Fundamentals of Electric Drives

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Lecture – 02

Dynamics of Electric Drives, Four Quadrant Operation, Equivalent Drive Parameters

Hello and welcome to our lecture on the fundamentals of electric drives! In our previous session, we provided a brief introduction to electric drives and explored their numerous advantages. Today, we will discuss the dynamics of electric drives, examining how they operate and respond under various conditions. Let's get started!

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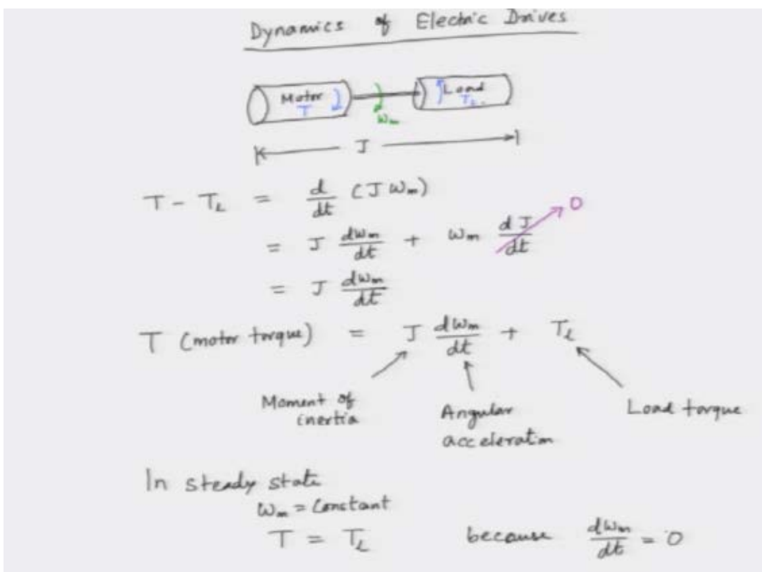

Now, let's explore the relationship between speed and torque in our system. We have a motor that is mechanically coupled to a load, and we need to examine how these components interact. In this setup, the motor produces a torque T in one direction, while the load generates an opposing torque, denoted as T_l. This load torque resists the motion of the motor.

The speed of the system is aligned with the direction in which the motor is attempting to drive the load, indicating that both speed and torque are acting in the same direction. To establish the dynamic equation for this system, we start by noting that the total moment of inertia is represented by J. As this is a rotational system, the motor is responsible for driving the mass of the load.

The fundamental dynamic equation governing this relationship can be expressed as follows:

$$T - T_l = \frac{d}{dt}(J \cdot \omega_m)$$

Here, ω_m represents the speed of the motor-load combination, and $J \cdot \omega_m$ is the angular momentum of the system. The left-hand side of the equation signifies the net torque acting on the system, while the right-hand side represents the rate of change of angular momentum.

If we expand the derivative on the right-hand side, we obtain two distinct terms:

$$T - T_l = J\frac{d\omega_m}{dt} + \omega_m \frac{dJ}{dt}$$

This formulation captures the essence of the dynamics in our electric drive system, illustrating how the motor and load interact to affect the overall motion.

In typical scenarios, the moment of inertia is considered constant over time, which allows us to simplify our analysis by assuming that the rate of change of inertia is zero. Consequently, we can set the term $\frac{dJ}{dt} = 0$. As a result, the equation simplifies to:

$$T - T_l = J\frac{d\omega_m}{dt}$$

From this, we can also express the motor torque T as:

$$T = J\frac{d\omega_m}{dt} + T_l$$

Here, J represents the moment of inertia, while $\frac{d\omega_m}{dt}$ is referred to as the angular acceleration. The term ω_m denotes the speed of the motor, and its derivative gives us the angular acceleration. The term T_l signifies the load torque.

This equation is crucial as it holds true under both steady-state and transient conditions. Now, let's examine what happens in the steady-state scenario. In this state, the motor speed ω_m remains constant. If the speed is constant, we can conclude that:

$$T = T_l$$

This is because $\frac{d\omega_m}{dt} = 0$; there is no angular acceleration in the steady state. Thus, when the motor operates at a constant speed, we refer to this condition as steady state. In this context, the inertial torque $J\frac{d\omega}{dt}$ becomes zero, indicating that the motor torque is precisely counterbalanced by the load torque. Consequently, the speeds of the motor and the load remain in equilibrium, with their respective torques opposing each other.

