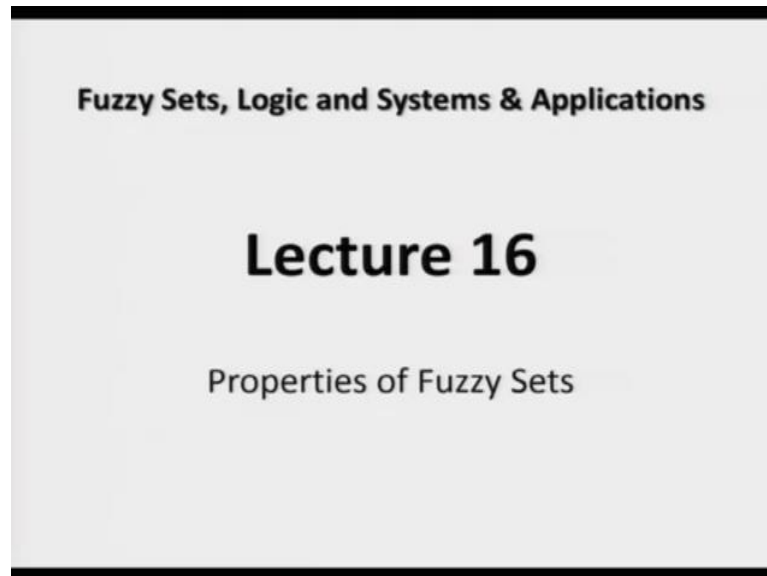


**Fuzzy Sets, Logic and Systems and Applications**  
**Prof. Nishchal K. Verma**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Kanpur**

**Lecture – 16**  
**Properties of Fuzzy Sets**

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Welcome to lecture number 16 of Fuzzy sets, Logic and Systems and Applications. So, here we are in the continuation of Properties of classical and Fuzzy Sets where we have seen that the properties listed here all are holding good for classical sets. Whereas, we are here seeing that some of the properties like law of contradiction, law of excluded middle, absorption of complement, these three properties out of all mentioned here are not holding good for fuzzy sets.

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| Property                   | CLASSICAL SETS   | FUZZY SETS   |
|----------------------------|--|--|
| Law of Contradiction ✓     | $A \cap \bar{A} = \phi$  | $A \cap \bar{A} \neq \phi$   |
| Law of Excluded Middle ✓   | $A \cup \bar{A} = X$   | $A \cup \bar{A} \neq X$  |
| Idempotency ✓              | $A \cap A = A, A \cup A = A$   | $A \cap A = A, A \cup A = A$   |
| Involution ✓               | $\bar{\bar{A}} = A$  | $\bar{\bar{A}} = A$  |
| Commutativity ✓            | $A \cap B = B \cap A, A \cup B = B \cup A$   | $A \cap B = B \cap A, A \cup B = B \cup A$   |
| Associativity ✓            | $(A \cup B) \cup C = A \cup (B \cup C)$<br>$(A \cap B) \cap C = A \cap (B \cap C)$                   | $(A \cup B) \cup C = A \cup (B \cup C)$<br>$(A \cap B) \cap C = A \cap (B \cap C)$                   |
| Distributivity ✓           | $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$<br>$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ | $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$<br>$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ |
| Absorption ✓               | $A \cup (A \cap B) = A$<br>$A \cap (A \cup B) = A$   | $A \cup (A \cap B) = A$<br>$A \cap (A \cup B) = A$   |
| Absorption of Complement ✓ | $A \cup (\bar{A} \cap B) = A \cup B$<br>$A \cap (\bar{A} \cup B) = A \cap B$                         | $A \cup (\bar{A} \cap B) \neq A \cup B$<br>$A \cap (\bar{A} \cup B) \neq A \cap B$                   |
| DeMorgan's Laws            | $\overline{A \cup B} = \bar{A} \cap \bar{B}$<br>$\overline{A \cap B} = \bar{A} \cup \bar{B}$         | $\overline{A \cup B} = \bar{A} \cap \bar{B}$<br>$\overline{A \cap B} = \bar{A} \cup \bar{B}$         |

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So, so far we have covered Law of Contradiction, Law of Excluded Middle, Idempotency property, Involution, Commutativity, Associativity, Distributivity, Absorption, Absorption of complement.

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### 10. Demorgan's Law of Union

For crisp sets  $A$  and  $B$ ,

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

For fuzzy sets  $A$  and  $B$ ,

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

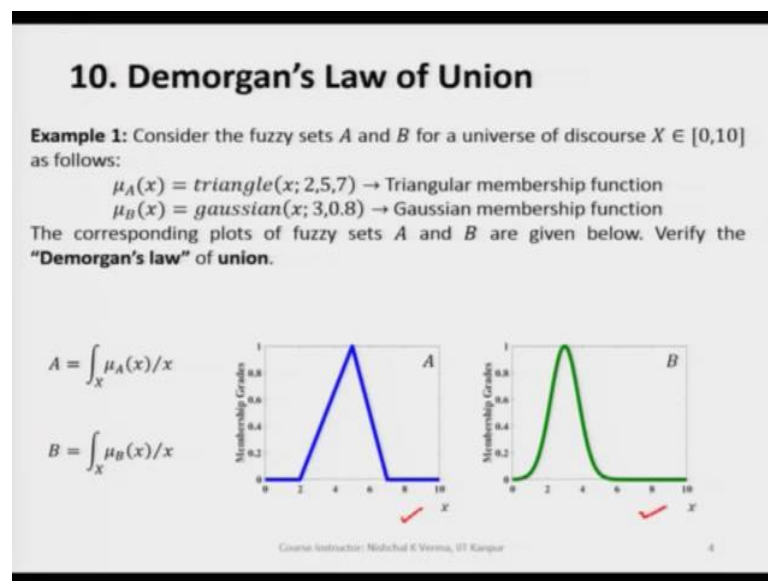
This is called the "Demorgan's law" of union.

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So, in the continuation in this lecture today, we will discuss DeMorgan's law with respect to fuzzy sets. And when we see DeMorgan's law of union is defined here for crisp sets  $A$  and  $B$  as when we take the complement of  $A$  union  $B$ , this is going to be the intersection of  $A$  complement and  $B$  complement.

So, this is holding good for crisp sets we all know. Let us see if we take fuzzy sets whether the DeMorgans law of union hold good for fuzzy sets or not. Yes it holds good for fuzzy sets as well. So, if we take two fuzzy sets  $A$  and  $B$  and we take the complement of  $A \cup B$ , we are going to get or we are getting the fuzzy set which is a complement  $A \cap \bar{B}$ . So, DeMorgan's law for fuzzy sets also hold good. So, let us now understand DeMorgan's law for fuzzy sets better by taking a couple of examples.

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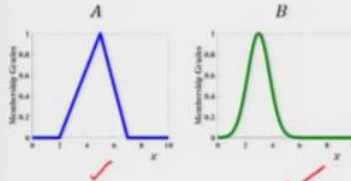


So, here we have example which has two fuzzy sets  $A$  and  $B$ , two continuous fuzzy sets  $A$  and  $B$ ;  $A$  is here and  $B$  is here.

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### 10. Demorgan's Law of Union

**Solution:** For fuzzy sets  $A$  and  $B$  with the universe of discourse  $X \in [0,10]$ , the **Demorgan's law of union** is given by  $\overline{A \cup B} = \bar{A} \cap \bar{B}$ . The membership function values of  $\overline{A \cup B}$  and  $\bar{A} \cap \bar{B}$  are defined as

$$\overline{A \cup B} = 1 - \max[\mu_A(x), \mu_B(x)] \forall x \in X$$
$$\bar{A} \cap \bar{B} = \min[\mu_{\bar{A}}(x), \mu_{\bar{B}}(x)] \forall x \in X$$


$A \cup B = ?$

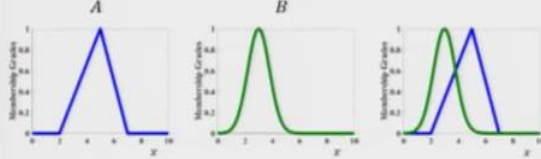
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So, let us now take the  $\overline{A \cup B}$  which is mentioned over here and then let us see whether this is equal to or this is exactly the same as the  $\bar{A} \cap \bar{B}$ . So, we first find the  $A \cup B$ . So, since  $A$  is here  $B$  is here fuzzy set a fuzzy set  $B$ , let us find  $A \cup B$ .

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### 10. Demorgan's Law of Union

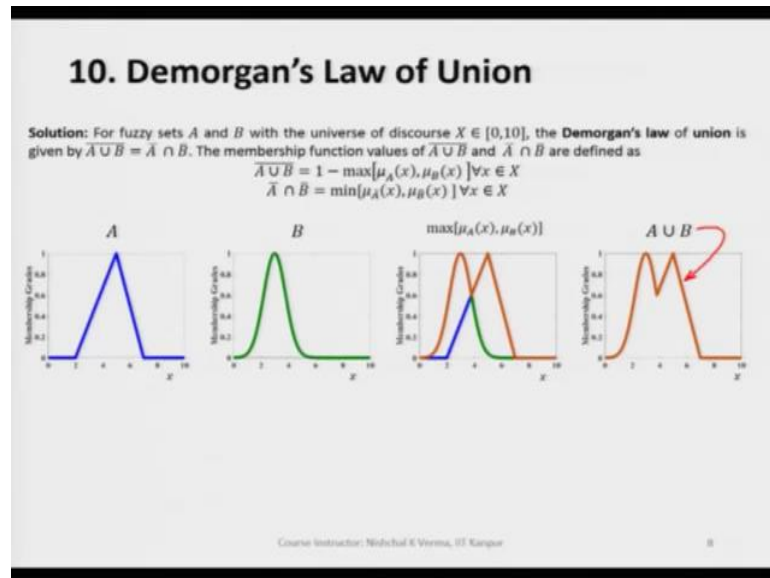
**Solution:** For fuzzy sets  $A$  and  $B$  with the universe of discourse  $X \in [0,10]$ , the **Demorgan's law of union** is given by  $\overline{A \cup B} = \bar{A} \cap \bar{B}$ . The membership function values of  $\overline{A \cup B}$  and  $\bar{A} \cap \bar{B}$  are defined as

$$\overline{A \cup B} = 1 - \max[\mu_A(x), \mu_B(x)] \forall x \in X$$
$$\bar{A} \cap \bar{B} = \min[\mu_{\bar{A}}(x), \mu_{\bar{B}}(x)] \forall x \in X$$


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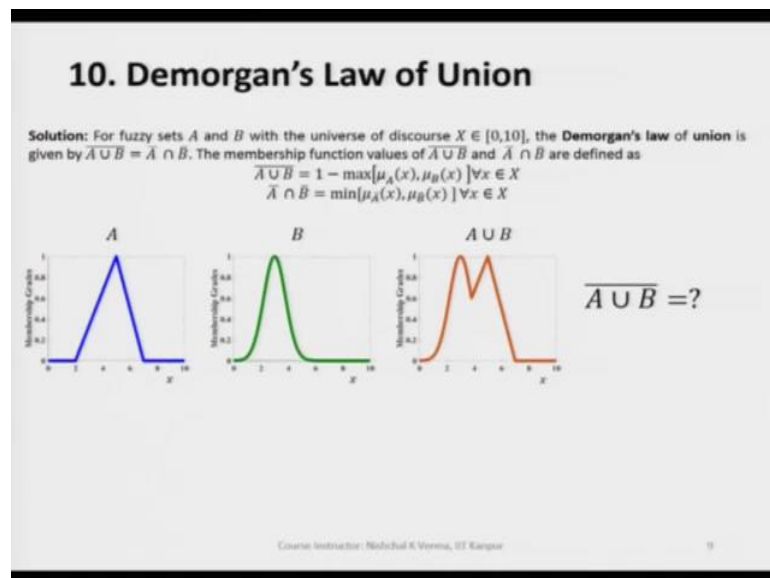
So, to find  $A \cup B$ , we know we superimpose these two fuzzy sets on each other and we apply max criteria and when we applied max criteria we get  $A \cup B$  as a result here.

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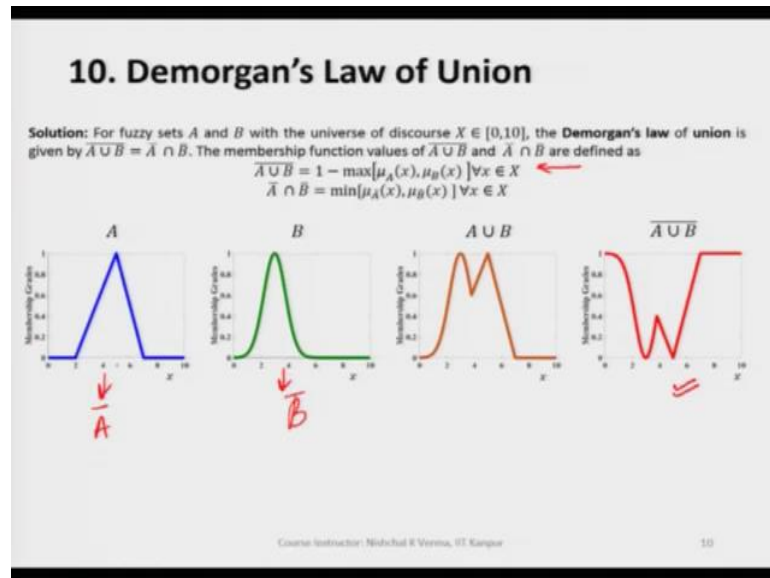


So, this is  $A \cup B$ . Now, we have to find the complement of this fuzzy set which will be the  $\overline{A \cup B}$ .

