Fuzzy Sets, Logic and Systems and Applications
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Lecture – 25
S-norm Operators

So, welcome to lecture number 25 of Fuzzy Sets, Logic and Systems and Applications. In this lecture we will study the S-norm Operators this also known as T-conorm operators.

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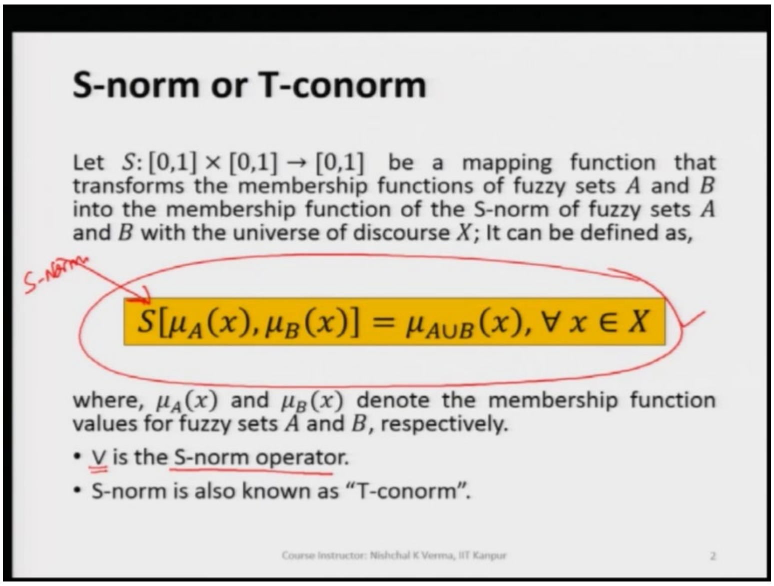

So, S-norm or T-conorm operators are nothing but they are the mapping function that transforms the membership function of fuzzy sets A and B into the membership function of the S-norm of fuzzy set A and B with the universe of discourse capital X. Can be defined as you see here the formula,

$$S[\mu_A(x), \mu_B(x)] = \mu_{A \cup B}(x), \forall x \in X$$

so S is just the operator and which is called the S-norm it is also called as T-conorm as I just mentioned.

So, the S of the two membership values $S[\mu_A(x), \mu_B(x)] = \mu_{A \cup B}(x)$. So here when we are taking the S-norm of two membership values that are coming from the two fuzzy sets A and B this is returning us the membership value which is termed as the $\mu_{A \cup B}(x)$. Why?

Because this we are getting by taking the union of the two fuzzy sets as far as the membership values are concerned.

So, here let us also understand that as in case of T-norm we have seen we were using open triangle symbol. So, here in for S-norm we will use this triangle inverted open triangle which is here. So, either S or inverted open triangle is used for the S-norm operator. S-norm is also known as T-conorm as we have already discussed.

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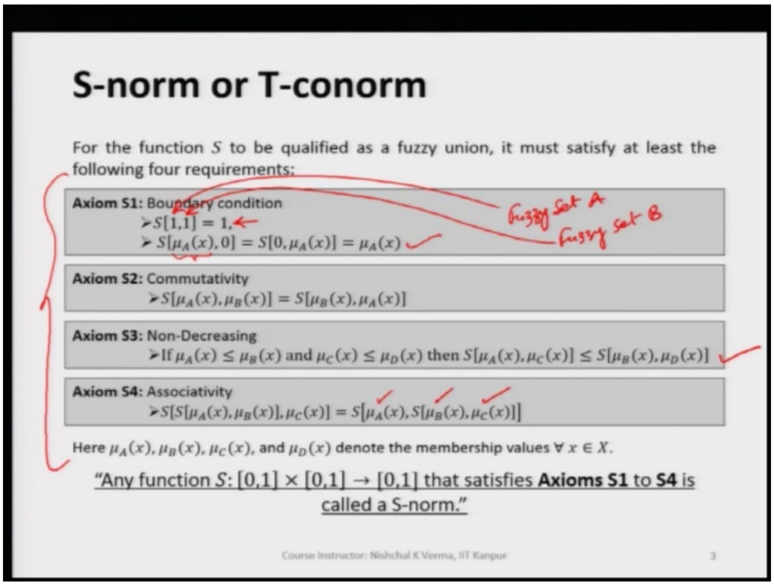

Now, on the same lines as we have discussed the T-norms in last couple of lectures. So, we have here 4 axioms. So, any operator if it satisfies the all 4 axioms S1, S2, S3, S4, this will qualify to be called as the S-norm or T-conorm. So, for any function S to be qualified as a fuzzy union I am saying here fuzzy union because S-norm normally is giving us the fuzzy union for its basic norms. Like we have seen in the basic T-norm is the intersection.

So, here the basic S-norm is the union. So, here we can say there is a function S to be qualified as a fuzzy union it must satisfy at least the following four requirements these are the four requirements and these requirements are the axioms, axioms S1, S2, S3, S4.

So, let me just quickly go through all these 4 axioms one by one. First one is the boundary condition. So, here we have 1, 1. So, these 1,1 are the highest values of the membership function of particular fuzzy set or I would say membership values of the fuzzy set. So, let this be the membership value of a fuzzy set A and this is another fuzzy set let us say B. So, both the membership values are coming from two different fuzzy sets.

