Fundamentals of Electric Drives

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Module No # 04

Lecture No # 18

Operation of Induction Motor from Non-sinusoidal Supply

Hello and welcome to this lecture on the fundamentals of electric drives! In our last session, we delved into the intriguing topic of how harmonics impact the performance of induction motors. We began our analysis of the various harmonics present in a typical non-sinusoidal waveform. Today, we will continue to explore this critical aspect of electric drives and its implications for motor operation.

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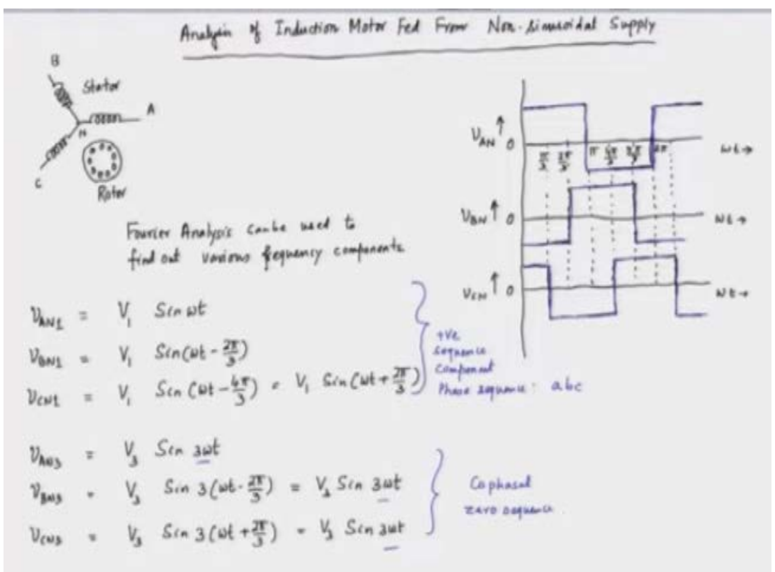

Let's continue from where we left off. Here, we have three square waves representing the output from a three-phase inverter. The first square wave is for phase A, and similarly, we have a corresponding square wave for phase B, which is shifted by 120◦ or $\frac{2\pi}{3}$ radians behind phase A. Additionally, we have phase C, denoted as V_{CN}, which is also a square wave, shifted from phase B by the same 120◦.

Thus, the three phase voltages, V_{AN}, V_{BN}, and V_{CN}, are all square waves supplied from a square wave inverter. Now, when we apply these three-phase voltages to an induction motor, we see that the motor consists of three-phase windings. These windings are designated as phase A, phase B, and phase C in the stator. For illustrative purposes, let's assume we have a neutral point in our setup.

As we apply the voltages between one phase and the neutral, we have the three-phase voltages active in the system. The rotor, which could be a short-circuited rotor, typically takes the form of a cage rotor or squirrel cage rotor.

When these three-phase voltages are applied to the stator of the induction motor, the resulting square waveforms contain numerous harmonics. Importantly, we can easily deduce that these square waveforms exhibit only odd harmonics. By employing Fourier analysis, we can break down these waveforms into their various frequency components, revealing the fundamental frequency and all the associated harmonics.

Now, let's examine the first harmonic, which is the fundamental component. We can denote this as V_{AN1}, represented by the expression $V_1 \sin(\omega t)$. Moving on to phase B, the fundamental for this phase, or the first harmonic, can be expressed as $V_{BN1} = V_1 \sin\left(\omega t - \frac{2\pi}{3}\right)$. This shift accounts for the 120° phase difference from phase A. Similarly, for phase C, we have the expression for the fundamental component as $V_{CN1} = V_1 \sin\left(\omega t - \frac{4\pi}{3}\right)$, which can also be rewritten as $V_1 \sin\left(\omega t + \frac{2\pi}{3}\right)$.

Now that we've covered the fundamental components, we can employ Fourier analysis to uncover the various harmonic components present in our waveforms. In this case, we are only dealing with odd harmonics due to the quarter-wave symmetry of the square wave.

Let's discuss the third harmonic next. The third harmonic can be expressed as $V_{AN3} = V_3 \sin(3\omega t)$. For phase B, this can be represented as $V_{BN3} = V_3 \sin\left(3\omega t - \frac{2\pi}{3}\right)$. Since phase B is shifted by 120° from phase A, this can be simplified to $V_{BN3} = V_3 \sin(3\omega t)$. For phase C, the third harmonic is expressed as $V_{CN3} = V_3 \sin\left(3\omega t + \frac{2\pi}{3}\right)$.