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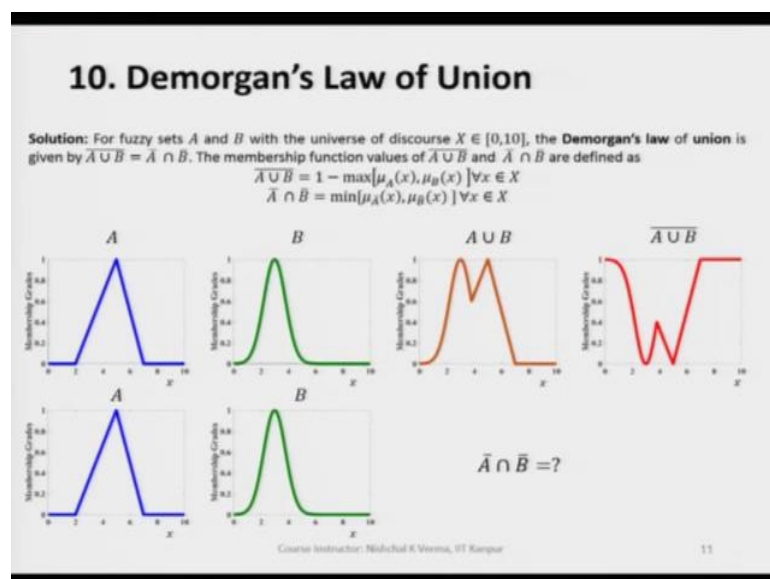


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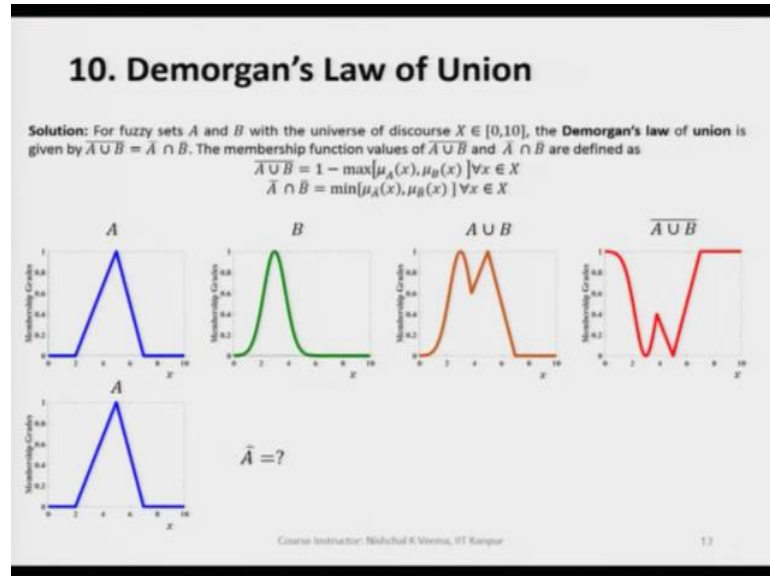
So, let us see how does it look like. So, when we have  $\overline{A \cup B}$ , we just subtract this, we apply the criteria mentioned here, we subtract this from one. So, we apply for getting A the  $\overline{A \cup B}$ , we get 1 minus max of all the corresponding values of membership functions of A and membership function of fuzzy set B for respective generic variable values in the universe of discourse. So, we see that we have the complement of A union B like this. Now let's find out the  $\bar{A}$  here,  $\bar{A}$  here and  $\bar{B}$  here.

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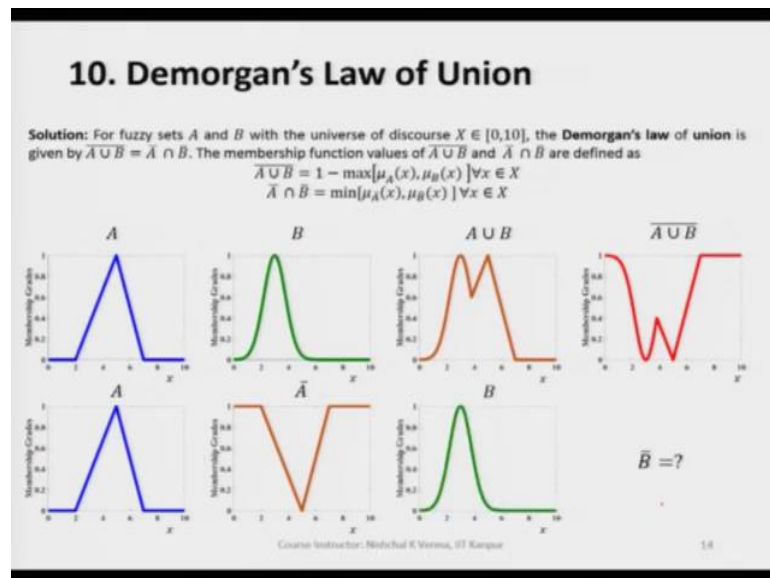


Let us find out from these two fuzzy sets and see what are we going to get when we take the intersection of these two fuzzy sets.

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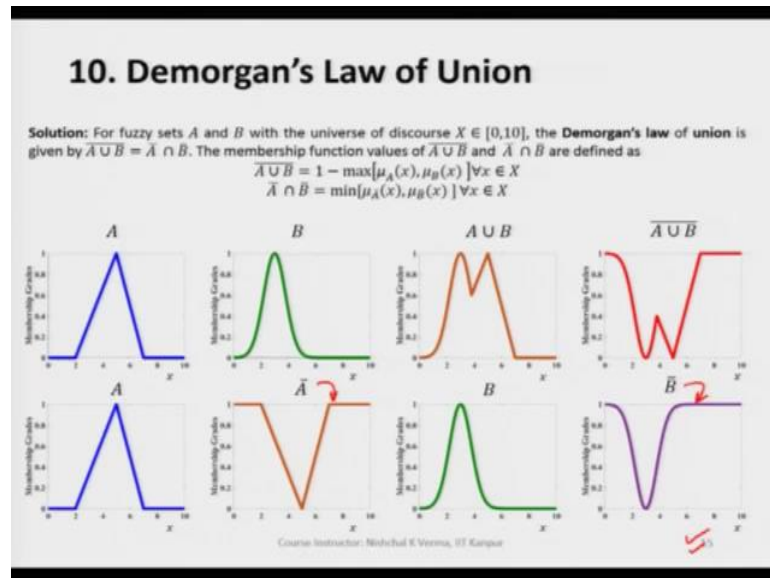


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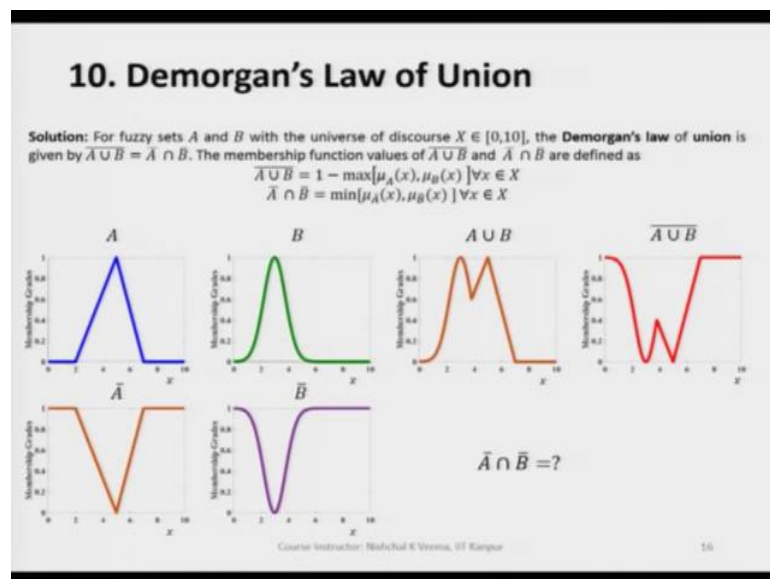
So,  $\bar{A}$  is here is complement of  $A$ , I mean  $\bar{A}$  and then complement of  $B$  that is  $\bar{B}$  which is here.

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So, the  $\bar{B}$ , this is  $\bar{A}$  we are interested in finding the intersection of these two complements.

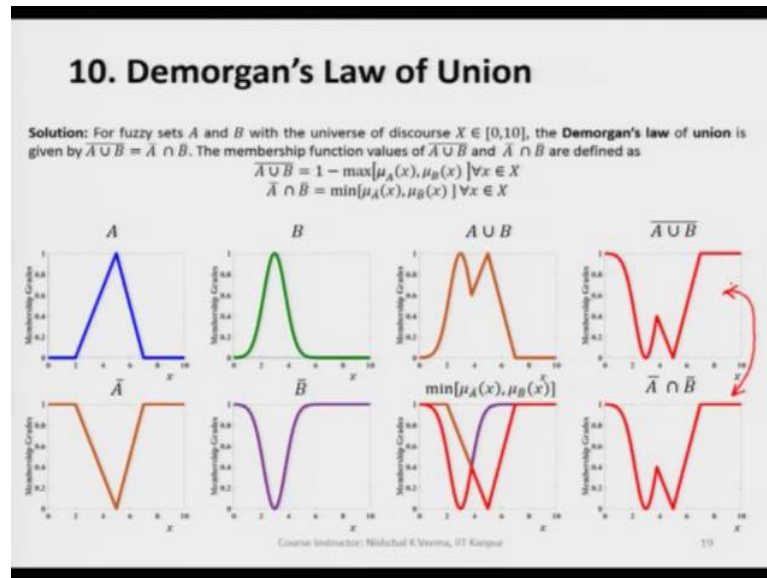
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And let us see what are we going to get when we take the intersection of these two.

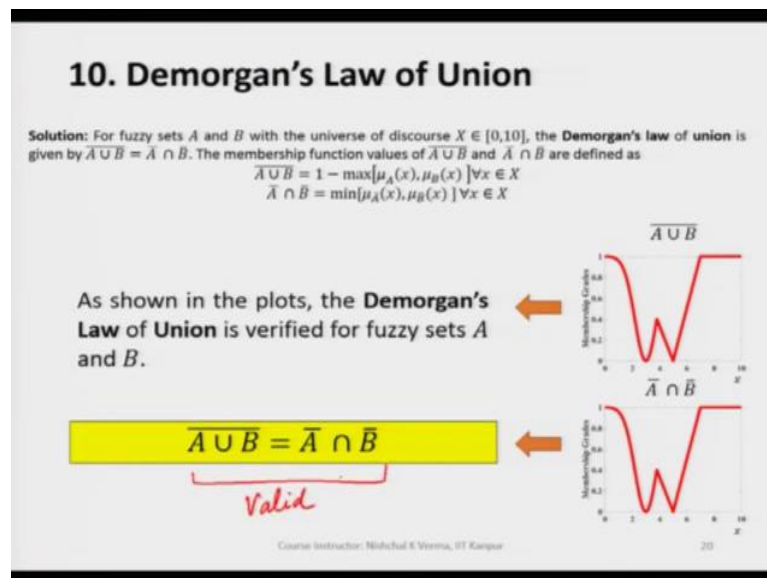


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So, we super impose these two fuzzy sets on each other here as we have done and since we are taking intersection when we apply main criteria we are going to get here this fuzzy set as the outcome. So, we clearly see here that when we take the  $\overline{A \cup B}$  and when we take  $\overline{A} \cap \overline{B}$ , we see that these two are same.

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So, this way we can since the outcome both the outcomes are equal. So, we can say that DeMorgan's Law of Union holds good for fuzzy sets  $A$  and  $B$  and that's how this relation is valid for fuzzy sets.

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### 10. Demorgan's Law of Union

**Example 2:** Let  $A$  and  $B$  are two fuzzy sets given below for the universe of discourse  $X = \{1,2,3,4\}$ .

$$A = 0.7/1 + 0.5/2 + 0.1/3 + 0.6/4$$

$$B = 0.8/1 + 0.3/2 + 0.7/3 + 0.5/4$$

Verify the **Demorgan's law of Union** for the given fuzzy sets.

**Solution:** For the fuzzy sets, the **Demorgan's law of union** is defined as

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

The elements of above expression are being defined as

**LHS:**  $\overline{A \cup B} = 1 - \max[\mu_A(x), \mu_B(x)] \quad \forall x \in X$

**RHS:**  $\bar{A} \cap \bar{B} = \min[\mu_{\bar{A}}(x), \mu_{\bar{B}}(x)] \quad \forall x \in X$

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Now, let us take an example here with discrete fuzzy sets and see what is happening. So, here also when we take fuzzy set  $A$  and fuzzy set  $B$ , and these two fuzzy sets are discrete fuzzy sets. And when we compute the complement of  $A \cup B$  as it is shown here.

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### 10. Demorgan's Law of Union

$$A = 0.7/1 + 0.5/2 + 0.1/3 + 0.6/4$$

$$B = 0.8/1 + 0.3/2 + 0.7/3 + 0.5/4$$

**LHS:**  $\overline{A \cup B} = 1 - \max[\mu_A(x), \mu_B(x)] \quad \forall x \in X$

**RHS:**  $\bar{A} \cap \bar{B} = \min[\mu_{\bar{A}}(x), \mu_{\bar{B}}(x)] \quad \forall x \in X$

$$A \cup B = \max(0.7,0.8)/1 + \max(0.5,0.3)/2 + \max(0.1,0.7)/3 + \max(0.6,0.5)/4$$

$$= 0.8/1 + 0.5/2 + 0.7/3 + 0.6/4$$

$$\rightarrow \overline{A \cup B} = (1 - 0.8)/1 + (1 - 0.5)/2 + (1 - 0.7)/3 + (1 - 0.6)/4$$

$$= \underline{0.2/1 + 0.5/2 + 0.3/3 + 0.4/4} \quad \text{--- (I)}$$

$$\bar{A} = 0.3/1 + 0.5/2 + 0.9/3 + 0.4/4$$

$$\bar{B} = 0.2/1 + 0.7/2 + 0.3/3 + 0.5/4$$

$$\rightarrow \bar{A} \cap \bar{B} = \min(0.3,0.2)/1 + \min(0.5,0.7)/2 + \min(0.9,0.3)/3 + \min(0.4,0.5)/4$$

$$= \underline{0.2/1 + 0.5/2 + 0.3/3 + 0.4/4} = \overline{A \cup B}$$

Hence,  $\overline{A \cup B} = \bar{A} \cap \bar{B}$

The **Demorgan's Law of Union** is verified for given fuzzy sets  $A$  and  $B$ .

