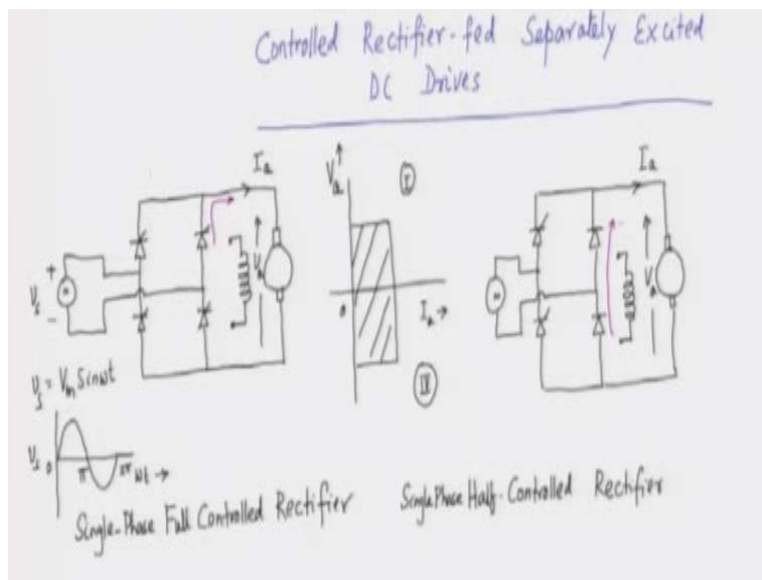


**Fundamentals of Electric Drives**  
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**Lecture – 08**

**Speed Control of Separately Excited DC Motor Using Controlled Rectifiers**

Hello and welcome to this lecture on the fundamentals of electric drives. In the last session, we explored the characteristics of DC motors, with a particular focus on shunt motors and series motors. Today, we will delve into the topic of speed control for a separately excited DC motor powered by a controlled rectifier. Let's now examine how we can effectively manage the motor's speed by utilizing controlled rectifiers.

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Now, let's explore the different types of controlled rectifiers we have. Primarily, there are two types: the full-controlled rectifier and the half-controlled rectifier. Let's start by looking at the topology of these two rectifiers.

For the full-controlled rectifier, we use a thyristor bridge, composed of silicon-controlled rectifiers (SCRs). This bridge feeds the armature of a DC motor, while the field winding is separately

excited. The input to this bridge comes from an AC supply.

Now, in this setup, the current flowing through the system is denoted by  $I_a$ , which represents the output current, and the output voltage across the bridge is denoted by  $V_a$ . In this case, it's important to note that the current can only flow in one direction because we are using controlled rectifiers (SCRs), which can only deliver current in a forward direction. This means current flows from the SCR to the load in one direction only, without any reverse current flow. However, the output voltage  $V_a$  can be either positive or negative due to the AC input supply.

The input voltage, denoted as  $V_s$ , is an AC waveform and can be mathematically expressed as:

$$V_s = V_m \sin(\omega t)$$

If we plot this input voltage, we get an alternating sinusoidal waveform. On the graph, the x-axis represents  $\omega t$ , and the y-axis shows the voltage. At key points, such as  $\pi$  and  $2\pi$ , we can observe the alternating nature of the supply voltage.

Now, although the input voltage is alternating, the output of this full-controlled rectifier will have an average DC component. This average output voltage can be positive or negative, depending on the firing angle of the SCRs, which is what makes it a full-controlled rectifier. And since we are working with a single-phase input, we refer to this setup as a single-phase full-controlled rectifier.

If we plot the average voltage and current on a plane, with voltage on the y-axis and current on the x-axis, this converter operates in two quadrants: the 1st and the 4th. The voltage can be both positive and negative, but the current can only be positive. Therefore, this is a two-quadrant converter, capable of operating in the 1st and 4th quadrants.

Now, let's examine another type of controlled rectifier. In this configuration, we again have four devices, but unlike the full-controlled rectifier, only two of these devices are SCRs, while the other two are diodes. This circuit also feeds the armature of a separately excited DC motor, with the field winding being separately excited. The current through the armature is denoted as  $I_a$ , and we observe the average current and voltage at the output. The input supply here, as before, is an AC source.