If we place this in parentheses, we note that $V_{CN3} = V_3 \sin(3\omega t + 2\pi)$. Since the sine function is periodic with a period of 2π, we can simplify this to just $V_3 \sin(3\omega t)$.

What we observe here is that the fundamental component represents a positive sequence, where the phase sequence follows the order of A, B, and C. This pattern repeats, creating a sequence of A, B, C, and so on.

Now, let’s turn our attention to the third harmonic. When we analyze it, we notice that all three phases, A, B, and C, share the same phase of 3ω t. Consequently, we can conclude that these are in phase with each other, indicating that this represents a zero-sequence component. This means that phases A, B, and C are all aligned in the same phase.

Next, let’s take a closer look at the fifth harmonic, which, like the others, will also consist of only odd harmonics present in our analysis.

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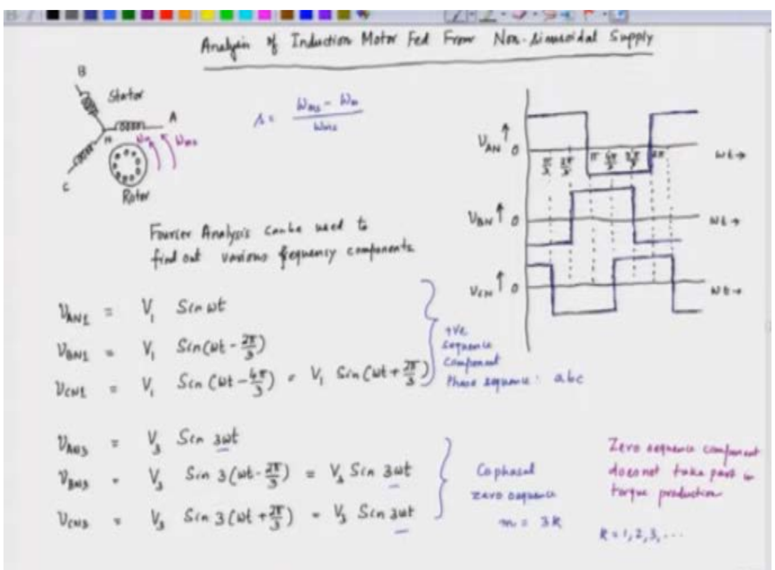

The fifth harmonic has a distinct characteristic, represented as V_{AN5}. We can employ Fourier transform techniques to determine the amplitude of the fifth harmonic, denoted as $V_5 \sin(5\omega t)$. For phase B, the expression for the amplitude of the fifth harmonic is $V_{BN5} = V_5 \sin\left(5\omega t - \frac{2\pi}{3}\right)$. Simplifying this yields $V_{BN5} = V_5 \sin\left(5\omega t - \frac{10\pi}{3}\right)$, which, upon further simplification, resolves to

$V_5 \sin(5\omega t)$ due to the periodic nature of the sine function.

Now, for phase C, the fifth harmonic voltage can be expressed as $V_{CN5} = V_5 \sin\left(5\omega t + \frac{2\pi}{3}\right)$. This indicates that phase C is shifted 120° ahead of 5ω t. Thus, we can rewrite this as $V_{CN5} = V_5 \sin\left(5\omega t - \frac{2\pi}{3}\right)$.

At this point, we observe that the fifth harmonic represents a negative sequence component. This is evident because phase B is leading phase A by $\frac{2\pi}{3}$, while phase C is lagging behind phase A by the same amount. Therefore, the phase sequence for the fifth harmonic is A, C, B.

Now, let's move on to the seventh harmonic. The voltage for the seventh harmonic can be denoted as $V_{AN7} = V_7 \sin(7\omega t)$. For phase B, the expression becomes $V_{BN7} = V_7 \sin\left(7\omega t - \frac{2\pi}{3}\right)$, which simplifies to $V_{BN7} = V_7 \sin\left(7\omega t - \frac{14\pi}{3}\right)$. When we simplify this further, it becomes $V_7 \sin\left(7\omega t - \frac{2\pi}{3}\right)$.

For phase C, we express the seventh harmonic voltage as $V_{CN7} = V_7 \sin\left(7\omega t + \frac{2\pi}{3}\right)$. Simplifying this, we have $V_{CN7} = V_7 \sin\left(7\omega t + \frac{2\pi}{3}\right)$, ensuring that we keep our phase angles within the 0 to 360° range.