*Valid*

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$A \cup B$  the  $\overline{A \cup B}$  we are getting the fuzzy set point  $2/1 + 0.5/2 + 0.3/3 + 0.4/4$  as a result. And when we find the  $\bar{A} \cap \bar{B}$  so, we find here as a result  $0.2/1 + 0.5/2 + 0.3/3 + 0.4/4$ .

So, we see that if we take this as the outcome here of the  $\overline{A \cup B}$  and we see that this is exactly equal to the same as when we take the  $\bar{A} \cap \bar{B}$ . So, we see that the DeMorgan's Law of union here also is valid when we take the discrete fuzzy sets  $A$  and  $B$ .

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**10. Demorgan's Law of Union**

For crisp sets  $A, B$  and  $C$ ,

$$\overline{A \cup B \cup C} = \bar{A} \cap \bar{B} \cap \bar{C} \quad \checkmark$$

For fuzzy sets  $A, B$  and  $C$ ,

$$\overline{A \cup B \cup C} = \bar{A} \cap \bar{B} \cap \bar{C} \quad \checkmark$$

This is called the "Demorgan's law" of union.

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Now, let's so so, earlier what we were seeing was the De Morgan's law of union and this we verified by taking examples of fuzzy sets  $A$  and  $B$ . Now we are extending this to three fuzzy sets. So, we all know that for crisp sets also when we take the  $\overline{A \cup B \cup C}$ , this is equal to the  $\bar{A} \cap \bar{B} \cap \bar{C}$ .

So, this is same as when we use instead of crisp sets we take fuzzy sets  $A, B$  and  $C$ , we see that the DeMorgan's law of union is valid here or holds good here. So, in other words we can say for fuzzy sets  $A, B, C$  when we take the complement of  $A \cup B \cup C = \bar{A} \cap \bar{B} \cap \bar{C}$ .

So, these two are valid. So, this is valid for crisp here means the DeMorgan's law of union is valid for crisp as well as fuzzy sets when we take more than two fuzzy sets. For two fuzzy set we have already seen. So, when we increase the number of fuzzy sets here also this is valid.

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### 10. Demorgan's Law of Union

**Example 3:** Consider the fuzzy sets  $A$ ,  $B$  and  $C$  for a universe of discourse  $X \in [0,10]$  as follows:

- $\mu_A(x) = \text{triangle}(x; 2,5,7) \rightarrow$  Triangular membership function
- $\mu_B(x) = \text{gaussian}(x; 3,0.8) \rightarrow$  Gaussian membership function
- $\mu_C(x) = \text{trapezoid}(x; 4,6,8,9) \rightarrow$  Trapezoidal membership function

The corresponding plots of fuzzy sets  $A$ ,  $B$  and  $C$  are given below. Verify the "Demorgan's law" of union.

$A = \int_X \mu_A(x)/x$   
 $B = \int_X \mu_B(x)/x$   
 $C = \int_X \mu_C(x)/x$

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So, let us now take an example and understand better. So, see here we have  $A$  fuzzy set,  $B$  fuzzy set and  $C$  fuzzy set.

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### 10. Demorgan's Law of Union

**Solution:** For fuzzy sets  $A, B$  and  $C$  with the universe of discourse  $X \in [0,10]$ , the Demorgan's law of union is given by  $\overline{A \cup B \cup C} = \bar{A} \cap \bar{B} \cap \bar{C}$ . The membership function values of  $\overline{A \cup B \cup C}$  and  $\bar{A} \cap \bar{B} \cap \bar{C}$  are defined as

$$\overline{A \cup B \cup C} = 1 - \max[\mu_A(x), \mu_B(x), \mu_C(x)] \forall x \in X$$

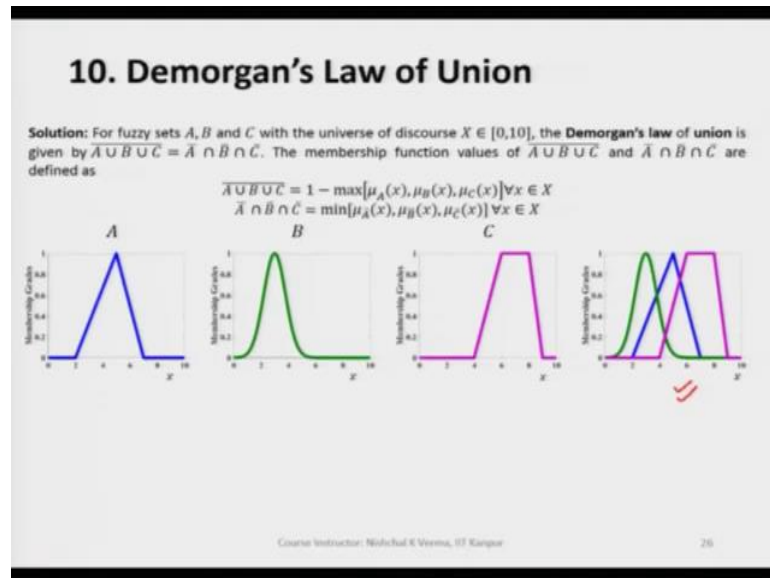
$$\bar{A} \cap \bar{B} \cap \bar{C} = \min[\mu_{\bar{A}}(x), \mu_{\bar{B}}(x), \mu_{\bar{C}}(x)] \forall x \in X$$

$\overline{A \cup B \cup C} = ?$

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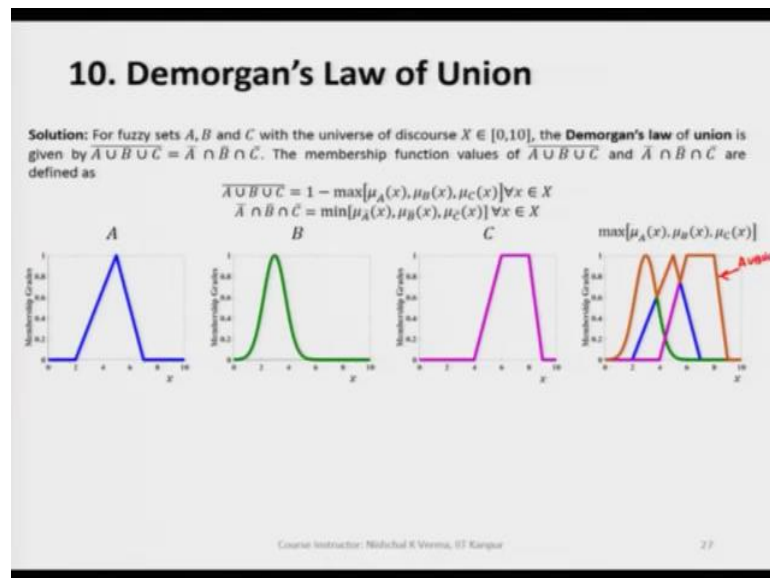
And let us now, take the complement of  $A \cup B \cup C$ . So, for getting this, we super impose these  $A, B, C$  fuzzy sets on each other.

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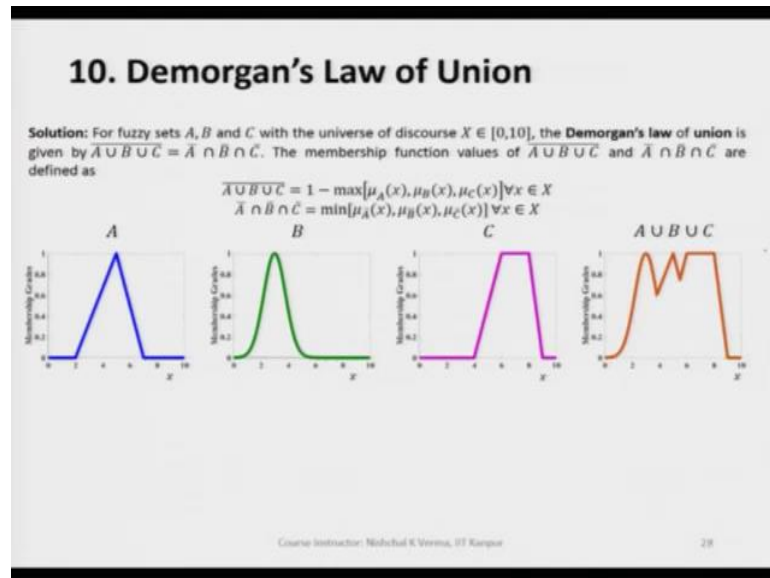
And when we have done this, we see that this looks like this. So, we have to apply the max criteria.

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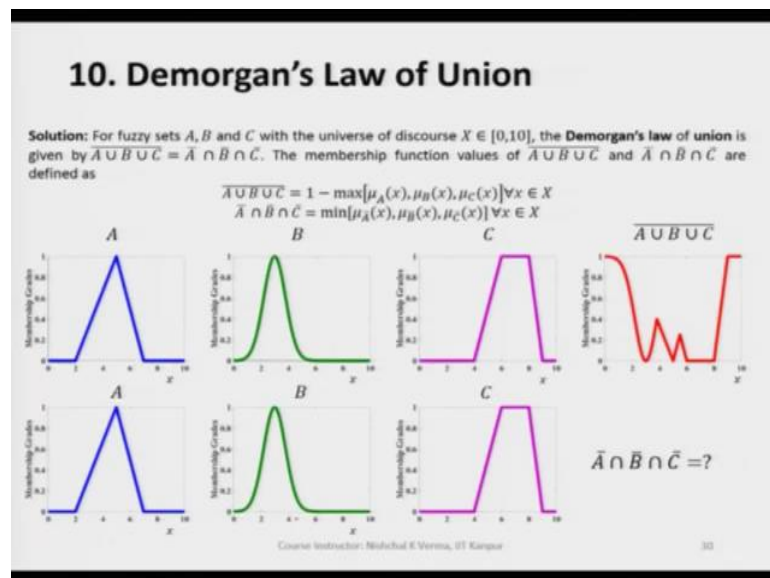
And when we apply max criteria, we are going to get this as which is shown by the red color. So, this is  $A \cup B \cup C$ .

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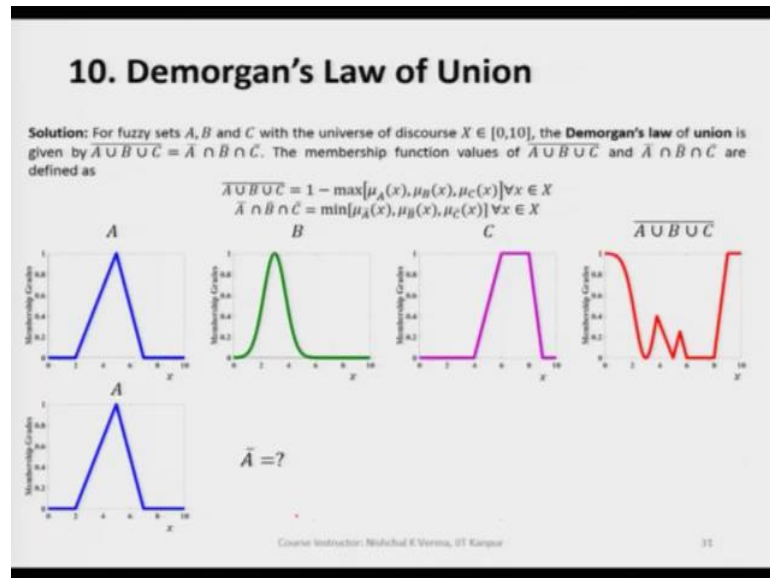
Now when we take the complement of this, so this is exactly the  $\bar{A} \cap \bar{B} \cap \bar{C}$ . And when we take the complement of this, we are going to get this as the fuzzy set.

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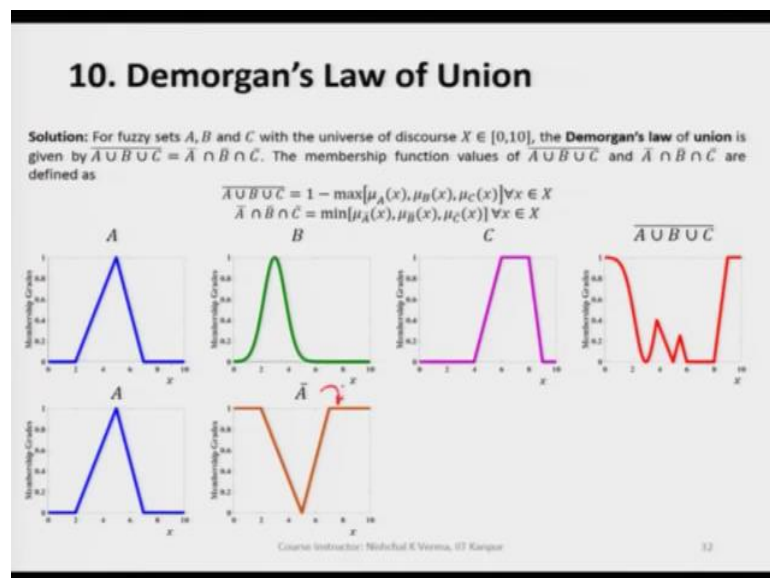


Now, let's take the  $A \cap B \cap C$ . So, we have  $A$  here as fuzzy set  $B, C$ .

