Fundamentals of Electric Drives

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Lecture - 05

Load Equalization, Characteristics of DC Motor

Hello and welcome to this lecture on fundamental of electric drives. In the last lecture, we were discussing about the load equalization by connecting a flywheel. Today, we will continue from that lecture.

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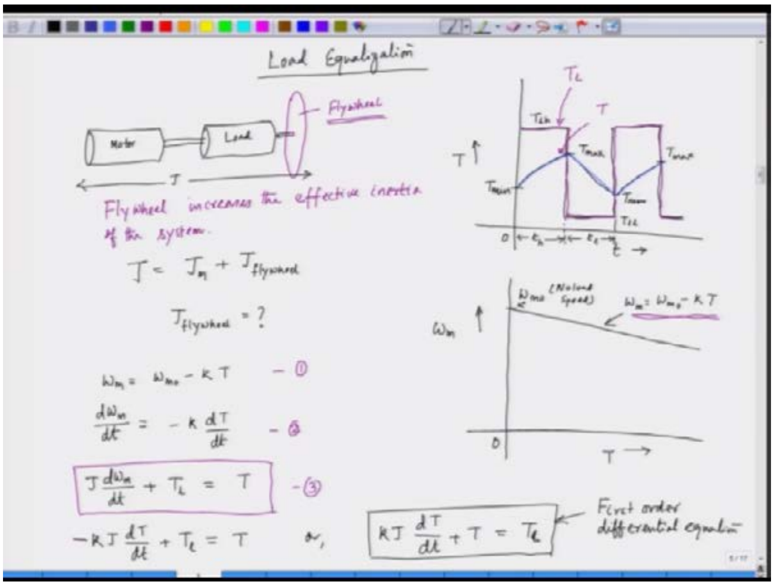

Now, in this scenario, we have connected a flywheel to the motor-load combination. Here is the flywheel, and the load torque exhibits a rectangular pattern; it is indeed a pulsating load torque. Our objective is to ensure that the motor torque fluctuates between a minimum value T_{min} and a maximum value T_{max}. We have been provided with the motor characteristics.

The motor has a speed-torque characteristic, which is represented as a straight line. The equation describing this linear relationship is given by:

$$\omega_m = \omega_{m0} - kT$$

where T represents the torque. In this equation, ω_m is the motor speed, and ω_{m0} is the no-load speed, the speed at which the motor operates when there is no load applied.

Now, we need to utilize this motor characteristic to determine the inertia of the flywheel. Let's write down the dynamic equation. Starting with the motor characteristic:

$$\omega_m = \omega_{m0} - kT$$

we can differentiate this with respect to time to find:

$$\frac{d\omega_m}{dt}$$

It is important to remember that ω_{m0} represents the no-load speed, which is the speed achieved when the load is zero.

Now, since this is a constant quantity, the derivative will be equal to zero. What we derive from this is that -k is a constant, leading us to the term $\frac{dT}{dt}$, and we will apply our equation. The equation we have is:

$$J\frac{d\omega_m}{dt} + T_l = T$$

This equation is quite well-known in the context of load and motor dynamics. Here, $J\frac{d\omega_m}{dt}$ represents the inertial torque, T_l is the load torque, and T is the motor torque.

Next, we substitute for $\frac{d\omega_m}{dt}$ using our earlier equation. Let’s denote this as Equation 1 and the motor equation as Equation 2. We can now derive Equation 3. By substituting for $\frac{d\omega_m}{dt}$ from Equation 2, we arrive at the expression:

$$-kJ\frac{dT}{dt} + T_l = T$$

Rearranging this, we can write it in a more conventional form:

$$kJ\frac{dT}{dt} + T = T_l$$

Ultimately, we have derived this equation. This represents a first-order differential equation in terms of T, the motor torque. Consequently, we need to solve this equation. Let's explore how we can approach solving it.