So, if we take the S-norm of the these two fuzzy sets that is going to give us 1 as a result. So, please understand that this m S-norm is applied on membership values. So, here S of 1 and 1 is going to give us 1 and then we have here another condition the $S[\mu_A(x), 0]$ means if we apply the S-norms on two values two membership values one membership value is $\mu_A(x)$ and the other one is 0.

So, if this kind of condition happens like any membership value like $\mu_A(x)$ you have and it is with 0 lowest possible membership value. So, if this is with 0 what is going to be returned here is the $\mu_A(x)$. So, it means that if we have any membership value which is with 0, if we take the S-norm of these two values we are going to get the value which is with 0.

We are not getting 0 we are getting $\mu_A(x)$ which is here this can also be written as S of or S-norm of 0 comma $\mu_A(x)$. And then we have the commutatively property if we take the $S[\mu_A(x), \mu_B(x)] = S[\mu_B(x), \mu_A(x)]$. So, this is the second axiom S2.

Now the third one is non decreasing, if we have $\mu_A(x) \leq \mu_B(x)$ and $\mu_C(x) \leq \mu_D(x)$ then S-norm of $\mu_A(x)$ and $\mu_C(x)$ will be less than or equal to the S-norm of $\mu_B(x)$ and $\mu_D(x)$. So, this condition is the non decreasing condition.

Now the 4th axiom S4 is the associativity condition. So, here we have the$S[S[\mu_A(x), \mu_B(x)]\mu_C(x)] = S[\mu_A(x), S[\mu_B(x), \mu_C(x)]]$. So, this needs to be clearly understood for three different membership values and these are coming from three different fuzzy sets A B and C. And this is needless to say here that the generic variable here is coming from the universe of discourse. So, that way now we can say any function S if it satisfies all 4 axioms will be called as S-norm.

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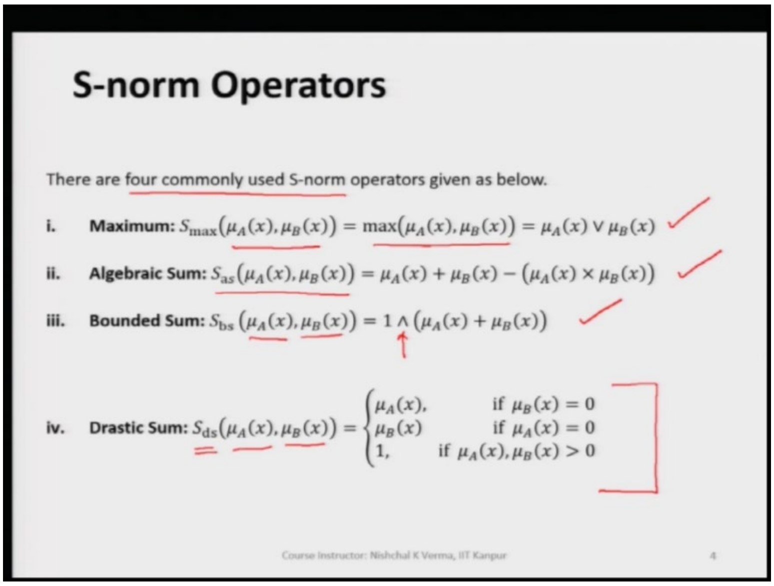

So, let us now see as to how many commonly S-norm operators exist. So, we have four commonly used S-norm operators and these are maximum, algebraic sum, bounded sum, drastic sum. So, $S - norm$ or I would say the S-norm as maximum is represented by S_{max}. So, S_{max} of the two membership values $S_{max}(\mu_A(x), \mu_B(x)) = \max(\mu_A(x), \mu_B(x)) = \mu_A(x) \vee \mu_B(x)$.

So, we simply take the maximum of these two and get the S_{max}. So, this way we get the S-norm of these two membership values. And here in this case we can also use the inverted triangle as the symbol this is $\vee$ symbol. So, which is used here you can see. So, either we write $\max(\mu_A(x), \mu_B(x))$ or we write $\mu_A(x) \vee \mu_B(x)$.

So, this is how the maximum S-norm is computed. This is also called the basic S-norm. So, now in the next you see algebraic sum defined by $S_{as}(\mu_A(x), \mu_B(x)) = \mu_A(x) + \mu_B(x) - (\mu_A(x) \times \mu_B(x))$. So, this way we can compute the algebraic sum and the representation here the symbol here is S_{as}.

S is capital as is both are is small and these are in subscript of S. Then the third one comes as the bounded sum S operator is presented by capital S, small b, small s as subscript. We operate this on two membership values $S_{bs}(\mu_A(x), \mu_B(x)) = 1 \wedge (\mu_A(x) + \mu_B(x))$.

The fourth one and the last one here is the drastic sum as the S-norm, this is represented by capital S and small d small s as the subscript. So, here it is S_{ds}.

$$S_{ds}(\mu_A(x), \mu_B(x)) = \begin{cases} \mu_A(x) & if\ \mu_B(x) = 0 \\ \mu_B(x) & if\ \mu_A(x) = 0 \\ 1 & if\ \mu_A(x), \mu_B(x) > 0 \end{cases}$$

So, this way we can compute the drastic sum. So, we see here that we have these four S-norm operators and depending upon the need we use these criterias to find the four S-norm operators minimum, algebraic sum, bounded sum, drastic sum. And as I mentioned the first one is the maximum which is the basic S-norm operator when nothing is said we normally use the maximum S-norm operators we take the max of the two values.

