Fuzzy Sets, Logic and Systems and Applications
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Lecture – 17
Distance between Fuzzy Sets

So, welcome to lecture number 17 of Fuzzy Sets, Logic and Systems & Applications. So, in this lecture today we will be discussing the distance between two fuzzy sets. That means, if there are two fuzzy sets A and B and then, we learn to find the distance between these two fuzzy sets A and B.

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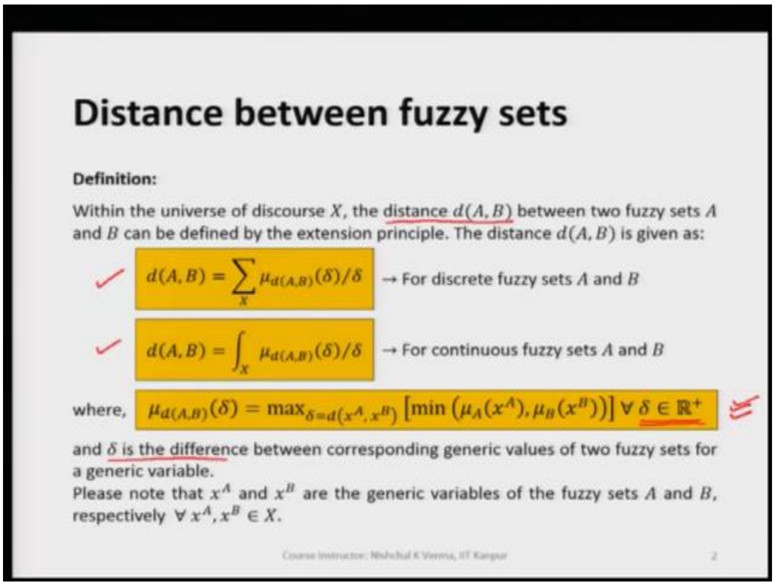

So, if we have two fuzzy sets say A and B means there are two fuzzy sets A and B, within the universe of discourse X, so the distance which is represented by d here the distance $d\,(A, B)$. So, here the distance between these two fuzzy sets can be defined by the extension principle and this distance can be given by this formula here and here we have two formulae like one is for discrete fuzzy sets A and B. Let's say we have discreet fuzzy sets then we use this formula to find the distance between two fuzzy sets A and B.

So as we all know that we represent the discreet fuzzy set by this representation.

$$d(A, B) = \sum_X \mu_{d(A,B)}(\delta)/\delta$$

I will tell you what the δ is, so let us first understand here that this is the formula for finding the discreet fuzzy set A and B the distance in between.

So, here if we have let's say two continuous fuzzy sets A and B like here in this case, where we will be having the distance between A and B fuzzy sets represented by

$$d(A,B) = \int_X \mu_{d(A,B)}(\delta)/\delta$$

And since we are using here, so let me tell you first that we have the resultant and this resultant in case of discreet fuzzy set we have the discreet fuzzy set as a result. Means the distance will be a discreet fuzzy set. So, in a way we can say, the distance is itself a fuzzy set. So, this distance between A and B or in other words if I say the distance between two discreet fuzzy sets A and B is going to give us a fuzzy set which is again a discreet fuzzy set. And this is represented by $\sum_X \mu_{d(A,B)}(\delta)/\delta$.

And similarly in case of a continuous fuzzy sets A and B we have the resultant, the distance that we compute we find is nothing but again a continuous fuzzy set. And this is represented by the fuzzy set which is described as $\int_X \mu_{d(A,B)}(\delta)/\delta$.

So, in this two expressions, one is for the discreet and other one is for continuous both includes $\mu_{d(A,B)}(\delta)$. So, how to find this $\mu_{d(A,B)}(\delta)$ is here. So, if we see here this expression is helping us in finding the $\mu_{d(A,B)}(\delta) = max_{\delta=d(x^A,x^B)}[\min(\mu_A(x^A), \mu_B(x^B))]\forall\delta \in \mathbb{R}^+$. Of course, because we have that we are computing the distance and that is how this delta is coming out to be the real positive value. So that is why this delta is coming from the set of positive real value.

So, it is also mentioned here that, the delta because we already have seen that how we can find the $\mu_{d(A,B)}(\delta)$, which is needed when we are computing the distance between two fuzzy sets. Now what is delta? So, delta is nothing but it is the difference between the corresponding generic variables of a two fuzzy sets A and B that we have taken for a generic variable.