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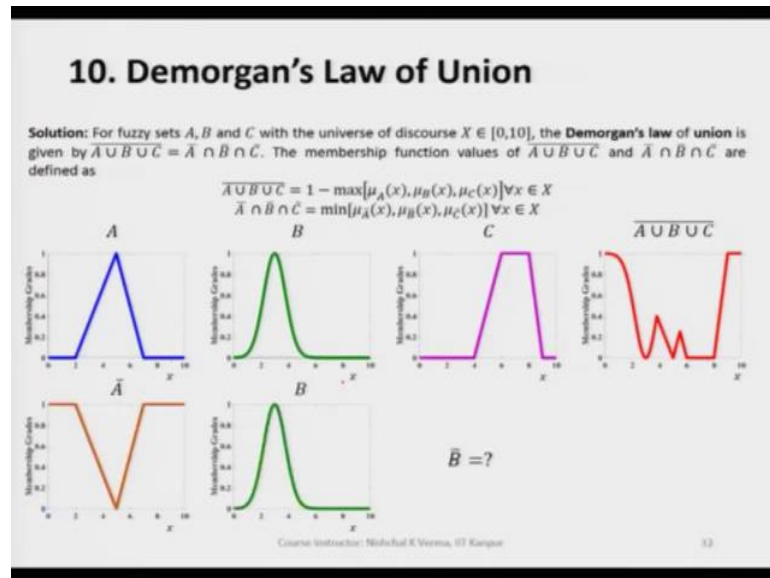


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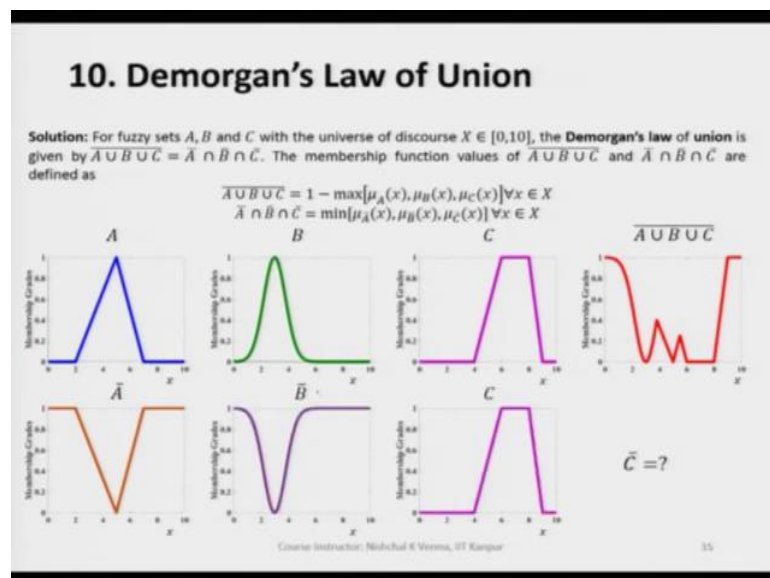


So, let us take the intersection of  $\bar{A}$ . So, A complement we see here, this is  $\bar{A}$ .

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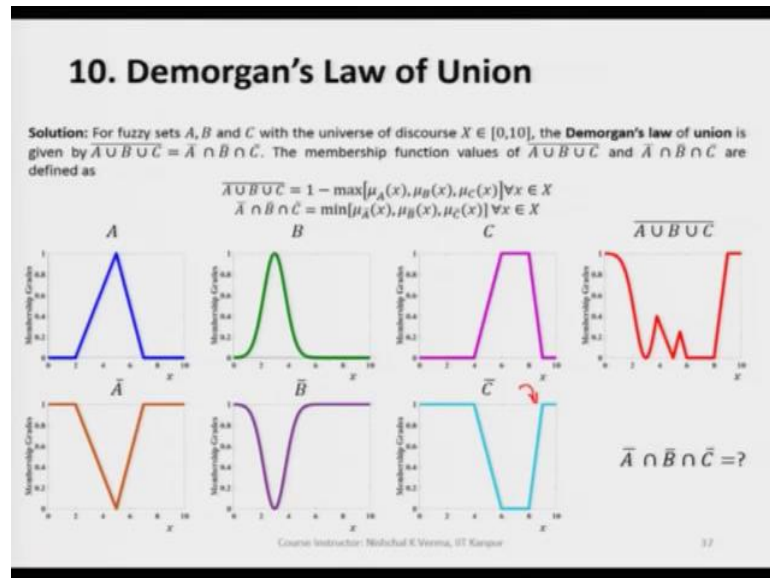
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And then we have  $\bar{B}$  and then we have here  $\bar{C}$ .

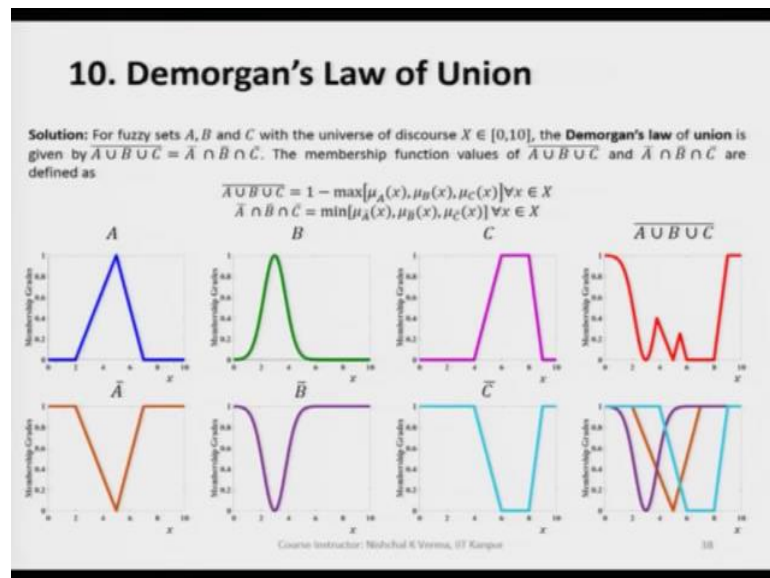


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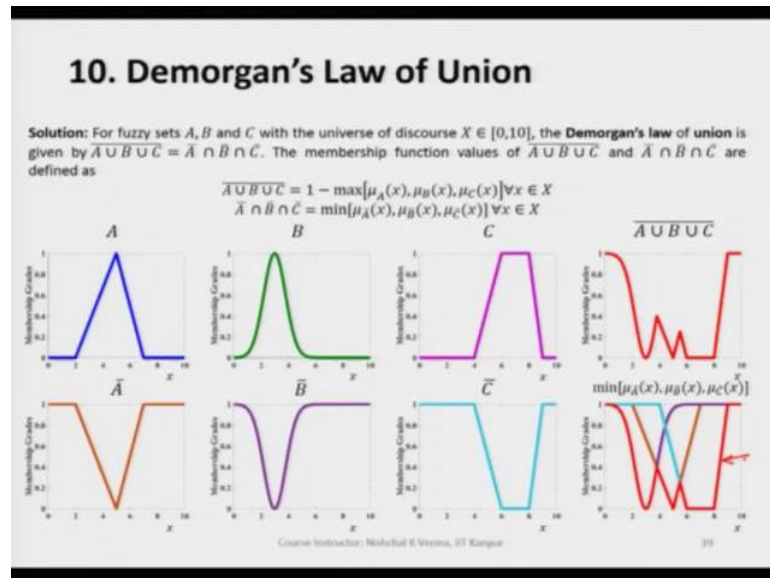
So, now we are interested in finding the intersection of these compliments. Let us superimpose these three compliments on each other which is here.

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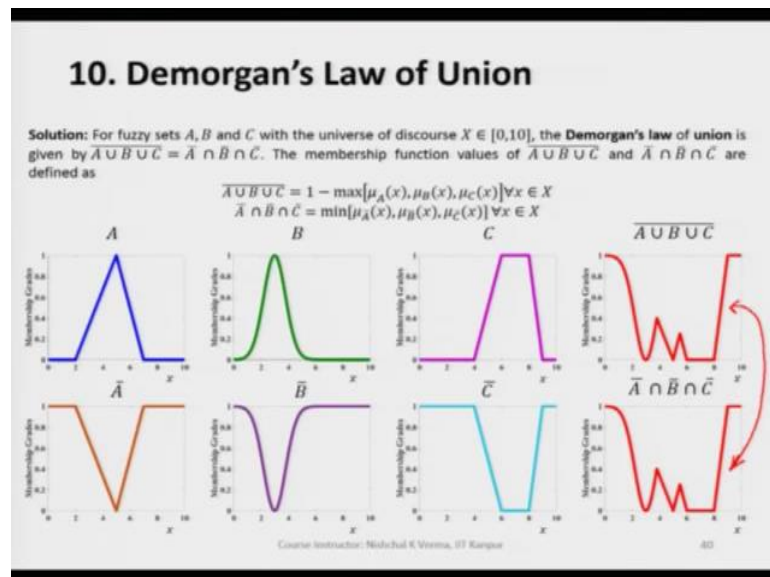
And since we are interested in intersection, we have to apply the min criteria.

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When we apply min criteria, we see the result which is shown by red color which is here.

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So, we see here very clearly that these two outcomes are equal, means when we take the  $\overline{A \cup B \cup C} = \bar{A} \cap \bar{B} \cap \bar{C}$ .

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### 10. Demorgan's Law of Union

**Solution:** For fuzzy sets  $A, B$  and  $C$  with the universe of discourse  $X \in [0,10]$ , the **Demorgan's law of union** is given by  $\overline{A \cup B \cup C} = \bar{A} \cap \bar{B} \cap \bar{C}$ . The membership function values of  $\overline{A \cup B \cup C}$  and  $\bar{A} \cap \bar{B} \cap \bar{C}$  are defined as

$$\overline{A \cup B \cup C} = 1 - \max[\mu_A(x), \mu_B(x), \mu_C(x)] \forall x \in X$$

$$\bar{A} \cap \bar{B} \cap \bar{C} = \min[\mu_{\bar{A}}(x), \mu_{\bar{B}}(x), \mu_{\bar{C}}(x)] \forall x \in X$$

As shown in the plots, the **Demorgan's Law of Union** is verified for fuzzy sets  $A, B$  and  $C$ .

$\overline{A \cup B \cup C} = \bar{A} \cap \bar{B} \cap \bar{C}$

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So, this way we can say that DeMorgan's law of union is holding good for fuzzy sets  $A, B$  and  $C$ . Now the same can be checked by taking three discrete fuzzy sets  $A, B$  and  $C$ .

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### 10. Demorgan's Law of Union

**Example 4:** Let  $A, B$  and  $C$  are three fuzzy sets given below for the universe of discourse  $X = \{1,2,3,4\}$ .

$$A = 0.7/1 + 0.5/2 + 0.1/3 + 0.6/4$$

$$B = 0.8/1 + 0.3/2 + 0.7/3 + 0.5/4$$

$$C = 0.4/1 + 0.7/2 + 0.3/3 + 0.9/4$$

I  
II  
III

Verify the **Demorgan's law of Union** for the given three fuzzy sets.

**Solution:** For fuzzy sets  $A, B$  and  $C$ , the **Demorgan's law of union** will be given as

$$\overline{A \cup B \cup C} = \bar{A} \cap \bar{B} \cap \bar{C}$$

The elements of above expression are being defined as

**LHS:**  $\overline{A \cup B \cup C} = 1 - \max[\mu_A(x), \mu_B(x), \mu_C(x)] \forall x \in X$

**RHS:**  $\bar{A} \cap \bar{B} \cap \bar{C} = \min[\mu_{\bar{A}}(x), \mu_{\bar{B}}(x), \mu_{\bar{C}}(x)] \forall x \in X$

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So, let's now see what is happening when we take three discrete fuzzy sets. So, let me just write it just represent this fuzzy set by I here and then by II here is by III here. So, these three are three discrete fuzzy sets.

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### 10. Demorgan's Law of Union

$A = 0.7/1 + 0.5/2 + 0.1/3 + 0.6/4$   
 $B = 0.8/1 + 0.3/2 + 0.7/3 + 0.5/4$   
 $C = 0.4/1 + 0.7/2 + 0.3/3 + 0.9/4$

**LHS:**  $\overline{A \cup B \cup C} = 1 - \max[\mu_A(x), \mu_B(x), \mu_C(x)] \quad \forall x \in X$   
**RHS:**  $\bar{A} \cap \bar{B} \cap \bar{C} = \min[\mu_{\bar{A}}(x), \mu_{\bar{B}}(x), \mu_{\bar{C}}(x)] \quad \forall x \in X$

$A \cup B \cup C = \max(0.7, 0.8, 0.4)/1 + \max(0.5, 0.3, 0.7)/2 + \max(0.1, 0.7, 0.3)/3 + \max(0.6, 0.5, 0.9)/4$   
 $= 0.8/1 + 0.7/2 + 0.7/3 + 0.9/4$

$\Rightarrow \overline{A \cup B \cup C} = (1 - 0.8)/1 + (1 - 0.7)/2 + (1 - 0.7)/3 + (1 - 0.9)/4$   
 $= 0.2/1 + 0.3/2 + 0.3/3 + 0.1/4$

$\bar{A} = 0.3/1 + 0.5/2 + 0.9/3 + 0.4/4$   
 $\bar{B} = 0.2/1 + 0.7/2 + 0.3/3 + 0.5/4$   
 $\bar{C} = 0.6/1 + 0.3/2 + 0.7/3 + 0.1/4$

$\bar{A} \cap \bar{B} \cap \bar{C} = \min(0.3, 0.2, 0.6)/1 + \min(0.5, 0.7, 0.3)/2 + \min(0.9, 0.3, 0.7)/3 + \min(0.4, 0.5, 0.1)/4$   
 $= 0.2/1 + 0.3/2 + 0.3/3 + 0.1/4 = \overline{A \cup B \cup C}$

Hence,  $\overline{A \cup B \cup C} = \bar{A} \cap \bar{B} \cap \bar{C}$

The Demorgan's Law of Union is verified for given fuzzy sets A, B and C.