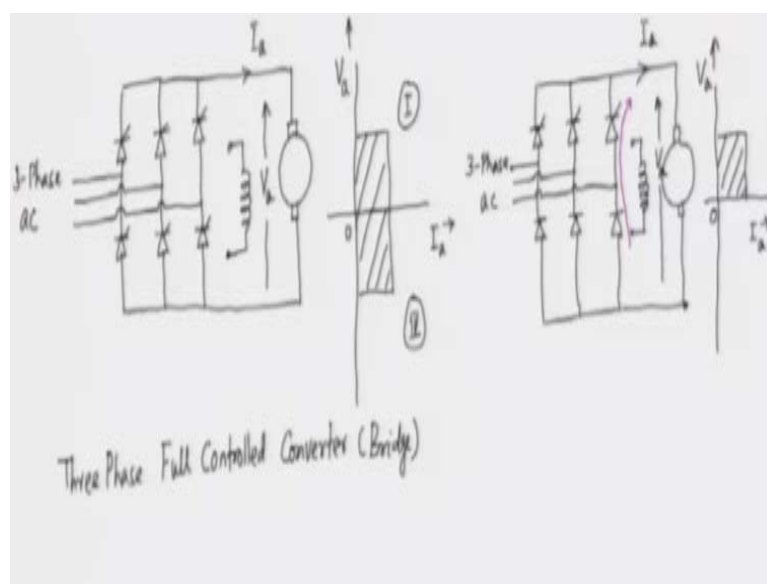
This type of rectifier is known as a half-controlled rectifier, and since we are working with a single-phase AC input, it is referred to as a single-phase half-controlled rectifier.

Now, let's consider the operating quadrants of this rectifier. Due to the presence of the two diodes in the circuit, these diodes provide a freewheeling path. This means that whenever the output voltage  $V_a$  tends to go negative, these diodes become forward biased, effectively clamping the voltage and preventing it from becoming negative. As a result, the output voltage  $V_a$  can never drop below zero, which is why this setup is called a half-controlled converter.

In terms of operating quadrants, this converter can only operate in the 1st quadrant. To visualize this, if we plot the operation of the converter on a VI plane, with  $V_a$  on the y-axis and  $I_a$  on the x-axis, the operation is restricted solely to the 1st quadrant. This is because the voltage can never be negative, and naturally, the current also cannot be negative due to the unidirectional nature of the SCRs and diodes.

Single-phase converters like these are typically used in lower power applications, specifically when the required power is less than 10 kilowatts. For higher power applications, where the power exceeds 10 kilowatts or more, we generally use three-phase converters to handle the increased load and efficiency requirements.

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In the case of a three-phase converter, we can also have similar topologies as those seen in single-phase systems. Here, we are dealing with three-phase converters, specifically the bridge converter topology. All the devices in this bridge are SCRs (Silicon Controlled Rectifiers), and this setup feeds the armature of a DC machine. The field winding, as usual, is separately excited, while the input to the converter is a three-phase AC supply. The output of this converter provides the armature current, denoted as  $I_a$ , and the average output voltage, denoted as  $V_a$ .

If we take a closer look at the input, it consists of a three-phase balanced voltage supply. The output voltage,  $V_a$ , can either be positive or negative, depending on the operating conditions. Just as in the single-phase converter, the three-phase converter can also operate in both the 1st and 4th quadrants. When we examine the operating quadrants, with voltage on the y-axis and current on the x-axis, we observe that this converter operates as a two-quadrant converter, meaning it can function in both the 1st quadrant (positive voltage and positive current) and the 4th quadrant (negative voltage and positive current).

Thus, this configuration is referred to as a three-phase full-controlled converter, and due to its bridge structure, it is commonly known as a bridge converter.