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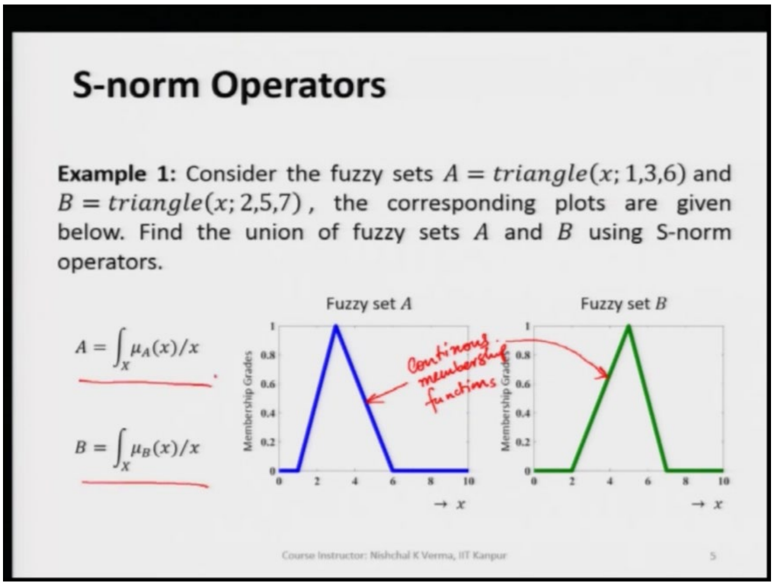

So, let us take a simple example here and let us find the S-norm of these membership values which are coming from the fuzzy set A and B. So, since these membership values are coming from the continuous membership functions of the respective fuzzy sets A and B.

So, here this fuzzy set A and membership function here is a continuous membership function this is continuous I can write here this is a continuous membership function. So, we can find any membership value with respect to its generic variable value for both the cases here. So, these are the two membership functions that is coming out from a fuzzy set A and fuzzy set B and there is needless to say that both the fuzzy sets here are continuous fuzzy sets and these are defined by this two expressions.

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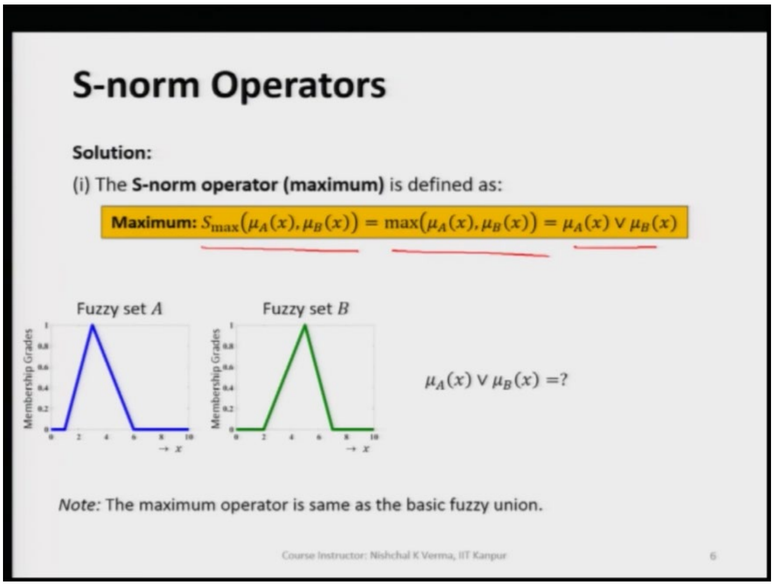

So, let us now find the four S-norms of the respective membership values from the continuous fuzzy sets A and B. So, let us first find the S-norm of these two fuzzy set. So, as per the definition S max is nothing, but S max of any two membership value is going to give us the max of these two membership values. So, let us see what is happening here in this case. So, since we have here the continuous membership functions. So, you can get any membership value for the corresponding generic variable value.

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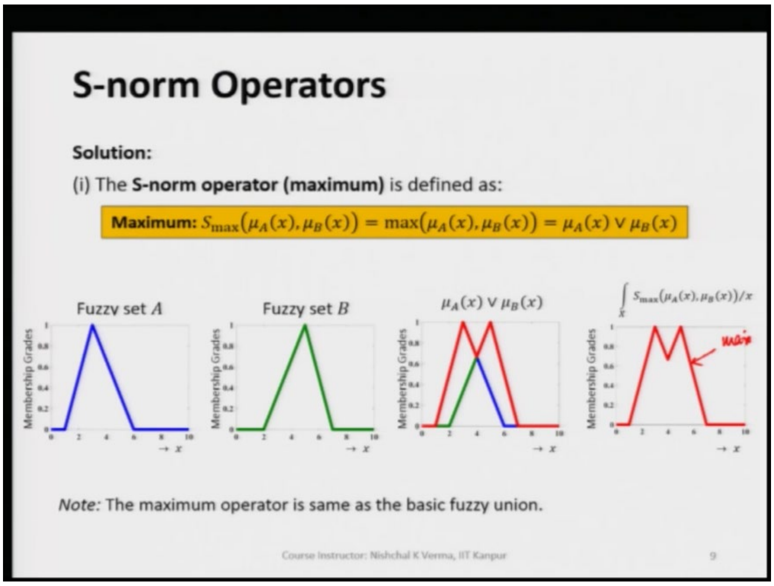

So, here let us now super impose these two fuzzy sets because we are taking the maximum of the two membership values corresponding to the generic variable value throughout and within the universe of discourse. Here we are interested in max because we are computing the max S-norm operator.

