Fundamentals of Electric Drives

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Lecture - 40

Duty Cycle of Traction Drives, Distance between Two Stops, Calculation of Total Tractive Effort and Drive Rating

Hello and welcome to this lecture on the fundamentals of electric drives! In our previous session, we explored the concept of traction drives and examined the speed-time curve of a locomotive as it travels from one station to another. Today, we will build upon that knowledge by deriving expressions for both the distance covered and the drive rating of a locomotive during its journey. So, let's take a closer look at our speed-time curve, which will serve as a foundation for our calculations.

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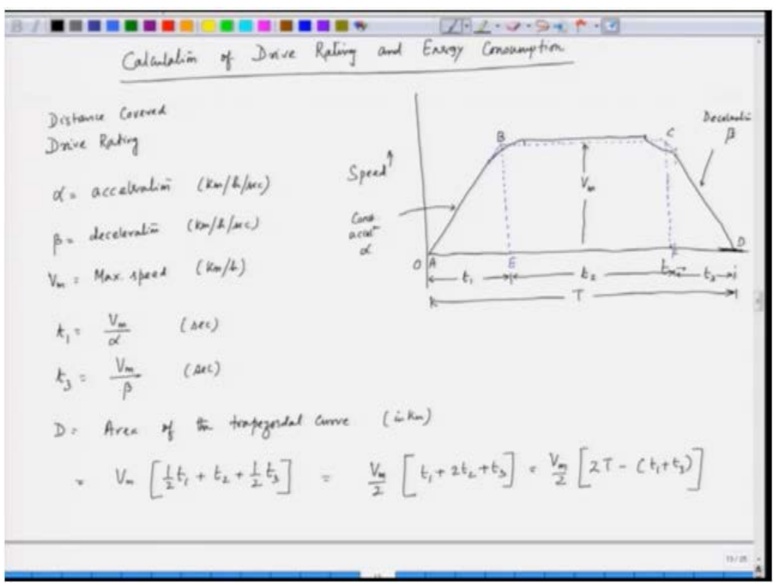

We have defined several key phases in our analysis: constant torque, constant power, free running, coasting, and braking. To better understand the relationship between speed and time, we will approximate the speed-time curve as a trapezoid. Let's label the various sections of this trapezoidal curve as A, B, C, and D.

To illustrate this, we can drop perpendicular lines from the curve to the time axis, labeling these points E and F. This gives us a clear representation of the speed-time curve. Here, we denote α as the acceleration, measured in kilometers per hour per second. In this context, we have a constant acceleration represented by the value α.

We also need to consider the deceleration during the braking phase, which we will denote as β, also in kilometers per hour per second. For reference, we define the time intervals as follows: t_1 corresponds to the time from point A to point B, t_2 is the duration of constant speed, and t_3 represents the braking period. The total time of travel is denoted by T.

To find the distance traveled, we must calculate the area under the curve. This area corresponds to the distance. The angle associated with this trapezoid represents the acceleration. We can express t_1 as the time required to reach maximum speed V_m, which is measured in kilometers per hour. Thus, we can determine that:

$$t_1 = \frac{V_m}{\alpha}$$

This gives us the time taken to accelerate from A to E. Similarly, for the braking phase, t_3 is defined as the time taken to decelerate from V_m to a stop:

$$t_3 = \frac{V_m}{\beta}$$

The area D under the trapezoidal curve can be expressed as:

$$D = \frac{V_m}{2}(t_1 + t_2 + t_3)$$

To simplify this expression, we factor out $\frac{V_m}{2}$:

$$D = \frac{V_m}{2}(t_1 + 2t_2 + t_3) = \frac{V_m}{2}(2T - t_1 + t_3)$$

Next, we can substitute the values of t_1 and t_3 into this equation:

$$D = \frac{V_m}{2}\left(2T - \frac{V_m}{\alpha} + \frac{V_m}{\beta}\right)$$

This allows us to further simplify our calculation of the distance covered during the locomotive's journey.

