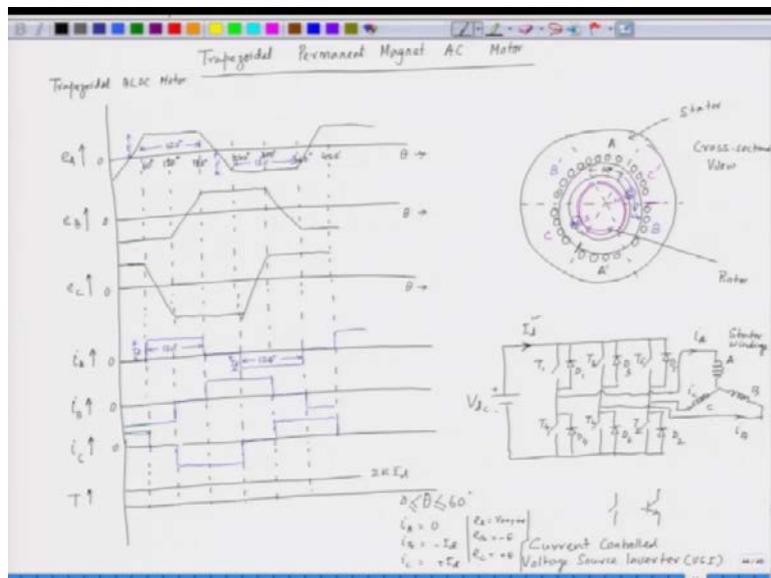


**Fundamentals of Electric Drives**  
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**Lecture - 36**

**Trapezoidal Permanent Magnet AC Motor, Derivation of Power and Torque, Closed-Loop Control of Trapezoidal BLDC Motor, Introduction to Switched Reluctance Motor**

Hello and welcome to this engaging lecture on the fundamentals of electric drives! In our last session, we explored the fascinating topic of the trapezoidally excited permanent magnet AC motor. During that discussion, we observed that the back electromotive forces (EMFs) in Phases A, B, and C exhibit a trapezoidal shape. We also examined the rotor, which is equipped with permanent magnets, specifically focusing on a two-pole rotor structure featuring a North Pole and a South Pole. Now, let's continue to build on our previous discussion and discuss this intriguing subject!

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In this segment, we observe the induced EMFs for Phase A, Phase B, and Phase C. The intervals are as follows: 60 degrees, 120 degrees, 180 degrees, 240 degrees, 300 degrees, and then finally 360 degrees. Beyond that, we continue the cycle with 420 degrees and so on. This angle, denoted as  $\theta$ , represents the rotor angle, and we are plotting the induced EMFs against this rotor angle. It's important to note that this system is also referred to as a trapezoidal brushless DC

(BLDC) motor due to the trapezoidal nature of the induced EMFs.

Now, let's consider the current flowing in Phase A, Phase B, and Phase C. The currents in these phases are quasi-rectangular in nature. Let's take a closer look at the waveform of the currents. We can plot the current waveforms for  $i_A$ ,  $i_B$ , and  $i_C$ . Here,  $i_A$  is the current flowing in Phase A,  $i_B$  in Phase B, and  $i_C$  in Phase C.

Each of these intervals is 60 degrees, allowing us to visualize the phase current waveforms of this motor. Starting with Phase A, the current takes on a quasi-rectangular shape, lasting for a duration of 120 degrees. This resembles a quasi-rectangular or quasi-square waveform.

This duration corresponds to the time when the induced EMF in Phase A is constant. As long as the induced EMF remains steady, the current also stays consistent for the same 120 degrees. Consequently, the Phase A current is positive during this period when the induced EMF is positive. When the induced EMF turns negative, the Phase A current also becomes negative for another 120 degrees.

For the sake of clarity, let's denote the amplitude of the induced EMF as  $E$  and the amplitude of the current as  $I_a$ . This same principle applies to Phase B as well: the current is positive when the induced EMF is positive, and it turns negative when the induced EMF dips into the negative range. Thus, we can see a similar pattern in the nature of the current for Phase B.

Now, let's turn our attention to Phase C. In Phase C, the current behaves similarly to what we've seen in the other phases. Specifically, the current is negative when the induced EMF is negative, and it shifts to positive when the induced EMF becomes positive. This clearly illustrates the nature of the current in Phase C. Thus, we have our various phase currents:  $i_A$ ,  $i_B$ , and  $i_C$ . As we have observed, these phase currents resemble quasi-square waveforms, remaining positive when the induced EMF is positive and turning negative when the induced EMF is negative.