So, here we will take the maximum of the corresponding membership values within the universe of capital X for the generic variable. So, when we are interested in max we super impose these two membership functions. So, let us see what we are getting. So, when we take the max of the two membership values throughout we are getting the outer envelope here. So, this is represented by the red color.

So, when we take the max the S_{max} is coming out to be like this. So, this is the max of the two. So, since we have multiple membership values these two fuzzy sets are continuous fuzzy sets. So, we have the continuous membership function as the outcome. So, this way we see that we have computed the S_{max}.

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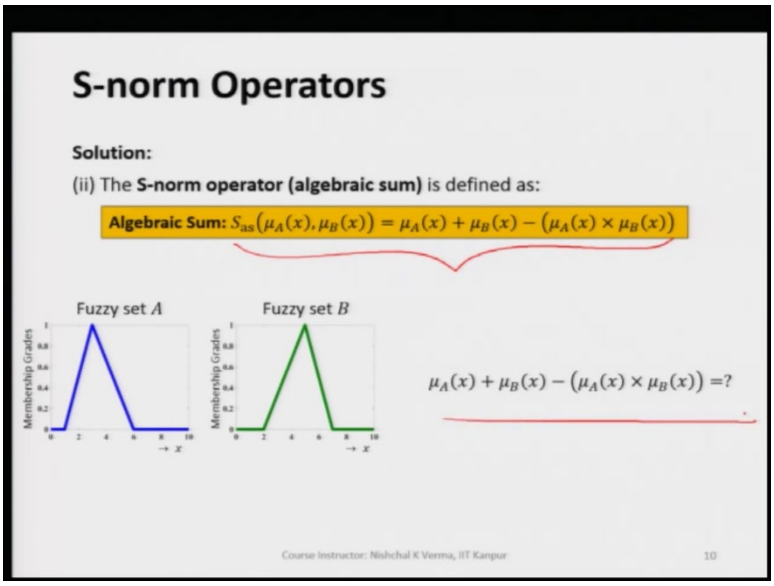

Now, let us look at the S_{as}. So, when we apply this $S_{as}(\mu_A(x), \mu_B(x)) = \mu_A(x) + \mu_B(x) - (\mu_A(x) \times \mu_B(x))$ for the algebraic sum. So, what we are doing here is when we are interested in finding the algebraic sum of the membership values $\mu_A(x)$ and $\mu_B(x)$. So, we first add them together and then subtract the multiplication of these two. So, here when we applied this formula for each and every pair of the membership values let us see what we are getting.

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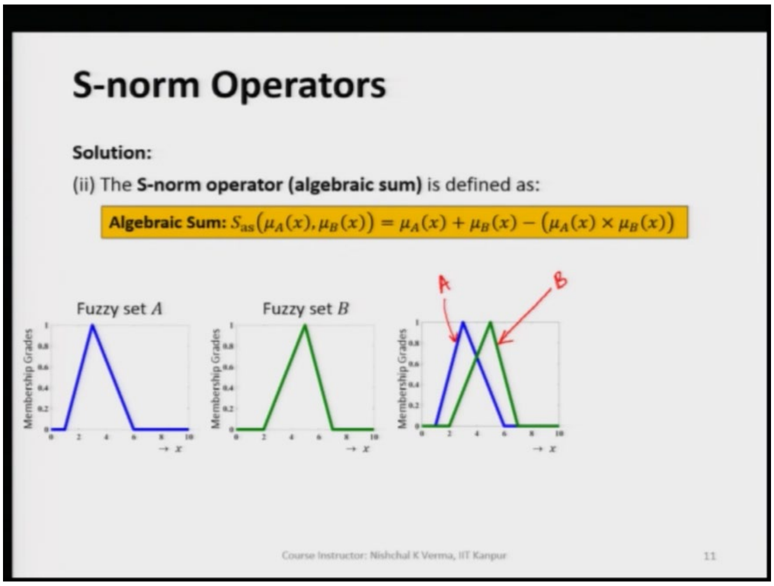

So, when we do that we have you see here this is the first fuzzy set A, this the second fuzzy set B and this is also characterized by corresponding membership functions that we can call as $\mu_A(x)$ and $\mu_B(x)$.

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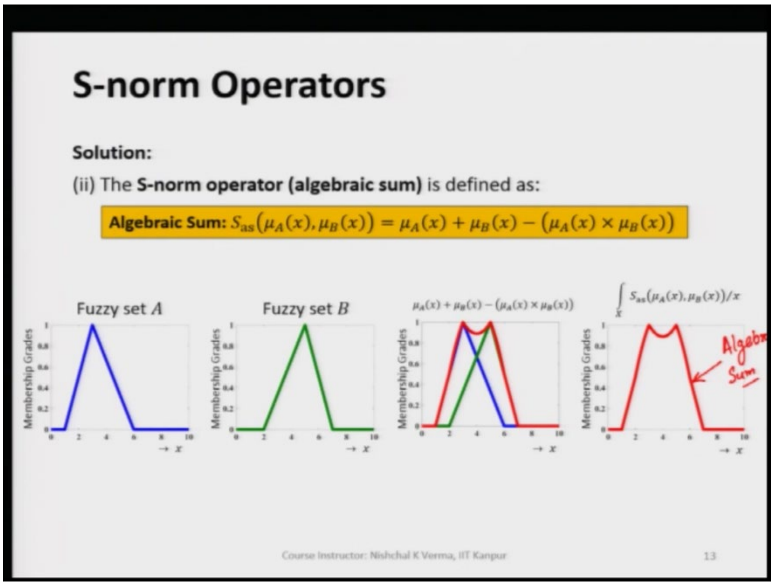

So, when we apply this formula here means for all the pairs of the membership values corresponding to the generic variable values when we take the S_{as} apply this formula we are going to get the membership function as a result which is represented by the red color, you can see here. So, this is algebraic sum this is the outcome of the algebraic sum. So, here we see that the algebraic sum is different from the max S-norm operator.