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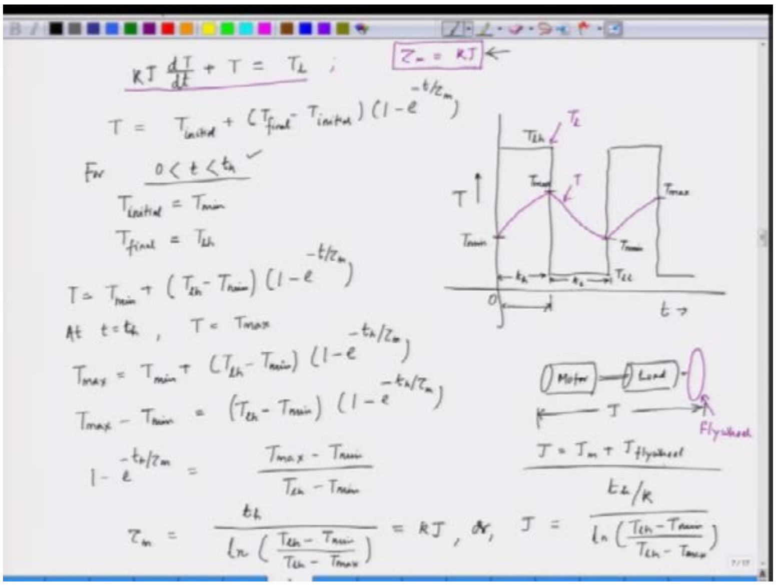

We have the equation:

$$kJ\frac{dT}{dt} + T = T_l$$

This equation needs to be solved. Upon solving it, we arrive at the following solution:

$$T = T_{\text{initial}} + T_{\text{final}} - T_{\text{initial}} \cdot \left(1 - e^{-\frac{T}{\tau_m}}\right)$$

Here, we notice that a time constant is involved. In this equation, the time constant τ_m is defined as $k \cdot J$. Thus, we can express the solution as:

$$T = T_{\text{initial}} + T_{\text{final}} - T_{\text{initial}} \cdot \left(1 - e^{-\frac{T}{\tau_m}}\right)$$

Now, we need to determine what T_{initial} and T_{final} are. If we revisit the pulsating torque that we illustrated earlier, we can clarify this further. As shown before, this is a pulsating load torque that fluctuates between high and low values. We designate the high value as T_{lh} and the low value as T_{ll}.

Our goal is for the motor's behavior to align with this torque variation. Thus, if we plot the motor torque T alongside the load torque T_l, we want the motor torque to remain confined between T_{min} and T_{max}. This means that even though the load experiences significant fluctuations, the variation in motor torque is more or less smoothed out, staying within the limits of T_{min} and T_{max}.

This curve represents an exponential relationship, oscillating between T_{min} and T_{max}. We refer to the duration during which the torque is high as t_h and the duration during which the torque is low as t_l. The equation

$$kJ\frac{dT}{dt} + T = T_l$$

is applicable to the motor behavior as it varies between T_{min} and T_{max}. For the initial time interval, let's assume it ranges from 0 to t_l.

At this point, we can define $T_{initial}$ and T_{final}. So, what is $T_{initial}$? Observing the curve, we find that $T_{initial}$ corresponds to T_{min}, the minimum value of torque. Hence, we can conclude that $T_{initial} = T_{min}$.

Next, let's consider T_{final}. If the system continues to operate under the condition of high load torque, T_{final} will ultimately reach T_{lh}. Thus, we can say $T_{final} = T_{lh}$.

Now, substituting these values into the earlier equation, we obtain:

$$T = T_{\min} + T_{lh} - T_{\min} \cdot \left(1 - e^{-\frac{T}{\tau_m}}\right)$$

This equation is valid for the time interval between 0 and t_l.

When we reach $t = t_l$ and $t = t_h$, we find that at $t = t_h$, the torque becomes T_{max}. Thus, we can express this relationship at the high torque interval, which is crucial for understanding the dynamics of our system.

When the time reaches the point where the torque actually attains T_{max}, we can express this relationship as:

$$T_{\max} = T_{\min} + T_{lh} - T_{\min} \cdot \left(1 - e^{-\frac{t_h}{\tau_m}}\right)$$

This equation serves as a foundation for determining the inertia. Rearranging the various terms, we have:

$$T_{\max} - T_{\min} = T_{lh} - T_{\min} \cdot \left(1 - e^{-\frac{t_h}{\tau_m}}\right)$$

This can be further simplified to:

$$1 - e^{-\frac{t_h}{\tau_m}} = \frac{T_{\max} - T_{\min}}{T_{lh} - T_{\min}}$$

From this, after some algebraic manipulation, we can isolate the time constant τ_m. Thus, we find:

$$\tau_m = \frac{t_h}{\ln\left(\frac{T_{lh} - T_{\min}}{T_{lh} - T_{\max}}\right)}$$

This expression gives us the value of the time constant τ_m. Since we know that the time constant is defined as kJ, we can equate:

$$\tau_m = kJ$$

From this, we can derive the value of J as follows:

$$J = \frac{t_h}{k \cdot \ln\left(\frac{T_{lh} - T_{\min}}{T_{lh} - T_{\max}}\right)}$$

This is how we can calculate the inertia of the system.