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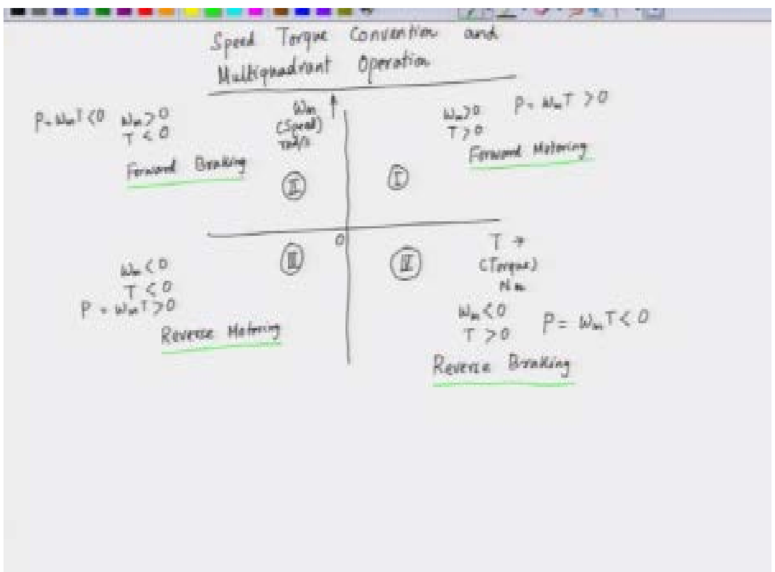

Now, let's delve into the speed-torque convention and the multi-quadrant operation of electric drives. We have established that in electric drives, two key mechanical variables come into play: speed and torque. It is essential to control both the speed and torque of the motor, as these outputs are critical to the motor's performance.

When we represent these two variables on a graph, we can plot them on an xy-plane. In this convention, speed is typically placed on the y-axis, while torque is represented on the x-axis. Therefore, when we draw the speed-torque characteristic, we have speed on the y-axis and torque on the x-axis.

Let's denote the mechanical speed as ω_m and torque as T. In the SI system, speed is expressed in radians per second, while torque is represented in Newton-meters. This gives us our origin point, and the graph is divided into four quadrants.

In the first quadrant, both speed and torque are positive. When we consider power, which is the product of speed and torque, this too is positive. Hence, we refer to operation in the first quadrant as forward motoring.

Now, moving to the second quadrant, we find that while the speed remains positive, the torque becomes negative. This is because the second quadrant lies on the left-hand side of the graph, where torque takes on negative values.

When the torque is negative, the power, which is defined as the product of speed and torque, also becomes negative. This scenario occurs in what we refer to as the second quadrant, known as forward braking. Here, the negative power indicates that energy is flowing from the motor back to the source, causing the motor to act as a generator. Consequently, we describe this condition as forward braking since the motor is effectively decelerating. In this quadrant, the torque is reversed, reinforcing the concept of forward braking.

Moving on to the third quadrant, we observe that both speed and torque are negative. In this case, since both variables are negative, the power, which is still the product of speed and torque, becomes positive. This positive power flow indicates that energy is moving from the source to the motor, which is described as reverse motoring. Despite the negative speed, the motor is actively motoring, thus the term "reverse motoring" is used to characterize this situation.

In the fourth quadrant, the dynamics change once again. Here, we find that the speed is negative while the torque remains positive. As a result, we must consider the power, which is still the product of speed and torque; in this case, it results in negative power. When power is negative, it indicates braking, and given that the speed is also negative, we refer to this phenomenon as reverse braking. Thus, we see how the interplay of speed and torque across these quadrants defines the operational modes of the motor.

The first quadrant characteristic is defined as reverse braking. When we refer to it as braking, we imply that the power is negative, indicating that the motor is functioning like a generator. Conversely, when we speak of motoring, the power becomes positive, signifying that the energy flows from the electric source to the motor.

Now, let's explore an example of this multi-quadrant operation. We have four distinct quadrants: Quadrant 1 represents forward motoring, Quadrant 2 is designated for forward braking, and Quadrant 3 corresponds to reverse motoring. This framework illustrates how the motor can operate in various modes depending on the torque and speed conditions.