And please note that we have x_A and x_B and these are the generic variables of the fuzzy sets A and B respectively. So, we can say for every x_A and x_B and these values for every

x_A, $x_B \in X$ which is nothing but the universe discourse. So, I hope this is alright to you and with this we can manage to find the distance between any two fuzzy sets.

So, now we know that we can compute the membership values of the corresponding generic variable values by this equation by this relation and delta as I already mentioned that this is nothing but the difference between the corresponding generic values of two fuzzy sets for a generic variable. So, let us now make this thing clear or more understandable by taking one example here. And this way we will be able to understand as to how we can find the distance between two fuzzy sets.

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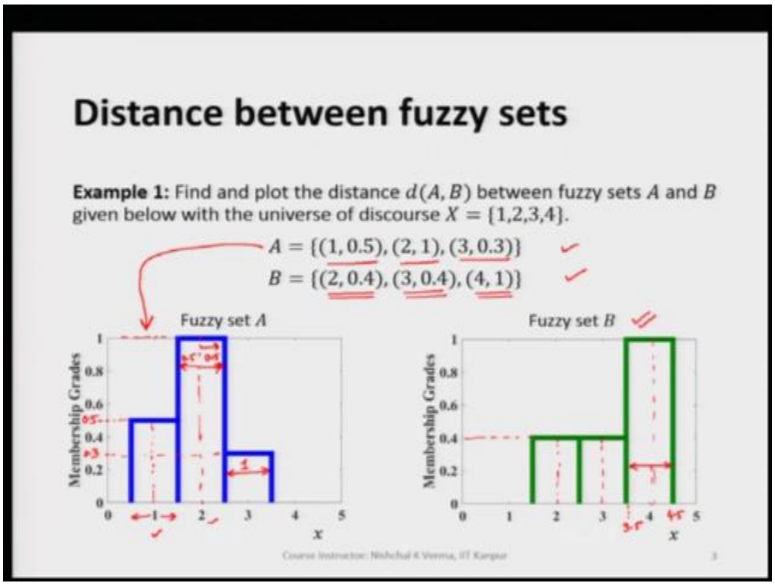

So, here I am going to take two discrete fuzzy sets A and B. So, we have the first fuzzy set here as A and then the second is B these two fuzzy sets are discreet. As we already we can see here and we can represent it this fuzzy set like this.

So let us first take fuzzy set A here. So, A discreet fuzzy set is having three elements, so first element we have here is the $1, 0.5$. So, $1, 0.5$ means, we have 1 as the generic variable value and 0.5 here is the corresponding membership value. Similarly, the second element here of the discreet fuzzy set A $2, 1$ so, this 2 is a generic variable value and 1 is the corresponding membership value.

The third element here is 3 , 0.3, so $3, 0.3$ is nothing but we have three as the generic variable value and 0.3 as the it is corresponding membership value. Now let us draw this fuzzy set here and this will look like this. So, we clearly see here that we have three elements in the discreet fuzzy set A and then we draw this fuzzy set we have add 1 as the generic variable value; we have 0.5 here this is 0.5, so 0.5 here is the corresponding membership value. So, it means that at 1 we have 0.5 membership value level. So, that is how we have a drawn a line over here at the level 0.5. And then for the second element here and second element has 2 as generic variable value and for this we have its corresponding membership value as 1. So, that is how we have its level as 1.

So, please note that this is a discreet fuzzy set and for representing this we have a taken range plus minus 0.5 as the generic variable value. So, that is how at one we have at 1 we can see here at 1 here we have 0.5 and it goes this side and this side both side 0.5 at the same level. This is just for the representation purpose and this will help us in understanding this computation of distance between two fuzzy sets a little better.

So, this 0.5 plus minus 0.5 variation has been taken for all the points all the elements of the discrete fuzzy set. And this way when we see here for the second element also here we have its membership value 1 and you see here this side also 0.5 and this side also 0.5 it remains the same here.

Similarly for the third element of the fuzzy set discrete fuzzy set A; we have the 0.3, here the 0.3 as the membership value and here also we have 0.5 plus side and minus side means left side and right side both. Or in other words we can say the width of this column is what is taken as 1 and mean of this has been taken as the generic variable value. Similarly this logic will is applied to plot all the discreet fuzzy sets for a distance computation purpose.

