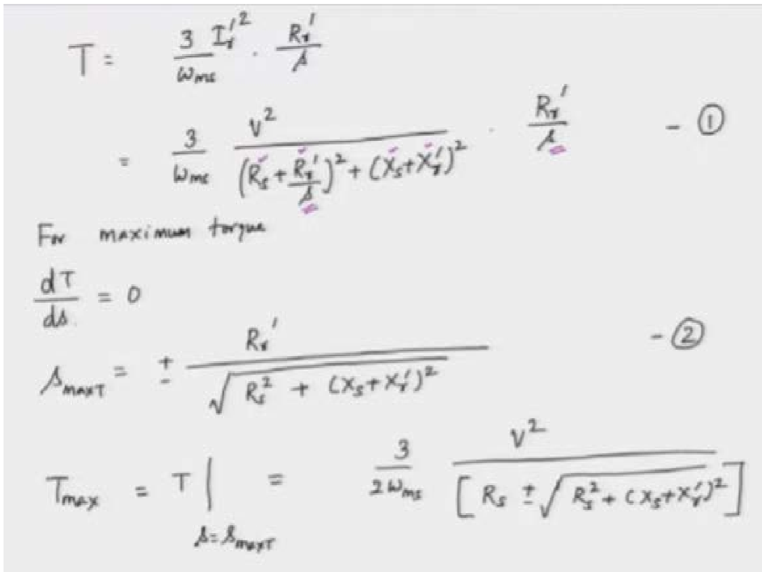

The torque output, denoted as T, is expressed as:

$$T = \frac{3I_r^2}{\omega_{ms}} \cdot \frac{R_r'}{S}.$$

Here, I_r^2 can be represented as $3 \cdot \frac{V^2}{\left(R_s+\frac{R_r'}{S}\right)^2}$. This equation reflects our quest for the absolute value of I_r. Thus, we can express I_r^2 in terms of the applied voltage V, the stator resistance R_s, and the rotor resistance referred to the primary side, R_r', along with the slip S. The complete equation becomes:

$$I_r^2 = \frac{V^2}{\left(R_s + \frac{R_r'}{S}\right)^2 + (X_s + X_r')^2}.$$

We can substitute this back into our torque equation, which allows us to write the torque of the three-phase induction motor in a more detailed form:

$$T = \frac{3 \cdot \frac{V^2}{\left(R_s + \frac{R_r'}{S}\right)^2 + (X_s + X_r')^2}}{\omega_{ms}} \cdot \frac{R_r'}{S}.$$

This equation represents the average torque output of the three-phase induction motor.

Next, to analyze the performance characteristics, we must determine if there is a maximum torque available. To find the maximum torque, we recognize that the only variable on the right-hand side of the equation is the slip S. The applied voltage, synchronous speed, frequency, stator resistance, rotor resistance, and the reactances are all fixed parameters in this scenario.

Thus, to find the maximum torque, we set the derivative of the torque with respect to slip $\frac{dT}{dS}$ equal to zero:

$$\frac{dT}{dS} = 0.$$

By performing this differentiation and simplifying the resulting expression, we can derive the slip at which maximum torque occurs, given by:

$$S_{max\,T} = \frac{R_r'}{\sqrt{R_s^2 + (X_s + X_r')^2}}.$$

This equation provides the slip for maximum torque, where $S_{max\,T}$ can take on both positive and negative values.

We find that the maximum slip is given by:

$$S_{max\,T} = \pm \frac{R_r'}{\sqrt{R_s^2 + (X_s + X_r')^2}}.$$

This implies that there are both positive and negative slip values. In fact, an induction motor can operate with a positive slip, indicating that it is running at a speed slower than the synchronous speed, or it can also function with a negative slip, suggesting that it is running faster than the synchronous speed.

Now, if we substitute this slip value into our previous torque equation, let's refer to that as Equation 1, and we label our derived slip equation as Equation 2, we can determine the maximum torque T_{max}.

To express this mathematically, we can write:

$$T_{max} = T \text{ at } S = S_{max\,T}.$$

When we substitute $S = S_{max\,T}$ into Equation 1, we arrive at the expression for maximum torque:

$$T_{max} = \frac{3V^2}{R_s \pm \sqrt{R_s^2 + (X_s + X_r')^2}} \cdot \frac{1}{2\omega_{ms}}.$$

Thus, the maximum value of torque can be represented as:

$$T_{max} = \frac{3V^2}{2\omega_{ms}} \cdot \frac{1}{R_s \pm \sqrt{R_s^2 + (X_s + X_r')^2}}.$$

With this equation, we can now proceed to plot the torque-slip characteristics of the induction motor. This plot will provide valuable insights into how torque varies with slip, offering a comprehensive understanding of the motor's performance across different operational conditions.

