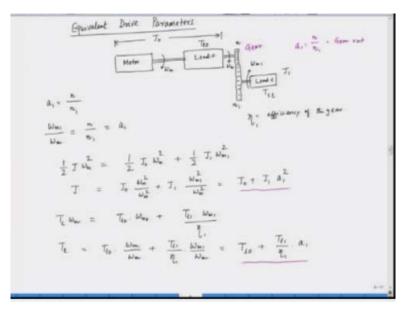
Fundamentals of Electric Drives Prof. Shyama Prasad Das Department of Electrical Engineering Indian Institute of Technology - Kanpur Lecture – 03

Equivalent Drive Parameters, Friction Components, Nature of Load Torque

Hello and welcome to this lecture on the fundamentals of electric drives! In our previous session, we discussed the equivalent drive parameters when a gear is positioned between the motor and the load. Today, we will continue to build on that foundation and explore further aspects of electric drives.

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We have observed that when we have a motor connected to a load through a gear, the equivalent parameters manifest in specific ways. For instance, the inertia is represented as $J = J_0 + J_1 a_1^2$, where a_1 signifies the gear ratio. Specifically, a_1 is defined as the ratio of the number of teeth on the motor gear (n) to the number of teeth on the load gear (n₁).

Furthermore, the torque perceived by the motor can be expressed as:

$$\mathbf{T}_{L0} + \frac{T_{L1}}{\eta_1} \cdot a_1$$

where η_1 denotes the efficiency of the first gear. This discussion pertains to the context of a

rotational system, specifically addressing the behavior of a rotational load.

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$$\begin{aligned} \mathcal{J} &= \mathcal{J}_{b} + \frac{a_{1}^{2}}{2}\mathcal{J}_{i} + \frac{a_{k}^{2}}{2}\mathcal{J}_{k} + \cdots \\ \mathcal{T}_{l} &= \mathcal{T}_{ls} + \frac{a_{i}}{2}\mathcal{T}_{i} + \frac{a_{k}}{2}\mathcal{T}_{k} + \cdots \\ \hline \mathcal{T}_{i} &= \mathcal{T}_{is} + \frac{a_{i}}{2}\mathcal{T}_{i} + \frac{a_{k}}{2}\mathcal{T}_{k} + \cdots \\ \hline \mathcal{T}_{i} &= \mathcal{T}_{is} + \frac{a_{i}}{2}\mathcal{T}_{i} + \frac{a_{k}}{2}\mathcal{T}_{k} + \cdots \\ \hline \mathcal{T}_{i} &= \mathcal{T}_{i} + \frac{a_{k}}{2}\mathcal{T}_{i} + \frac{a_{k}}{2}\mathcal{T}_{i} + \frac{a_{k}}{2}\mathcal{T}_{i} + \frac{a_{k}}{2}\mathcal{T}_{i} + \frac{a_{k}}{2}\mathcal{T}_{i} + \frac{a_{k}}{2}\mathcal{T}_{k} + \cdots \\ \hline \mathcal{T}_{i} &= \mathcal{T}_{i} + \frac{a_{k}}{2}\mathcal{T}_{i} +$$

When the load is rotating, and we have multiple gears in play, we can express the gear ratios as a_1 , a_2 , and so forth. The effective inertia seen by the motor is given by the equation:

$$J = J_0 + a_1 J_1^2 + a_2 J_2^2 + a_3 J_3^2 + \cdots,$$

where each term represents the contribution of each gear. Similarly, the equivalent load torque can be represented as:

$$T_L = T_{L0} + \frac{a_1 T_{L1}}{\eta_1} + \frac{a_2 T_{L2}}{\eta_2} + \cdots,$$

with η_1 and η_2 denoting the efficiencies of gears 1 and 2, respectively. This formulation can be extended to accommodate any number of gears.

Now, let's shift our focus to the translational system. But what exactly do we mean by a translational system? Unlike rotational systems, a translational system does not rotate; instead, it moves in a linear direction. A prime example of this is a lift that goes up and down, demonstrating translational motion. However, behind the scenes, there is a motor responsible for converting rotational motion into translational motion.

Currently, we will discuss the coupling of a translational load with a rotating motor. In this scenario, we have a motor coupled to a load, which we will refer to as load 0. This load is a

rotational load, and it is linked through a mechanical coupling to a conversion system that transforms rotational motion into linear motion transmission. Additionally, we have a mass involved, which can move up or down at a velocity denoted as v_1 .

This mass, denoted as m_1 , exerts a force F_1 . The combination of the motor and the load has an effective inertia represented as J_0 , referring specifically to the first load, which is the zero-th load. The motor rotates at a speed of ω_m , while load 0 exerts an opposing load torque denoted as T_{L0} . The motor torque, represented as T_L , is what we need to determine, alongside the linear velocity of this mass, which is denoted as v_1 .