So here also if you see for fuzzy set B here, fuzzy set B we have three elements again and these three elements are this is the first element and this is the second element and third element here. So, in the first element we see we have $2, 0.4, 2$ is the generic variable value and its corresponding membership value is 0.4. So, we can clearly see here that we have at 2 we have 0.4 as the membership value. Similarly the second element we have 3 and it is a corresponding membership value is 0.4.

So, 3 here is the generic variable value and 0.4 again means there is both 2 and 3 are the same level. So, 3 here will have 0.4 as it is corresponding membership value. Similarly the third element here is $4, 1$. So, this 4 is the generic variable value and the corresponding membership value here is 1. So, we can see and with the same logic that each bar each column here is of width 1 and keeping its mean add the corresponding generic variable value. So, this means that we have this point as point 4.5 and this point we will have 3.5. So here also the same logic is applied.

So now, we can clearly see that we have two fuzzy sets A and B and each fuzzy sets; I mean both the fuzzy sets A and B are discreet and these two fuzzy sets have three elements each. So now, our aim is to find the distance between these two fuzzy sets these two discrete fuzzy sets, let us see how we can manage to find the distance $d(A, B)$.

So, for this as I have already explained how to find the $\mu_{d(A,B)}$, means the membership values of the resulting discrete fuzzy set here and then the corresponding generic variable value which is nothing but delta. So, let,s now go to the solution and for computing the fuzzy distance of the these two fuzzy these two discrete fuzzy sets A and B.

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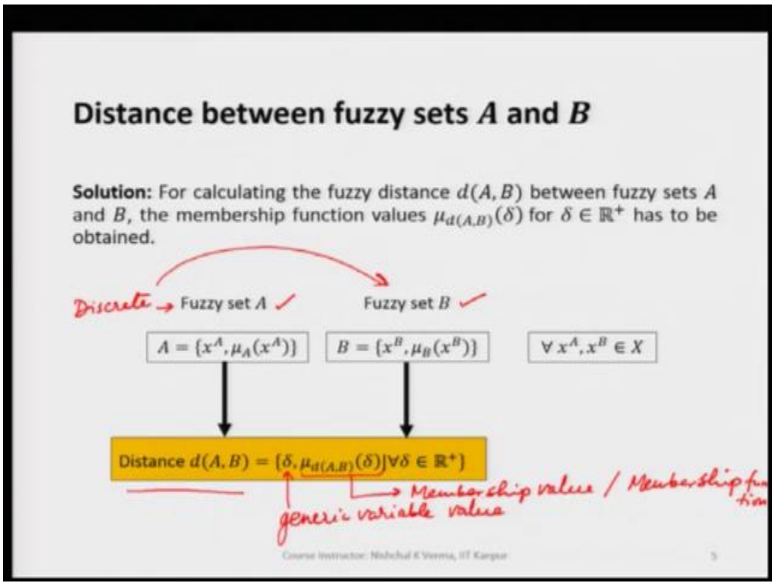

So, we first need to find the δ and let us see how we can manage to find the δ in each and every case. And as I mentioned here already that when we consider two fuzzy sets let us say A and B and both these are the discreet fuzzy sets, we represent the fuzzy set here A is equal to its generic variable value x_A. So, x_A is nothing but this is the generic variable values taken from the fuzzy set A. Or in other words again we can say that x_A or all those generic variable values which are present in fuzzy set A and $\mu_A(x_A)$ here is the corresponding membership values.

So similarly if we apply the same logic, we represent here the fuzzy set B and both these fuzzy sets are discrete fuzzy set which is very clearly visible. So, these both these fuzzy sets are discrete fuzzy sets. So, let us now understand this also that the resulting fuzzy set here that which is nothing but the distance $d(A,B)$ will have two components one is the generic variable value.

This is generic variable value which is here delta and it's corresponding membership value. So, we can say here this $\mu_{d(A,B)}(\delta)$ is a membership value. But if we if we have multiple values or if this is a general function we can say its a membership function.