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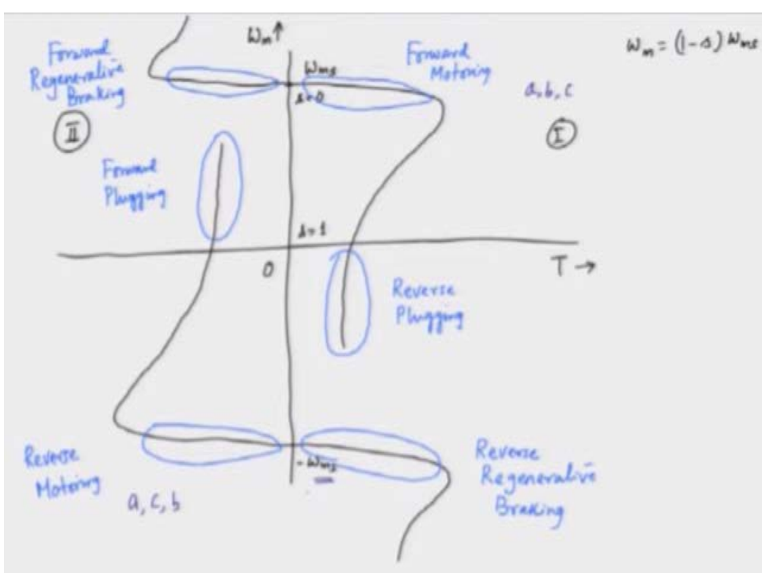

If we were to illustrate the torque-slip or torque-speed characteristics, we can plot the data derived from Equation 1 to create a visual representation. On the X-axis, we will have the torque, while on the Y-axis, we will depict the speed. It's essential to note that the speed and slip are related through the equation $\omega_m = (1 - s)\ \omega_{ms}$. This relationship means that as slip changes, so does the speed.

Let’s identify some critical points on this graph. We will denote the no-load speed as ω_{m0}, and here we will have the synchronous speed, which is represented as ω_{ms}. Typically, if we were to plot Equation 1 for various slip values, we would observe a characteristic curve. At this point, the slip value is equal to 0, while here it reaches 1, and beyond this, we see a slip value greater than 1, entering the realm of negative slip.

The area where slip is less than 1 is referred to as the forward motoring region. This is the zone where the motor generally operates, known as forward motoring. If we were to increase the motor's speed beyond the synchronous speed, we would enter the generating operation phase, commonly referred to as forward regenerative braking.

In this context, the section representing negative slip pertains to regenerative braking. When the motor operates within this range, it functions as a generator, and any energy produced will be fed back into the AC supply.

Now, let’s consider the effect of changing the phase sequence of the motor. For instance, if the initial phase sequence is A, B, C, and we alter it to A, C, B, we effectively change the direction of the rotating magnetic field. In this case, instead of having a positive ω_{ms}, the rotating magnetic field will now have a value of $-\omega_{ms}$. Consequently, this shift places us in the reverse motoring region, specifically within the third quadrant of the graph, which we will refer to as the reverse regenerative braking region. This demonstrates how altering the phase sequence not only changes the direction of the magnetic field but also transitions the operational mode of the motor.

In our analysis, we begin with the first quadrant, which represents the motoring operation of the induction motor. Moving to the second quadrant, we observe the characteristics associated with forward braking. This region is particularly notable for a phenomenon known as forward plugging, where the slip is indeed positive, indicating that the motor is still functioning in a forward direction while experiencing a braking effect.

As we extend our examination into the fourth quadrant, we encounter the concept of reverse plugging. This completes the torque-speed characteristic curve for an induction motor, encompassing all operational modes: forward motoring, forward regenerative braking, forward plugging, reverse motoring, reverse regenerative braking, and reverse plugging. The basis for these classifications stems from Equation 1, which illustrates that by assigning a negative value to the synchronous speed, or the speed of the rotating field, we can analyze the motor's behavior during reverse operations.

Now, let’s delve into the typical behavior of the motor. Generally, we supply the induction motor with a three-phase balanced supply. For instance, consider a scenario where the voltage is set at 400 volts; in this case, the phase voltage is 230 volts, with a phase sequence of A, B, and C. However, it's crucial to note that in real-world applications, this supply voltage may contain harmonics. These harmonics can significantly affect the motor's performance and must be considered in our analysis moving forward.

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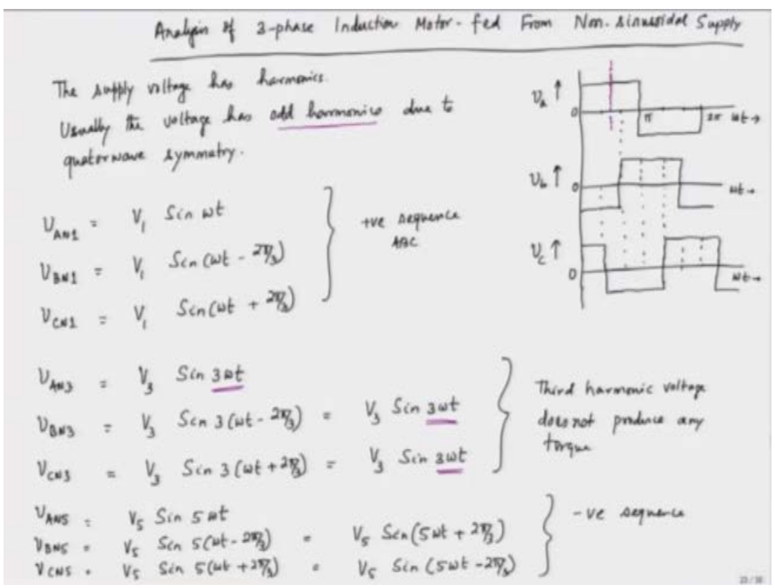