It's important to note that this inertia is a combined inertia. We have a motor, a load, and a flywheel in our setup. Therefore, we can express the combined inertia J as:

$$J = J_m + J_{\text{flywheel}}$$

By understanding the total value of J and having knowledge of the motor inertia as well as the load inertia, we can effectively determine the inertia of the flywheel.

The inertia of the flywheel can be expressed as the difference between the combined inertia and the inertia of the motor. Thus, we have:

$$J_{\text{flywheel}} = J - J_m$$

This relationship ensures that the actual torque of the motor can fluctuate between T_{min} and T_{max}. This process is known as load equalization, where we connect a flywheel to help balance the load torque between these two values.

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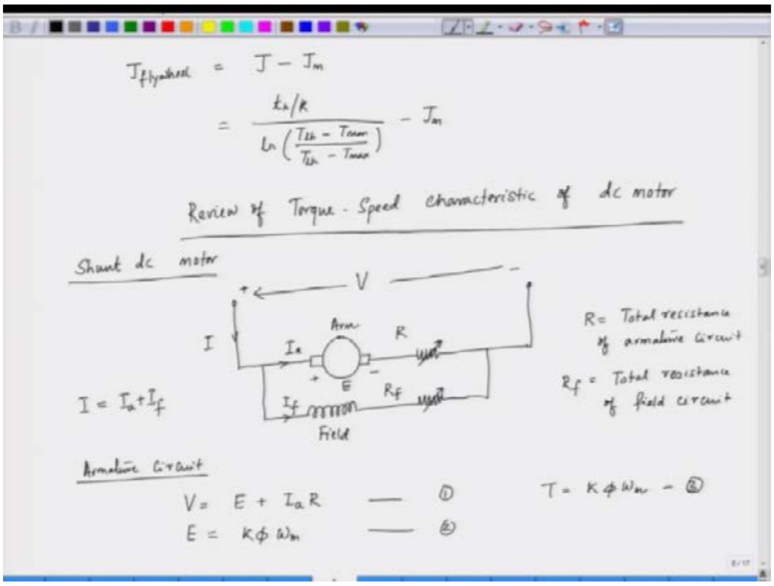

Now, let's take a moment to review the characteristics of a DC motor. We have already studied the fundamentals of DC motors, but we'll briefly revisit their characteristics, starting with the shunt motor.

In a shunt motor, we have the armature and a resistance placed in the armature circuit, as well as a field circuit with its own resistance. Specifically, we denote the resistance of the armature as R and the resistance of the field circuit as R_f.

When we apply a DC supply to both the armature and the field, the motor draws a current, denoted as I. This current is divided, with a portion flowing to the armature and the remaining portion supplying the field.

Let's clarify the components involved in our discussion. Here, R represents the total resistance of the armature circuit, while R_f denotes the total resistance of the field circuit.

We can derive some fundamental equations. First, let's consider the current drawn from the DC power supply, denoted as I. This current is the sum of the armature current I_a and the field current I_f:

$$I = I_a + I_f$$

In the armature circuit, we can express the applied voltage V in relation to the motor's back electromotive force (EMF), E. The equation can be written as follows:

$$V = E + I_a \cdot R$$

Now, what is this back EMF? The back EMF E is given by the formula:

$$E = k \cdot \phi \cdot \omega_m$$

where ω_m represents the motor speed.

Additionally, we have a torque equation that describes the torque produced by the motor:

$$T = k \cdot \phi \cdot \omega_m$$

These three equations encapsulate the core dynamics of a DC motor. To summarize, we have:

1. The voltage equation:

$$V = E + I_a \cdot R$$

2. The back EMF equation:

$$E = k \cdot \phi \cdot \omega_m$$

3. The torque equation:

$$T = k \cdot \phi \cdot \omega_m$$

Our next step is to derive the speed-torque characteristic of a shunt DC motor using these foundational equations.