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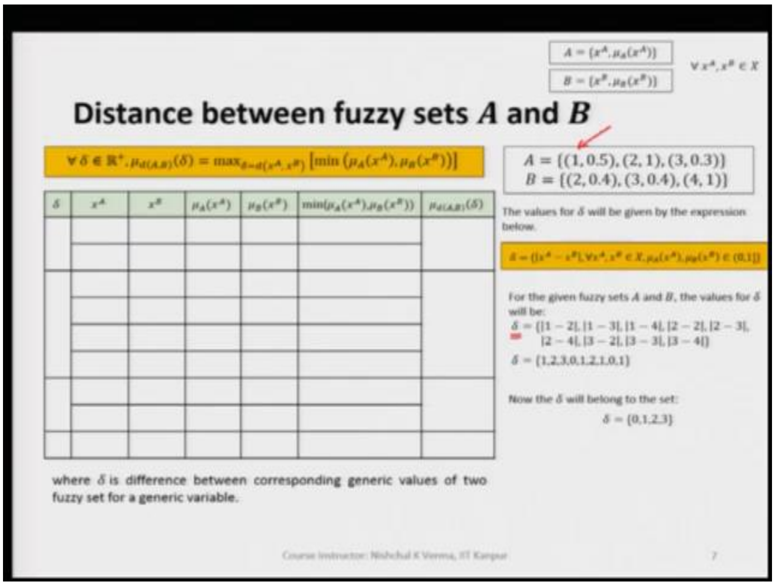

So, now let us move ahead and find the distance between two discreet fuzzy sets A and B. So, as we have already seen that we have two for fuzzy sets A and B like this; in a fuzzy set, we have three elements one is 1,0.5. The second element we have 2,1 and the third element here is 3,0.3. Similarly, we discrete fuzzy sets we have three elements the first element 2,0.4 the second element is 3,0.4 the third element here is 4,1.

And as I mentioned we can find here a δ by this expression and δ is nothing but all possible differences in the generic variable values of the discrete fuzzy sets A and B. So, let us now first find out the delta set, delta set is a set which has all the possible values of $\delta \in \mathbb{R}^+$ means positive real value. So, since we have two fuzzy sets A and B, so let us take all the generic variable values from A and B both so for computing the δ. So, we see here that we have first generic variable here is 1 from A fuzzy set and then the second fuzzy set B we have and the generic variable value here is we have generic variable value as 2.

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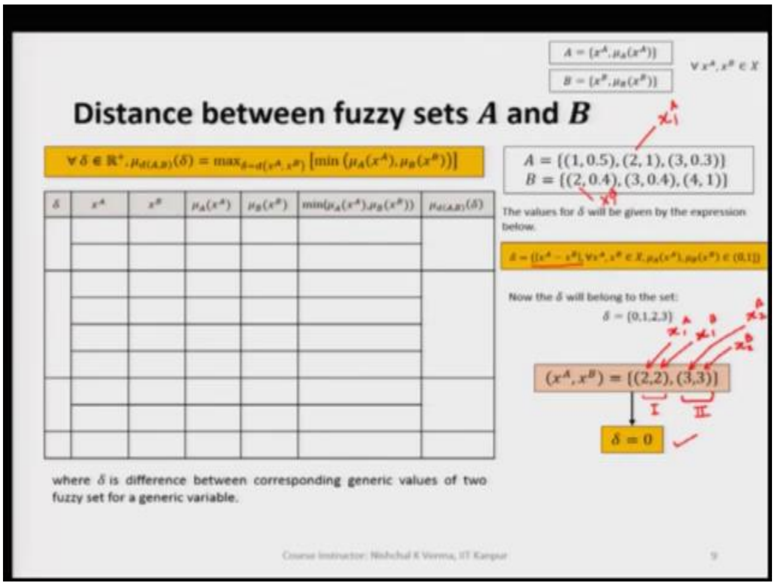

So, in a way what we are doing here is, we are finding the difference in the corresponding generic variable values and which are termed as the values of delta. Once again I would like to mention that all the δ values will be positive values, because we are going to compute the distance. So, let us first start from 0 as the difference, so let us first find out what are the combinations of the generic variable values from A and B which are going to give us the difference the $\delta = 0$.

So we see that here we have for 0 we have here if we take $2, 2$ and $3, 3$ combination, $2, 2$ means this is from A and this is from B this is from A this is from B fuzzy set. So, and we have 2 here from A fuzzy set and 2 here from B fuzzy set. So, this is generic variable this is my x_A I can write here as x_1 x_A 1 and here I have x_B 1 and this value is going to give us delta is equal to 0. So, we are interested here to first find the difference which is the least and least difference is 0, because there cannot be any value of δ below 0. Because we are interested in finding out the distance and distance will be only a real positive value. So, the least difference is going to be $\delta = 0$.