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And we are interested in the  $\overline{A \cup B \cup C}$  first. So, when we find this as this is mentioned here so, A the  $\overline{A \cup B \cup C} = 0.2/1 + 0.3/2 + 0.3/3 + 0.1/4$ . So, this is what is the discrete fuzzy set which is outcome of the  $\overline{A \cup B \cup C}$ . Now let us find out the  $\bar{A} \cap \bar{B} \cap \bar{C}$ .

So, first we need to find out the  $\bar{A}$ . So, this  $\bar{A}$ , then this is our  $\bar{B}$  and this is our  $\bar{C}$ . So, these three are the fuzzy sets or the complements of A fuzzy set B fuzzy set C fuzzy sets respectively. Now if we take their intersection their intersection means the  $\bar{A} \cap \bar{B} \cap \bar{C}$ . So, means we have three fuzzy sets, these three fuzzy sets are complements of A, B and C separately. And when we take intersection we of course, we apply the min criteria and then when we apply min criteria, we are getting a fuzzy set which is A discrete fuzzy set and this is  $0.2/1 + 0.3/2 + 0.3/3 + 0.1/4$  and when we see this fuzzy set is exactly same as we have got here.

So, this fuzzy set is same as the fuzzy set which we have got out of the  $\overline{A \cup B \cup C}$ . So, this way we can say that the DeMorgan's Law of union for discrete fuzzy set A, B and C is holding good.

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### 10. Demorgan's Law of Intersection

For crisp sets  $A$  and  $B$ ,

$$\overline{A \cap B} = \overline{A} \cup \overline{B} \quad \checkmark$$

For fuzzy sets  $A$  and  $B$ ,

$$\overline{A \cap B} = \overline{A} \cup \overline{B} \quad \checkmark$$

This is called the "Demorgan's law" of intersection.

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So, first when we take crisp fuzzy set we all know that DeMorgan's law of intersection is valid holding good and when we take fuzzy sets  $A$  and  $B$ , here also this is holding good means this is valid. So, when we take the  $\overline{A \cap B} = \overline{A} \cup \overline{B}$ . So, this is called the DeMorgan's law of intersection and let us now understand this better by taking couple of examples. So, first we will take two fuzzy sets  $A$  and  $B$  here. We have  $A$  here  $A$  fuzzy set here and  $B$  fuzzy set here.

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### 10. Demorgan's Law of Intersection

**Example 5:** Consider the fuzzy sets  $A$  and  $B$  for a universe of discourse  $X \in [0,10]$  as follows:

$\mu_A(x) = \text{triangle}(x; 2,5,7) \rightarrow$  Triangular membership function  
 $\mu_B(x) = \text{gaussian}(x; 3,0.8) \rightarrow$  Gaussian membership function

The corresponding plots of fuzzy sets  $A$  and  $B$  are given below. Verify the "Demorgan's law" of intersection.

$A = \int_X \mu_A(x)/x$   
 $B = \int_X \mu_B(x)/x$

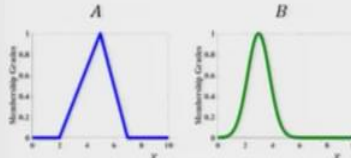
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And let us see how this DeMorgan's law of intersection is holding good.

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### 10. Demorgan's Law of Intersection

**Solution:** For fuzzy sets  $A$  and  $B$  with the universe of discourse  $X \in [0,10]$ , the **Demorgan's law of intersection** is defined as  $\overline{A \cap B} = \overline{A} \cup \overline{B}$ . The membership function values of  $\overline{A \cap B}$  and  $\overline{A} \cup \overline{B}$  are defined as

$$\overline{A \cap B} = 1 - \min[\mu_A(x), \mu_B(x)] \forall x \in X$$
$$\overline{A} \cup \overline{B} = \max[\mu_{\overline{A}}(x), \mu_{\overline{B}}(x)] \forall x \in X$$


$A \cap B = ?$

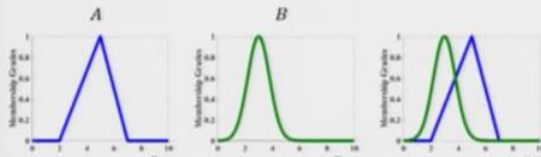
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So, to check that let us first find the complement of  $A \cap B$ . So, here first find the intersect  $A \cap B$ . So, let us first super impose A and B on each other as it is shown here.

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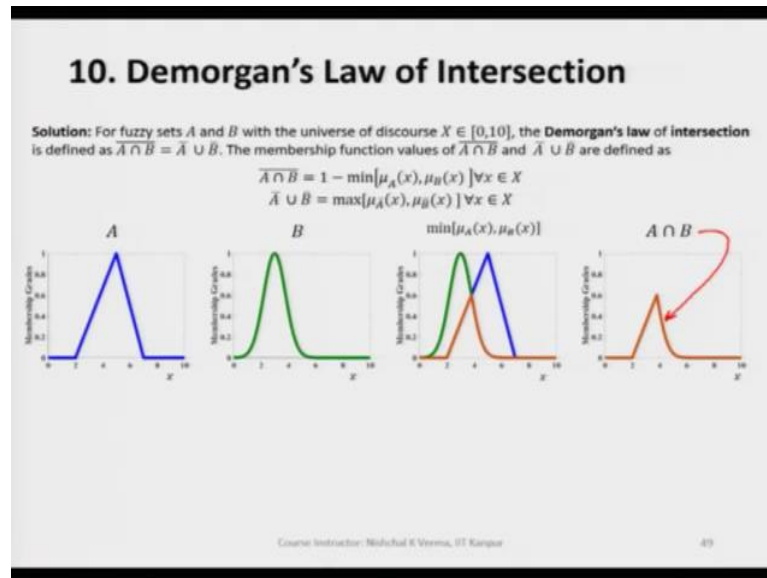
### 10. Demorgan's Law of Intersection

**Solution:** For fuzzy sets  $A$  and  $B$  with the universe of discourse  $X \in [0,10]$ , the **Demorgan's law of intersection** is defined as  $\overline{A \cap B} = \overline{A} \cup \overline{B}$ . The membership function values of  $\overline{A \cap B}$  and  $\overline{A} \cup \overline{B}$  are defined as

$$\overline{A \cap B} = 1 - \min[\mu_A(x), \mu_B(x)] \forall x \in X$$
$$\overline{A} \cup \overline{B} = \max[\mu_{\overline{A}}(x), \mu_{\overline{B}}(x)] \forall x \in X$$


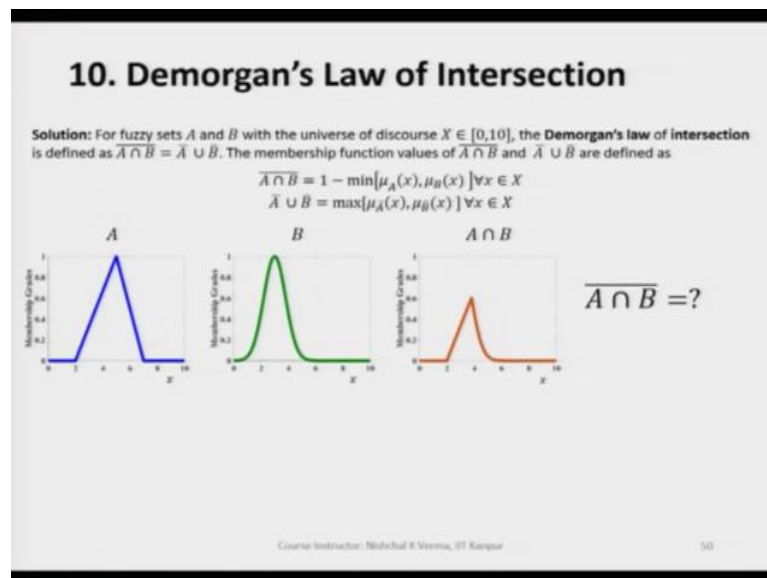
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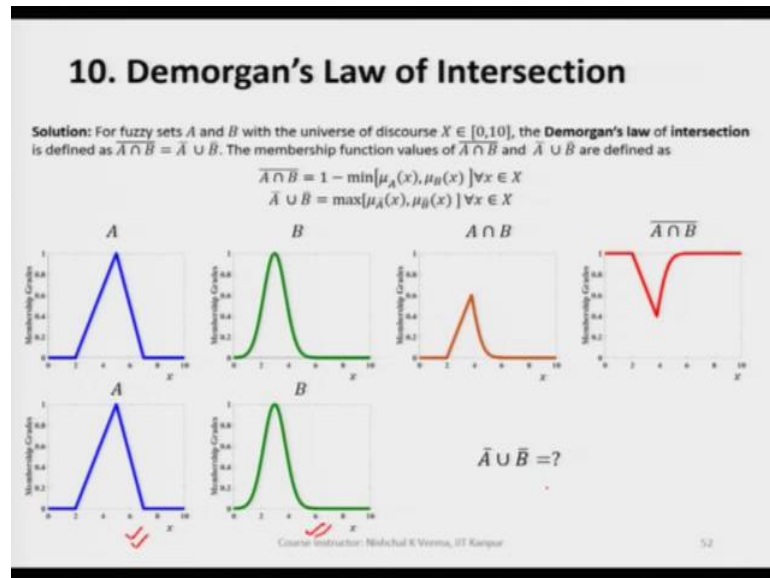
And then since we are taking the intersection of course, we will be applying the min criteria and then we see here we are getting this fuzzy set as  $A \cap B$ .

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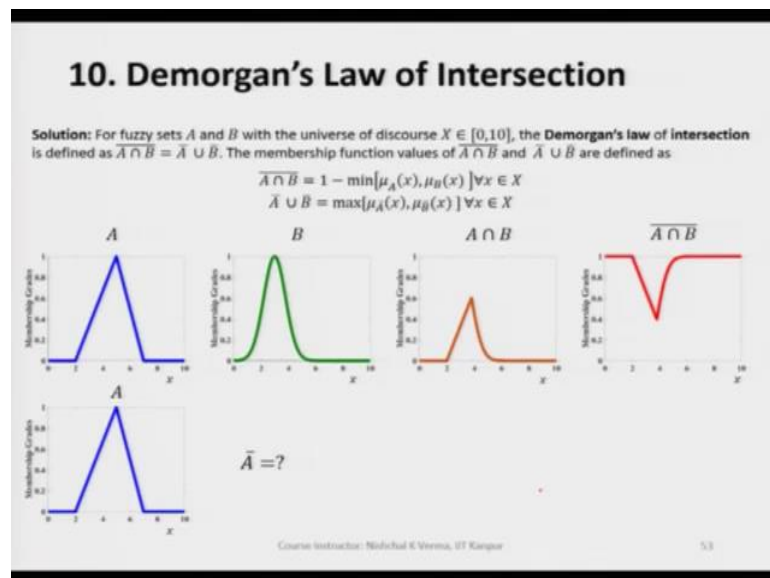
Now, let us take the complement of this outcome; let us take the  $\overline{A \cap B}$ . So, let us see what are we going to get we had to subtract the respective membership values from one and if we do that we are going to get this fuzzy set as the  $\overline{A \cap B}$ .

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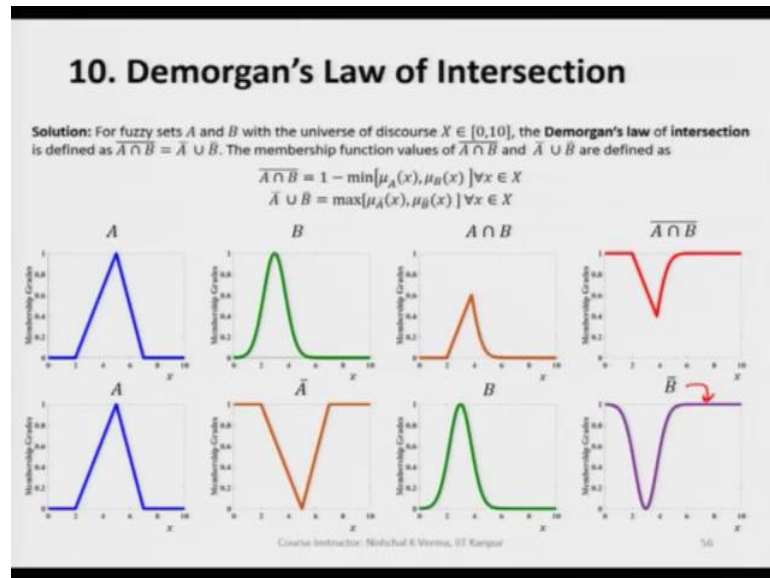
Now, let us find the  $\overline{A}$  and  $\overline{B}$  and then take their union and let us see whether this equal to the  $\overline{A \cap B}$  or not. So, let us now start. So,  $A$  fuzzy set is here and  $B$  fuzzy set is here. Let's take the  $\overline{B}$ .

