Fundamentals of Electric Drives

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Module No # 04

Lecture No # 19

Stator Current of Induction Motor with Non-Sinusoidal Supply, Operation of Induction Motor with Unbalanced Voltage Supply

Hello, and welcome to this lecture on the fundamentals of electric drives. In our previous session, we discussed the effects of harmonics on the operation of an induction motor, and today, we will continue exploring that topic in further detail. Let's build on what we've covered and deepen our understanding of how harmonics impact motor performance.

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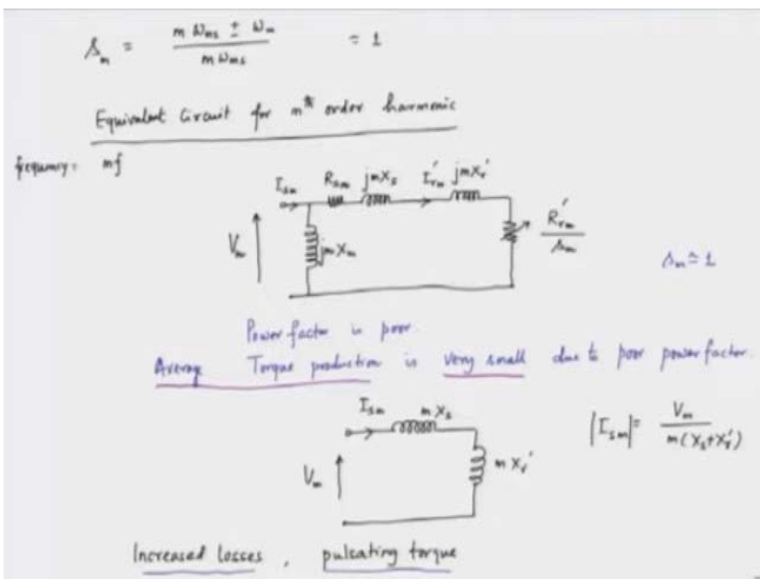

As we've previously discussed, harmonics primarily increase the losses in the motor and introduce pulsating torque, but they do not contribute to any significant average torque. In fact, the contribution to average torque from harmonics is negligible, meaning that the overall average torque remains very small. Now, when the motor is powered by a non-sinusoidal supply, calculating the total current becomes essential. The total current will be the sum of the fundamental current along with the various harmonic components. Each harmonic adds its own component to the overall current, which collectively influences the motor's performance.

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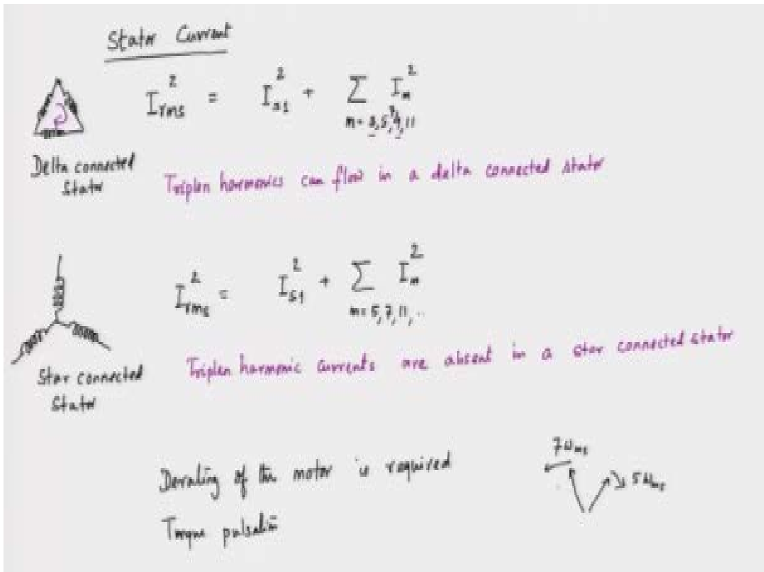

So, when harmonics are present in the system, the total stator current can be represented as the root mean square (RMS) current, denoted as I_{rms}. The square of the total RMS current is equal to the square of the fundamental component plus the sum of the squares of the various harmonic components. Mathematically, we can express this as:

$$I_{\text{rms}}^2 = I_s^2 = I_{s1}^2 + \sum_{m=3,5,9,11,\ldots} I_m^2$$

where I_{s1} represents the fundamental component, and I_m represents the harmonic components.

