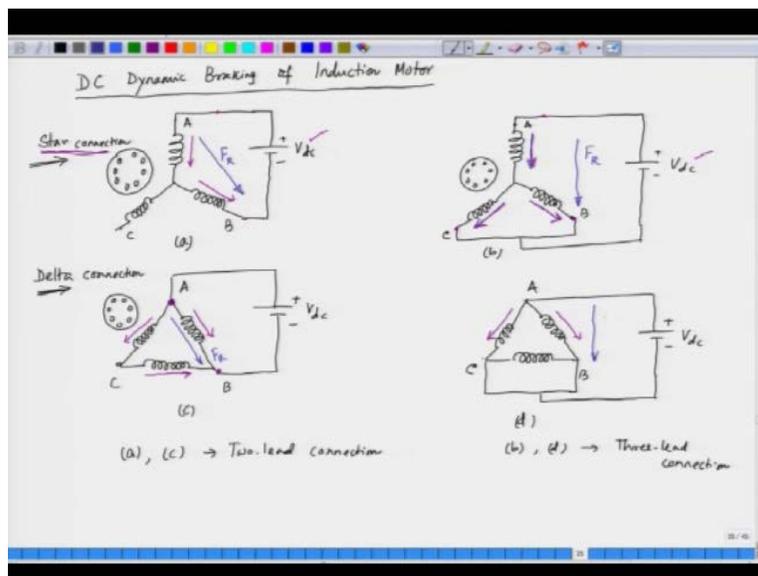


**Fundamentals of Electric Drives**  
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**Lecture-22**

**Analysis of DC Dynamic Braking of Induction Motor**

Hello and welcome to this lecture on the fundamentals of electric drives! In our previous lecture, we discussed the concept of DC dynamic braking for induction motors. To break an induction motor, we apply a DC voltage to the stator instead of the usual AC. This application of DC causes the motor to gradually slow down until it ultimately reaches a speed of zero. Now, let's explore how we achieve this DC dynamic braking in an induction motor.

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Now, let's examine our circuit, which features two types of stator connections, specifically, a star-connected stator. In this configuration, we remove the AC supply and apply DC to the stator, utilizing just two leads: A and B. The phase C lead is left open, meaning we do not apply any voltage to it.

Alternatively, we can set up another configuration by applying the voltage between phase A and

phase B, while short-circuiting phases B and C. In this setup, we again apply DC to the stator.

When we do this, we create a DC magnetic field, which generates several torque effects. Phase A generates a magnetomotive force (MMF) in one direction, while phase B produces an MMF in the opposite direction. The vector addition of these two MMFs results in a net MMF directed accordingly. It's important to note that although we are adding two DC MMFs vectorially, the resulting MMF is not rotating; rather, we have a stationary field.

Now, when we switch to a three-lead connection, where all three phases A, B, and C are connected, we see that each phase produces its own MMF. Phase A generates an MMF in one direction, phase B produces an MMF along a different axis, and phase C produces its MMF along yet another axis.

Now, if we analyze these three magnetomotive forces (MMFs) vectorially, we identify the MMF for phase A, the MMF for phase B, and the MMF for phase C. When we add these three vectors together, we arrive at a resultant MMF, which we denote as  $F_R$ . This resultant MMF is aligned along a specific axis, as we've discussed earlier.

In a three-lead connection, we also observe this resultant MMF,  $F_R$ . Furthermore, we can arrange a delta-connected stator, which is simply a matter of how we configure the phase connections.

With a delta-connected stator, we can create similar circuits. For a two-lead connection, we apply the DC voltage between phases A and B, leaving phase C disconnected. As a result, the current flows through phases A and B, with part of the current directed in one way and another part in a different direction. The resultant of these three currents aligns along the A-B axis, forming the resultant current.

Now, when we consider a three-lead connection, the configuration looks like this: we have the current coming from the DC supply, and this current flows into the system. Since phases B and C are short-circuited, they won't carry any current, leading to a resultant that points in a specific direction. This is our resultant MMF. At this stage, the motor is operational, but it is important to note that the MMF we are dealing with is now a DC field.

To better understand the situation, let's draw the equivalent circuit of an induction motor under DC dynamic braking conditions. In this circuit, we start with the rotor as our primary component.

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Motor speed =  $\omega_m$

$\omega_{ms}$  = Synchronous speed when a balanced ac voltage is applied in the three phase stator.

$E_r$  = Induced emf in the rotor in standstill condition when a normal three phase voltage is applied in the stator.