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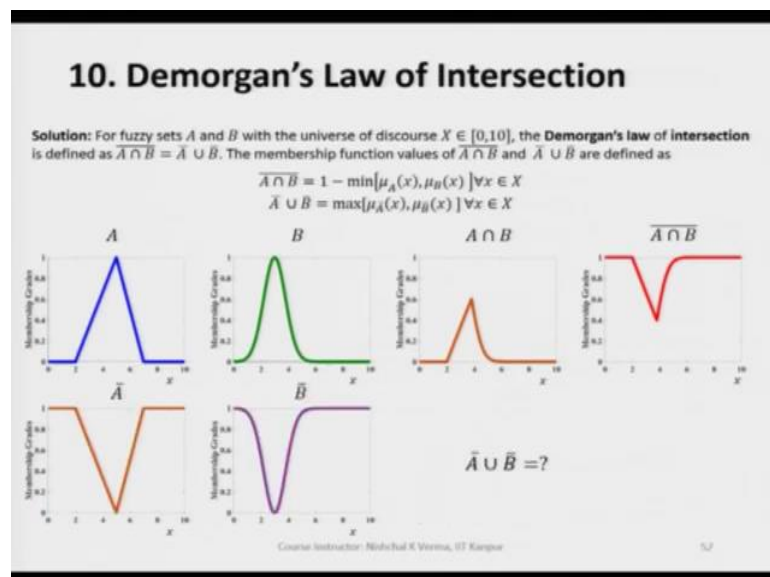


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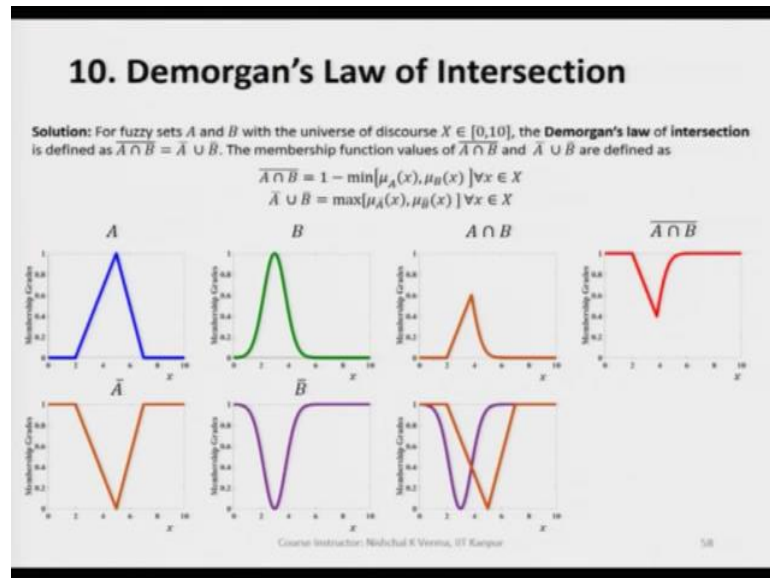


Let us get the  $\overline{A}$ ,  $\overline{B}$  is here. This is  $\overline{A}$  and  $\overline{B}$  is here, now we are interested in the  $\overline{A} \cup \overline{B}$ .

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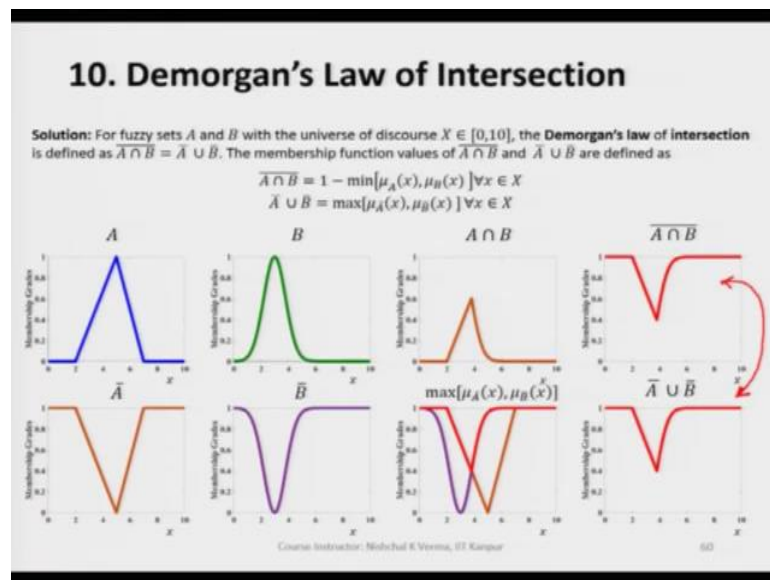


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It means we have to super impose  $\overline{A}$  and  $\overline{B}$  on each other like this and then since we are interested in the union, we have to apply the max criteria and when we apply max criteria, we are going to get here this as the outcome.

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So, this is the outcome of the  $\overline{A} \cup \overline{B}$  and if we see it is very clear that these two outcomes are same. So, what does this mean? This means that the  $\overline{A \cap B} = \overline{A} \cup \overline{B}$ .

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### 10. Demorgan's Law of Intersection

**Solution:** For fuzzy sets  $A$  and  $B$  with the universe of discourse  $X \in [0,10]$ , the **Demorgan's law of intersection** is defined as  $\overline{A \cap B} = \overline{A} \cup \overline{B}$ . The membership function values of  $\overline{A \cap B}$  and  $\overline{A} \cup \overline{B}$  are defined as

$$\overline{A \cap B} = 1 - \min[\mu_A(x), \mu_B(x)] \forall x \in X$$

$$\overline{A} \cup \overline{B} = \max[\mu_{\overline{A}}(x), \mu_{\overline{B}}(x)] \forall x \in X$$

As shown in the plots, the **Demorgan's Law of Intersection** is verified for fuzzy sets  $A$  and  $B$ .

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

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So, this way you see here we can say that the DeMorgan's law of intersection holds good for continuous fuzzy sets. Now let us check the same for discrete fuzzy sets.

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### 10. Demorgan's Law of Intersection

**Example 6:** Let  $A$  and  $B$  are two fuzzy sets given below for the universe of discourse  $X = \{1,2,3,4\}$ .

$$A = 0.7/1 + 0.5/2 + 0.1/3 + 0.6/4 \quad \checkmark$$

$$B = 0.8/1 + 0.3/2 + 0.7/3 + 0.5/4 \quad \checkmark$$

Verify the **Demorgan's law of Intersection** for the given fuzzy sets.

**Solution:** For the fuzzy sets, the **Demorgan's law of intersection** is defined as

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

The elements of above expression are being defined as

**LHS:**  $\overline{A \cap B} = 1 - \min[\mu_A(x), \mu_B(x)] \forall x \in X$

**RHS:**  $\overline{A} \cup \overline{B} = \max[\mu_{\overline{A}}(x), \mu_{\overline{B}}(x)] \forall x \in X$

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So, we see here if we take two fuzzy sets  $A$  and  $B$  and we see that we have on the same lines as we have done in the previous example, we take the  $\overline{A \cap B}$ . So,  $A \cap B$  is here  $\overline{A \cap B}$ .

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### 10. Demorgan's Law of Intersection

$$A = 0.7/1 + 0.5/2 + 0.1/3 + 0.6/4$$

$$B = 0.8/1 + 0.3/2 + 0.7/3 + 0.5/4$$

LHS:  $\overline{A \cap B} = 1 - \min[\mu_A(x), \mu_B(x)] \quad \forall x \in X$

RHS:  $\overline{A \cup B} = \max[\mu_{\bar{A}}(x), \mu_{\bar{B}}(x)] \quad \forall x \in X$

$$A \cap B = \min(0.7, 0.8)/1 + \min(0.5, 0.3)/2 + \min(0.1, 0.7)/3 + \min(0.6, 0.5)/4$$

$$= 0.7/1 + 0.3/2 + 0.1/3 + 0.5/4$$

→  $\overline{A \cap B} = 0.3/1 + 0.7/2 + 0.9/3 + 0.5/4$

→  $\bar{A} = 0.3/1 + 0.5/2 + 0.9/3 + 0.4/4$

→  $\bar{B} = 0.2/1 + 0.7/2 + 0.3/3 + 0.5/4$

→  $\bar{A} \cup \bar{B} = \max(0.3, 0.2)/1 + \max(0.5, 0.7)/2 + \max(0.9, 0.3)/3 + \max(0.4, 0.5)/4$

$$= 0.3/1 + 0.7/2 + 0.9/3 + 0.5/4 = \overline{A \cap B}$$

Hence,  $\overline{A \cap B} = \bar{A} \cup \bar{B}$

The Demorgan's Law of Intersection is verified for given fuzzy sets A and B.

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So, this is coming out as  $0.3/1 + 0.7/2 + 0.9/3 + 0.5/4$ . So, this is what we are getting as the fuzzy set discrete fuzzy set which is coming as the outcome when we take the  $\overline{A \cap B}$ . Now let us take the union of the complements the complements of A and B. So, we have  $\bar{A}$  here  $\bar{B}$  here and I can show it like this by this arrow  $\bar{A}, \bar{B}$ .

And when we take union of these two fuzzy sets when we take union of these two fuzzy sets, so we see that we are getting  $0.3/1 + 0.7/2 + 0.9/3 + 0.5/4$ . If we see here this outcome is same as the outcome of the  $\overline{A \cap B}$ .

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### 10. Demorgan's Law of Intersection

For crisp sets A, B and C,

$$\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C} \quad \checkmark$$

For fuzzy sets A, B and C,

$$\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$$

This is called the "Demorgan's law" of intersection.

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Now, if we take more than two fuzzy sets so, let us see what is happening of course, here also the DeMorgan's law of intersection is holding good. So, and if you talk of crisp sets  $A$  and  $B$ ,  $A$ ,  $B$  and  $C$  these crisp sets the DeMorgan's law of intersection is satisfied. So, we do not have to care about crisp sets because when we for crisp sets when we see the DeMorgan's law of intersection this is valid. So, so this valid when we say valid means when we take the  $\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$ .

So, this is valid for crisp sets and when we take fuzzy sets  $A$ ,  $B$  and  $C$  here also this is valid this DeMorgan's law of intersection is valid. So, this means that if we have  $A$  fuzzy set  $B$  fuzzy set  $C$  fuzzy set and if we take the  $\overline{A \cap B \cap C}$  this is going to be same as the complement of  $\bar{A} \cup \bar{B} \cup \bar{C}$  and this is called the DeMorgan's law of intersection as I just mentioned.

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**10. Demorgan's Law of Intersection**

For crisp sets  $A$ ,  $B$  and  $C$ ,

$$\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$$

For fuzzy sets  $A$ ,  $B$  and  $C$ ,

$$\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$$

This is called the "Demorgan's law" of intersection.

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So, let us now understand this also better by taking one example here and here we take again same as other examples we take the three fuzzy sets  $A$  and  $B$ ,  $A$ ,  $B$  and  $C$  fuzzy sets. These fuzzy sets are continuous fuzzy sets. So, here this  $A$  fuzzy set  $A$  continuous fuzzy set  $B$  continuous fuzzy set and  $C$  continuous fuzzy sets  $A$  is triangular  $B$  is Gaussian fuzzy set and  $C$  is trapezoidal fuzzy set.

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### 10. Demorgan's Law of Intersection

**Solution:** For fuzzy sets  $A, B$  and  $C$  with the universe of discourse  $X \in [0,10]$ , the **Demorgan's law of intersection** is given by  $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$ . The membership function values of  $\overline{A \cap B \cap C}$  and  $\overline{A} \cup \overline{B} \cup \overline{C}$  are defined as

$$\overline{A \cap B \cap C} = 1 - \min[\mu_A(x), \mu_B(x), \mu_C(x)] \forall x \in X$$
$$\overline{A} \cup \overline{B} \cup \overline{C} = \max[\mu_{\overline{A}}(x), \mu_{\overline{B}}(x), \mu_{\overline{C}}(x)] \forall x \in X$$

$\overline{A \cap B \cap C} = ?$

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So, our intention now is to find the  $\overline{A \cap B \cap C}$ , how do we get that? So, for getting this first, we have to find  $A \cap B \cap C$  and then we take the complement of the outcome.

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### 10. Demorgan's Law of Intersection

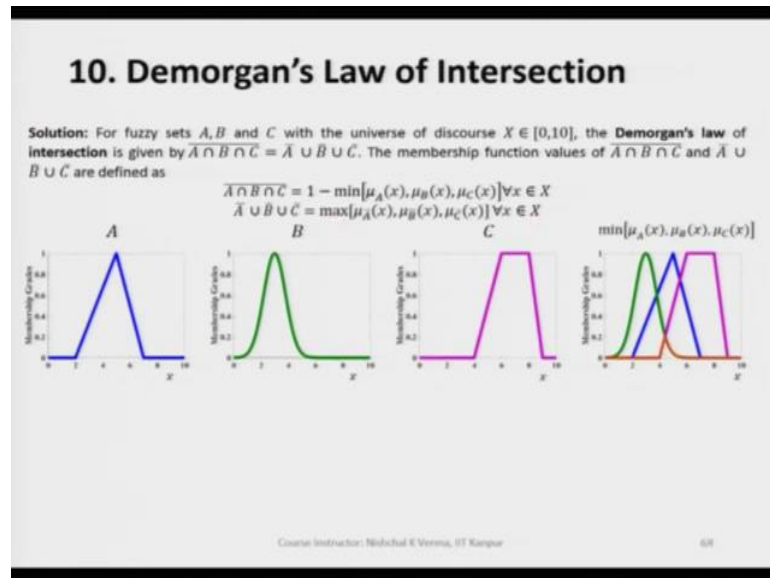
**Solution:** For fuzzy sets  $A, B$  and  $C$  with the universe of discourse  $X \in [0,10]$ , the **Demorgan's law of intersection** is given by  $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$ . The membership function values of  $\overline{A \cap B \cap C}$  and  $\overline{A} \cup \overline{B} \cup \overline{C}$  are defined as

$$\overline{A \cap B \cap C} = 1 - \min[\mu_A(x), \mu_B(x), \mu_C(x)] \forall x \in X$$
$$\overline{A} \cup \overline{B} \cup \overline{C} = \max[\mu_{\overline{A}}(x), \mu_{\overline{B}}(x), \mu_{\overline{C}}(x)] \forall x \in X$$

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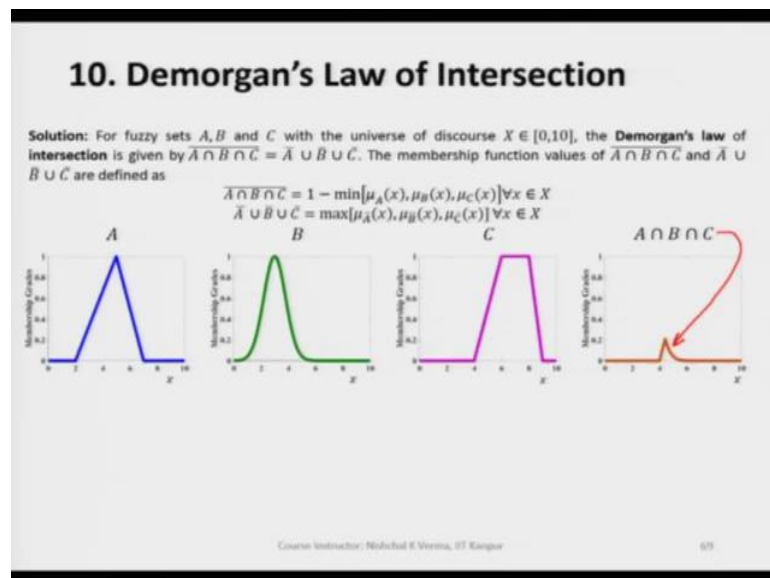
So, if we have  $A$  here;  $A$  fuzzy set here  $B$  fuzzy set here  $C$  fuzzy set here, we can super impose these three fuzzy sets on each other which is here and which is shown here and then apply the min criteria.