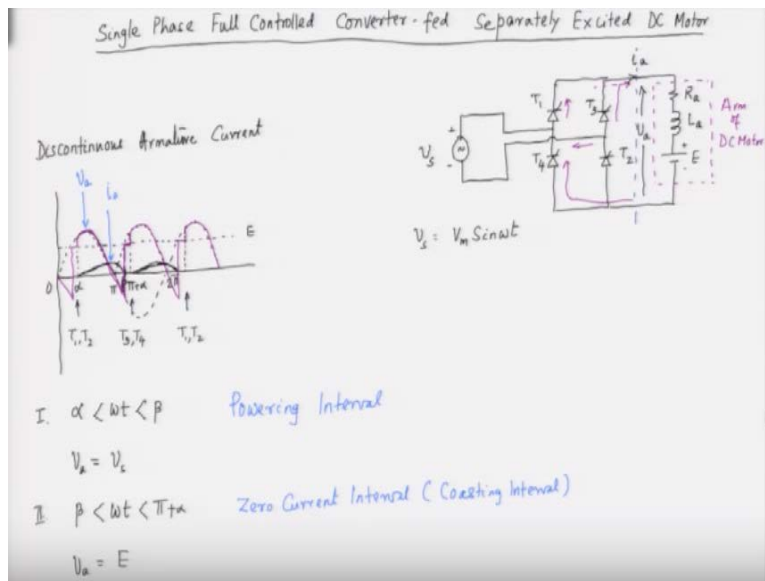
Similarly, we can also implement a half-controlled three-phase converter. In this case, the converter consists of six devices, three of which are SCRs (controlled rectifiers), while the other three are diodes (uncontrolled rectifiers). The diodes act as the uncontrolled rectifiers. The input to this half-controlled converter is again a three-phase AC supply, and the output feeds the armature of a DC motor, with the field winding being separately excited. The armature current is represented by  $I_a$ , and the armature voltage is represented by  $V_a$ .

This half-controlled configuration is often used when full control over the output is not required or when cost efficiency is a priority, as fewer controlled devices (SCRs) are needed compared to the fully controlled bridge converter. If we examine the operation of this converter, it is clear that it only functions in a single quadrant. When we plot the voltage and current on the xy-plane, we can see that this converter is limited to operating in the 1st quadrant, and that is precisely why it is referred to as a half-controlled converter. The voltage cannot become negative because of the freewheeling path that exists between one diode and one SCR.

Whenever the voltage attempts to become negative, the freewheeling action kicks in, forming a path through the diode and SCR. This freewheeling mechanism causes the voltage to drop down to zero, meaning that the output voltage,  $V_a$ , can have a minimum value of zero but can never go below that. As a result, the voltage remains positive, and the converter operates strictly in the 1st quadrant, or quadrant 1, which is why this configuration is termed a three-phase half-controlled converter. Given its bridge topology, it is sometimes referred to as a three-phase half-controlled bridge converter.

Now, let's explore how we can analyze the performance of a separately excited DC motor when it is powered by a single-phase full-controlled converter. This analysis will help us understand the detailed operation of DC motors under such rectifier-fed conditions.

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We are feeding a separately excited DC motor from a single-phase full-controlled converter. Let's begin by drawing the circuit diagram to visualize the setup. We have a single-phase bridge with four SCRs, and to simplify the motor's representation, we replace it with its equivalent circuit. This equivalent circuit of the motor includes the armature, which consists of resistance, inductance, and a back EMF.

This is the armature representation of the DC motor, and the input to the converter is an AC source

with a voltage magnitude denoted as  $V_s$ . Now,  $V_s$  is the instantaneous value of the AC supply voltage, which can be expressed as:

$$V_s = V_m \sin(\omega t)$$

Here, we are working with a single-phase, full-controlled bridge converter. The output voltage of the converter can be controlled by adjusting the firing or triggering angle  $\alpha$  of the SCRs. The current through the armature is denoted as  $I_a$ , and the voltage across the armature is  $V_a$ . When we use lowercase letters like  $i_a$  and  $v_a$ , we are referring to the instantaneous values of the current and voltage, respectively.

This converter can operate in two distinct modes: continuous current operation and discontinuous current operation. We will explore both modes in separate parts of the lecture.

Let's first focus on the discontinuous current operation. In this mode, the armature current becomes discontinuous, meaning the current  $I_a$  drops down to zero at certain intervals. Now, to better understand this, let's visualize the output voltage waveform. The input to the converter is a sine wave, so when we plot the waveform, we'll see the typical sinusoidal pattern: starting at 0, reaching  $\pi$ , then continuing to  $2\pi$ ,  $3\pi$ , and so on.