= Induced emf in the rotor under dc dynamic braking when an equivalent stationary field is produced by the stator - and the rotor speed is  $\omega_{ms}$

If the rotor speed is  $\omega_m < \omega_{ms}$ , the rotor induced emf =  $\frac{\omega_m E_r}{\omega_{ms}} = s E_r$  where  $s = \frac{\omega_m}{\omega_{ms}}$

$\Delta = \frac{\omega_{ms} - \omega_m}{\omega_{ms}} = 1 - \frac{\omega_m}{\omega_{ms}}$  or,  $\frac{\omega_m}{\omega_{ms}} = 1 - \Delta = s$

The diagram shows an equivalent circuit with a Stator winding connected to a source E. The Rotor winding is connected in series with its resistance  $R_r$  and reactance  $sX_r$ . The rotor current is  $I_r$ . A separate diagram shows a rotor with a downward arrow labeled  $F_r$  and a curved arrow labeled  $\omega_m = \omega_{ms}$ .

In this equivalent circuit, we have the rotor circuit represented here, complete with rotor reactance and rotor resistance. Additionally, we have the stator circuit, which remains isolated from the rotor. At this point, let's denote the motor speed as  $\omega_m$ , measured in radians per second.

Now, when we apply a normal three-phase voltage to the stator, a rotating magnetic field is established, achieving a synchronous speed denoted as  $\omega_{ms}$ . This rotating field is generated when a balanced AC voltage is supplied to the three-phase stator. Under these conditions, if the rotor remains stationary, an induced electromotive force (EMF), labeled  $E_r$ , occurs within the rotor.

Thus, we can state that  $E_r$  represents the standstill induced EMF that results from applying a normal three-phase voltage to the stator of an induction machine. It's crucial to recognize that when this normal three-phase voltage is applied, the stator's magnetomotive force (MMF) rotates at the synchronous speed of  $\omega_{ms}$ .

The relative velocity between the stator's magnetomotive force (MMF) and the rotor is given as  $\omega_{ms}$ . Now, when we apply a DC voltage, this generates a DC MMF. Consequently, the same induced EMF will also appear in the stator when the rotor speed is  $\omega_{ms}$ . This implies that the induced EMF in the rotor under DC dynamic braking is equivalent to the situation where a stationary field is produced by the stator.

So, we can assert that the induced EMF in the rotor during DC dynamic braking occurs when an equivalent stationary field is generated by the stator, and importantly, the rotor speed is not zero; it is, in fact,  $\omega_{Ms}$ . This means that braking is initiated while the rotor is still rotating at this speed. In this scenario, since the rotor is moving at  $\omega_{Ms}$  and the field remains stationary, the relative speed between the MMF and the rotor continues to be  $\omega_{Ms}$ .

Now, let's visualize the rotor, which may be a cage rotor. As we've established, there exists a resultant field that is a DC field. At this point, if the rotor is rotating at a speed of  $\omega_M$ , which could differ from  $\omega_{Ms}$ , we need to determine the induced EMF,  $E_R$ , in that case.

When the rotor speed,  $\omega_M$ , is not equal to  $\omega_{Ms}$  or is less than  $\omega_{Ms}$ , we can express the induced EMF in the rotor as  $E_R \cdot \frac{\omega_M}{\omega_{Ms}}$ . This relationship is quite intuitive: when the rotor speed equals  $\omega_{Ms}$ , the induced EMF is simply  $E_R$ . However, when the rotor speed is different, specifically when it is less than  $\omega_{Ms}$ , the induced EMF becomes  $\omega_M \cdot \frac{E_R}{\omega_{Ms}}$ .

We can also represent this as  $S \cdot E_R$ , where  $S$  is defined as  $\frac{\omega_M}{\omega_{Ms}}$ . Additionally, we can relate  $S$  to the slip of the induction machine. The slip is calculated as

$$S = \frac{\omega_{Ms} - \omega_M}{\omega_{Ms}}$$

When we simplify this, we find that

$$S = 1 - \frac{\omega_M}{\omega_{Ms}}$$

This allows us to express  $\frac{\omega_M}{\omega_{Ms}}$  as  $1 - s$ , which leads us to the conclusion that at any rotor speed  $\omega_M$ , the induced EMF can be expressed as  $S \cdot E_R$ .