But how does this current interact with the motor? The motor itself is a three-phase motor, and it utilizes an inverter to supply this quasi-square current to the stator windings. Let's take a closer look at the three-phase inverter responsible for this process. Here, we have our three-phase inverter, which consists of six switches. These switches connect to the respective stator phases: Phase A, Phase B, and Phase C.

In this configuration, Phase A is positioned here, Phase B is situated here, and Phase C is over here. The inverter operates with a direct current (DC) supply, denoted as  $V_{dc}$ . The current flowing through this system is  $I_d$ . Each switch within the inverter is typically a transistor and is paired with anti-parallel diodes to facilitate proper current flow. This entire setup represents a voltage source inverter (VSI) structure. Ultimately, the inverter efficiently feeds the stator of the trapezoidal excited permanent magnet AC motor, ensuring optimal performance and control.

Let's focus on Phase A and examine how Phase C is supplied from this point, while Phase B is fed from the opposite side. We have the phase currents represented as  $i_A$  for Phase A,  $i_B$  for Phase B, and  $i_C$  for Phase C. The switches we are discussing here are essentially transistors; they function as transistor switches within a current-controlled voltage source inverter (VSI).

To clarify, we are dealing with a current-controlled VSI. In this setup, we have six transistors labeled  $T_1, T_2, T_3, T_4, T_5,$  and  $T_6$ . Correspondingly, there are six diodes denoted as  $D_1, D_2, D_3, D_4, D_5,$  and  $D_6$ . This current-controlled VSI efficiently supplies power to the stator windings of the brushless DC (BLDC) motor. The current is regulated to maintain a quasi-square wave form, as depicted in the accompanying diagram.

Now, let's analyze the torque equation. Specifically, we want to derive the torque equation during the interval where the rotor angle  $\theta$  ranges from 0 to 60 degrees. During this interval, we observe that  $i_A$  equals 0. Meanwhile, the current  $i_B$  is  $-I_d$ , and the current  $i_C$  is  $+I_d$ .

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Handwritten mathematical derivation on a whiteboard showing the torque equation for a trapezoidal BLDC motor during the interval  $0 \leq \theta \leq 60^\circ$ .

For  $0 \leq \theta \leq 60^\circ$ :

$$i_A = 0 \quad e_A = \text{Varying}$$

$$i_B = -I_d \quad e_B = -E$$

$$i_C = +I_d \quad e_C = +E$$

$$P = e_A i_A + e_B i_B + e_C i_C = P_m$$

$$P_m = (-E)(-I_d) + (E)(I_d) = 2EI_d$$

$$T = \text{Torque} = \frac{P_m}{\omega_m} = \frac{2EI_d}{\omega_m} = 2kI_d$$

For  $60^\circ \leq \theta \leq 120^\circ$ :

$$i_A = +I_d \quad e_A = -E$$

$$i_B = -I_d \quad e_B = -E$$

$$i_C = 0 \quad e_C = \text{Varying}$$

$$P_m = 2EI_d$$

$$T = 2kI_d$$

Advantage of Trapezoidal BLDC Motor: Reduced Torque Ripple.

As for the induced electromotive forces (EMFs), we can summarize the variations as follows: the induced EMF  $e_A$  is fluctuating,  $e_B$  is  $-E$ , and the induced EMF of Phase C,  $e_C$ , is  $+E$ . This analysis sets the stage for determining the torque equation within this specific angular range.

For the interval of  $\theta$  from 0 to 60 degrees, we observe the following values:  $i_A = 0$  and  $e_A$  is varying. Specifically,  $i_B = -I_d$  and  $e_B = -E$ ; for  $i_C$ , we find that  $i_C = +I_d$  and  $e_C = +E$ . It's evident that during the first 60 degrees,  $i_A$  remains at 0. To summarize, we have  $i_A = 0$ ,  $i_B = -I_d$ ,  $i_C = +I_d$ ,  $e_B = -E$ , and  $e_C = +E$ .