Let’s delve into the analysis of a three-phase induction motor powered by a non-sinusoidal supply, specifically one that contains harmonics. In this scenario, if we use an inverter to supply the motor, we may observe that the waveforms for phases A, B, and C are not sinusoidal. For instance, consider the voltage for phase A; instead of a smooth sine wave, it manifests as a square wave.

This waveform can be illustrated as follows: [insert visual representation here].

Now, when we shift our focus to phase B, it is important to note that it lags phase A by 120 degrees. This results in a waveform for phase B that appears like this: [insert visual representation here]. The x-axis represents ω t. Furthermore, phase C is shifted from phase B by another 120 degrees, starting from this point, leading to its unique waveform: [insert visual representation here]. Consequently, all three input voltages, phases A, B, and C, are square waves, which inherently means they contain harmonics beyond the fundamental frequency.

To analyze these harmonics, we can employ a Fourier series, allowing us to separate the fundamental frequency from the third harmonic, fifth harmonic, seventh harmonic, and so on. The square wave will predominantly consist of all odd harmonics. This occurrence is attributed to the phenomenon known as quarter-wave symmetry. In simpler terms, if we examine one-fourth of the cycle, we notice that the left-hand side is a mirror image of the right-hand side; this symmetry is crucial for our analysis.

Due to this quarter-wave symmetry, the supply voltage will be rich in odd harmonics. For instance, let's consider the voltage for phase A. The fundamental component can be expressed as $V_1 \sin(\omega t)$. In the case of phase B, the fundamental voltage is given by $V_1 \sin\left(\omega t - \frac{2\pi}{3}\right)$, while for phase C, it can be written as $V_1 \sin\left(\omega t + \frac{2\pi}{3}\right)$.

Now, when we turn our attention to the triplen harmonics, particularly the third harmonic, we can express the voltage for phase A's third harmonic as $V_{an3} = V_3$, where V_3 is the amplitude of the third harmonic voltage. This amplitude can be accurately determined through Fourier analysis.

Let's explore the nature of the third harmonic voltages in a three-phase induction motor. We start with the expression for the third harmonic voltage for phase A, which can be represented as $V_{AN3} = V_3 \sin(3\omega t)$. This indicates that the frequency of the third harmonic is three times that of the fundamental frequency.

Moving on to phase B, the third harmonic voltage can be expressed as $V_{BN3} = V_3 \sin\left(3\omega t - \frac{2\pi}{3}\right)$. In essence, this maintains the same frequency, confirming that it is $V_3 \sin(3\omega t)$ but with a phase shift. For phase C, we have $V_{CN3} = V_3 \sin\left(3\omega t + \frac{2\pi}{3}\right)$, which once again corresponds to $V_3 \sin(3\omega t)$ with a different phase shift.

The crucial point to note here is that the third harmonic voltages, V_{AN3}, V_{BN3}, and V_{CN3}, are essentially in phase with one another. This alignment means that when these voltages are applied, they do not create a rotating magnetic field. Consequently, the third harmonic voltage does not contribute to torque production in the motor.

Now, let's turn our attention to the positive sequence of voltages, which corresponds to the order a, b, c. We can similarly analyze the fifth and seventh harmonics. For the fifth harmonic, we can denote the voltages as V_{AN5}, V_{BN5}, and V_{CN5}, which can be expressed as follows:

- $V_{AN5} = V_5 \sin(5\omega t)$
- $V_{BN5} = V_5 \sin\left(5\omega t - \frac{2\pi}{3}\right)$
- $V_{CN5} = V_5 \sin\left(5\omega t + \frac{2\pi}{3}\right)$

Upon simplifying the expressions for the fifth harmonic, we find that the voltage for phase B becomes $V_{BN5} = V_5 \sin\left(5\omega t + \frac{2\pi}{3}\right)$, while for phase C, it simplifies to $V_{CN5} = V_5 \sin\left(5\omega t - \frac{2\pi}{3}\right)$.

This configuration results in what we refer to as a negative sequence, meaning the order of rotation is altered from a, b, c to a, c, b. Thus, the fifth harmonic voltages will create a rotating magnetic field in the opposite direction.

In summary, this exploration highlights how various harmonics influence the electromagnetic fields within the air gap of the induction motor. In our next class, we will continue our discussion on the effects of these harmonics on the operation of induction motors, further deepening our understanding of this complex topic. So we will continue discussing this harmonics the effect of harmonics on the operation of induction motor in the next class.