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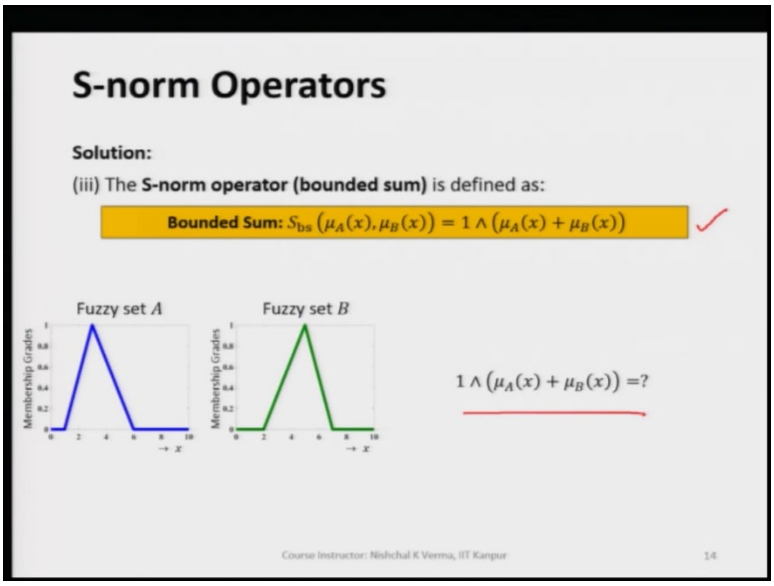

Now, the $S_{bs}$when we are taking the bounded sum of the corresponding membership values what we are getting here after applying this formula $S_{bs}(\mu_A(x), \mu_B(x)) = 1 \wedge (\mu_A(x) + \mu_B(x))$, you see here this is nothing but the you have to take the minimum of 1 and the sum of the two membership values $\mu_A(x)$ and $\mu_B(x)$.

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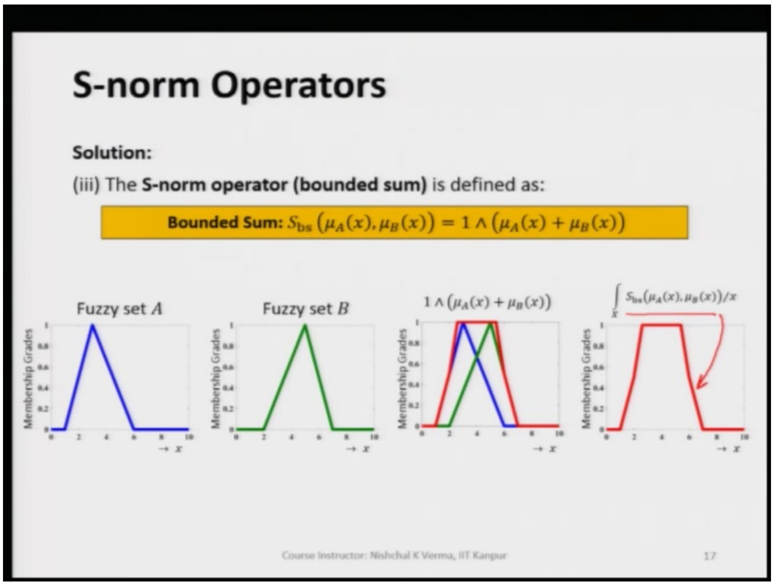

So, when we compute this for each and every pair of membership values corresponding to its generic variable values for fuzzy set A and fuzzy set B we are going to get this thing this is plotted by the red color this is the outcome of the membership values. So, since the fuzzy set that we have taken is continuous fuzzy sets. So, the corresponding membership values are also coming from the respective membership functions $\mu_A(x)$ $\mu_B(x)$.

So, when we are finding the bounded sum, as the S-norm the result is also going to be returned as the continuous membership function and this is computed by taking the bounded sum as S norm. You can see here very clearly and this is how it can be represented. So, this is actually nothing, but the fuzzy set as a whole and this is characterized by the membership function which has been shown by the red color.

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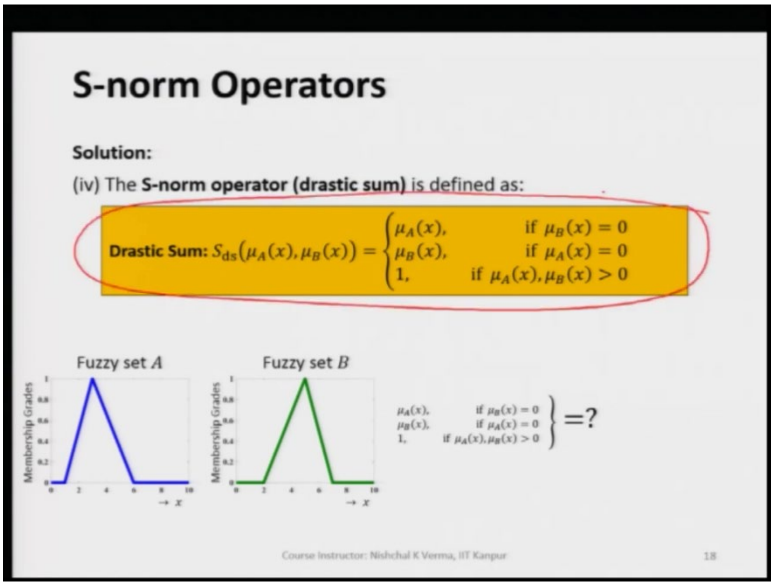

Now, we have the drastic sum, now when we apply the S_{ds}let us see what we are getting here.