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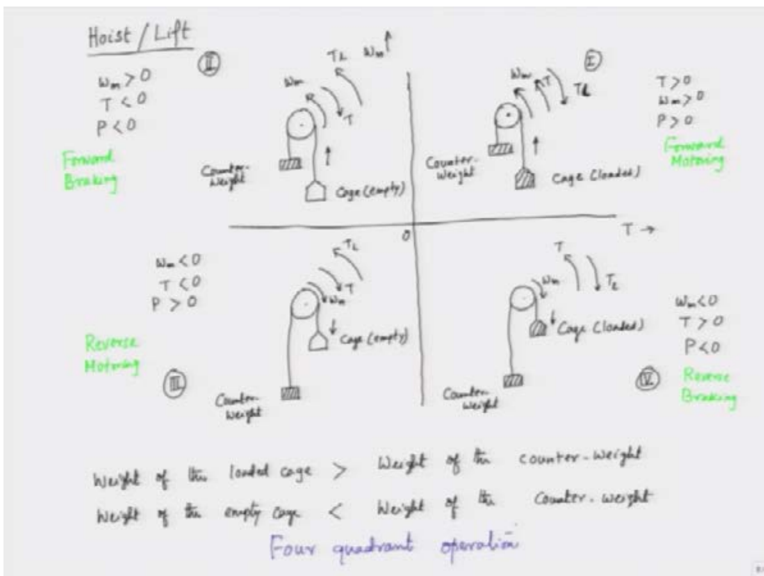

Quadrant 4 is characterized as reverse braking. Now, let’s delve into a practical example of this four-quadrant operation. Consider a hoist, which can simply be referred to as a lift that transports either a person or materials. In examining the hoist, we aim to understand its operation across the four different quadrants.

To visualize this, we establish our origin, with speed represented on the y-axis and torque on the x-axis. Picture a pulley mechanism with a loaded hoist. Additionally, there is always a counterweight, this is the counterweight in our setup. The hoist's cage, which carries the load, is balanced by this counterweight, and the motor is coupled with the pulley.

In Quadrant 1, which we’ll identify as our first quadrant, we are focused on lifting the loaded cage. To achieve this, the motor must rotate in the anticlockwise direction, representing the speed of the motor. Our condition here is that the weight of the loaded cage is greater than the weight of the counterweight. This implies that the load torque is directed downwards.

The load torque is attempting to drive the system in a clockwise direction, and it inherently opposes the motor torque. If we denote the motor torque as T, it acts in the opposite direction to the load torque T_l. Here, T and ω_m (the speed of the motor) are aligned in the same direction. Thus, both T and ω_m are positive, leading to positive power output. This situation exemplifies forward motoring, a concept we have already discussed in detail.

In the second quadrant, we again have the pulley mechanism to which the motor is fixed, but this time, the cage is unloaded, there’s no material inside. The counterweight remains in place, and our condition here is that the weight of the empty cage is less than the weight of the counterweight. So, we have the empty cage here, along with the counterweight, and the motor continues to rotate in the anticlockwise direction as we attempt to lift the empty cage.

The motion occurs as follows: this is the speed of the motor, denoted as ω_m. However, the load torque acts in the downward direction, opposing the motor torque. Thus, the motor torque works against the load torque. While ω_m remains positive, indicating that we are still lifting the cage, the torque has now reversed due to the empty nature of the cage. As a result, the power becomes negative, which we refer to as forward braking.

Having established forward motoring in the first quadrant, we now recognize forward braking in the second quadrant.

Now, let’s transition to the third quadrant. In this quadrant, which we identify as Quadrant 3, we again observe our pulley mechanism. Here, we are focused on loading the cage with the counterweight in place while the cage itself remains empty. In this scenario, we are lowering the empty cage.

The speed of the motor is now in the clockwise direction. The counterweight is heavier than the empty cage, resulting in a load torque acting in the anticlockwise direction. The motor torque, as before, will oppose the load torque, thus functioning in the opposite direction. In this third quadrant, the speed is negative, indicating that we are now lowering the cage. This dynamic illustrates how the system operates differently across the quadrants, depending on the loading conditions and motor behavior.

In this scenario, the torque is also negative, which results in the power being positive. Since power is the product of torque and speed, we refer to this situation as reverse motoring, essentially, motoring in the reverse direction.

Now, let’s examine the fourth quadrant. Here, we have our pulley mechanism, and we are focused on lowering a loaded cage. This cage is no longer empty; it is now carrying a load, with the counterweight positioned accordingly. As we lower this loaded cage, the speed of the motor rotates in the clockwise direction, which is considered negative in this context.

Given that the loaded cage exerts more weight, the load torque acts in a downward direction. Consequently, the motor torque must oppose this load torque, resulting in the motor torque acting in the opposite direction. In this quadrant, we observe that while the speed is negative, the torque remains positive. However, because the speed is reversed and negative, the power is also negative, which we identify as reverse braking.