We have a back EMF generated by the motor, and the SCR is triggered at an angle  $\alpha$ , which occurs in every half cycle. This results in a rectified waveform. For example, in one half-cycle, we trigger the SCR at  $\alpha$ , and the corresponding value for the next trigger point is  $\pi + \alpha$ . When we trigger the SCR, the current rises from zero, initiating conduction. Now, let's look at the behavior of the circuit, which is an RLE circuit, meaning it consists of resistance  $R$ , inductance  $L$ , and the motor's back EMF  $E$ .

When the SCRs are triggered, let's say SCRs T1 and T2 are triggered in the positive half cycle, since they are forward biased during this period. In contrast, T3 and T4 are triggered during the negative half cycle. At the instant  $\alpha$  is reached, T1 and T2 are triggered, entering into conduction. This allows current to flow, starting from zero and rising. However, the current does not persist indefinitely, it decreases and eventually returns to zero before the next pair of SCRs are triggered.

At  $\pi + \alpha$ , we trigger the next pair of SCRs, T3 and T4, in the negative half cycle. Similarly, the

current again starts from zero and decreases back to zero before the next half cycle begins. This periodic behavior continues as T1 and T2 are triggered again, maintaining a cyclical operation.

The current waveform, denoted as  $I_a$ , reflects this periodic behavior. The current begins at zero, rises, and then returns to zero before repeating the process with each subsequent triggering of SCRs. The point where the current returns to zero is referred to as the extinction angle.

This angle is referred to as  $\beta$ . Now, let's discuss the output voltage. The output voltage will indeed be a rectified voltage, but it will have a specific waveform. When we trigger the SCRs, the output voltage becomes identical to the input voltage. This happens because, once the SCRs are triggered, they enter conduction, effectively connecting the input directly to the output. Thus, the current flows through the SCRs and returns back to the source.

For instance, when we trigger SCRs T1 and T2, the output voltage  $V_a$  becomes equal to the source voltage  $V_s$ . This continues up to  $\omega t = \pi$ , but the current continues to flow beyond  $\omega t = \pi$  due to inductive effects, maintaining the continuity of the current. The current does not drop to zero until the angle  $\beta$  is reached. As a result, the output voltage  $V_a$  remains the same as  $V_s$  until the current reaches zero at  $\beta$ .

Once the current becomes zero at  $\beta$ , the circuit is effectively disconnected, the source is no longer linked to the load. At this point, the output voltage becomes equal to the back EMF of the motor, as there is no more current flowing through the circuit to maintain  $V_s$ . The voltage now follows the back EMF curve.

Then, at  $\pi + \alpha$ , SCRs T3 and T4 are triggered. The current is redirected through this new pair of SCRs, T3 and T4, causing the current to flow once more through the circuit. As a result, the voltage across the load reverses direction, reflecting the alternating nature of the input supply.

This entire process is periodic, meaning the output voltage follows this pattern repeatedly, giving rise to a waveform where the voltage follows the source during conduction and the back EMF when the current ceases. The waveform of the output voltage will reflect this alternating behavior, maintaining a periodic structure due to the nature of the input AC supply and the switching of the SCRs. Thus, the output voltage will exhibit a pattern that repeats consistently over time.

So, the output voltage  $V_a$  consists of two distinct intervals: one is the powering interval, and the other is the zero current interval or coasting interval. Let's break this down.

During the interval when  $\alpha < \omega t < \beta$ , the output voltage  $V_a$  is identical to the input voltage  $V_s$ . This phase is known as the powering interval. In this period, the SCRs are conducting, and the circuit is actively delivering power, resulting in  $V_a = V_s$ .

Then, we have another interval where  $\beta < \omega t < \pi + \alpha$ . In this small interval, the output voltage becomes equal to the motor's back EMF. This is referred to as the zero current interval or sometimes as the coasting interval. During this coasting phase, there is no current flowing, and the voltage across the armature  $V_a$  is purely the back EMF of the motor, as the input source is disconnected.

So essentially, we observe two different operating modes: the powering interval, where  $V_a = V_s$ , and the coasting interval, where  $V_a = \text{back EMF}$ . In the following analysis, we will examine both intervals separately, starting with the powering interval.