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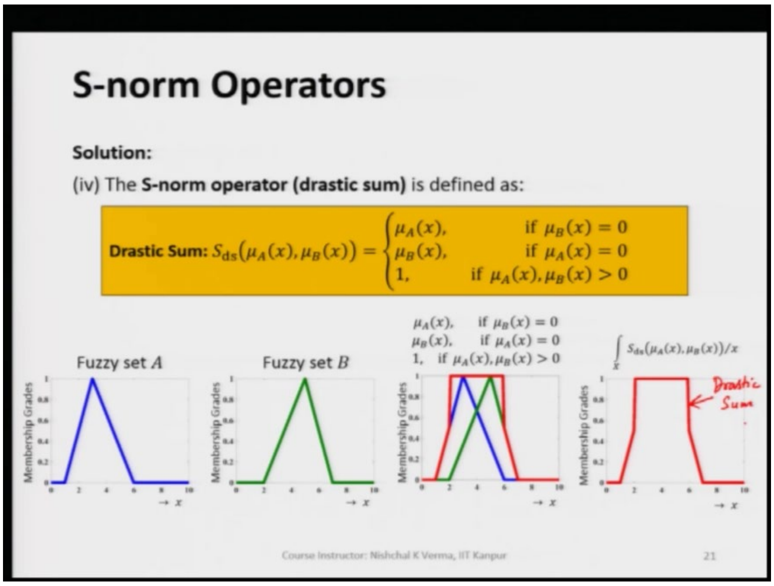

When we apply this formula we see it is returning us $\mu_A(x)$ when we have $\mu_B(x) = 0$. So, similarly when we apply this criteria $\mu_B(x)$ when we have $\mu_A(x) = 0$. So, this way we are getting this portion and we get 1 when we have $\mu_A(x)$ and $\mu_B(x) > 0$. So, we see here this portion, this portion we are getting for the intermediate values. So, let us now make it little better and see that we are getting here this as the outcome of the drastic sum of two membership functions.

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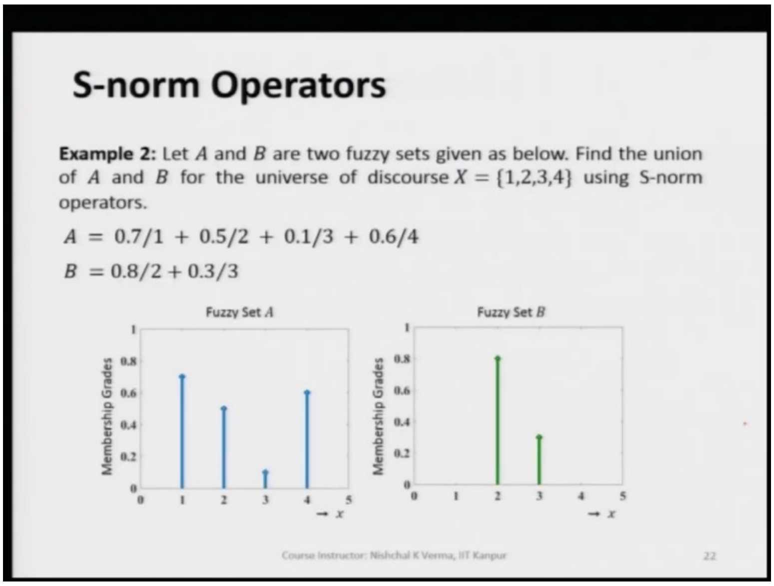

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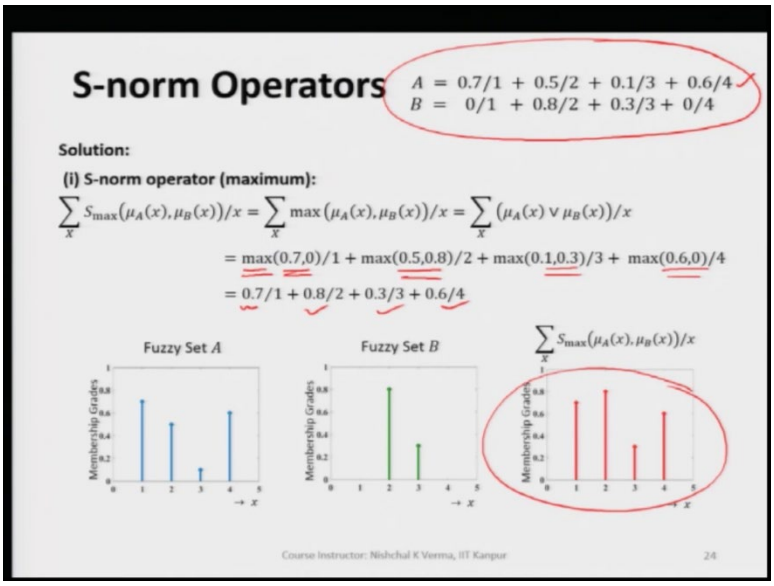

So, this way we have seen how the different S operators are giving us different results when we apply the different S-norm. So, first S-norm that we have discussed was here as the maximum, you can see here I am quickly going back here S-norm and then algebraic sum on the same set of fuzzy sets, and then you see here the bounded sum here, and this is what is being returned as the outcome when we apply bounded sum and then we have the drastic sum.

