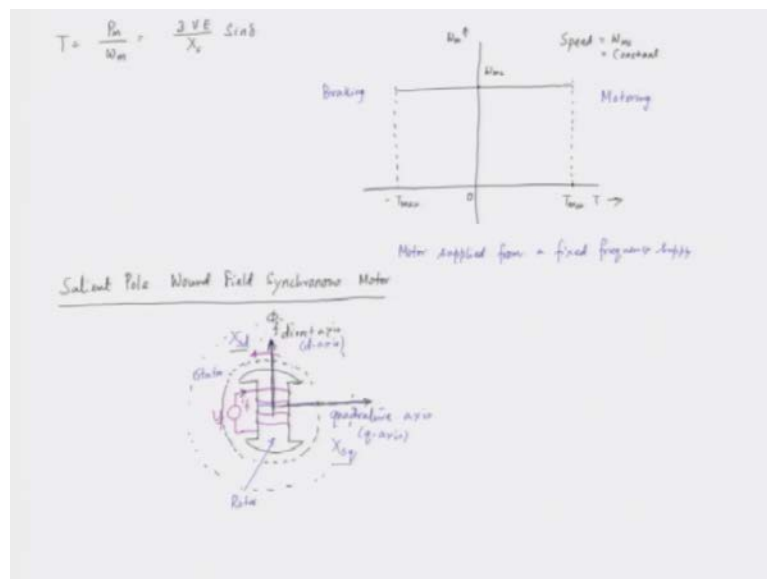


**Fundamentals of Electric Drives**  
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**Lecture 32**

**Phasor diagram of Salient pole synchronous motor, Expression of power and torque for a salient pole synchronous motor, Synchronous reluctance motor, Open-loop V/f control of synchronous motor**

Hello and welcome to this lecture on the fundamentals of electric drives! In our previous session, we explored the salient pole synchronous motor and learned that it features two distinct synchronous reactances: one aligned along the direct axis and the other along the quadrature axis. Today, we will take the next step and draw the phasor diagram for the salient pole synchronous motor. Let's dive in!

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In this discussion, we are focusing on the two synchronous reactances present in the salient pole synchronous motor. We have the direct axis, denoted as  $X_{sd}$ , and the quadrature axis, represented as  $X_{sq}$ . To accurately illustrate this, we need to draw the phasor diagram for the salient pole synchronous motor.

In this system, the field excitation generates flux in the direct axis, which we denote as  $\Phi_f$ . Surrounding the rotor is a cylindrical stator, which houses the windings. As the field rotates at

a certain speed, it induces an electromotive force (EMF) in the stator conductors, represented as E.

With that background in mind, we can begin constructing the phasor diagram, starting with the direct axis. Let's proceed!

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$$\vec{V} = \vec{E} + jI_{sd}X_{sd} + jI_{sq}X_{sq}$$

$$P_{in} = 3VI_s \cos\theta = P_{out} = P_m$$

$$I_s \cos\theta = I_{sq} \cos\delta + I_{sd} \cos(90^\circ + \delta)$$

$$= I_{sq} \cos\delta - I_{sd} \sin\delta \quad \text{--- (1)}$$

$$I_{sd}X_{sd} = V \cos\delta - E$$

$$I_{sd} = \frac{V \cos\delta - E}{X_{sd}} \quad \text{--- (2)}$$

$$I_{sq}X_{sq} = V \sin\delta$$

$$I_{sq} = \frac{V \sin\delta}{X_{sq}} \quad \text{--- (3)}$$

Substituting the values of  $I_{sd}$  &  $I_{sq}$  from (2) & (3) in (1)

$$I_s \cos\theta = \frac{V \sin\delta \cos\delta}{X_{sq}} - \frac{V \cos\delta - E}{X_{sd}} \sin\delta = \frac{V \sin 2\delta}{2X_{sq}} - \frac{V \cos 2\delta}{2X_{sd}} - \frac{E \sin\delta}{X_{sd}}$$

We are now focusing on the axes of the field pole, which we represent using a pole structure. Here, we have the direct axis, or d axis, and perpendicular to it is the quadrature axis, denoted as the q axis.

Next, let's consider the stator current, represented as  $I_s$ . The current and the magnetomotive force (MMF) are aligned in the same direction, where MMF is simply the product of the current and the number of turns in the winding. The stator current  $I_s$  can be resolved into two components: one along the d axis and the other along the q axis. The component along the d axis is referred to as  $I_{sd}$ , the direct axis stator current, while the component along the q axis is labeled  $I_{sq}$ , the quadrature axis stator current.