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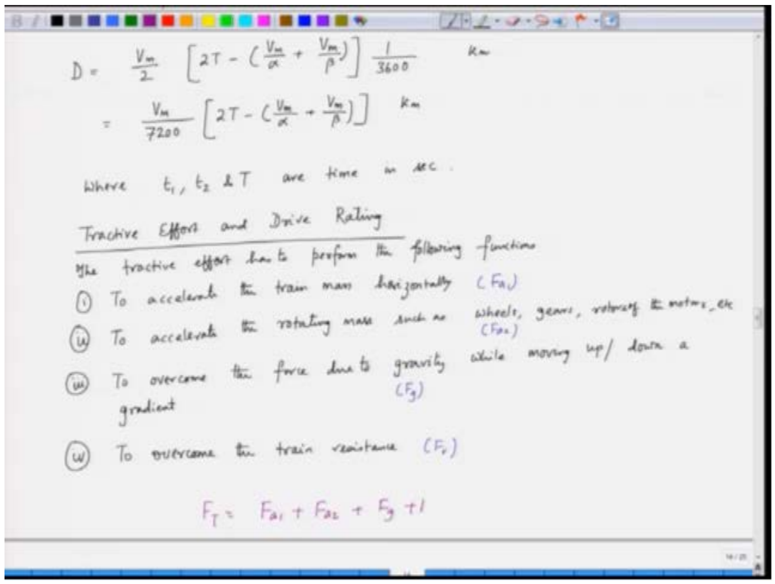

The expression we have derived for the distance traveled is given by:

$$D = \frac{V_m}{7200}\left(2T - \frac{V_m}{\alpha} + \frac{V_m}{\beta}\right)$$

In this formula, V_m is measured in kilometers per hour, and since we are working with time in seconds, we need to convert V_m to kilometers by dividing by 3600. This conversion ensures that our final expression accurately represents the distance in kilometers. Thus, the factor of $\frac{1}{3600}$ is applied to our calculations, leading us to the simplified form in kilometers.

It's important to note that t_1, t_2, and T are all expressed in seconds, while the acceleration values α and β are in kilometers per hour per second. Therefore, when we seek to determine the distance covered between the two stops, we arrive at the final expression:

$$D = \frac{V_m}{7200}\left(2T - \frac{V_m}{\alpha} + \frac{V_m}{\beta}\right) \quad \text{(in kilometers)}$$

Next, we must address the concept of drive rating, which requires us to calculate the tractive effort. As the train moves, several components contribute to the overall tractive effort. In a steady state, these include train resistance, frictional forces, internal friction, external friction, and wind resistance. However, we also need to consider the situation when the train is accelerating. The train does not simply maintain a constant speed; it accelerates from a standstill to its maximum velocity.

So, let's discuss the tractive effort required when the train is accelerating from zero speed to a certain speed. Understanding these dynamics will help us accurately calculate the necessary tractive effort during acceleration.

Let's discuss the tractive effort and drive rating in detail. The tractive effort must accomplish several critical functions. First and foremost, it needs to accelerate the train's mass horizontally—that is, it has to enable the train to move forward. Additionally, the tractive effort must also accelerate the rotating masses, which include components such as the wheels, gears, and motor rotors.

Furthermore, when the train is traversing a gradient, the tractive effort must be sufficient to overcome the gravitational force. This means it must counteract the effects of gravity when the train is moving uphill or downhill. Lastly, the tractive effort must also be capable of overcoming train resistance, which can stem from various factors including friction and air resistance.

To quantify these functions, we can denote the different components of tractive effort as follows:

- To accelerate the train horizontally, we will call the tractive effort F_{a1}.
- To accelerate the rotating masses such as wheels, gears, axles, and motor rotors, we will refer to this as F_{a2}.
- The tractive effort required to overcome the gravitational force will be represented as F_g.
- Lastly, the force needed to counteract train resistance will be labeled F_r.