Now, let's examine the equation $V = E + I_a \cdot R$. We can rearrange this to express the back EMF E as:

$$E = V - I_a \cdot R$$

What exactly is this back EMF? The back EMF is defined as:

$$E = k \cdot \phi \cdot \omega_m$$

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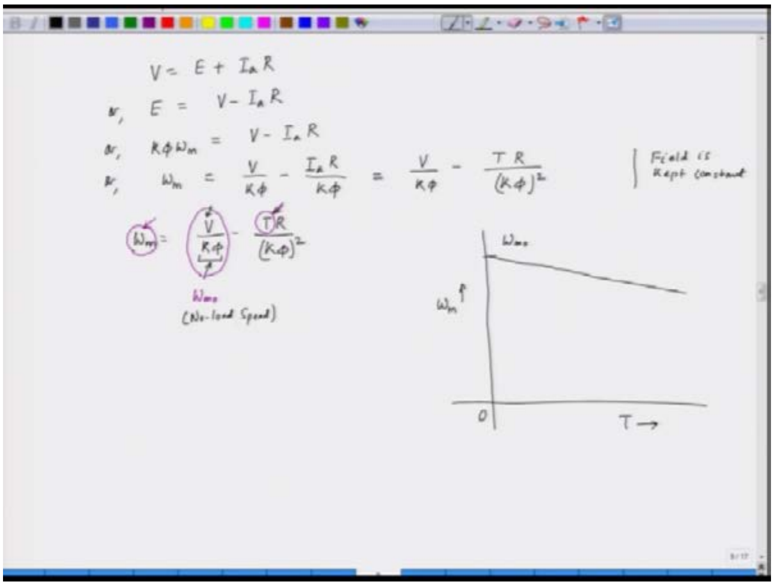

Therefore, we can write:

$$k \cdot \phi \cdot \omega_m = V - I_a \cdot R$$

From this, we can isolate the speed of the motor ω_m:

$$\omega_m = \frac{V}{k \cdot \phi} - \frac{I_a \cdot R}{k \cdot \phi}$$

Next, substituting I_a with the expression $I_a = \frac{T}{k \cdot \phi}$, we derive the following equation:

$$\omega_m = \frac{V}{k \cdot \phi} - \frac{T \cdot R}{(k \cdot \phi)^2}$$

In this scenario, we have kept the field current constant, which means the flux φ remains unchanged. Thus, this equation represents the relationship for the speed of the motor:

$$\omega_m = \frac{V}{k \cdot \phi} - \frac{T \cdot R}{(k \cdot \phi)^2}$$

This equation describes a straight line when we plot ω_m on the y-axis against torque T on the x-axis. Here, V is the applied voltage, and $k \cdot \varphi$ is constant due to our assumption of constant flux.

Since $k \cdot \varphi$ remains constant, both T and ω_m are treated as variables in our plot. Essentially, we are graphing speed versus torque. The first term in the equation represents the no-load speed ω_{m0}, while $\frac{R}{(k \cdot \phi)^2}$ indicates the slope of this characteristic curve.

Thus, we obtain a straight-line graph where ω_{m0} denotes the no-load speed, and the slope of the characteristic is given by $\frac{R}{(k \cdot \phi)^2}$.

This characteristic applies to both shunt DC motors and separately excited DC motors. Now, the next step is to discuss how we can control the speed of a shunt DC motor or a separately excited DC motor.