Now, let's determine the power delivered to the motor over this interval. Since one of the phases, Phase A, is not conducting—its current is effectively zero—we can derive the power as follows:

The power  $P_m$  can be expressed as:

$$P_m = e_A \cdot i_A + e_B \cdot i_B + e_C \cdot i_C.$$

Given that  $i_A = 0$ , this term vanishes. Thus, the power simplifies to:

$$P_m = e_B \cdot i_B + e_C \cdot i_C.$$

Substituting in the values, we get:

$$P_m = (-E) \cdot (-I_d) + (+E) \cdot (+I_d) = E \cdot I_d + E \cdot I_d = 2E \cdot I_d.$$

We see that the power is a function of  $I_d$ , the DC link current. This current ultimately determines the power injected into the stator windings.

If we want to calculate the torque, we can use the relationship between mechanical power and rotor speed:

$$\text{Torque} = \frac{P_m}{\omega_m} = \frac{2E \cdot I_d}{\omega_m}.$$

This can be expressed in terms of a constant  $K$ :

$$\text{Torque} = 2K \cdot I_d.$$

What we've established here is that the torque of the motor is directly proportional to the current in the windings, specifically  $I_d$ . Therefore, to increase the torque, we can simply

increase  $I_d$ , and conversely, to decrease the torque, we reduce  $I_d$ .

Now, let's take a look at the plot of the torque in this diagram.

Let's take a look at the torque waveform that we can plot here. As you can see, the torque remains constant during this interval because when  $I_d$  is constant, the torque will also stay constant. This observation holds true for the range from 0 to 60 degrees.

Now, if we shift our focus to the interval where  $\theta$  varies from 60 to 120 degrees, we encounter a different scenario. In this case, we can analyze the values of  $i_A$ ,  $i_B$ , and  $i_C$  alongside their corresponding induced EMFs,  $e_A$ ,  $e_B$ , and  $e_C$ .

For the interval from 60 to 120 degrees, we find that  $i_A = +I_d$ ,  $i_B = -I_d$ , and  $i_C = 0$ . Consequently, the induced EMFs are  $e_A = +E$ ,  $e_B = -E$ , and  $e_C$  varies throughout this interval. Just like we determined for the first interval, we can see that the mechanical output remains  $2 \times I_d$ . When we calculate the torque, we find that it is given by:

$$\text{Torque} = 2K \cdot I_d.$$

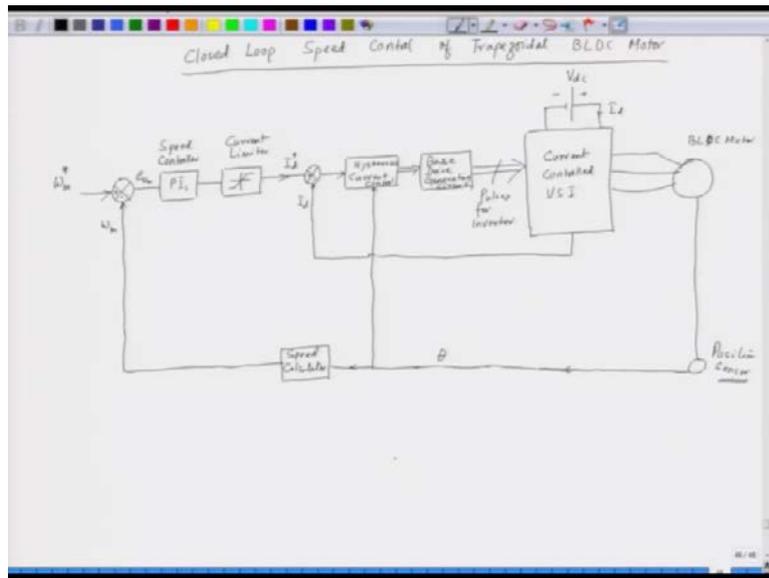
Notably, the torque does not depend on the interval itself; it fundamentally relies on the current. As long as the current is constant, the torque will also remain constant. Therefore, when we plot the torque waveform for this interval, we observe that it is a consistent waveform, remaining equal to  $2K \cdot I_d$ .

This type of motor we're discussing is a brushless DC motor, specifically a trapezoidal brushless DC motor. The currents being quasi-rectangular in nature allow us to maintain minimal torque ripple, which is a significant advantage of this drive. The trapezoidal BLDC motor stands out due to its reduced torque ripple.

This characteristic makes these motors particularly well-suited for applications in electric vehicles, such as electric scooters and low-cost electric cars. The trapezoidally excited brushless DC motors are favored in electric vehicles precisely because of their low torque ripple.