So, the total tractive effort can be expressed as:

$$\text{Tractive Effort} = F_{a1} + F_{a2} + F_g + F_r$$

Now, let's take a closer look at each of these components to evaluate their contributions to the overall tractive effort required for efficient train operation.

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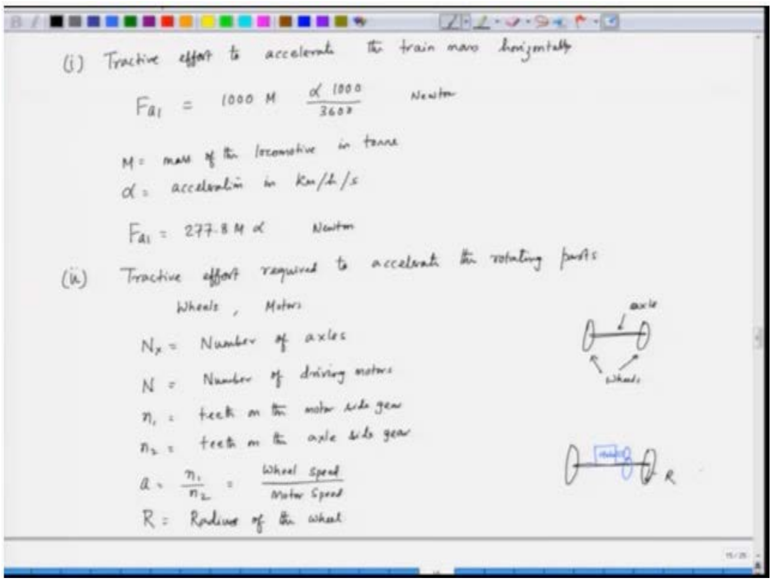

To begin with, we need to evaluate the tractive effort required to accelerate the train's mass horizontally. The expression for this tractive effort can be given as:

$$F_{a1} = 1000 \times M \times \alpha \times \frac{1000}{3600}$$

where F_{a1} is expressed in Newtons (N). Here, M represents the mass of the locomotive in tonnes, and since 1 tonne equals 1000 kg, we multiply by 1000 to convert it. The variable α denotes the acceleration, measured in kilometres per hour per second.

To convert this acceleration into metres per second squared, we must convert kilometres into metres by multiplying by 1000, and we need to convert hours into seconds by multiplying the hour component by 3600. Therefore, we can simplify F_{a1} to:

$$F_{a1} = 277.8 \times M \times$$

with the unit being Newtons (N).

Next, we need to consider the tractive effort required to accelerate the rotating parts of the train. The primary rotating parts include the wheels and the motors. For our calculations, we will neglect the moment of inertia of the axles, as this can be effectively combined with that of the wheels.

Let's denote N_x as the number of axles, with each axle supporting two wheels. Thus, the total number of wheels is $2 \times N_x$. When discussing the driving motors, we denote the number of driving motors as N.

Now, each motor drives its corresponding wheel via an axle. In our configuration, we have two gears: one on the motor side and another on the axle side. We can denote the number of teeth on the motor side gear as n_1 and on the axle side gear as n_2.

The gear ratio a is defined as:

$$a = \frac{n_1}{n_2}$$

This ratio indicates that the wheel speed is the same as the axle speed, as the wheel is directly connected to the axle. However, since the motor is not directly connected to the wheel but rather to the axle via a fixed gear, we can express the relationship between the wheel speed and motor speed accordingly.

Lastly, let R be the radius of the wheel, expressed in metres. Each wheel has its own radius, which plays a crucial role in the overall dynamics of the tractive effort. This radius R is an essential parameter in our calculations, as it influences the torque transmitted from the motor to the wheels.