We also have the field current  $I_f$  directed along the d axis. This field current generates an induced EMF, denoted as E, which is the standard induced EMF we previously observed in cylindrical rotor machines. To calculate the total induced EMF E, we need to account for two voltage drops: one across the d axis reactance and another across the q axis reactance.

Starting with the d axis, we identify the reactance drop as  $j I_{sd} X_{sd}$ . Here, the reactance drop is

at a right angle to the current  $I_{sd}$ , which is directed as shown. Therefore, we include a  $j$  torque to represent this phase difference.

Next, we consider the reactance drop along the  $q$  axis, represented as  $j I_{sq} X_{sq}$ . Again, this reactance drop is perpendicular to the current  $I_{sq}$ .

To find the total voltage  $V$ , we combine these two phasors. Thus, the expression for voltage  $V$  becomes the vector sum of the induced EMF  $E$ , the  $d$  axis reactance drop  $j I_{sd} X_{sd}$ , and the  $q$  axis reactance drop  $j I_{sq} X_{sq}$ .

In summary, the total voltage can be expressed as:

$$V = E + jI_{sd}X_{sd} + jI_{sq}X_{sq}$$

Since we are dealing with two distinct reactances, it is crucial to multiply each reactance by its corresponding current to obtain the total voltage  $V$ .

Finally, the torque angle, defined as the angle between the induced EMF  $E$  and the terminal voltage  $V$ , is noteworthy. Additionally, the angle between the voltage  $V$  and the current is known as the power factor angle.

This completes our phasor diagram!

We can determine the input power, denoted as  $P_{input}$ , using the equation:

$$P_{input} = 3VI_s \cos \theta.$$

In this context, we've disregarded losses, which allows us to equate this to the output power  $P_{out}$ , which is also equivalent to the mechanical power. Our next step is to express  $I_s \cos \theta$ , in a more usable form. The term  $I_s \cos \theta$ , represents the projection of  $I_s$  along  $V$  and can be expressed in terms of the current components:

$$I_s \cos \theta = I_{sq} \cos \delta + I_{sd} \cos(90^\circ + \delta)$$

Since the angle between the  $d$  axis and the  $q$  axis is  $90^\circ$ , we can simplify this further to:

$$I_s \cos \theta = I_{sq} \cos \delta - I_{sd} \sin \delta.$$

Now, let's find the expression for  $I_{sd}$ . From our phasor diagram,  $I_{sd}$  appears at a specific point,

and we can derive its value by analyzing the voltage components. If we take the projection of the voltage along the q axis and subtract the induced EMF  $E$  from  $V$ , we obtain:

$$I_{sd}X_{sd} = V \cos \delta - E$$

Thus, we can express  $I_{sd}$  as:

$$I_{sd} = \frac{V \cos \delta - E}{X_{sd}}$$

Let's label this as Equation 1. Now, for Equation 2, we need to consider the  $I_{sq}$  component. Similarly, we find the reactance drop for  $I_{sq}$ :

$$I_{sq}X_{sq} = V \sin \delta,$$

From this, we can express  $I_{sq}$  as:

$$I_{sq} = \frac{V \sin \delta}{X_{sq}}$$

We'll label this as Equation 3. Now that we have Equations 1, 2, and 3, we can substitute the values of  $I_{sd}$  and  $I_{sq}$  from Equations 2 and 3 into Equation 1.

Substituting gives us:

$$I_s \cos \theta = I_{sq} \cos \delta - I_{sd} \sin \delta,$$

Replacing  $I_{sq}$  and  $I_{sd}$  yields:

$$I_s \cos \theta = \left( \frac{V \sin \delta}{X_{sq}} \right) \cos \delta - \left( \frac{V \cos \delta - E}{X_{sd}} \right) \sin \delta,$$

We can now simplify this. The product of  $\sin \delta$  and  $\cos \delta$  can be expressed as:

$$\sin \delta \cos \delta = \frac{1}{2} \sin 2 \delta,$$

Substituting this back in allows us to condense the equation:

$$I_s \cos \theta = \frac{V}{X_{sq}} \cdot \frac{1}{2} \sin 2 \delta - \frac{V}{X_{sd}} \cdot \frac{1}{2} \sin 2 \delta - \frac{E}{X_{sd}} \sin \delta,$$

Thus, we arrive at a final expression for  $I_s \cos \theta$ . Knowing the value of  $I_s \cos \theta$  now enables us to compute the power output. The equation for the output power  $P_m$  is given by:

$$P_m = 3VI_s \cos \theta,$$

By substituting the derived expression for  $I_s \cos \theta$  in terms of voltage, reactances, and  $\delta$ , we can simplify and ultimately derive the desired power expression.