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I. Powering Interval  $\alpha < \omega t < \beta$

$$V_a = V_s = R_a i_a + L_a \frac{di_a}{dt} + E$$

$$V_m \sin \omega t = R_a i_a + L_a \frac{di_a}{dt} + E$$

$$i_a = i_a (\text{Steady state}) + i_a (\text{Transient})$$

$$= \underbrace{\frac{V_m}{Z} \sin(\omega t - \theta)}_{\text{Steady state}} + \underbrace{A e^{-t/\tau_a}}_{\text{Transient}}$$

$$\theta = \tan^{-1} \frac{\omega L_a}{R_a} = \text{power factor angle}$$

$$\tau_a = \frac{L_a}{R_a}$$

In the powering interval, which occurs when  $\alpha < \omega t < \beta$ , several important things happen. First, during this interval, the input voltage is identical to the output voltage, meaning that the output voltage  $V_a$  is equal to the input voltage  $V_s$ .



Now, let's examine the armature voltage  $V_a$ . The armature voltage consists of three key components:

1. The voltage drop across the armature resistance  $R_a$ ,
2. The voltage drop across the inductance  $L_a$  due to the changing current  $\frac{di_a}{dt}$ , and
3. The motor's back EMF  $E$ .

Thus, we can express the armature voltage as:

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + E$$

Now, since the input voltage is sinusoidal, we can substitute for  $V_s$  as  $V_m \sin(\omega t)$ , where  $V_m$  is the peak voltage. So, the equation becomes:

$$V_m \sin(\omega t) = R_a i_a + L_a \frac{di_a}{dt} + E$$

This is a first-order linear differential equation, assuming that the back EMF  $E$  remains constant throughout the interval.

To solve this differential equation, we get two components in the solution for  $i_a$  (the armature current):

1. The steady-state component, which exists when  $t \rightarrow \infty$ ,
2. The transient component, which decays over time.

In the steady state, the inductive voltage becomes the inductive reactance multiplied by the current, reducing the circuit to an RLE (resistance-inductance-EMF) model. For the steady-state solution, we can write:

$$i_{a,\text{steady}} = \frac{V_m}{Z} \sin(\omega t - \theta) - \frac{E}{R_a}$$

where  $Z$  is the impedance and  $\theta$  is the phase angle. This represents the steady-state current.

In addition to the steady-state term, there is a transient component, which arises due to the RL nature of the circuit. The transient component has the form of an exponential decay:

$$i_{a,\text{transient}} = Ae^{-t/\tau}$$

where  $\tau = \frac{L_a}{R_a}$  is the time constant of the RL circuit.

Thus, the total solution for the armature current  $i_a$  during the powering interval consists of both the steady-state term and the transient term. The steady-state part describes the current as a function of the sinusoidal input, while the transient part captures the short-term effects of the RL circuit as it stabilizes.

As time  $t$  approaches infinity, the transient component of the solution will diminish to zero. This leaves us with the steady-state part, which consists of two significant voltages. The first is  $V_m$ , the applied voltage, expressed as  $V_m \sin(\omega t)$ , where  $V_m$  represents the peak voltage. Accompanying this is the angle  $\theta$ , known as the power factor angle, calculated as  $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$ . This angle reflects the relationship between the inductive reactance and resistance, indicating how the circuit's impedance affects the current.

The second voltage present in the steady-state is  $E$ , which represents the back EMF or DC voltage in the circuit. Notably,  $E$  opposes the flow of current, contrasting with the input voltage  $V_s$ , which facilitates current flow. Therefore, when considering  $E$  in the context of Ohm's Law, it can be expressed as  $-\frac{E}{R_a}$ .

In addition to these steady-state voltages, we also have the transient component given by  $Ae^{-\frac{t}{\tau}}$ , where  $\tau$  denotes the time constant of the circuit, defined as  $\frac{L_a}{R_a}$ . This time constant, which can also be represented as  $\tau_a$ , characterizes how quickly the circuit responds to changes in current.

To summarize, the overall solution to the differential equation comprises both a steady-state component and a transient component. The transient component will gradually approach zero as  $t$  tends to infinity, leaving us with only the steady-state response.

That concludes today's lecture. In our next session, we will focus on determining the constant  $A$ .