Moreover, these motors can be effectively controlled using a closed-loop system. To implement closed-loop control, we must incorporate feedback mechanisms. Let's discuss how this closed-loop control system operates.

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Let's delve into the closed-loop speed control of a trapezoidal BLDC motor. We begin by establishing a reference speed, denoted as  $\omega_m^*$ . This reference speed is then compared to the actual speed of the motor, allowing us to identify any discrepancies. The result of this comparison is fed into a PI controller, labeled PI1, which functions as our speed controller.

Following this, we introduce a current limiter to ensure that the motor operates within safe parameters. This limiter outputs the reference current,  $I_d^*$ , which serves as a critical input for controlling the inverter. The reference current  $I_d^*$  is then compared with the actual current,  $I_d$ , obtained from feedback on the DC link side.

At this stage, we implement hysteresis current control, which enables us to manage the triggering angle of the inverter effectively. This discussion pertains specifically to phase A; however, the principles apply across the inverter system. Our current-controlled voltage source inverter (VSI) feeds the brushless DC motor, ensuring optimal performance.

To further enhance our control system, we incorporate a position sensor that provides vital information about the rotor's position, denoted as  $\theta$ . This position data is essential for generating the reference current. The hysteresis current control mechanism utilizes the rotor position to calculate the speed,  $\omega_m$ , ensuring a responsive and efficient control strategy.

In this setup, the DC voltage  $V_{dc}$  is applied to the current-controlled VSI, where we monitor the current  $I_d$  through a current sensor. The output from the VSI is then directed to the brushless

DC motor. Additionally, we have a gate drive generating circuit, which is responsible for producing the necessary signals to control the transistor switches within the VSI.

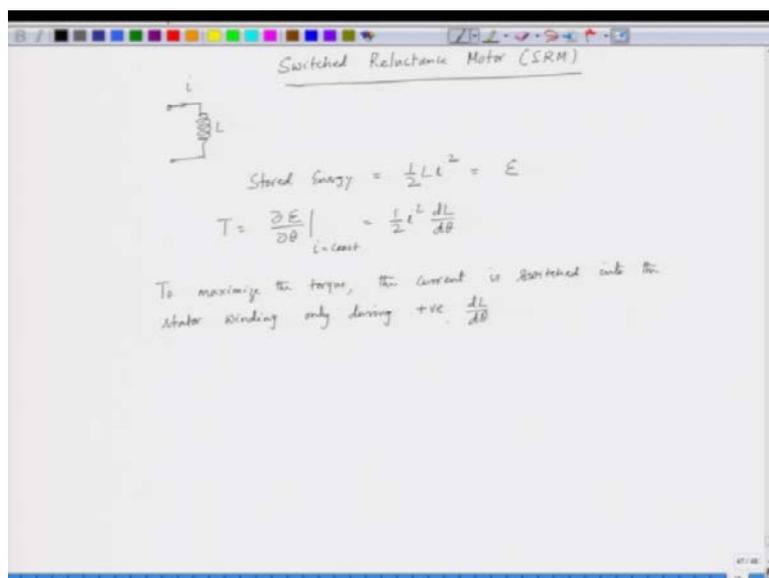
So, this closed-loop control strategy integrates speed monitoring, current limiting, and hysteresis control, ensuring precise regulation of the trapezoidal BLDC motor's performance.

Now, let's explore the triggering pulses supplied to the inverter, which are essential for the closed-loop speed control of a trapezoidally excited brushless DC motor. When there is a speed error, this discrepancy generates a reference current. This reference current is derived from the speed error and is utilized in conjunction with a hysteresis current controller. Here, the actual current  $I_a$  is compared to this reference, as it plays a crucial role in determining the motor's torque.

To enhance torque, it is imperative to increase  $I_a$ . This control is maintained within a specified hysteresis band, ensuring the inverter switches operate effectively. Moreover, these switches are regulated based on position feedback. This configuration highlights the self-controlled nature of the drive; for a trapezoidal excited BLDC motor, position feedback is essential and can be acquired through a position sensor.

This overview underscores the significance of brushless DC motors, particularly in electric vehicle applications, where they are increasingly popular due to their efficiency and reliability.