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$$P_m = 3VI_s \cos \theta$$

$$= 3 \left[ \frac{VE \sin \delta}{X_{sd}} + \frac{V^2 (X_{sd} - X_{sq})}{2X_{sd}X_{sq}} \sin 2\delta \right]$$

$$T = \frac{P_m}{\omega_{ms}} = \frac{P_m}{\omega_{ms}}$$

$$= \frac{3}{\omega_{ms}} \left[ \frac{VE \sin \delta}{X_{sd}} + \frac{V^2 (X_{sd} - X_{sq})}{2X_{sd}X_{sq}} \sin 2\delta \right]$$

↑ Synchronous Motor (Field) Torque
 ↑ Reluctance Torque

$X_{sd} \neq X_{sq}$   
 $X_{sd} > X_{sq}$

This expression can be formulated as:

$$P_m = 3 \cdot VE \sin \delta \cdot \frac{1}{X_{sd}} + \frac{V^2 \cdot (X_{sd} - X_{sq})}{2X_{sd}X_{sq} \sin 2\delta}$$

Let's designate this as Equation 4. From Equation 4, once we multiply by  $V$  and simplify, we arrive at an equation that represents the power output for a salient pole synchronous motor. Knowing this power allows us to compute the torque, as torque is defined as power divided by speed. Here, the speed refers to the synchronous speed.

Therefore, we can express the torque as:

$$T = \frac{P_m}{\omega_{ms}}$$

Substituting  $P_m$  into the equation gives us:

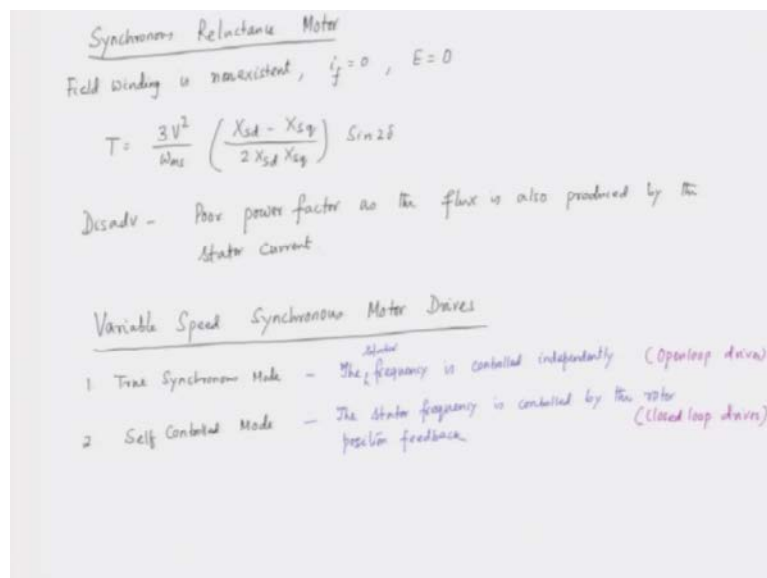
$$T = \frac{3}{\omega_{ms}} \left( VE \sin \delta \cdot \frac{1}{X_{sd}} + \frac{V^2 (X_{sd} - X_{sq})}{2X_{sd}X_{sq} \sin 2\delta} \right)$$

In the context of a salient pole synchronous machine, we observe that torque  $T$  is a function of  $\delta$ , specifically dependent on  $\sin \delta$  and  $\sin 2\delta$ . This is a result of the salient nature of the machine, leading to  $X_{sd} \neq X_{sq}$ . Consequently, the torque expression includes the term  $\sin 2\delta$ .

The first term represents the torque associated with the field winding, commonly referred to as the synchronous motor or field torque. The second term is known as the reluctance torque. Notably,  $X_{sd}$  is greater than  $X_{sq}$  for a wound field synchronous machine, which results in a positive reluctance torque for the salient pole synchronous motor.

Additionally, there exists a class of synchronous motors known as synchronous reluctance motors. In these machines, the field winding is removed, and they operate under the condition where the excitation EMF is zero. Now, let us delve into a discussion about synchronous reluctance motors.