Now, the configuration of the stator plays an important role in determining the behavior of these harmonics. If the stator is delta-connected, meaning the windings are connected in a delta configuration, all harmonics, including the triplen harmonics (multiples of 3 like 3rd, 9th, 15th), can circulate within the stator. These triplen harmonics will flow through the delta configuration, creating circulating currents that do not require a neutral path.

However, if the stator is star-connected, meaning the windings are connected in a star (or wye) configuration, there is no path for triplen harmonics to flow. This is because triplen harmonics are co-phasal, requiring a neutral path to circulate. In the absence of a neutral connection in a star-connected stator, these triplen harmonics are effectively blocked, and only the non-triplen odd harmonics, like the 5th, 7th, and 11th, will contribute to the stator current.

For the star-connected stator, the total RMS current can be represented as:

$$I_{\mathrm{rms}}^2 = I_{s1}^2 + \sum_{m=5,7,11,\ldots} I_m^2$$

To find the total harmonic current, we must calculate the equivalent harmonic circuit for each harmonic, sum the squares of the harmonic components, and take the square root. This will give us the total RMS current, which is inevitably higher than the fundamental component alone. As a result, this increased current causes additional losses in the motor, leading to reduced efficiency and the need for de-rating the motor.

De-rating means the motor cannot supply its full rated power due to increased losses from harmonic currents, especially core and copper losses. Furthermore, the interaction between different harmonics (e.g., the 7th harmonic rotating in the forward direction and the 5th harmonic in the backward direction) generates pulsating torque. This pulsating torque, combined with the fundamental component, leads to noisy operation of the motor.

Lastly, when discussing induction motors, it is essential to consider the impact of an unbalanced supply voltage. While we typically assume that the stator is supplied with balanced three-phase voltages, real-world scenarios often involve unbalanced supplies. The effect of such unbalanced voltages on motor operation is significant, and this is what we will delve into next.

The operation of an induction motor under unbalanced voltage conditions introduces some critical effects on the system. Let's imagine we have the stator of the induction motor, and the motor is being supplied by unbalanced voltages across phases A, B, and C. This means that the voltages are not perfectly shifted by 120° from each other, and their magnitudes might not be the same. So, instead of balanced supply voltages, we have an unbalanced set, with the voltages across each phase denoted as V_a, V_b, and V_c.

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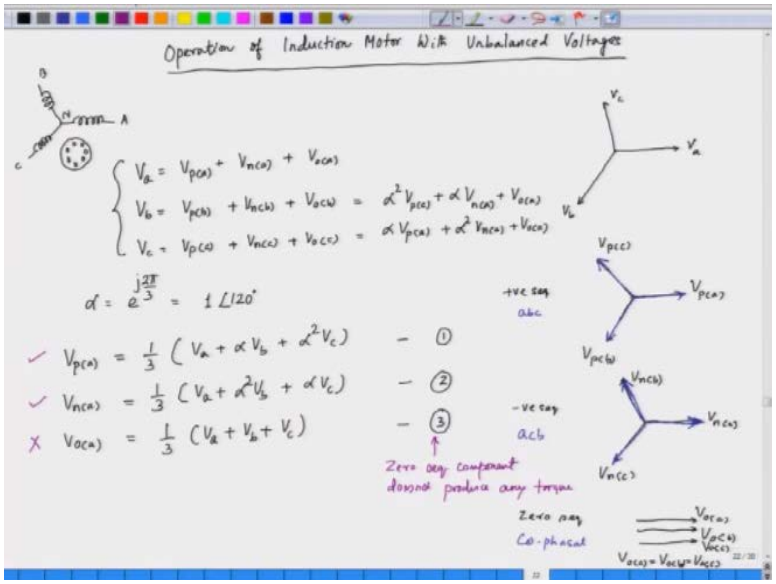

When the supply voltage is unbalanced, what happens? We can break this unbalanced voltage system into three distinct components:

1. Positive Sequence Component: These voltages correspond to a balanced three-phase set that rotates in the normal direction (A → B → C).