So, for $\delta = 0$, we can have two combinations 2, 2 means this is x_1^A and here we have x_1^B. Similarly here we have x_A it is the second value and here we have x_2^B. So, or we can otherwise we can represent these values, but here we are representing this way that the first value second value and both these the pair of these values are from set A and set B fuzzy set A fuzzy set B.

So, there are two pairs of the generic variable values $2, 2$ and $3, 3$, these two are going to give us this is the first pair and this is the second pair. These two pairs are going to give us $\delta = 0$ and δ is nothing but the difference which is mentioned over here. So, this is mod of $x_A - x_B$. So, these two pairs are going to give us $\delta = 0$ which is the least value of difference. So, when we start the first and foremost step here is to start with finding the delta is equal to 0. In many cases we may not have any such combinations for which we have $\delta = 0$.

So, if we do not find any such combination any such pair for which we get $\delta = 0$, we can simply move ahead for finding another or the incrementally the next least value of delta. For example, if we are not getting here any pair for which we are getting $\delta = 0$, we can immediately move ahead and try for getting $\delta = 1$.

So, similarly here also if we are not getting any pair any such pair for which we have $\delta = 1$, then we can move ahead and we will try to find any such combinations for which we have $\delta = 2$. And this way we can keep on increasing our steps or moving forward to find the next least values. But if we get here any pair like in this example we have 2,2, $3, 3$. So, two pairs of the generic variable values and these two pairs are resulting $\delta = 0$. So now, let us see what we should do and how should we account this value.

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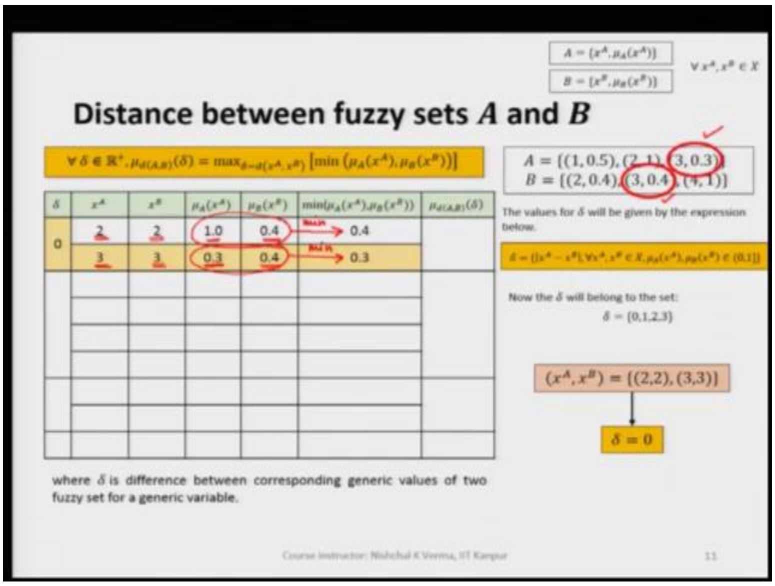

So, for $\delta = 0$ we have two pairs. So, that is how we are writing $\delta = 0$ you can see here. So, this is our $\delta = 0$; we write here we have a table and this table will finally help us in getting the resultant distance of the two fuzzy two discreet fuzzy sets. So, we have the $\delta = 0$ we will write in delta column 0 and since we are getting two pairs we will write those pairs here.

So, since we have got two pairs one pair is 2 2. So, we have written 2 here as x_A and the 2 here also as x_B. Why x_A and why x_B? Because this 2 the second column 2 is from discreet fuzzy set A and the this 2 this which is in the third column is corresponding to the B fuzzy set, so we have written here 2,2.

Similarly, now we have another pair which is 3,3 for which we are getting 0. So, we will write here 3,3 as x_A and x_B you can see here. So, let us now understand this as to how we are getting these values of x_A x_B for which the $\delta = 0$. Now another important point here is to list the corresponding membership values.

So, the corresponding membership value here $\mu_A(x_A)$, we are getting here as 1 and $\mu_B(x_B)$ we are getting 0.4. So, which all these values are coming from the fuzzy sets the discreet fuzzy sets that are given to us. So, we need not worry about anything we are just we need to just take the values from the given fuzzy sets and include in this table.