So, see how the outputs are different from each other. Now let us take one more example here earlier we took two continuous fuzzy sets A and B. Now, let us take two discrete fuzzy sets A and B and see what is the result what is the outcome has we changed the different kinds of S-norm.

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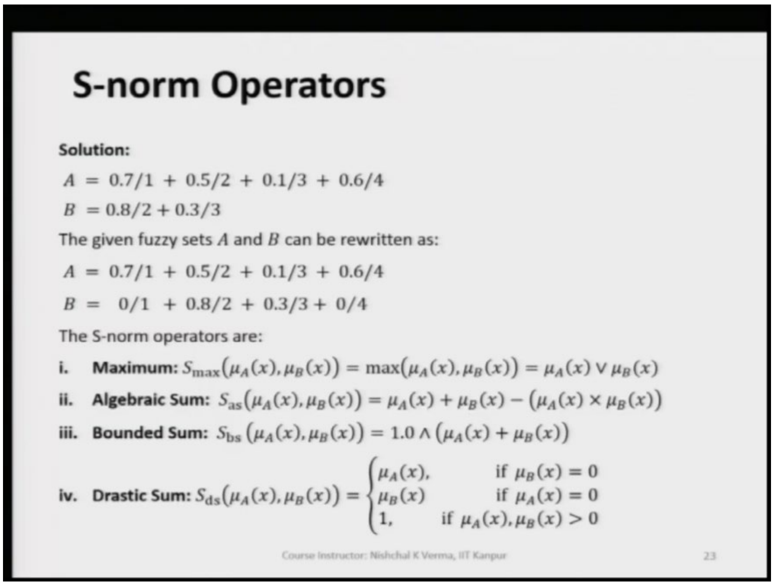

So, when we apply the maximum S-norm we see that we have here for the same example here we have the fuzzy set the discreet fuzzy set A and B like this, the A is $0.7/1 + 0.5/2 + 0.1/3 + 0.6/4$ here. And the B is another discrete fuzzy set which is 0 by 1 plus normally we do not write 0, 0 is written here just for the computation purpose, but otherwise when we write the fuzzy set discrete fuzzy set B here no element will be written for 0 membership value.

So, this will not be included, but here for competition purposes we have written. So, B is equal to $0/1 + 0.8/2 + 0.3/3 + 0/4$. So, if we take these examples here. So we see that when we are taking the maximum. So, here you see the max of all the pairs corresponding pairs are taken you will see here for all the elements. So, since we have 4 elements here in first discrete fuzzy set A and then second fuzzy set that is B which is also discreet fuzzy set it has only two non-zero membership value elements.

So, two more elements are added with the 0 membership value just for the computation purpose here because we had to take the max of the two membership values corresponding to the same generic variable value. So, for one when we take the max the membership value the max membership value is coming out to be 0.7 and then for 2 we are getting 0.8 for 3 we are getting 0.3 for 4 we are getting 0.6.

So, this way we are getting four elements of the resulting fuzzy set or fuzzy membership function. So, this is you see here are the discreet fuzzy set or discreet membership function when we take the max S-norm of the two fuzzy set we are getting the outcome like this. You can see here that we are getting the maximum of each pairs of the corresponding generic variable value.

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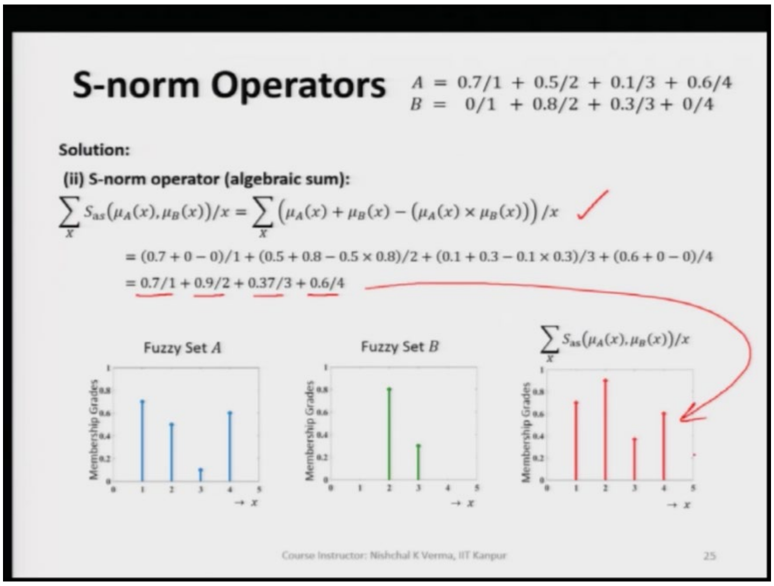

So, now let us use the same example here and take the S-norm algebraic sum. So, when we take algebraic sum when we apply the formula here we see that we are getting 4 elements after using this formula. And we can plot here we can just represent here all these four membership values for the corresponding pairs of the membership values that we have taken and again this is corresponding to the same generic variable values. So, we see that the algebraic sum S-norm when we do we are getting something different than what we have gotten for the max as the S-norm operator.