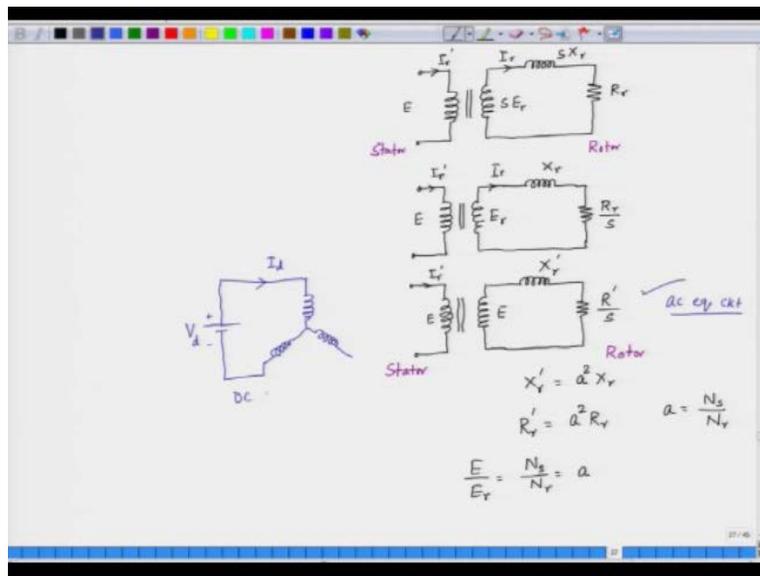
Now, concerning frequency, it is also proportional to the slip, given by

$$s \cdot F,$$

because the rate of change of flux linkage occurs at a frequency of  $s \cdot \omega_{Ms}$ .

Thus, we can complete the equivalent circuit with the induced EMF expressed as  $E_R \cdot s$ , where  $X_R$  represents the rotor reactance and the frequency in this context is  $s \cdot F$ . Therefore, the expression for the rotor resistance  $R$  and the rotor current  $I_R$  can be incorporated into the circuit. An induction machine functions similarly to a rotating transformer, where the stator's induced EMF, denoted as  $E$ , serves as the primary voltage. Thus, we can succinctly rewrite this equivalent circuit.

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What we have right now is an equivalent circuit resembling a transformer. In this circuit, we have an ideal transformer along with reactance and resistance components:  $R_R$ ,  $s \cdot X_R$ , and  $s \cdot E_R$ . The current flowing through this circuit is the rotor current, denoted as  $I_R$ , while the induced EMF is represented by  $E$ .

To maintain the same current, we modify the circuit by dividing both the induced voltage and the impedances by  $S$ . By doing this, we can create our modified equivalent circuit, where we still have  $E_R$  and the same rotor current  $I_R$ . However, we have adjusted the reactance to  $X_R$  because we have divided everything by  $S$ . Consequently, the resistance is expressed as  $\frac{R_R}{S}$ . This setup is quite similar to the rotor circuit of an induction machine, but here, we specifically use the capital  $S$  rather than the small  $s$ .

Next, we refer everything to the primary side of the circuit. When we do this, we can denote the

induced EMF as  $E$ , and we equalize the number of turns. Thus, we have  $E$  on the primary side, and the reflected current from the rotor, which we call  $I_{R'}$ , also appears in this configuration. This reflected current mirrors the earlier reflected current, maintaining the relationship with the stator circuit. Therefore, we consistently refer to the rotor circuit, ensuring that  $I_{R'}$  remains representative in both scenarios.

When we equalize the number of turns, we refer the parameters to the primary side, just as we would with a transformer. By doing so, we arrive at the following equivalent circuit. This circuit comprises  $\frac{R'_R}{s}$  and  $X_{R'}$ , and the induced EMF is represented as  $E$ . Here, we have equalized the number of turns, leading us to denote the reactance as  $X_{R'}$ . Now, what exactly is  $X_{R'}$ ? It can be expressed as  $A^2 \cdot X_R$ .

So, what does  $A$  represent? The turns ratio  $A$  is defined as the number of stator turns divided by the number of rotor turns per phase. Similarly, the resistance referred from the primary side is given by  $A^2 \cdot R_R$ , where  $A$  is defined as  $\frac{N_S}{N_R}$ . We can also express the relationship between the induced EMF as  $\frac{E}{E_R} = \frac{N_S}{N_R} = A$ . This shows that the induced EMF is proportional to the number of turns.

Now that we have this equivalent circuit, let's explore how the stator current is produced. We have the rotor present, and the stator current is generated not by a three-phase AC supply but rather by a DC current. To illustrate this, consider a simple two-lead connection for a star-connected stator. The stator current is produced by applying a DC voltage to the stator in the following manner: we have the voltage  $V_D$ , and this generates the stator current, denoted as  $I_D$ .

In our equivalent circuit, the stator current we've derived is essentially an AC current, meaning we can refer to this as an AC equivalent circuit. However, it is important to note that this is ultimately a DC circuit. Therefore, we must establish a relationship between the DC and AC circuits. To do this, we equate the magnetomotive forces (MMFs) or determine the equivalent MMF that would be produced in the case of a DC circuit. Now, let's equalize the MMFs for both DC and AC scenarios and derive the stator current in terms of the DC current.