Let's discuss the details of the moments of inertia involved in our system. First, we denote J_w as the moment of inertia of a single wheel, while J_m represents the moment of inertia of a single motor, specifically the rotor of the motor.

To calculate the total moment of inertia for all the wheels, we start with N_x, the number of axles. Since each axle has two wheels, the total number of wheels becomes $2 \times N_x$. Consequently, the total moment of inertia of all wheels can be expressed as:

$$J_1 = 2 \times N_x \times J_w$$

Now, moving on to the total moment of inertia of all motors, we have:

$$J_2 = N \times J_m$$

where N is the total number of motors, and J_m is the moment of inertia of a single motor.

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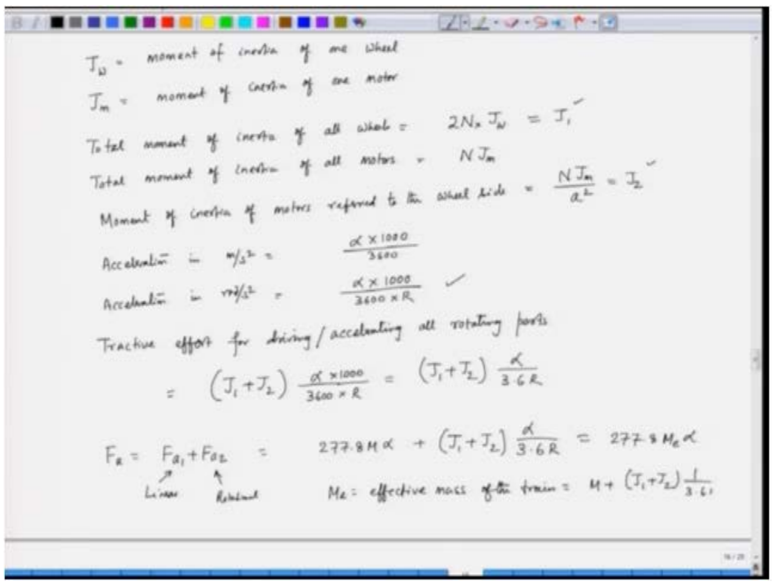

Next, we need to refer the moment of inertia of the motors to the wheels because the motors are positioned before the wheels in the drive system, connected through gears. To convert this inertia J_m to the wheel side, we apply the gear ratio. Thus, the referred moment of inertia of the motors to the wheel side can be expressed as:

$$J_{mref} = \frac{J_m}{a^2}$$

where a is the gear ratio, defined as the number of teeth on the motor side gear to the number of teeth on the axle side gear.

Now, the total moment of inertia of the motors referred to the wheels becomes:

$$J_2 = N \times \frac{J_m}{a^2}$$

In our calculations, we are effectively referring all variables to the wheel side, establishing J_1 and J_2 as our moments of inertia for wheels and motors, respectively.

Lastly, we need to consider the acceleration in our calculations. While we have been discussing acceleration in kilometres per hour per second, to convert this into metres per second squared, we apply the following conversion:

$$\text{Acceleration} = \alpha \times \frac{1000}{3600}$$

This conversion ensures that we maintain consistency in our units across the calculations.

When considering the acceleration in a rotating system, it's crucial to convert our linear acceleration into radians per second squared. This is done by dividing by the radius of the wheel, R, which is measured in meters. Thus, we establish that:

$$\text{Angular Acceleration} = \frac{\alpha \times \frac{1000}{3600}}{R}$$

Now, let's discuss the tractive effort required to drive or accelerate all the rotating parts. The total tractive effort for this purpose is given by the equation:

$$F_{a_{rot}} = (J_1 + J_2) \times \text{Angular Acceleration}$$

In our case, the angular acceleration can be expressed as:

$$\text{Angular Acceleration} = \frac{\alpha \times \frac{1000}{3600}}{R}$$

Thus, we can rewrite our tractive effort equation as:

$$F_{a_{rot}} = (J_1 + J_2) \times \frac{\alpha}{3.6R}$$

Now, let's consider the total acceleration. The total accelerating force F_a is the sum of the linear and rotational components:

$$F_a = F_{a_{lin}} + F_{a_{rot}}$$

The linear component is given by:

$$F_{a_{lin}} = 277.8 \times M \times \alpha.$$

and the rotational component as previously derived:

$$F_{a_{rot}} = (J_1 + J_2) \times \frac{\alpha}{3.6R}$$

Combining these two forces, we arrive at:

$$F_a = 277.8 \times M \times \alpha + (J_1 + J_2) \times \frac{\alpha}{3.6R}$$

This allows us to express the total force in a more compact form:

$$F_a = 277.8 \times M_e \times \alpha.$$

Where M_e represents the effective mass of the train, calculated as:

$$M_e = M + \frac{(J_1 + J_2)}{3.6R}$$

This effective mass takes into account both the linear acceleration and the angular acceleration of the rotating mass, resulting in a value that is typically about 10% higher than the actual mass of the train. Therefore, instead of using M, we should utilize M_e, which accurately reflects the dynamics involved in both linear and angular acceleration.

Lastly, we must consider the tractive effort required to overcome gradients in our calculations.

The tractive effort required to overcome the force due to gravity is a critical consideration in locomotive performance. Typically, we define the gradient G in terms of elevation over a distance of 1,000 meters. This means that G represents the change in elevation across this specified distance.

Imagine a track that is inclined, with our locomotive moving along it—either ascending or descending.

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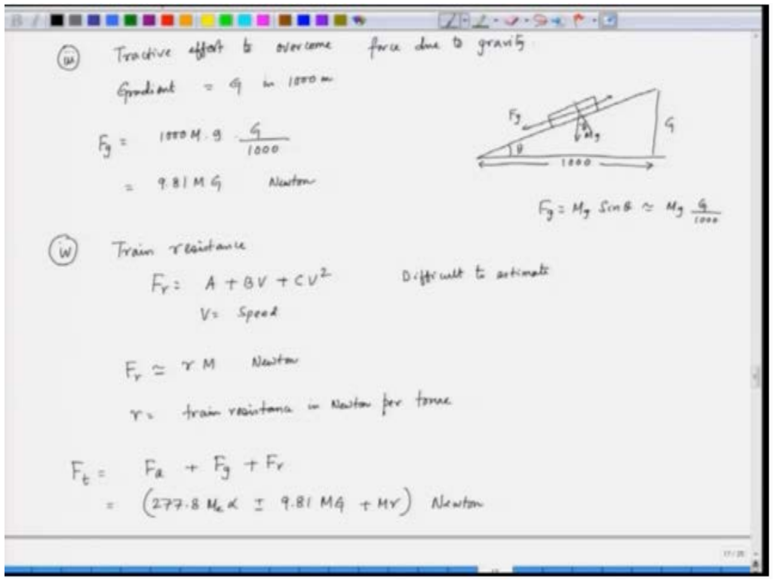

When the locomotive is moving uphill, the gravitational force acting on it can be expressed as Mg, where M is the mass of the locomotive. This mass has a component acting along the incline, which we can describe using the angle θ. Notably, this angle θ corresponds to the gradient, where we can define the gradient as the elevation over 1,000 meters. To find the gravitational force acting on the locomotive, we use the formula:

$$F_g = Mg \cdot \sin(\theta) \approx \frac{Mg \cdot G}{1000}$$

This simplifies to:

$$F_g = 9.81\, M \cdot G \quad \text{(in Newtons)}$$

This expression captures the gravitational force acting on the locomotive, which can either be positive (for an uphill gradient) or negative (for a downhill gradient).