To analyze this system, we will employ the same methodology as before. First, let's calculate the effective kinetic energy of the entire system. We have a rotational system involving load 0 and a translational system that moves vertically, either up or down, exhibiting linear motion. So, what is the effective kinetic energy of this entire system?

We can express the equivalent kinetic energy as:

$$KE_{\rm equiv} = \frac{1}{2}J\omega_m^2,$$

where J is the equivalent inertia and ω_m is the equivalent speed. The kinetic energy of the entire system can also be represented as the sum of the kinetic energies of its components:

$$KE_{\text{equiv}} = \frac{1}{2}J_0\omega_m^2 + \frac{1}{2}mv_1^2.$$

Here, the first term corresponds to the kinetic energy of the rotational load, while the second term represents the kinetic energy of the translational load, calculated as $\frac{1}{2}mv_1^2$. This formulation provides us with the effective kinetic energy of the system.

To determine the effective inertia J, we can rearrange this equation as follows:

$$J = \frac{J_0 \omega_m^2}{\omega_m^2} + \frac{m v_1^2}{\omega_m^2}$$

This allows us to express the effective inertia in terms of the individual components of the system.

We can simplify this expression to represent the effective inertia seen by the motor as:

$$J = J_0 + \frac{m \cdot v_1}{\omega_m^2}.$$

Here, J₀ signifies the original moment of inertia, while the additional term accounts for the translational load that is in motion with a velocity of v₁. The ratio of $\frac{v_1}{\omega_m^2}$ is utilized to transform the mass of the translational load into its equivalent rotational mass, effectively represented as a moment of inertia.

Now, let's discuss the effective load torque. To understand the effective load torque, we first need to evaluate the power perceived by the motor. The power seen by the motor can be expressed as:

$$P=T_L\cdot\omega_m.$$

This power consists of the contribution from load 0, represented as $T_{L0} \cdot \omega_m$, and the power associated with the translational load. The power corresponding to the translational load can be calculated as the force F₁ multiplied by the velocity v₁. Thus, we have:

$$P_{\text{trans}} = F_1 \cdot v_1.$$

However, we must account for the coupling efficiency, denoted as η_1 . Therefore, we divide by this coupling efficiency:

$$P_{\rm eff} = \frac{T_{L0} \cdot \omega_m + F_1 \cdot v_1}{\eta_1}$$

Next, we can express the effective load torque seen by the motor as:

$$T_L = \frac{T_{L0} \cdot \omega_m}{\omega_m} + \frac{F_1 \cdot v_1}{\eta_1 \cdot \omega_m} = T_{L0} + \frac{F_1}{\eta_1} \cdot \frac{v_1}{\omega_m}$$

This expression captures the effective load torque perceived by the motor when dealing with a translational load. In the case of a translational load, it is essential to take into account the ratio of linear velocity to angular velocity, as this ratio will facilitate the transformation of both the inertia and the load torque into their appropriate equivalents.

Now, when we discuss load torque, it's essential to examine its various components. We will be delving into the intricacies of load torque components. Imagine we have a motor driving a load; these two elements are mechanically coupled. The motor is rotating in one direction, generating

a motor torque, denoted as T, while the load presents an opposing torque, referred to as TL.

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So, what are the components of the load torque T_L ? It is typically composed of two primary parts. The first component is known as the friction component, denoted as T_F , and the second component is the useful load torque, represented as T_L itself.

To clarify further, T_F is the friction torque, while T_L is the useful load torque, which serves as a fixed and integral part of any mechanical system. It is crucial to understand that no mechanical system can function effectively without friction.

When we discuss friction, we recognize that there are several types contributing to the total friction torque T_F . This total friction can be categorized into several components: the first is static friction, denoted as T_S ; the second is Coulomb friction, represented as T_C ; the third type is viscous friction, referred to as T_V ; and the fourth type is windage friction, labeled as T_W .

In summary:

- Ts is the static friction,
- T_C is the Coulomb friction,
- Tv is the viscous friction,
- T_w is the windage friction.

These components collectively define the total friction torque T_F that impacts the performance of the mechanical system.

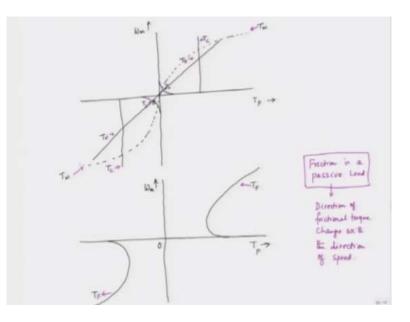
We primarily have four types of friction: static friction, Coulomb friction, viscous friction, and windage friction.