By analyzing the various voltages of the seventh harmonic across phases A, B, and C, we find that the phase sequence follows A, with phase B lagging behind phase A by $\frac{2\pi}{3}$ or 120°. This further highlights the relationships and dynamics present in the harmonic analysis of the induction motor.

Similarly, we find that phase C is leading phase A by $\frac{2\pi}{3}$ or 120°. In other words, we can also express it as phase C lagging behind phase A by $\frac{4\pi}{3}$ or 240°. Thus, in this scenario, the phase sequence remains positive, consistent with the fundamental sequence, which is A followed by B, followed by C.

Now, let's discuss a general formula for identifying harmonic orders. For any positive sequence component, if m represents the order of the harmonic, we can state that:

$$m = 6k + 1$$

Here, k can take values from 0, 1, 2, and so on. For negative sequence components, the relationship changes to:

$$m = 6k - 1$$

In this case, k cannot be 0; it must start from 1 and can continue with 2, 3, etc. When referring to the fundamental harmonic, we note that it represents the case where m = 0. Therefore, for harmonics, we can identify them as 1, 2, and so forth.

To summarize, when we have m = 6k - 1, we derive negative sequence components or harmonics that produce a negative sequence field. Conversely, when m = 6k + 1, we obtain positive sequence voltages, defining the order of the harmonics in this context.

Now, if we revisit the discussion regarding the triplen harmonics, we note that they are expressed by the formula:

$$m = 3k$$

where k can take values of 1, 2, 3, and so on. It’s important to highlight that triplen harmonics are all in phase, and they represent zero sequence voltages.

In our analysis, we've observed that certain harmonics produce zero sequence components, while others generate positive or negative sequence components. Out of these, we categorize the three sequence components as positive, negative, and zero, each playing a critical role in the overall behavior of the induction motor system.

The zero-sequence component does not produce a rotating field, which means it does not contribute to torque production. In essence, all harmonics that belong to the zero-sequence category are unable to participate in generating torque. Now, let’s explore what occurs when the rotor is rotating at a speed of ω_m, while we have a stator field rotating at its own speed.

The stator field moves from point A to point B to point C; this is how we visualize our stator field in motion. We refer to this as the synchronous speed, denoted as ω_{ms}. Meanwhile, the rotor speed follows this pattern as it rotates. To define the slip, we can express it mathematically as:

$$\text{Slip} = \frac{\omega_{ms} - \omega_m}{\omega_{ms}}.$$

This definition of slip is applicable when we have only one frequency present, specifically a positive sequence component. However, when we introduce both positive and negative sequence components, we encounter different slips associated with each sequence component, as well as variations for different harmonics. This complexity highlights the intricate dynamics at play in the operation of induction motors, particularly in scenarios where multiple frequency components are present.

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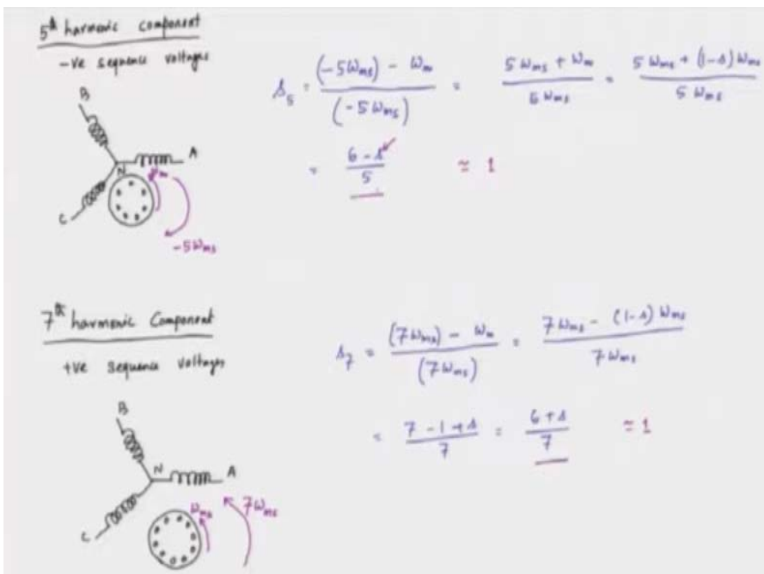

Now, let’s delve into what occurs to the field when we apply the fifth harmonic component. When the fifth harmonic is introduced, it generates a negative sequence component, resulting in negative sequence voltages. This means that the speed of the rotating field undergoes a significant transformation.