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Now, let's transition to another fascinating category of motors known as switched reluctance

motors. Unlike traditional motors, switched reluctance motors do not incorporate any rotor windings, making them exceptionally lightweight and suitable for applications in constrained environments, such as space. In the following discussion, we will delve into the construction and operating principles of switched reluctance motors, exploring their unique features and advantages.

This motor is commonly abbreviated as SRM, or switched reluctance motor. Let's consider a coil, for instance, which carries a current  $i$ , and we denote its inductance as  $L$ . The energy stored in this coil can be expressed as

$$E = \frac{1}{2} Li^2.$$

Now, if the inductance  $L$  varies as a function of position, we can derive a force or torque from this relationship. To calculate the torque, we consider the energy  $E$  and take its derivative with respect to the rotor position  $\theta$  while keeping the current constant. Thus, the torque  $\tau$  can be represented as

$$\tau = \left. \frac{dE}{d\theta} \right|_{i \text{ constant}}.$$

Substituting for  $E$ , we find

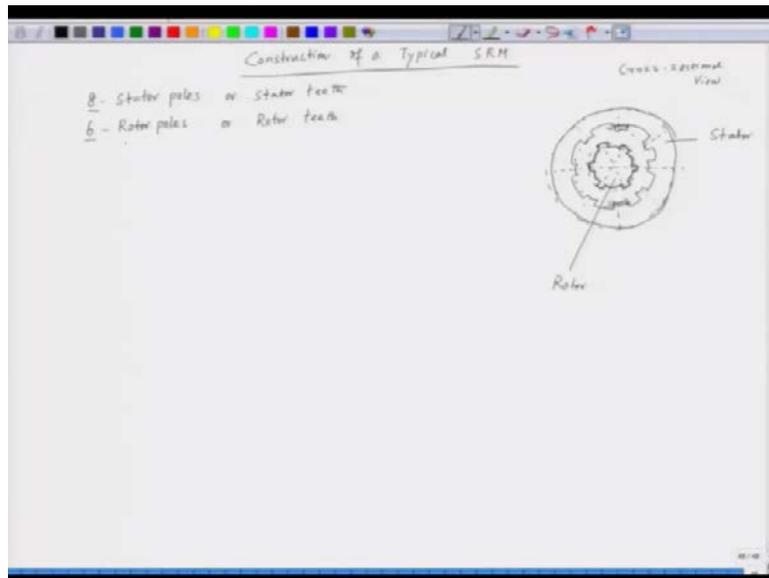
$$\tau = \frac{1}{2} i^2 \frac{dL}{d\theta}.$$

Here, we differentiate the inductance  $L$  with respect to  $\theta$  while maintaining a constant current. This illustrates that the torque produced by a switched reluctance motor is proportional to the square of the current  $i^2$  and also proportional to the rate of change of inductance with respect to the rotor position,  $\frac{dL}{d\theta}$ .

To maximize torque, it is crucial to switch the current in the stator winding only during intervals when  $\frac{dL}{d\theta}$  is positive. This strategy ensures that we achieve positive torque.

Now, let's explore the constructional principles of a typical switched reluctance motor. For this example, we will consider an SRM with an 8-pole stator and a 4-pole rotor, which provides a fascinating insight into its design and functionality.

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Let's delve into the structure of the switched reluctance motor (SRM). This motor features a stator with 8 stator poles, also known as stator teeth, and a rotor with 6 rotor poles or teeth. If we examine a cross-sectional view of the motor, we can visualize the rotor nestled within the stator, with the stator enveloping the rotor.

The rotor itself consists of 6 salient poles, giving it a distinctive structure without any windings. These salient poles are positioned as follows: there's one rotor pole here, another one there, and so on, until we reach the sixth rotor pole. On the other hand, the stator is divided into 8 parts, corresponding to its 8 poles. These stator poles also feature salient structures, and importantly, they are equipped with windings.

So, to summarize, we have 8 stator poles and 6 rotor poles. The stator is where the windings are located, while the rotor remains free of windings. As the rotor rotates, there will be variations in inductance, which we denote as  $\frac{dL}{d\theta}$ .

To optimize the torque generated by the motor, we must switch the current in the stator windings precisely when  $\frac{dL}{d\theta}$  is positive. This careful timing ensures that the current is present only during the intervals of positive inductance change, ultimately maximizing the torque output. In our next lecture, we will further explore the constructional features of this 8-pole stator and 6-pole rotor configuration, deepening our understanding of the switched reluctance motor's design and operational principles.