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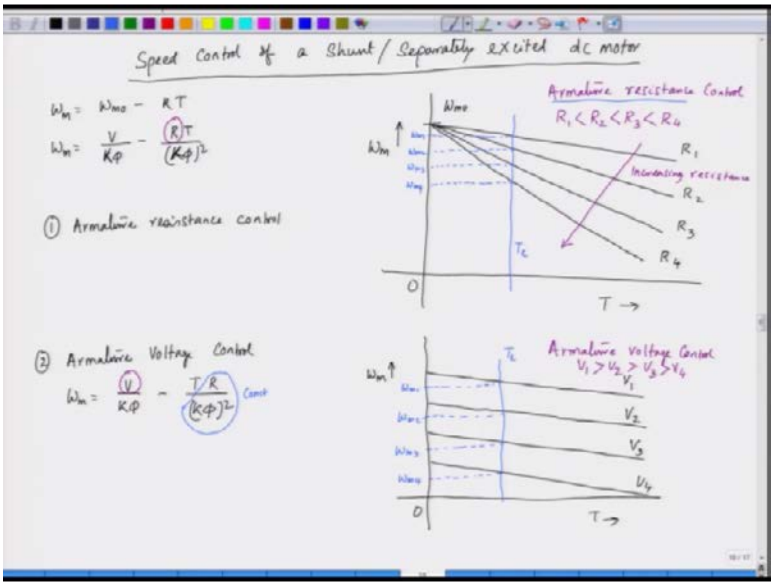

Now, let’s delve into the speed control of a shunt or separately excited DC motor. To illustrate this, we'll first draw the characteristic curve, with motor speed ω_m plotted on the y-axis and torque T on the x-axis. The characteristic exhibits a straight-line relationship, resembling the following:

$$\omega_m = \omega_{m0} - k \cdot T$$

Here, ω_{m0} represents the no-load speed, and we can express this relationship more precisely as:

$$\omega_m = \frac{V}{k \cdot \phi} - \frac{R \cdot T}{(k \cdot \phi)^2}$$

In this equation, k remains consistent throughout. Now, let's discuss the first method of speed control, which is the armature resistance method. By altering the armature resistance R, we essentially modify this parameter in our equation.

When we change R, the slope of the characteristic line will be affected, but it’s important to note that the no-load speed ω_{m0} remains unchanged. The characteristic curve will transform accordingly, shifting positions as we adjust the armature resistance.

As we increase the armature resistance, the characteristic will change in this manner:

- For R_1
- For R_2
- For R_3
- For R_4

In this scenario, we observe that $R_1 < R_2 < R_3 < R_4$, meaning we are effectively increasing the armature resistance. Consequently, this adjustment leads to a variation in the torque-speed characteristic.

Now, suppose we have a load torque T_l that remains nearly constant. This load torque can be represented on the same graph, which helps us understand the impact of changing armature resistance on the motor's performance under load conditions.

The intersection of the motor and load characteristics determines the operating point of the system. Initially, we have one speed, represented as ω_{m1}. As we change the armature resistance, the speed adjusts accordingly, leading us to a second speed, ω_{m2}. If we continue to increase the resistance, the speed will shift again, resulting in ω_{m3} and then ω_{m4}.

Thus, with each increment of armature resistance, we see a corresponding change in speed. This is how speed control can be achieved through the armature resistance method. However, it’s important to note that this method is not particularly efficient. The introduction of armature resistance leads to I^2R losses, which can impact overall performance.

Now, let’s explore an alternative method of speed control: armature voltage control.

In this approach, we manipulate the armature voltage. The torque-speed characteristic can be represented as ω_m versus torque T for a specific voltage value. When we alter the voltage, the equation modifies to:

$$\omega_m = \frac{V}{k \cdot \phi} - \frac{T \cdot R}{(k \cdot \phi)^2}$$

Here, we are adjusting the armature voltage, which in turn affects the no-load speed of the motor. As we change the voltage, we can observe the resulting variations in the speed, enhancing our control over the motor's operation.

When we adjust the voltage V, we observe that the no-load speed changes, while the slope of the characteristic remains constant. This slope is effectively a fixed quantity. As a result, when the no-load speed varies, we can plot a family of curves representing different voltage levels, such as V_1, V_2, V_3, and V_4. In this scenario, we see that $V_1 > V_2 > V_3 > V_4$.

Now, if we consider the load characteristic, we can identify various speeds: ω_{m1}, ω_{m2}, ω_{m3}, and ω_{m4}. As we change the voltage, the corresponding speeds also change. This method of speed control is more efficient than the armature resistance control because it minimizes I^2R losses; we are simply adjusting the voltage amplitude rather than adding resistance.

To implement armature voltage control effectively, we require a variable voltage power supply. If we have a DC power supply that allows us to vary the voltage, we can utilize this method, which proves to be more efficient than controlling speed through armature resistance.

That concludes today's lecture. We will continue our discussion in the next session.