To visualize this, consider our stator phases: A, B, and C. When we apply the fifth harmonic voltage, the rotation of the field progresses from phase A to phase C and then to phase B. This rotation follows a clockwise direction, indicating a negative rotation. We can express this rotational speed as $-5\omega_{ms}$, which represents the speed of the rotating field. Meanwhile, the rotor is functioning according to the normal convention, rotating at the speed ω_m.

Now, let's define the slip concerning the fifth harmonic. In this scenario, we multiply by 5 because we are dealing with a negative sequence component. Therefore, the slip is defined as:

$$\text{Slip} = \frac{\omega_{ms} - \omega_m}{\omega_{ms}}.$$

In this case, it simplifies to:

$$\text{Slip} = \frac{5\omega_{ms} - \omega_m}{5\omega_{ms}}.$$

This indicates that the rotation is in the negative direction, giving us $-\omega_m$. By definition, this can be expressed as $\omega_{ms} - \omega_m$ divided by the synchronous speed, leading us to:

$$\text{Slip} = \frac{5\omega_{ms} + \omega_m}{5\omega_{ms}}.$$

To better analyze this, we can replace ω_m using our previous expressions. By making simple mathematical adjustments, we find that:

$$\omega_m = (1 - s)\omega_{ms},$$

where ω_{ms} represents the synchronous speed, and ω_m is the rotor speed.

Now, substituting this value of ω_m into our equation yields:

$$\text{Slip} = \frac{5\omega_{ms} + (1 - s)\omega_{ms}}{5\omega_{ms}}.$$

After performing some cancellations, the ω_{ms} terms simplify, resulting in:

$$\text{Slip} = \frac{6 - s}{5}.$$

This expression captures the slip for the fifth harmonic component. By analyzing this equation, we can ascertain the value of slip from the normal rotation, demonstrating that for the fifth harmonic component, the slip is expressed as:

$$\text{Slip} = \frac{6 - s}{5}.$$

Now, let's delve into the details regarding the seventh harmonic component. When we analyze the slip associated with this harmonic, we observe that it yields a value that is close to 1, typically hovering around 1.1, which suggests it is slightly greater than 1 but not excessively distant from it.

Now, focusing on the seventh harmonic component, we recognize that it serves as a positive sequence component. This means that when we apply these positive sequence voltages to the stator of the induction motor, comprising phases A, B, and C, along with a neutral point, the rotor, designed with short-circuited bars in a squirrel cage configuration, will respond accordingly.

As the seventh harmonic voltage is introduced, the stator field rotates from phase A to phase B and then to phase C. The rotation of the seventh harmonic field occurs at a frequency of $7 \cdot \omega_{ms}$, where ω_{ms} represents the synchronous speed. The rotor generally rotates in the same direction due to the influence of the positive sequence component, which aligns with the fundamental component.

Now, let's calculate the slip for the seventh harmonic voltage. The slip is defined as the difference between the synchronous speed and the rotor speed, divided by the synchronous speed. Here, the synchronous speed is $7 \cdot \omega_{ms}$, so we express it mathematically as follows:

$$\text{Slip} = \frac{7 \cdot \omega_{ms} - \omega_m}{7 \cdot \omega_{ms}}.$$

We can substitute ω_m using the expression $\omega_m = (1 - s) \cdot \omega_{ms}$. When we do this substitution, we have:

$$\text{Slip} = \frac{7 \cdot \omega_{ms} - (1 - s) \cdot \omega_{ms}}{7 \cdot \omega_{ms}}.$$

Upon simplification, this yields:

$$\text{Slip} = \frac{7-(1-s)}{7} = \frac{6+s}{7}.$$

This value, $\frac{6+s}{7}$, indicates that the slip for the seventh harmonic component is still a quantity that is less than 1, reaffirming that it is not far removed from unity. In fact, it remains quite close to 1.

Therefore, we conclude that the slip for the higher-order harmonics, such as the seventh harmonic, tends to be close to 1, paralleling the slip observed for the fifth harmonic component.