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In a synchronous reluctance motor, the field winding is absent, which means that the field current  $I_f$  is equal to zero, resulting in an induced EMF  $E$  that also equals zero. Let's denote this observation as Equation 5. By substituting  $E = 0$  into our equations, the first term vanishes, leaving us with a new expression for torque in a synchronous reluctance machine. The torque  $T$  can then be expressed as:

$$T = \frac{3V^2}{\omega_{ms}} \cdot \frac{X_{sd} - X_{sq}}{2X_{sd}X_{sq}} \cdot \sin(2\delta)$$

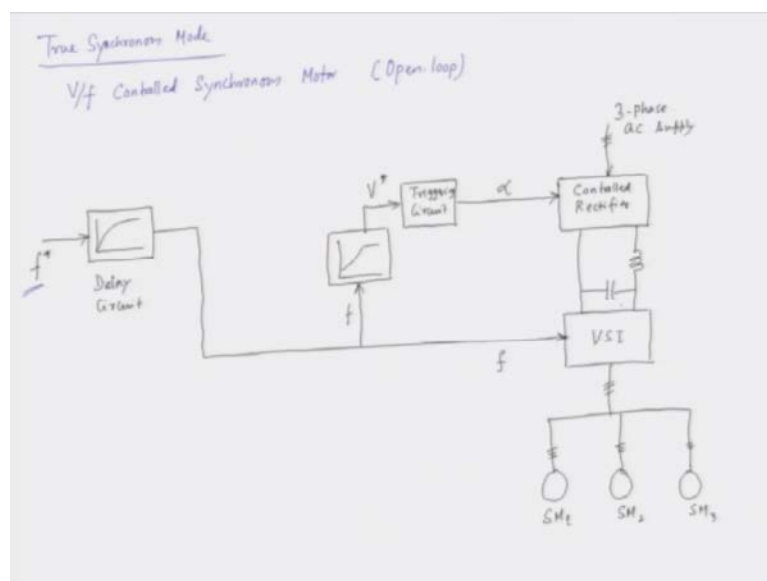
Notably, the torque does not depend on the field, as the field has been removed and the field current is zero. Instead, this torque becomes a function of the applied stator voltage  $V^2$  and varies with  $\sin(2\delta)$ . This phenomenon is quite intriguing, despite the absence of a field winding, the machine continues to develop torque.

However, one significant disadvantage of a synchronous reluctance motor is that, since the field flux is zero, the armature must generate the flux. As a result, the power factor tends to be poor. While the motor can indeed produce torque, it operates at a notably low power factor.

Having discussed the fundamental equations governing synchronous motors, both cylindrical and salient pole types, let's consider the implications of feeding these motors with a variable voltage and variable frequency supply. To achieve variable speed operation, we must alter the synchronous speed, which necessitates changing the supply frequency.

Now, there are two types of variable speed synchronous motor drives: one is referred to as true synchronous mode, and the other is known as self-controlled mode. To achieve variable speed in a synchronous machine, the key option is to vary the frequency. In the true synchronous mode, the frequency is controlled independently, allowing us to adjust it without affecting other parameters.

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On the other hand, in the self-controlled mode, the frequency is controlled as a function of the rotor speed. This means that the stator frequency is influenced by rotor position feedback. Typically, the drives operating in true synchronous mode are open-loop systems, lacking position feedback. In contrast, self-controlled mode drives operate as closed-loop systems, incorporating feedback mechanisms to enhance control and performance.

Let's delve into a scheme for the true synchronous mode, specifically focusing on the open-loop drive system in this configuration. In this case, we employ what's known as a V/f control drive in open-loop operation. To illustrate this, let's draw a block diagram.

At the outset, we have a reference frequency that can change independently. This reference frequency is fed into a delay block. Following this delay circuit, we have a Voltage Source Inverter (VSI) that supplies power to multiple synchronous motors, which we will denote as SM1, SM2, and SM3. The stator of each motor receives power from the VSI, forming a three-phase system.

To provide context, the DC link in this system is powered by a controlled rectifier. This DC link includes a DC link filter designed to eliminate ripple from the rectified output. The process begins with a three-phase AC input, which is rectified to produce a DC voltage, and subsequently, this DC voltage is converted back into an AC voltage using the inverter.

Now, the frequency output from the delay circuit is fed directly into the VSI. Here, we also have a V/f function generator that produces a frequency output. This generator is crucial, as its output gives rise to the reference voltage, denoted as  $V^*$ . Following this, a triggering circuit is employed, which provides the angle  $\alpha$  necessary for controlling the VSI's switching voltage.

In this system, the inverter output frequency is denoted as  $f$ . As we adjust the frequency, the voltage also changes accordingly, ensuring that the V/f ratio remains approximately constant. At higher frequencies, when operating at low speeds, we may need to introduce a resistor drop, similar to what is done in induction motor drives.

This drive configuration allows for effective control under true synchronous mode. When we modify the reference frequency, the actual frequency changes after a brief delay, and simultaneously, the voltage is adjusted to maintain a nearly constant V/f ratio. For today's lecture, we will conclude here. In our next session, we will continue our discussion on synchronous motor drives, exploring further nuances and applications.