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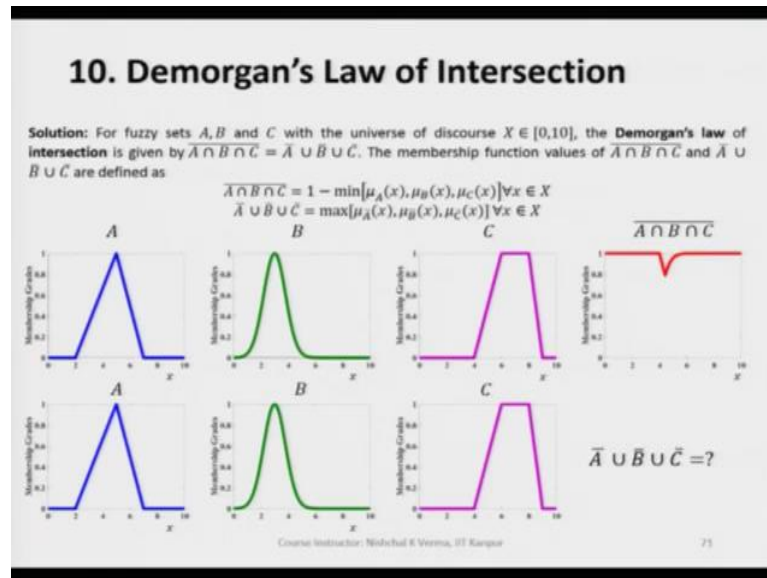


And when we apply min criteria, we get here fuzzy set as the outcome. So, this is nothing, but the intersection of  $A, B$  and  $C$  fuzzy sets.

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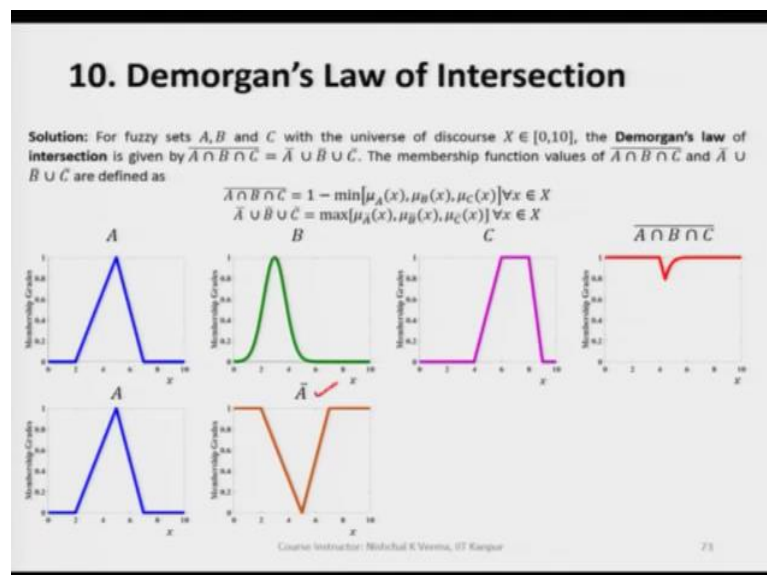


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Now, we are interested in complement of it. So, we have to take the complement when we take the complement, we are going to subtract all respective values of membership values all respective membership values from one. So, when we do that, we are going to get this fuzzy set as the outcome. So, this is nothing, but the  $\overline{A \cap B \cap C}$ . Now let us find out the  $\overline{A} \cup \overline{B} \cup \overline{C}$ . So, here we have  $\overline{A}$  which we have got from the fuzzy set  $A$ .

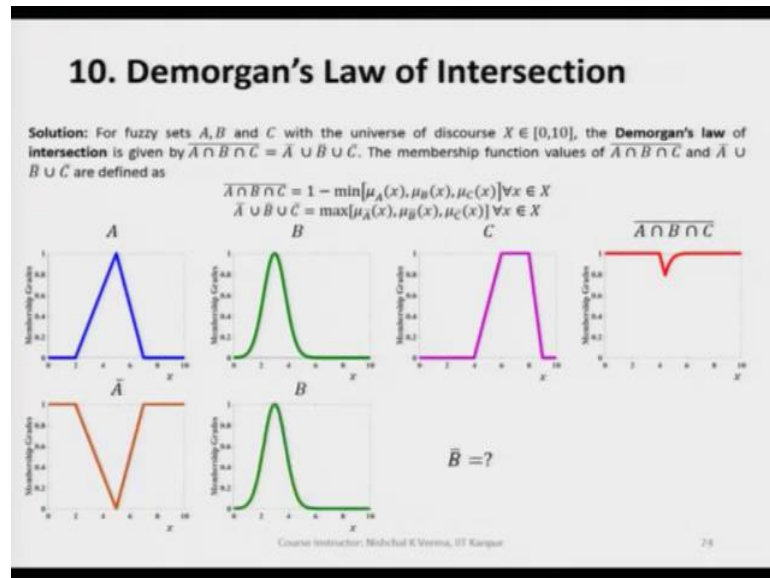
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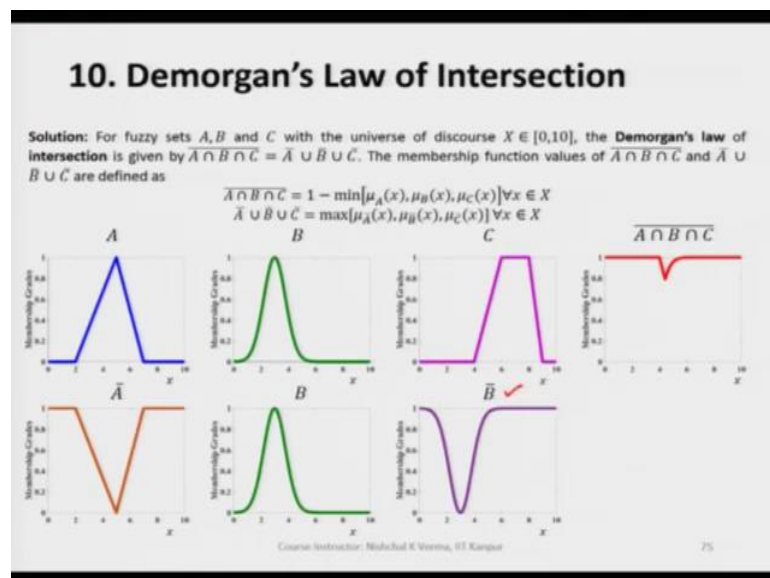
So, we have  $\overline{A}$  here and then let us find  $\overline{B}$ .



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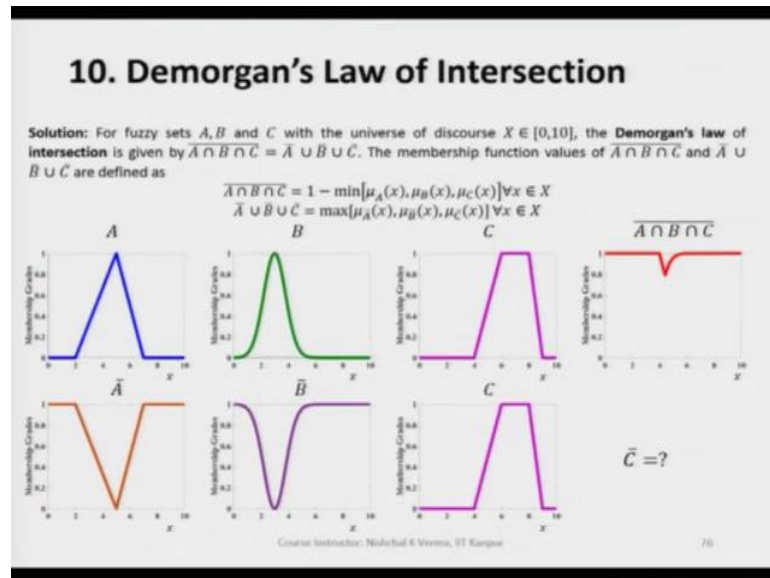


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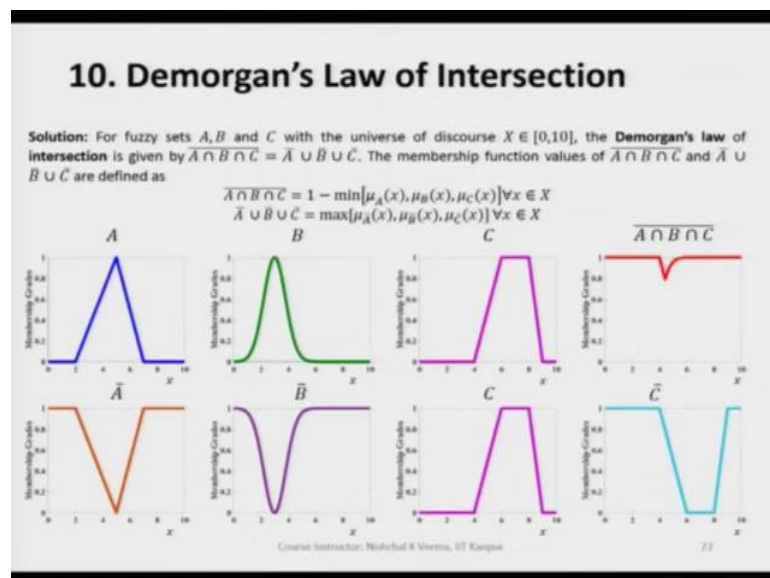


So,  $\overline{B}$  is here, we already know as to how we can find the complements. We subtract the respective membership values from one throughout the universe of discourse.

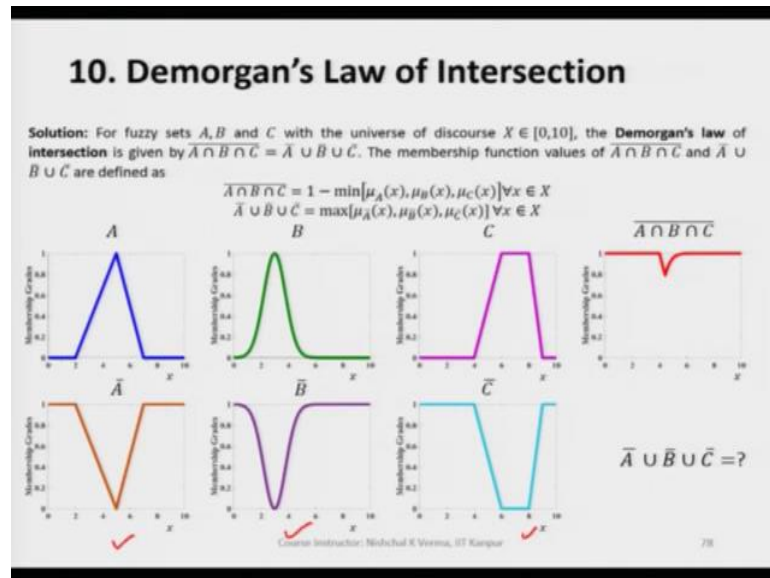
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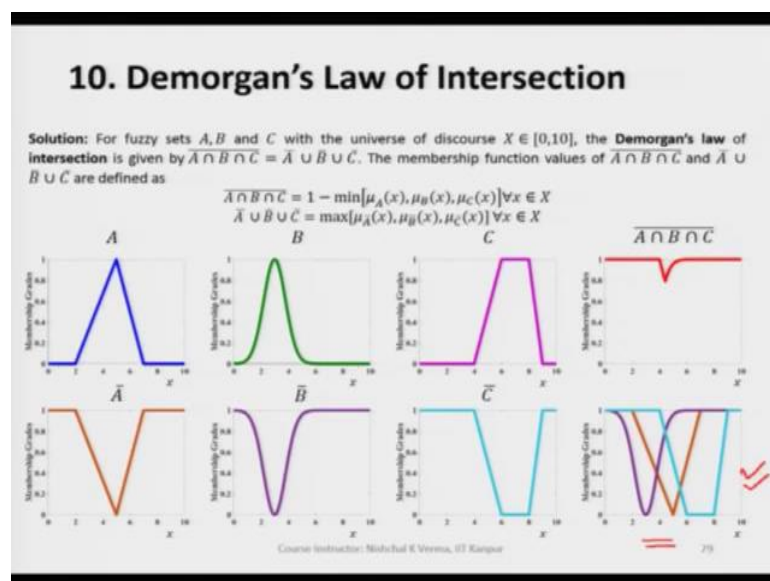
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We get the complement and so, when we have  $\overline{B}$ , now we see  $\overline{C}$ . So,  $\overline{C}$  is here. Now we have  $\overline{A}$ , we have  $\overline{B}$ , we have  $\overline{C}$ .