Next, we must consider train resistance, which can be a complex factor to estimate. The resistance experienced by the train is generally expressed as a function of its speed V. Due to the intricacies involved in accurately assessing train resistance, we often rely on an approximate formula. We typically express train resistance as a function of the mass of the train, noting that it is significantly smaller in magnitude compared to the force required for acceleration. Thus, we approximate train resistance as approximately 20 Newtons per tonne.

Consequently, we can express the train resistance R as:

$$R \approx r \cdot M \quad \text{(in Newtons)}$$

where r represents the train resistance in Newtons per tonne of mass.

Finally, to calculate the total tractive effort, we sum the accelerating effort, the force required to overcome the gradient, and the train resistance. Therefore, we arrive at the equation:

$$\text{Total Tractive Effort} = \text{Accelerating Effort} + \text{Gradient Force} + \text{Train Resistance}$$

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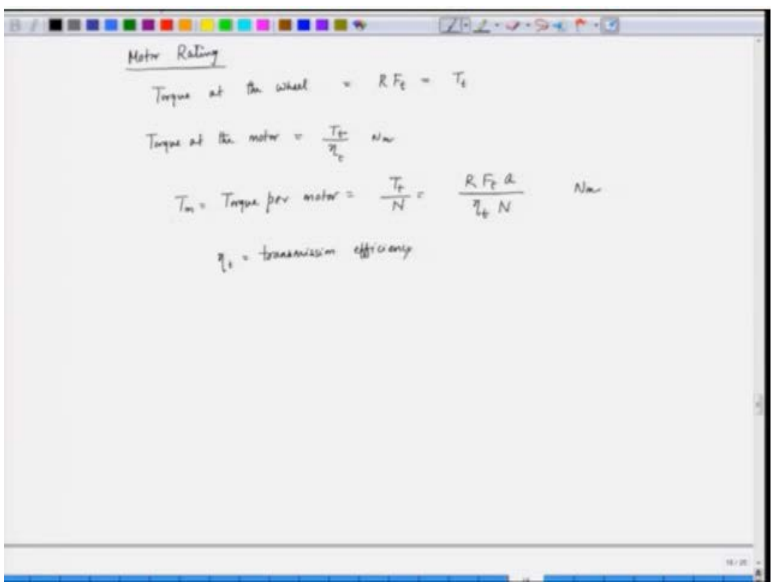

Substituting the expressions we've discussed, we get:

$$\text{Total Tractive Effort} = 277.8\, M_e \cdot \alpha + 9.81\, M \cdot G + r \cdot M$$

This equation provides a comprehensive understanding of the total tractive effort exerted by the train, enabling it to overcome various forces and continue moving forward effectively, powered by the motor's efforts.

To calculate the drive rating, we can proceed with the following steps. First, let's consider the torque at the wheel. The torque at the wheel can be expressed as:

$$\text{Torque}_{\text{wheel}} = R \times F_t$$

where R is the radius of the wheel and F_t is the tractive force. Next, we can determine the torque at the motor, which is related to the torque at the wheel by the transmission efficiency (η_t). Therefore, the torque at the motor can be calculated using the formula:

$$T_m = \frac{\text{Total Torque}}{\text{Number of Motors}} = \frac{R \cdot F_t}{\eta_t}$$

Here, T_m is the torque per motor, and its unit is Newton meters. The transmission efficiency η_t accounts for any losses in the system.

We previously discussed that the tractive effort is composed of several components: the effort to accelerate the train, the effort required to overcome the gradient, and the effort necessary to counteract train resistance. If we wish to determine the time taken to reach a certain speed, we must evaluate the tractive effort perceived by the train, which is represented by this essential expression.

It is crucial to remember this expression and apply it appropriately in relevant contexts, as it plays a vital role in our calculations.

In this lecture, we have covered how to calculate the distance traveled by the train, explored the various components of tractive effort, and finally evaluated the torque per motor, which is derived from the total torque divided by the number of motors. With that, we stop here for todays lecture and hopefully we have been able to touch upon the important topics in this course. So thank you very much.