2. Negative Sequence Component: These voltages form a balanced set that rotates in the opposite direction (A → C → B).

3. Zero Sequence Component: Here, the voltages in all three phases are in phase, meaning they are co-phasal.

Now, breaking down these components:

- Positive Sequence Component: For phase A, we denote this as V_{pa}, for phase B as V_{pb}, and for phase C as V_{pc}.
- Negative Sequence Component: Similarly, we have V_{na}, V_{nb}, and V_{nc} for phases A, B, and C, respectively.
- Zero Sequence Component: These are represented as V_{0a}, V_{0b}, and V_{0c}, and all of them are equal because they are co-phasal.

So, even though the original voltage system is unbalanced, we can express it as the sum of these positive, negative, and zero sequence components. For instance, the voltage V_a of phase A can be written as the sum of the positive, negative, and zero sequence components for that phase:

$$V_a = V_{\mathrm{pa}} + V_{\mathrm{na}} + V_{\mathrm{0a}}$$

Similarly, for phase B and phase C:

$$V_b = V_{\mathrm{pb}} + V_{\mathrm{nb}} + V_{\mathrm{0b}}$$

$$V_c = V_{\mathrm{pc}} + V_{\mathrm{nc}} + V_{\mathrm{0c}}$$

Now, focusing on the sequence of rotations:

- The positive sequence follows the usual sequence of A → B → C, with phase B lagging behind phase A by 120°, and phase C lagging behind phase B by another 120°.
- The negative sequence, on the other hand, reverses the direction of rotation. Here, phase C lags behind phase A by 120°, and phase B lags behind phase C by another 120°. So, the sequence is A → C → B, rather than A → B → C.
- The zero sequence is co-phasal, meaning all phases (A, B, and C) are in the same phase, rotating together without any phase shift.

We can further describe this mathematically by introducing the operator α, which is a rotation operator defined as:

$$\alpha = e^{j\frac{2\pi}{3}}$$

This represents a 120° phase shift. Using α, we can express the phase relationships. For example, phase B can be obtained by rotating phase A by 240° (i.e., α^2), and phase C can be expressed as another rotation by α.

Finally, for the zero sequence component, because $V_{\mathrm{0a}} = V_{\mathrm{0b}} = V_{\mathrm{0c}}$, there is no distinction between the phases, they all carry the same voltage.

The voltage V_{0a} is the same as V_{0b} and V_{0c}, meaning all zero-sequence components are identical.

Now, let's look at the third equation in this sequence analysis. It involves α V_{pa} for V_c, plus α^2 V_{na}, and V_{0a}. From this equation, we can determine the sequence components V_{pa}, V_{na}, and V_{0a}, which represent the positive, negative, and zero sequence components, respectively.

So, after analyzing the set of equations, we can rearrange and solve to obtain the individual sequence components. When simplified, the results are as follows:

The positive sequence component V_{pa} is given by:

$$V_{pa} = \frac{1}{3}(V_a + \alpha V_b + \alpha^2 V_c)$$

The negative sequence component V_{na} is:

$$V_{na} = \frac{1}{3}(V_a + \alpha^2 V_b + \alpha V_c)$$

The zero sequence component V_{0a} is:

$$V_{0a} = \frac{1}{3}(V_a + V_b + V_c)$$

These equations help us decompose an unbalanced set of voltages into positive, negative, and zero sequence components. This is essential because when we have an unbalanced voltage system, it's difficult to directly determine its impact on the induction motor's operation. By using sequence components, we break the unbalanced voltages V_a, V_b, and V_c into their respective components and find the sequence voltages using the three equations described above.

To explain further:

- The first equation gives us the positive sequence component using V_a + α V_b + α^2 V_c, representing the normal rotating field (A → B → C).
- The second equation gives us the negative sequence component as V_a + α^2 V_b + α V_c, representing the opposite rotating field (A → C → B).
- The third equation gives us the zero sequence component as V_a + V_b + V_c, which is co-phasal and does not contribute to rotation.

These three sequence components are applied to the induction motor. Since we assume a linear system (ignoring the effects of saturation), we can apply the principle of superposition. This means we can treat the voltages independently and analyze the effect of each sequence component on the motor's operation.

Now, we know that the zero sequence component does not contribute to torque production because it does not create a rotating field. As a result, we can eliminate the zero sequence component from further consideration and focus solely on the positive and negative sequence components. These two are the ones that influence the motor's operation, particularly during starting and normal functioning.