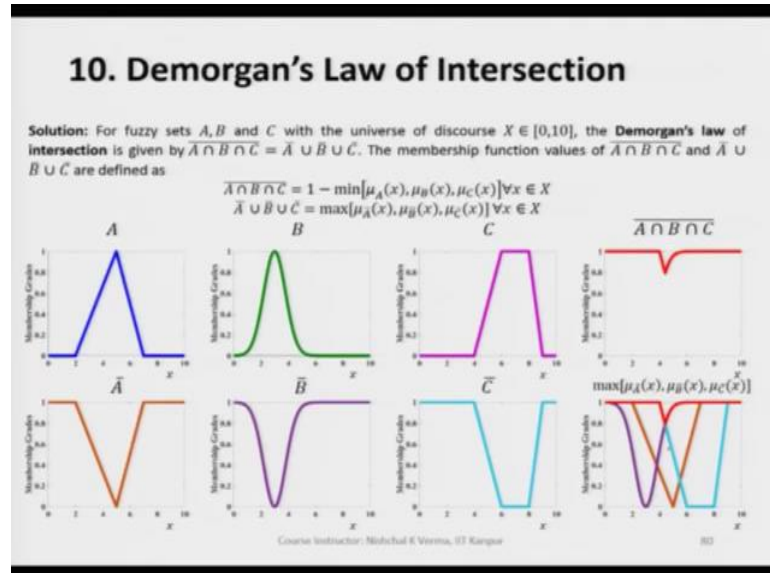
We are interested in their union. So, let us find the union of these three complements. How do we do that? So, we super impose each of these complements, each of these fuzzy sets the complement fuzzy sets on each other and since we are interested in the union, we apply the max criteria.

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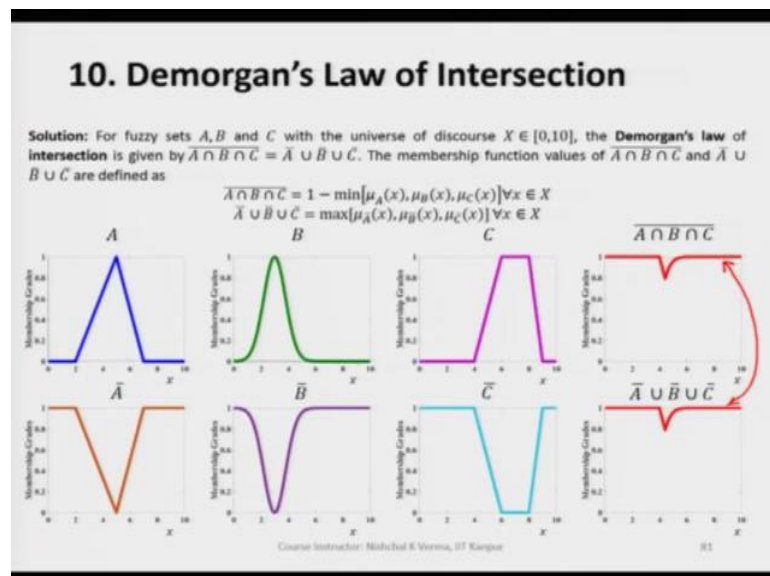
So, we can clearly see here that in this plot here, we have super imposed; we have super imposed all the three complements.

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And then when we apply the max criteria, we find the fuzzy set which is mentioned by which is denoted by the red color which is shown in red color.

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So, what we are getting here as the outcome of the  $\overline{A \cap B \cap C}$ , we see that this is exactly the same as what we have got. This is the fuzzy set exactly same as what we have got when we have taken the  $\overline{A \cap B \cap C}$ .

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### 10. Demorgan's Law of Intersection

**Solution:** For fuzzy sets  $A, B$  and  $C$  with the universe of discourse  $X \in [0,10]$ , the **Demorgan's law of intersection** is given by  $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$ . The membership function values of  $\overline{A \cap B \cap C}$  and  $\overline{A} \cup \overline{B} \cup \overline{C}$  are defined as

$$\overline{A \cap B \cap C} = 1 - \min[\mu_A(x), \mu_B(x), \mu_C(x)] \forall x \in X$$

$$\overline{A} \cup \overline{B} \cup \overline{C} = \max[\mu_{\overline{A}}(x), \mu_{\overline{B}}(x), \mu_{\overline{C}}(x)] \forall x \in X$$

As shown in the plots, the **Demorgan's Law of Intersection** is verified for fuzzy sets  $A, B$  and  $C$ .

$\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$

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So, this way we can say since the outcome is equal here for the continuous case when we have taken three fuzzy sets three continuous fuzzy sets, we can say that the DeMorgan's law of intersection is holding good for continuous fuzzy sets. So, in other words I can repeat here in other words we can say that if we have to find the union of the complements of  $A, B$  and  $C$  we can do this, we can find this by simply taking the  $\overline{A \cap B \cap C}$ . Or in other words again  $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$ . So, either way we can say.

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### 10. Demorgan's Law of Intersection

**Example 8:** Let  $A, B$  and  $C$  are three fuzzy sets given below for the universe of discourse  $X = \{1,2,3,4\}$ .

$$A = 0.7/1 + 0.5/2 + 0.1/3 + 0.6/4$$

$$B = 0.8/1 + 0.3/2 + 0.7/3 + 0.5/4$$

$$C = 0.4/1 + 0.7/2 + 0.3/3 + 0.9/4$$

Verify the **Demorgan's law of Intersection** for the given fuzzy sets.

**Solution:** For fuzzy sets  $A, B$  and  $C$ , the **Demorgan's law of Intersection** is defined as

$$\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$$

The elements of above expression are being defined as

**LHS:**  $\overline{A \cap B \cap C} = 1 - \min[\mu_A(x), \mu_B(x), \mu_C(x)] \quad \forall x \in X$

**RHS:**  $\overline{A} \cup \overline{B} \cup \overline{C} = \max[\mu_{\overline{A}}(x), \mu_{\overline{B}}(x), \mu_{\overline{C}}(x)] \quad \forall x \in X$

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Now, the same can be checked with three discrete fuzzy sets. So, we have taken here three discrete fuzzy sets. This is first fuzzy set, this is second fuzzy set is 3, this is third discrete fuzzy set. So, DeMorgan's law of intersection, let us now try to see what is happening when we take these three discrete fuzzy sets and see whether the DeMorgan's law of intersection is holding good or not.

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### 10. Demorgan's Law of Intersection

$$A = 0.7/1 + 0.5/2 + 0.1/3 + 0.6/4$$

$$B = 0.8/1 + 0.3/2 + 0.7/3 + 0.5/4$$

$$C = 0.4/1 + 0.7/2 + 0.3/3 + 0.9/4$$

**LHS:**  $\overline{A \cap B \cap C} = 1 - \min[\mu_A(x), \mu_B(x), \mu_C(x)] \quad \forall x \in X$

**RHS:**  $\overline{A} \cup \overline{B} \cup \overline{C} = \max[\mu_{\overline{A}}(x), \mu_{\overline{B}}(x), \mu_{\overline{C}}(x)] \quad \forall x \in X$

$$A \cap B \cap C = \min(0.7, 0.8, 0.4)/1 + \min(0.5, 0.3, 0.7)/2 + \min(0.1, 0.7, 0.3)/3 + \min(0.6, 0.5, 0.9)/4$$

$$= 0.4/1 + 0.3/2 + 0.1/3 + 0.5/4$$

$$\Rightarrow \overline{A \cap B \cap C} = (1 - 0.4)/1 + (1 - 0.3)/2 + (1 - 0.1)/3 + (1 - 0.5)/4$$

$$= 0.6/1 + 0.7/2 + 0.9/3 + 0.5/4$$

$$\overline{A} = 0.3/1 + 0.5/2 + 0.9/3 + 0.4/4$$

$$\overline{B} = 0.2/1 + 0.7/2 + 0.3/3 + 0.5/4$$

$$\overline{C} = 0.6/1 + 0.3/2 + 0.7/3 + 0.1/4$$

$$\overline{A} \cup \overline{B} \cup \overline{C} = \max(0.3, 0.2, 0.6)/1 + \max(0.5, 0.7, 0.3)/2 + \max(0.9, 0.3, 0.7)/3 + \max(0.4, 0.5, 0.1)/4$$

$$= 0.6/1 + 0.7/2 + 0.9/3 + 0.5/4 = \overline{A \cap B \cap C}$$

Hence,  $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$

The **Demorgan's Law of Intersection** is verified for given fuzzy sets  $A, B$  and  $C$ .

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So, for this we have to first find the  $\overline{A \cap B \cap C}$ . So, when we see that what we are getting here is the  $0.6/1 + 0.7/2 + 0.9/3 + 0.5/4$  here.

So, I can repeat the outcome here  $\overline{A \cap B \cap C}$  is coming out to be  $0.6/1 + 0.7/2 + 0.9/3 + 0.5/4$ . Now let us take the  $\bar{A} \cup \bar{B} \cup \bar{C}$ . So, when we take this union, we see that we are getting here as the  $0.6/1 + 0.7/2 + 0.9/3 + 0.5/4$ .

So, if we compare this fuzzy this discrete fuzzy set with this fuzzy set. So, we see that this is same as these two fuzzy sets are same as you know they both are same they both are these two fuzzy sets are equal. So, we can right here this fuzzy set is equal means  $\bar{A} \cup \bar{B} \cup \bar{C} = \overline{A \cap B \cap C}$ . So, we can clearly say that the DeMorgan's law of intersection is holding good for discrete fuzzy sets  $A, B$  and  $C$ .

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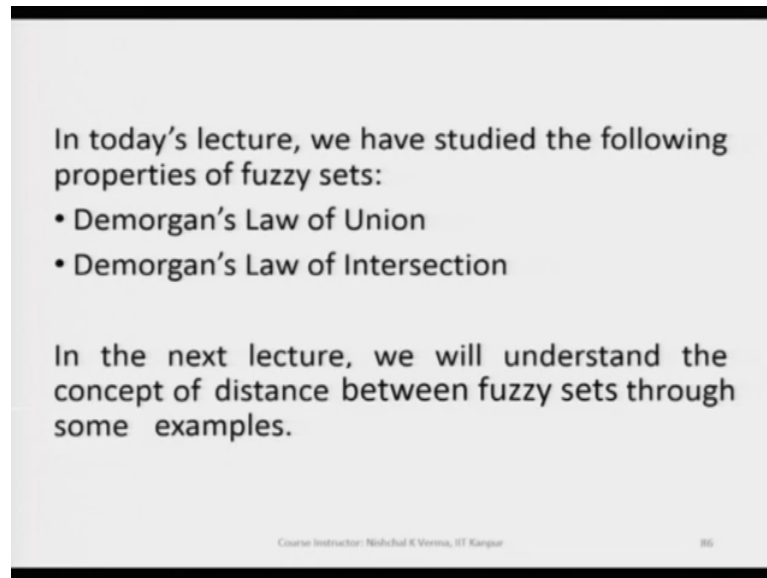
| Property ✓               | CLASSICAL SETS ✓   | FUZZY SETS ✓   |
|--------------------------|--|--|
| Law of Contradiction     | $A \cap \bar{A} = \phi$  | $A \cap \bar{A} = \phi$  |
| Law of Excluded Middle   | $A \cup \bar{A} = X$   | $A \cup \bar{A} = X$   |
| Idempotency              | $A \cap A = A, A \cup A = A$   | $A \cap A = A, A \cup A = A$   |
| Involution               | $\bar{\bar{A}} = A$  | $\bar{\bar{A}} = A$  |
| Commutativity            | $A \cap B = B \cap A, A \cup B = B \cup A$   | $A \cap B = B \cap A, A \cup B = B \cup A$   |
| Associativity            | $(A \cup B) \cup C = A \cup (B \cup C)$<br>$(A \cap B) \cap C = A \cap (B \cap C)$                   | $(A \cup B) \cup C = A \cup (B \cup C)$<br>$(A \cap B) \cap C = A \cap (B \cap C)$                   |
| Distributivity           | $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$<br>$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ | $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$<br>$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ |
| Absorption               | $A \cup (A \cap B) = A$<br>$A \cap (A \cup B) = A$   | $A \cup (A \cap B) = A$<br>$A \cap (A \cup B) = A$   |
| Absorption of Complement | $A \cup (\bar{A} \cap B) = A \cup B$<br>$A \cap (\bar{A} \cup B) = A \cap B$                         | $A \cup (\bar{A} \cap B) \neq A \cup B$<br>$A \cap (\bar{A} \cup B) \neq A \cap B$                   |
| DeMorgan's Laws          | $\overline{A \cup B} = \bar{A} \cap \bar{B}$<br>$\overline{A \cap B} = \bar{A} \cup \bar{B}$         | $\overline{A \cup B} = \bar{A} \cap \bar{B}$<br>$\overline{A \cap B} = \bar{A} \cup \bar{B}$         |

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So, this way we have seen that the DeMorgan's law is very much holding good for fuzzy sets as well. Now let us go through all the properties of sets once again. So, we have a table here and in the first column, we have listed all the properties and then the second column, we have the classical sets third column, we have fuzzy sets. So, this way we can say that out of all these properties that are listed here for classical sets and fuzzy sets, only three namely law of contradiction, law of excluded middle and absorption of complement.

These three they behave differently for classical sets and fuzzy sets rest all others are same as I mean in both the cases for classical set and fuzzy sets, they behave the same means they are holding they are valid for classical sets and fuzzy sets.

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In today's lecture, we have studied the following properties of fuzzy sets:

- Demorgan's Law of Union
- Demorgan's Law of Intersection

In the next lecture, we will understand the concept of distance between fuzzy sets through some examples.

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So, with this I stop here and in the next lecture, we will discuss the concept of fuzzy distance.

Thank you.