Static friction occurs only when motion has not yet commenced, specifically, when the motor speed is zero. This type of friction is significant in standstill conditions or during very low-speed scenarios, making it a crucial component when transitioning from a state of rest to motion. Thus, static friction is only present during the range from zero to low speeds.

On the other hand, Coulomb friction is unique in that it remains constant regardless of the speed. Whether the motor is at rest, operating at a medium speed, or running at high speed, Coulomb friction does not vary; it is independent of speed.

When we turn our attention to viscous friction, we find that it is directly proportional to the speed of the system. In contrast, windage friction behaves differently; it is proportional to the square of the speed. To summarize, while viscous friction increases linearly with speed, windage friction escalates with the square of the speed.

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Now, let's explore these four types of friction, static friction, Coulomb friction, viscous friction, and windage friction, by plotting a graph in the speed-torque plane. This will allow us to visualize the various friction components and their relationships more clearly. The components of these

frictions will be represented in this graphical analysis.

Let's begin by establishing our graph. We have the origin here, and we are plotting the friction components on this speed-torque plane. The y-axis represents speed, while the x-axis represents friction torque.

First, let's consider static friction. This frictional torque is present when the speed is zero, which indicates that static friction exists at that point. As the speed begins to increase, however, the static friction gradually decreases until it ultimately becomes zero. This behavior can be represented as Ts reaching zero as the speed approaches a positive value. Interestingly, when the speed is negative, the torque also becomes negative, reflecting the opposing direction of the frictional force. So, we have two plots for static friction: one for positive speeds and another for negative speeds.

Now, let's move on to Coulomb friction. Unlike static friction, Coulomb friction does not depend on speed; it remains nearly constant across the entire range of operation. Thus, we can describe T_C as independent of speed. Similarly, when the speed is negative, the Coulomb friction torque also becomes negative, illustrating its consistent nature regardless of speed.

Next, we have viscous friction. This type of friction is directly proportional to speed, which we can illustrate as a linear relationship on our graph. This line represents viscous friction T_V .

Finally, we come to windage friction. This friction behaves differently; it is proportional to the square of the speed. When we plot windage friction, we will observe a curve reflecting this quadratic relationship. As the speed increases, the windage torque escalates with the square of the speed, demonstrating its unique nature in comparison to the others.

Now, when we sum all these friction components together, we can derive the overall behavior of the frictional torque in our system. This comprehensive view allows us to understand how each type of friction contributes to the overall dynamics of the system as speed varies.

Now, let's take a closer look at the overall behavior of the frictional torque, which can be illustrated in a plot. Initially, we see that static friction gradually decreases until it eventually vanishes. On the negative side, static friction behaves similarly, increasing in the opposite direction. In this context, the y-axis represents speed, while the x-axis denotes torque, specifically the frictional torque. This plot captures the dynamic behavior of frictional torque.

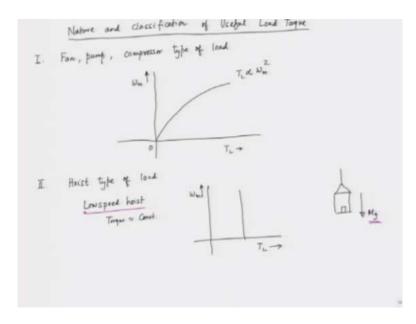
At rest, when the motor is stationary, all types of friction come into play. Here, static friction and Coulomb friction are present, while both viscous friction and windage friction are zero. As the speed begins to increase, we observe that viscous friction and windage friction start to rise, while Coulomb friction remains constant. In contrast, static friction gradually disappears. This relationship gives us a clear picture of how frictional torque behaves throughout different operating conditions.

It's important to note that friction is classified as a passive load. We refer to friction as a passive load because it responds to changes in speed direction; essentially, it always opposes motion. The defining characteristic of a passive load is that when the speed direction changes, the torque also reverses. Therefore, any torque that reverses with changes in speed direction is classified as a passive load.

Specifically, the direction of frictional torque is contingent upon the direction of speed. When the speed reverses, friction must also reverse to ensure that it continues to oppose that speed. This foundational understanding sets the stage for our discussion on the friction components of load torque.

Now, let's shift our focus to the useful load torque. We will explore the various types of useful load torque that exist in electric drives.

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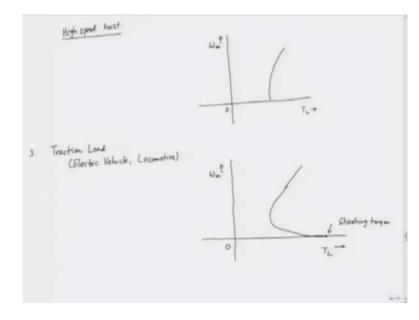
Let's delve into the nature and classification of useful load torque. When we refer to useful load

torque, it is essential to understand that this torque must perform some beneficial work. The first type of load we encounter is the fan type of load. Fans, as they rotate, generate airflow, which is incredibly useful in various applications. However, it's important to note that while the fan provides this wind, it also exerts a torque on the motor. Thus, we categorize this type of load as belonging to the fan, pump, and compressor categories.