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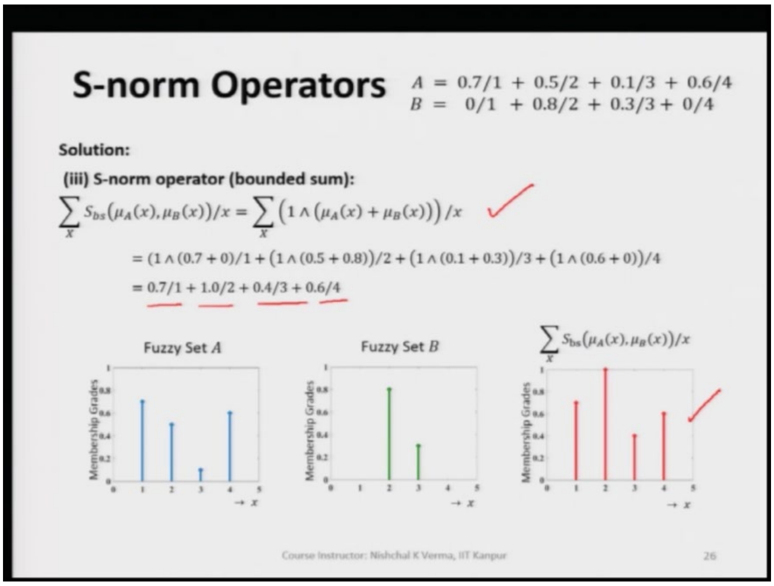

Now, let us go to the third one that is bounded sum and the bounded sum S-norm when we apply this formula which we have already discussed when we apply this we are again going to get 4 elements of the resulting fuzzy set. So, when we plot these membership values we see that we have the result again this is different from the previous one that is you see here the algebraic sum. So, all these are very similar, but little different.

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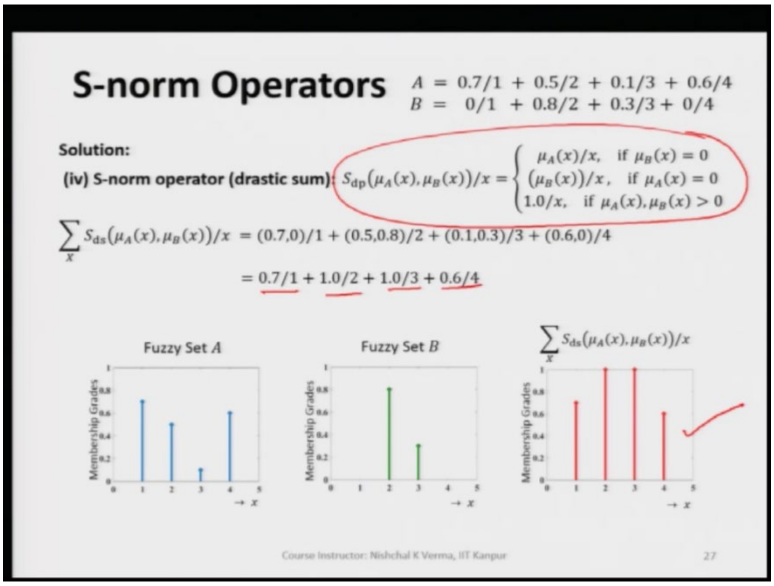

Now, the fourth one is the drastic sum S-norm when we apply this formula which we have already discussed we are getting 4 elements out of this drastic sum of discrete fuzzy set A and discrete fuzzy set B. So, we see that all these four membership values of the resulting fuzzy set is plotted here as you shown in red color. So, this way we can see that how the different S-norms are giving us the different results for the same fuzzy sets. By now we have understood that we have S-norm operators or T-conorm operators as the max first.

So, first one is a max I would say the S_{max} and then we haveS_{as}, and then we have the S_{bs}, and then lastly here the S_{ds}.

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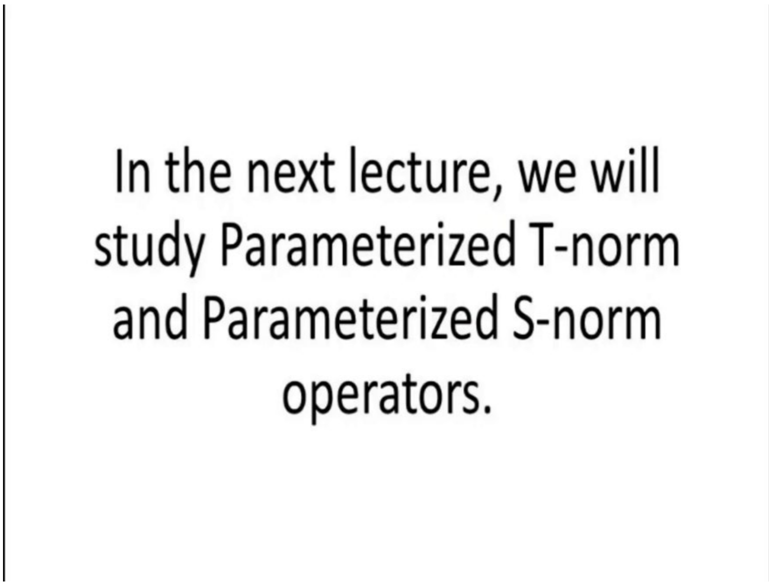

So, in today's lecture we have seen S-norm operators, in the next lecture we will study parameterized T-norm operators and parameterized S-norm operators.

Thank you.