These four quadrants demonstrate the different operational modes: Quadrant 1 represents forward motoring, Quadrant 2 is forward braking, Quadrant 3 is reverse motoring, and Quadrant 4 signifies reverse braking. When a drive is capable of operating seamlessly across all four quadrants, we refer to it as a four-quadrant drive, and we describe this entire operation as a four-quadrant operation.

This four-quadrant operation signifies that the drive can effectively operate in forward motoring, forward braking, reverse motoring, and reverse braking, all within the speed-torque plane. Now, let's consider a scenario where the motor and the load are not directly coupled but are instead connected through gears or a pulley system. We need to determine how to calculate the combined equivalent inertia and torque of this entire combination as seen by the motor.

Today, we will be discussing the equivalent drive parameters. To start, let's consider a motor that is directly coupled to a load. This load will be referred to as Load 0. The motor is connected directly to this load, which exerts a torque denoted as T_{L0}. The motor's speed is represented by ω_m, and the moment of inertia for this combined system is J_0.

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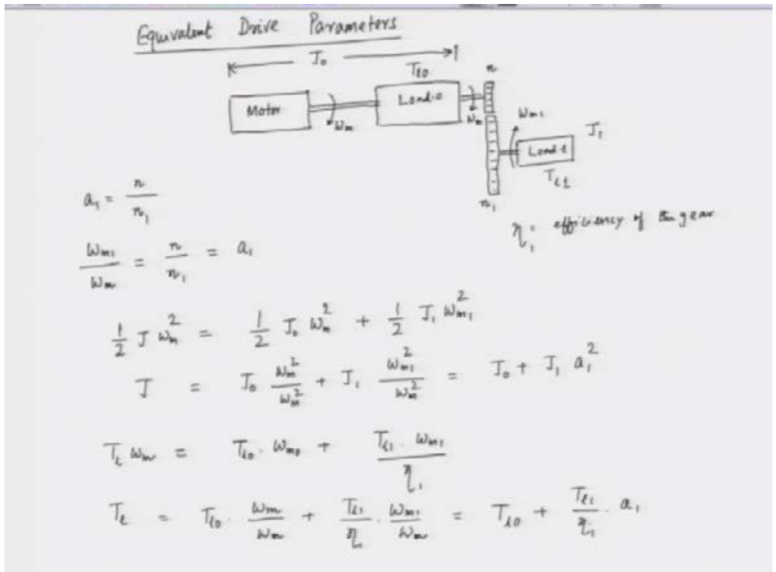

Now, this motor-load combination is connected to another system via mechanical gears. In this setup, we have Load 1, which is driven by the motor through a gear mechanism. The gear on the motor side has n teeth, while the gear on the load side has n_1 teeth. Consequently, the motor speed ω_m drives the load at a different speed, denoted as ω_{m1}. This discrepancy arises because the gear ratio is not equal to 1.

The inertia of Load 1 is represented as J_1, and the torque exerted by Load 1 is indicated as T_{L1}. Given this configuration, we need to determine the equivalent inertia that the motor perceives and the equivalent torque it experiences as a result of this gear coupling. Understanding these parameters is crucial for analyzing the performance and dynamics of the entire system.

Let's begin by discussing the gear ratio. The gear ratio is defined as the number of teeth on the motor side (n) divided by the number of teeth on the load side (n_1), expressed as $\frac{n}{n_1}$. When we consider the speed ratio, we can relate it to the motor speeds: $\frac{\omega_{m1}}{\omega_m} = \frac{m}{m_1}$.

This means that the speed of the system is inversely proportional to the number of teeth on the gears. If the number of teeth is greater, the speed will be lower; conversely, if the number of teeth is fewer, the speed of that particular system will be higher. Therefore, we can state that the ratio of the speeds is the inverse of the number of teeth, thus giving us $\frac{\omega_{m1}}{\omega_m} = \frac{m}{m_1}$, which we will refer to as a_1.

Next, let's explore the equivalent inertia of the entire system. To determine the equivalent inertia, we must consider the kinetic energy of the entire system. When the motor rotates, the load also rotates, resulting in a certain amount of kinetic energy. The equivalent kinetic energy can be expressed as:

$$KE = \frac{1}{2} J \omega_m^2.$$

In this case, the total kinetic energy consists of the kinetic energy from both parts: the first part, or the 0th part, has a kinetic energy of

$$KE_0 = \frac{1}{2} J_0 \omega_m^2,$$

and the second part contributes with

$$KE_1 = \frac{1}{2} J_1 \omega_{m1}^2.$$

Thus, the equation that represents the total kinetic energy of the entire system is:

$$KE = \frac{1}{2} J_0 \omega_m^2 + \frac{1}{2} J_1 \omega_{m1}^2.$$

To find the equivalent moment of inertia J from this equation, we can manipulate it by moving ω_m^2 to the right-hand side:

$$J = \frac{J_0 \omega_m^2}{\omega_m^2} + \frac{J_1 \omega_{m1}^2}{\omega_m^2}.$$

This formulation allows us to calculate the equivalent inertia seen by the motor in relation to the entire system, giving us a clearer understanding of its dynamics and energy distribution.