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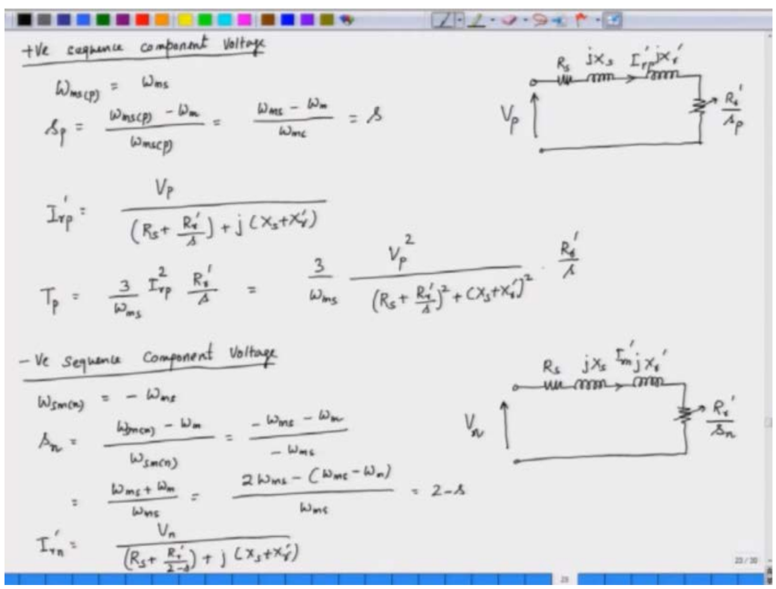

Let's now examine the positive sequence component in more detail. When we apply the positive sequence component voltage, the equivalent circuit remains the same as the standard circuit used for analyzing the induction motor. This circuit consists of the stator resistance, stator reactance, rotor reactance, and rotor resistance. In this case, we are applying V_p, which is the phase voltage, across the stator.

In this equivalent circuit, we have:

- The stator resistance R_s,

- The stator reactance X_s,
- The rotor resistance R_r', and
- The rotor reactance X_r'.

The slip associated with the positive sequence component is denoted as s_p, which is essentially the same as the normal slip of the induction motor. The positive sequence voltage generates a positive sequence rotating field, and the synchronous speed for this field is ω_{ms}, the same as the synchronous speed of the induction motor.

The slip for the positive sequence component is given by:

$$s_p = \frac{\omega_{ms} - \omega_m}{\omega_{ms}}$$

where ω_{ms} is the synchronous speed, and ω_m is the rotor speed. Since $\omega_{ms} = \omega_{msp}$, this simplifies to the standard slip s, as expected for the positive sequence component.

Now, we can derive the rotor current I_{rp} for the positive sequence component. This current is given by the following expression:

$$I_{rp} = \frac{V_p}{R_s + \frac{R_r'}{s_p} + j(X_s + X_r')}$$

Next, the torque for the positive sequence component can be calculated. The expression for torque is:

$$T_p = \frac{3I_{rp}^2 R_r'}{s_p \omega_{ms}}$$

Substituting I_{rp} into the equation, the torque becomes:

$$T_p = \frac{3V_p^2}{\left(R_s + \frac{R_r'}{s_p}\right)^2 + (X_s + X_r')^2} \cdot \frac{R_r'}{\omega_{ms}}$$

Since the torque produced by the positive sequence component is positive, it contributes to the normal operation of the induction motor.

Now, let's consider the negative sequence component. When we apply the negative sequence component voltage, we use a similar equivalent circuit as before. However, this time the sequence is negative. The negative sequence component generates a field that rotates in the opposite direction compared to the positive sequence field.

In this case, the synchronous speed of the negative sequence field is $-\omega_{ms}$, i.e., it rotates in the reverse direction. Therefore, the slip for the negative sequence component is:

$$s_n = \frac{\omega_{ms} - \omega_m}{\omega_{ms}}$$

Substituting $\omega_{ms} = -\omega_{ms}$ into the equation, we get:

$$s_n = \frac{-\omega_{ms} - \omega_m}{-\omega_{ms}} = \frac{\omega_{ms} + \omega_m}{\omega_{ms}}$$

This simplifies to:

$$s_n = 2 - s$$

Thus, the slip for the negative sequence component is 2 - s.