Now, when we take the equivalent of an AC circuit and equalize the magnetomotive force (MMF),

let's begin by drawing a two-lead connection for a star-connected stator. We establish a connection like this, where we apply the voltage between two terminals, perhaps between phase A and phase B. The voltage applied here is  $V_D$ , and the current flowing through the winding is denoted as  $I_D$ .

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Equivalent ac current by equalizing mmf:

Two-lead connection  
Star-connected stator

$V_d$   $I_d$   $N_s$   $N_s$   $N_s$

$F_R$   $N_s I_d$   $120^\circ$   $N_s I_d$

$$F_R = \sqrt{(N_s I_d)^2 + (N_s I_d)^2 - 2(N_s I_d)^2 \cos 120^\circ}$$

$$= \sqrt{(N_s I_d)^2 + (N_s I_d)^2 + (N_s I_d)^2} = \sqrt{3} N_s I_d$$

$F_s = \text{Peak value of mmf}$

$$= \frac{3}{2} N_s I_s \sqrt{2}$$

$$\sqrt{3} N_s I_d = \frac{3}{2} \sqrt{2} N_s I_s \Rightarrow I_s = \sqrt{\frac{2}{3}} I_d$$

$I_a$   $I_b$   $I_c$   $I_s = \text{rms value}$

In this case, the number of turns per phase is  $N_s$ . So, what MMF is produced here? The MMF generated will be a combination of the MMFs produced by both phases. If we visualize the MMF vectorially, one MMF vector points in one direction, while the other MMF vector points in another direction. The magnitude of each MMF is given by  $N_s \cdot I_D$ .

The resultant of these two MMFs forms the third side of a triangle, which we can depict here. In this triangle, we have an angle of  $120^\circ$  because we are dealing with a three-phase stator. Now, let's calculate  $F_R$ , the resultant MMF. The resultant MMF can be expressed as:

$$F_R = \sqrt{N_s I_D^2 + N_s I_D^2 - 2N_s I_D \cdot N_s I_D \cdot \cos(120^\circ)}.$$

In this case, since the triangle is isosceles, we know both sides are  $N_s I_D$ . Now, using the fact that  $\cos(120^\circ) = -\frac{1}{2}$ , we can substitute this value into our equation.

So, the equation simplifies to:

$$F_R = \sqrt{N_S I_D^2 + N_S I_D^2 + N_S I_D^2} = \sqrt{3N_S I_D^2}.$$

This ultimately results in:

$$F_R = \sqrt{3}N_S I_D.$$

Now, when we apply a normal three-phase AC supply to the stator of an induction motor, each phase carries a current equal to  $I_S$ .

Now, when we apply a normal three-phase AC supply, we have a three-phase stator in this configuration. The currents in the three phases, phase A, phase B, and phase C, are balanced. We denote the currents as  $I_A$ ,  $I_B$ , and  $I_C$ . Let's assume the RMS value of these currents is  $I_S$ .

If the RMS value of the phase current is  $I_S$ , the magnetomotive force (MMF) produced by this configuration is a rotating MMF. This means that instead of being stationary, the MMF is rotating at a speed of  $\omega_{Ms}$ . We denote this rotating MMF as  $F_S$ , which represents the stator MMF.

The peak value of the stator MMF can be expressed as:

$$F_S = \frac{3}{2}N_S I_S \sqrt{2},$$

where  $N_S$  is the number of turns per phase. Now, if we want to equate this to the MMF produced in the DC dynamic braking scenario, we can express the induced MMF  $F_R$  as:

$$F_R = \sqrt{3}N_S I_D,$$

where  $I_D$  is the DC current. By equating these two expressions for MMF, we arrive at the following relationship:

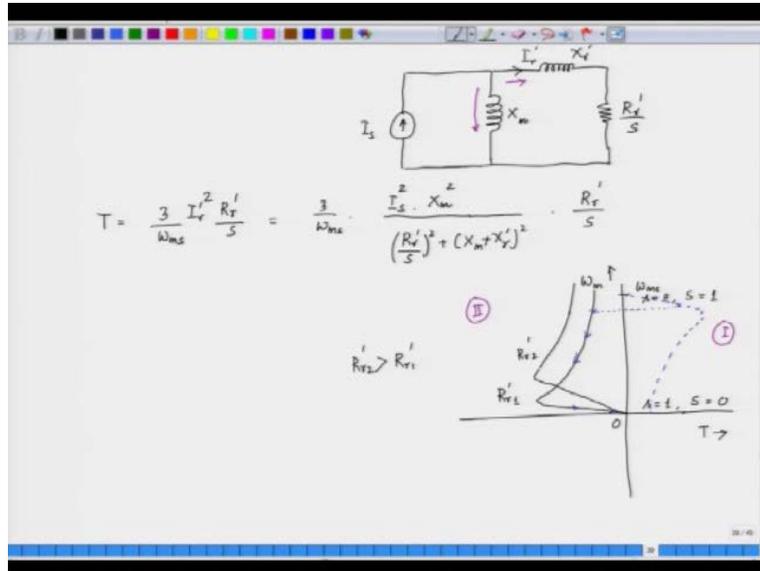
$$\sqrt{3}N_S I_D = \frac{3}{2}N_S \sqrt{2} I_S.$$

By simplifying this equation, we find:

$$I_S = \frac{\sqrt{2}}{3} I_D.$$

This derivation is crucial for understanding the concept of DC dynamic braking. It indicates that if we excite the stator with a DC current, we can achieve an equivalent AC current represented by the equation  $I_S = \frac{\sqrt{2}}{3} I_D$ . This relationship will be essential for our further discussions on dynamic braking.

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Now, our stator is excited by an AC source, denoted as  $I_s$ . In this configuration, we have a magnetizing branch characterized by the magnetizing reactance  $X_m$ . Additionally, we have the rotor leakage reactance, represented as  $X_r'$ , and the rotor resistance, expressed as  $\frac{R_r}{S}$ . The current flowing into the rotor is  $I_r'$ .

Since the stator is excited by an AC current source, we can derive the torque equation accordingly. The torque produced in this system can be expressed as:

$$T = \frac{3}{\omega_{Ms}} I_r'^2 \frac{R_r}{S}$$

This is derived from the AC equivalent circuit. To find  $I_r'$ , we need to analyze the current divider circuit, where  $I_r'$  is determined by:

$$I_r' = I_s \frac{X_m}{\left(\frac{R_r}{S}\right)^2 + X_m^2 + X_r'^2}$$

In this case, we are squaring the total impedance in the denominator. Thus, the equation for torque production becomes:

$$T = \frac{3}{\omega_{Ms}} I_s^2 \frac{X_m^2 R_r}{\left(\frac{R_r}{S}\right)^2 + X_m^2 + X_r'^2}$$

This torque represents the braking torque, and in this scenario, S is defined as 1 - s.

Now, if we plot the torque-speed characteristic curve, we can visualize it on a graph with the torque on the vertical axis and the speed on the horizontal axis. In this context, the braking torque appears as a negative value, indicating that it opposes the rotor's motion. This characteristic is critical for understanding the performance of the induction motor under dynamic braking conditions.

In this discussion, we observe the torque characteristics of an induction motor during dynamic braking. Let's consider the normal synchronous speed, denoted as  $\omega_{Ms}$ . At this synchronous speed, the slip S is equal to 0, indicating that the motor operates efficiently. However, when the motor speed decreases to 0, the slip becomes 1, which changes the characteristics significantly.

The torque-speed characteristic under these braking conditions contrasts sharply with that of a standard induction motor. Instead of the typical curve that begins at the synchronous speed, the braking characteristic starts from this point and exhibits an entirely different nature.

We can visualize this on a graph with two quadrants. The first quadrant represents the motoring region, where the motor operates normally, while the second quadrant denotes the braking region. In this second quadrant, the torque assumes negative values.

If the motor was initially functioning with a forward torque-speed characteristic and we suddenly apply dynamic braking using DC voltage, the torque transitions to a negative value at the same speed. This results in the motor decelerating; as the torque remains negative, the speed of the motor will gradually decrease until it settles at 0. Importantly, it's noteworthy that at 0 speed, the braking torque diminishes to zero, eliminating any possibility of the speed becoming negative.

To enhance the braking torque, we can introduce additional rotor resistance into the circuit. For example, if we denote the rotor resistance as  $R_{r1}$  for one configuration, increasing this rotor resistance to  $R_{r2}$  will yield a higher braking torque while maintaining the same peak torque. This adjustment is particularly feasible in slip ring induction motors.

In summary, we have examined how an induction motor can be effectively braked using DC dynamic braking by applying a DC voltage to the stator. This process allows the motor to decelerate from a finite speed to a complete stop, with the braking torque equaling zero when the speed reaches 0. This behavior is illustrated in the torque-speed characteristic, which resides in the second quadrant.

With that, we conclude today's lecture on DC dynamic braking. We will continue this discussion in our next session.