We can express the total inertia seen by the motor as:

$$J = J_0 + J_1 \cdot a_1^2,$$

where a_1 is the ratio $\frac{\omega_{m1}}{\omega_m}$. Now, let’s shift our focus to the power perceived by the motor. The power seen by the motor can be represented as:

$$P = T_L \cdot \omega_m.$$

In this scenario, we have two different loads. Load 0 is directly coupled to the motor, and its power can be expressed as $T_{L0} \cdot \omega_m$.

When mechanical gears are involved, it’s essential to consider efficiency, which is never 100%. We denote the efficiency of the gear as η or η_1. Consequently, the power seen by the motor is greater than the power delivered to Load 1. This can be articulated as:

$$P = \frac{T_{L1} \cdot \omega_{m1}}{\eta_1}.$$

Now, if we manipulate this equation by bringing ω_m to the right-hand side, we have:

$$T_L = \frac{T_{L0} \cdot \omega_{m0}}{\omega_m} + \frac{T_{L1}}{\eta_1} \cdot \frac{\omega_{m1}}{\omega_m}.$$

Since ω_{m0} is equivalent to ω_m, we can simplify this to:

$$T_L = T_{L0} + \frac{T_{L1}}{\eta_1} \cdot a_1.$$

This expression gives us the equivalent load torque seen by the motor.

Now, if we consider multiple couplings connected to the motor, each with varying gear ratios, denoted as a_1, a_2, a_3, a_4, and corresponding efficiencies η_1, η_2, η_3, η_4, we can derive a general equation. This equation will allow us to represent the equivalent inertia and equivalent torque for the entire system in a comprehensive manner. Thus, we can elegantly express the dynamics of multiple couplings and their effects on the motor's performance.

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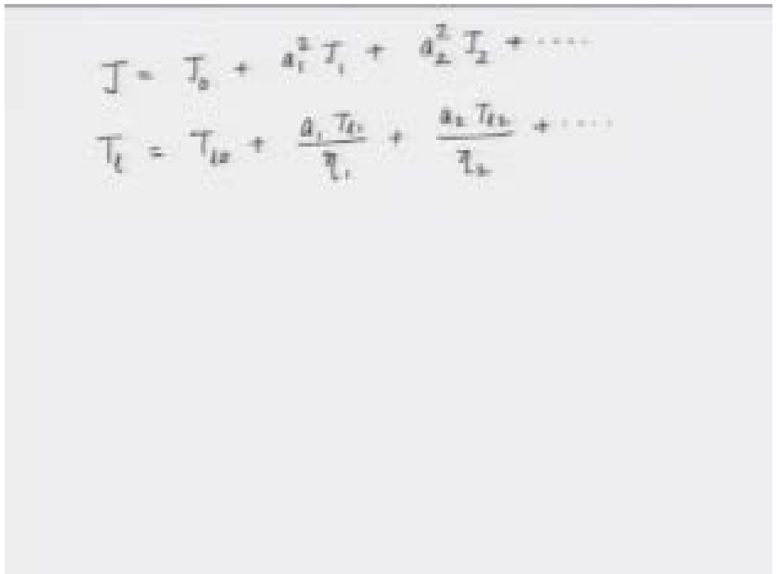

The general equation for the equivalent inertia J can be expressed as:

$$J = J_0 + a_1^2 J_1 + a_2^2 J_2 + \cdots,$$

where a_1, a_2, etc., represent the gear ratios. In a similar fashion, the equivalent load torque can be formulated as:

$$T_L = T_{L0} + \frac{a_1 T_{L1}}{\eta_1} + \frac{a_2 T_{L2}}{\eta_2} + \cdots,$$

where η_1 and η_2 are the efficiencies of gears 1 and 2, respectively.

We have already explored how to derive the equivalent moment of inertia and equivalent load torque when the motor is coupled with the load through gears.

With this understanding, we will conclude today's lecture. We will continue our discussion in the next session.