The rotor current for the negative sequence component, I_{rn}, can now be expressed as:

$$I_{rn} = \frac{V_n}{R_s + \frac{R_r'}{2 - s_n} + j(X_s + X_r')}$$

By analyzing both the positive and negative sequence components, we can better understand how each sequence affects the operation of the induction motor. The positive sequence contributes to normal torque production, while the negative sequence, due to its opposite rotating field and different slip, can cause undesirable effects such as increased losses and reduced motor efficiency. This analysis highlights the importance of understanding and managing sequence components, especially in cases of unbalanced voltage conditions.

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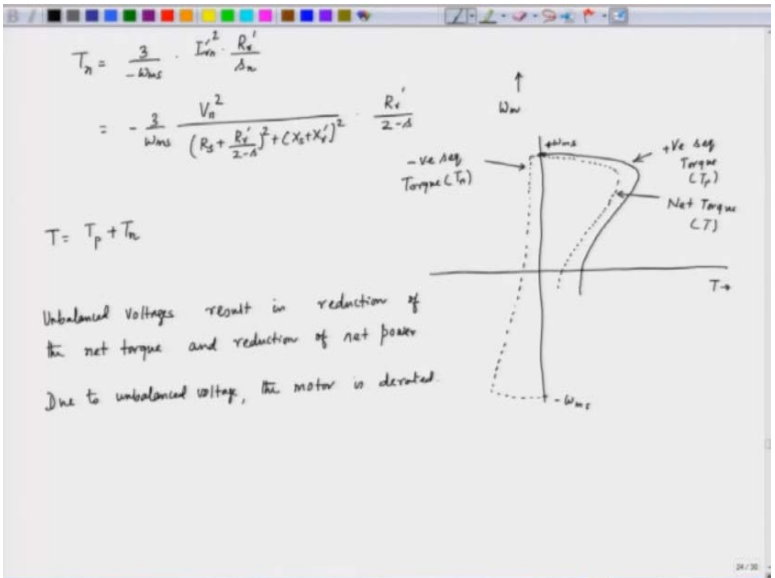

Let's now consider the torque produced by the negative sequence component. The torque for the negative sequence is given similarly to the positive sequence, but with key differences due to the opposing direction of rotation. The expression for the negative sequence torque is:

$$T_n = \frac{3}{-\omega_{ms}} \cdot I_{rn}^2 \cdot \frac{R_r'}{s_n}$$

This can be expanded as:

$$T_n = -\frac{3V_n^2}{\left(R_s + \frac{R_r'}{2-s}\right)^2 + (X_s + X_r')^2} \cdot \frac{R_r'}{2-s}$$

Here, the negative sign indicates that the torque produced by the negative sequence component is negative because the field rotates in the opposite direction. The synchronous speed for this sequence is also negative, which contributes to the negative torque.

Now, let's visualize this in terms of the torque-speed characteristics. On the y-axis, we have speed, and on the x-axis, torque. The positive sequence voltage generates the typical torque-speed curve, with the synchronous speed at ω_{ms} and the corresponding positive sequence torque T_p.

On the other hand, the negative sequence voltage generates a torque-speed curve with a negative synchronous speed, represented as $-\omega_{ms}$, and a corresponding negative torque T_n.

Now, the total torque experienced by the motor is the sum of these two torques: T_p from the positive sequence and T_n from the negative sequence. When we combine these two, the overall torque, or net torque T, is given by:

$$T = T_p + T_n$$

Because T_n is negative, the net torque is reduced compared to the torque that would have been generated by the positive sequence component alone. This reduction in torque means that under unbalanced voltage conditions, the motor's total torque is lower, leading to a decrease in net power output as well.

Thus, the unbalanced voltage supply results in:

1. A reduction in net torque,

2. A reduction in net power output.

As a consequence, the motor's performance is compromised, and it can no longer supply its full rated power. This necessitates derating the motor, meaning it has to operate at a lower power rating than under balanced voltage conditions.

In summary, this analysis demonstrates the impact of an unbalanced voltage supply on the operation of an induction motor. The presence of both positive and negative sequence components causes a reduction in torque and power, requiring the motor to be derated. This concludes our discussion for this lecture. We'll continue with further details in the next session.