So, we have 2,2 as the generic variable values from the first pair and then corresponding membership values. Now what next? Next is that, let us find the minimum of these two membership values. So, minimum of these two membership values means here. So, we need to find the minimum of membership values which is here. So, minimum of 1.0 and 0.4 so we have 0.4.

So, next is the second pair here $3, 3$ we have corresponding membership values as 0.3 and 0.4 we can clearly see here its mentioned its encircled here. So, we see here that we have $0.3, 0.4$ or the membership values. Now when we take the minimum of these two values we are going to get what? We are going to get 0.3. So, this are the minimum of these two minimum, so this is what we need to do.

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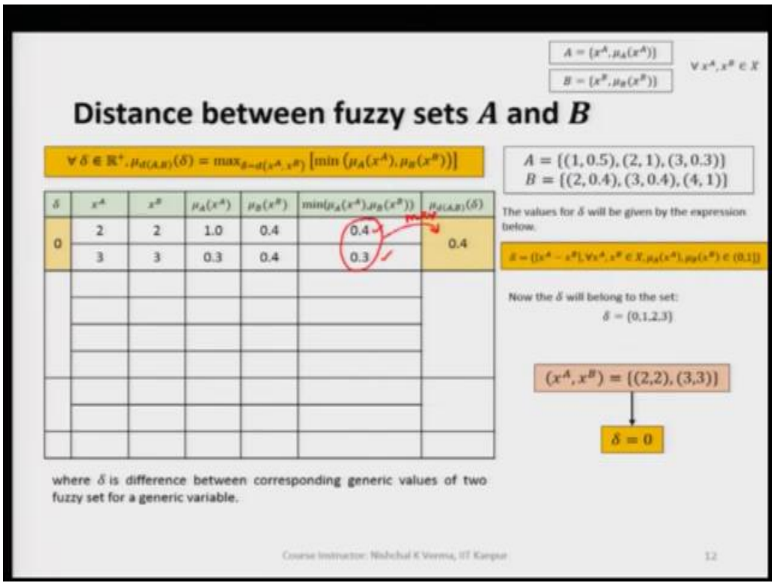

And then the next step here is, for 0 as the δ, so we need to find the maximum of these two. So, here please understand that we have two pairs of the generic variable values from fuzzy set A and fuzzy set B and these two pairs are for $\delta = 0$. So, when we have these 0.4 and 0.3 0.4 and 0.3 as the min of these two we take the max of these two and we find 0.4. So, by now we have understood as to how we can compute the delta and this way we can go ahead and we can compute delta is equal to $1, 2, 3$ if these deltas exist.

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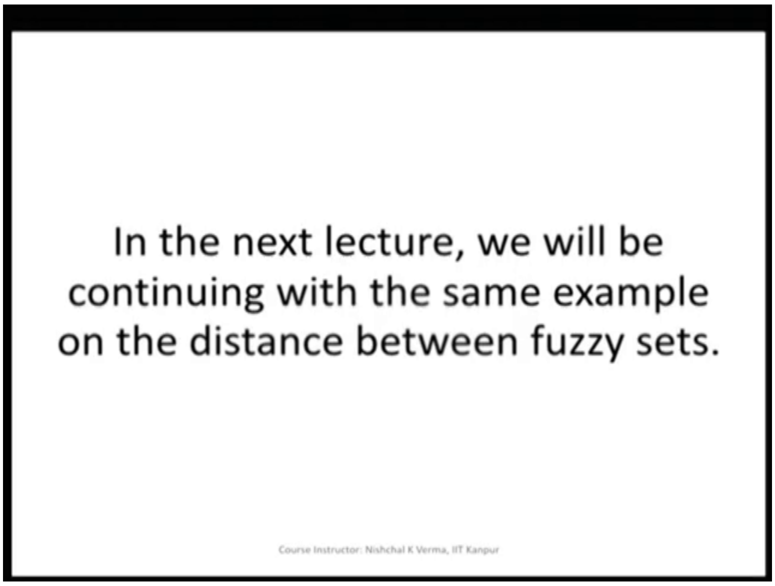

So, since the time is up. And, I will continue the rest of the discussion with respect to this example in the next class.

Thank you very much.