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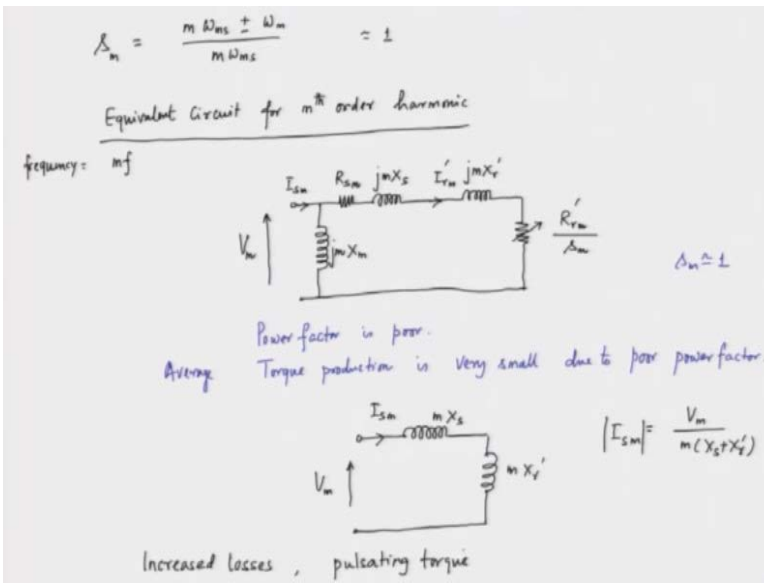

We can derive a general formula to determine the behavior of any harmonic m. For the mth harmonic, we express this as follows:

$$\text{Slip} = \frac{m \cdot \omega_{ms} \pm \omega_m}{m \cdot \omega_{ms}}.$$

Here, the plus sign corresponds to the negative sequence component, while the minus sign is used for the positive sequence component. Notably, for harmonics of higher order, the slip tends to be close to 1. Now, let's explore the implications of this on the operation of the induction motor.

To analyze the effects on the induction motor's performance, we need to utilize the equivalent circuit specifically for the mth order harmonic. It’s essential to clarify that our discussion will not cover the fundamental component but will focus exclusively on the mth order harmonic. The frequency associated with this harmonic is m times the base frequency f.

Next, we construct the equivalent circuit, which includes elements such as the stator resistance, the stator leakage reactance, the rotor leakage reactance, and the rotor resistance. In this equivalent circuit, the stator resistance is denoted for the mth order harmonic, and we represent the reactance as X_s. However, since we are dealing with the mth order harmonic, the reactance will adjust to $m \cdot X_s$.

For the rotor, the reactance is denoted as X_r', which becomes $m \cdot X_r'$ for this context. Regarding the rotor resistance R, we refer it from the stator side, adjusting it to account for the slip s_m associated with the mth order harmonic.

Additionally, we need to consider the magnetizing reactance, which we denote as $m \cdot X_m$, and we include a j to signify that this is indeed a reactance. The input voltage across the phases is denoted as V_m.

In this configuration, we identify the input current as I_{sm} and the rotor current as I_{rm}. It's crucial to note that when m is a large quantity, the equivalent circuit predominantly behaves as a reactive circuit. This indicates that the reactive components significantly influence the overall performance of the induction motor under these harmonic conditions.

The magnetizing reactance is indeed substantial, allowing us to represent it primarily as a reactance. We previously discussed that the slip s_m is close to 1, indicating that the slip is relatively high in this scenario. Given that the rotor resistance is a small quantity, we can neglect the resistance of the circuit in comparison to the reactance. Furthermore, since we are dealing with a reactive circuit, the power factor becomes quite poor.

This poor power factor results in negligible torque production; therefore, the average torque generated is very minimal due to this unfavorable power factor. Consequently, harmonics do not significantly contribute to any average torque production. As a result, we can simplify this equivalent circuit to primarily consist of the reactance of the stator and the reactance of the rotor, while we can disregard the magnetizing reactance due to its considerable size.

In this simplified model, the input voltage is denoted as V_m, and we can express the current I_{sm} as follows:

$$I_{sm} = \frac{V_m}{m \cdot X_s + X_r'},$$

where we focus on the magnitude of the current.

Now, what is the effect of these harmonics? The presence of harmonics leads to an increase in the losses within the system. These losses rise significantly, and additionally, we experience torque pulsations. The interaction of various rotating fields creates pulsating torque, further complicating the motor's operation.

In summary, the effects of harmonics on the operation of the induction motor include increased losses and pulsating torque. It's important to note that there is very little average torque contribution from these harmonic components. Instead, the harmonic components primarily contribute to heating the motor by elevating both core losses and copper losses. Consequently, the motor must be de-rated when the input supply is not sinusoidal. So we stop here today lecture we will continue out discussion in the next lecture.