Now, if we examine the load torque behavior, denoted as T_L versus ω_m (where ω_m represents speed), we find that the useful load torque T_L for fans, pumps, and compressors is proportional to the square of the speed. In other words, we can express this relationship as $T_l \propto \omega_m^2$.

Next, we move on to the second type of load torque, known as hoist load torque. Here, we have a cage being lifted against the force of gravity. This scenario introduces a constant gravitational torque; however, it can be further classified into two subtypes: low-speed hoist and high-speed hoist.

For a low-speed hoist, the torque remains nearly constant and is independent of the speed. When we plot the load torque against speed, placing speed on the y-axis and torque on the x-axis, we observe that the torque for a low-speed hoist is virtually constant, not varying significantly with speed. This constancy arises because the gravitational pull Mg (where M is mass and g is the acceleration due to gravity) remains unchanged regardless of the speed.



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As the speed increases, we transition into the realm of high-speed hoisting. Here, the

characteristics of the load torque change, leading to different operational behaviors.

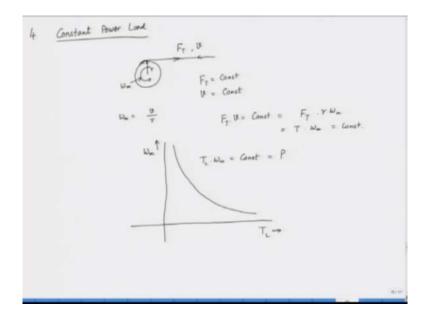
In high-speed hoisting, the situation is somewhat different. We observe the load torque plotted against the speed, ω_m . For high-speed hoisting, the torque initially remains constant. However, as the speed increases, both viscous friction and windage friction come into play. Consequently, for high-speed hoisting, we see the torque profile rising as the speed increases.

Now, let's turn our attention to another type of load: the traction load. This load is primarily associated with electric vehicles and locomotives, commonly referred to as traction loads. So, how does the traction load behave with respect to speed?

When we discuss traction loads, it's important to note that maximum torque is typically generated at startup. At this moment, the machine faces significant static friction that must be overcome. This high level of static friction characterizes the traction load, and as the speed increases, this friction decreases.

As the speed continues to rise, we also encounter additional factors like windage friction. Nevertheless, traction loads are particularly characterized by this considerable amount of static friction, which must be surmounted initially. As a result, the torque required at startup is predominantly due to overcoming static friction. Therefore, we can say that the starting torque is primarily influenced by this static friction component, leading to a dynamic interaction as the vehicle accelerates.

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We also encounter other types of loads; for instance, consider a constant power load. A perfect example of this is a coiler drive, which is responsible for winding cloth onto a roll. As the cloth is being coiled, the diameter of the roll increases. In this scenario, we have the coiler mechanism rotating, and the material being coiled also rotates, generating tension, denoted as F_T. This setup is associated with a linear velocity, v.

It's essential to maintain F_T as constant throughout this process. This means both the tension and the linear velocity must remain constant. The coiler also has an angular velocity, represented as ω_m . Now, what is ω_m ? Simply put, it can be defined as $\frac{v}{r}$, where r is the radius of the coil. Therefore, we can express the relationship as $T \cdot v$ being constant, which leads us to F_T . Since v is equal to $r \cdot \omega_m$, we can derive that $F_T \cdot r = \text{Torque} \cdot \omega_m$, which is equal to a constant value.

For a constant power drive, the power remains unchanged. Power is defined as the product of torque and speed, so when we keep both torque and speed constant, we obtain a constant power load. The speed-torque characteristic for this type of load can be depicted graphically, with speed plotted on the y-axis and torque on the x-axis. The relationship looks like this: $T_L \cdot \omega_m$ remains constant. When we assert that the product of torque and speed is constant, we indeed describe a constant power drive, where P remains unchanged.

In summary, we've explored various types of useful load torques. We began by discussing the frictional component, then moved on to the fan type load, where the load torque is proportional to the square of the speed. Next, we examined the low-speed hoist, where the torque remains primarily constant, followed by the high-speed hoist, where torque increases with speed.

We also covered the traction load, characterized by a significant amount of starting torque; as the speed increases, we have to account for factors like viscous friction and windage friction. Finally, we looked at the constant power load, where the product of torque and speed needs to remain constant. So, we stop here for today's lecture, we will continue in the next class.