Fuzzy Sets, Logic and Systems and Applications

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Lecture – 42
Linguistic Hedges and Negation/ Complement and Connectives

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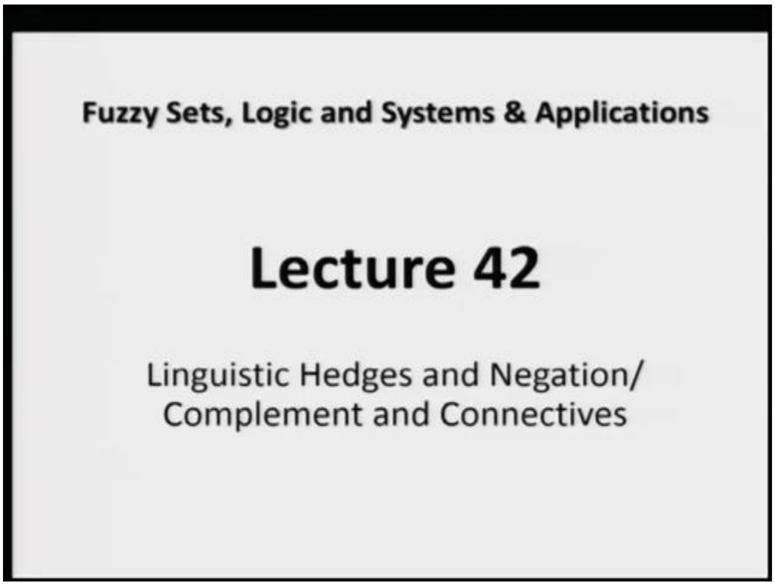

Welcome to lecture number 42 of Fuzzy Sets, Logic and Systems and Applications. In this lecture we will discuss some examples on Linguistic Hedges and then we will discuss the Negation, Complement and Connectives.

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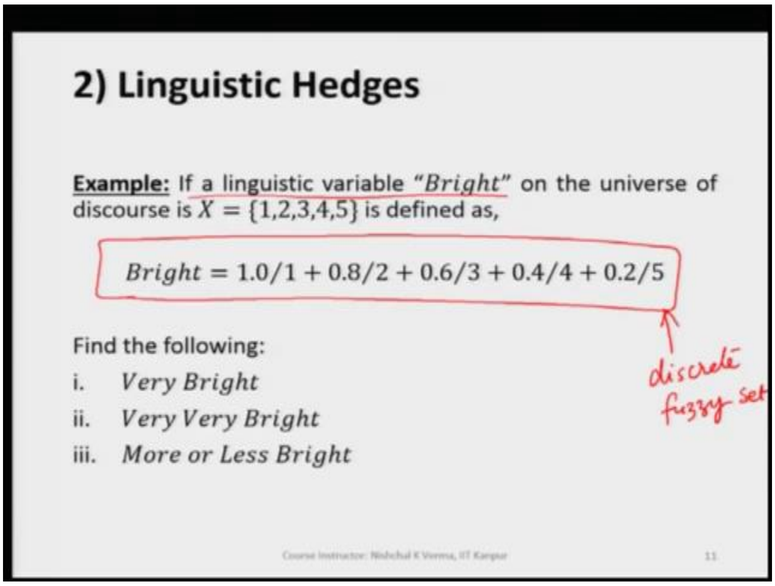

So, let us take an example here to understand this better. So, if we have a linguistic variable bright. So, we have a bright and as I already mentioned that this bright is termed as a primary term, this bright basically is a fuzzy value which is represented by a fuzzy set.

So, if we have bright, simply bright here and bright here is a discrete fuzzy set, this is a discrete fuzzy set. So, just to make you understand we have taken this example. So, let us now convert this fuzzy set here which is characterized by a membership values along with the corresponding generic variable values 1, 2, 3, 4, 5. So, let us now using the given $bright$ a discrete fuzzy set let us now find $very\ bright$ and then $very\ very\ bright$ and then $more\ or\ less\ bright$.

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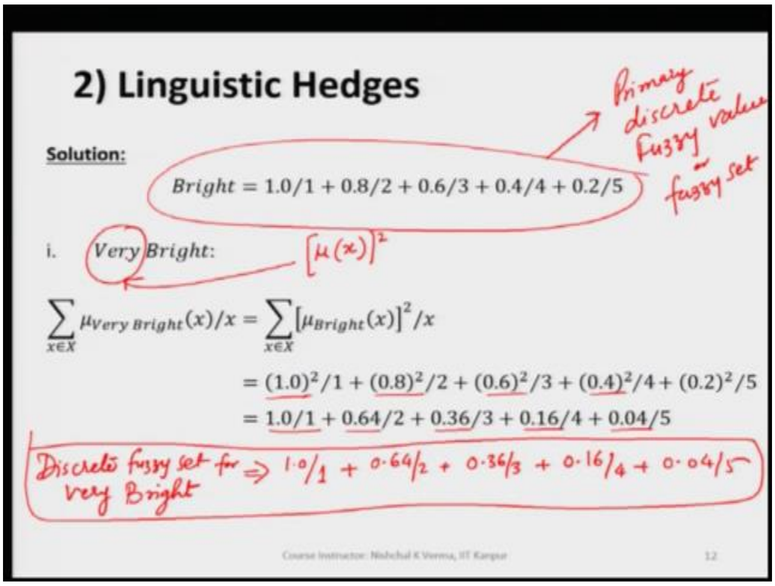

So, since we have been given here the primary discrete set we can write primary discrete fuzzy value or fuzzy set all these names can be used interchangeably. So, a $bright$ fuzzy set has been given, now let us convert this fuzzy set into $very\ bright$. So, when let us say we are supposed to use $very\ bright$ word in order to communicate something.

So, how can we make use of this $bright$ fuzzy set and we can generate a new fuzzy set using the fuzzy set $bright$ for $very\ bright$. So, $very\ bright$ as I have already mentioned $very$ when $very$ word comes here this means we have to raise the power here by 2 on the membership function and if it is a discrete fuzzy set it is all the membership values are squared. So, $very\ bright$ is very simple to get here.

So, since we already have $1/1 + 0.8/2 + 0.6/3 + 0.4/4 + \ 0.2/5$. So, here what we have to do for $very\ bright$ is because we are only adding the adjective $very$ here on the $bright$, $bright$ is already given. So, this $very$ term has to simply increase the power of increase the membership value, that means the we have to square the values of the membership. And let us see what we are getting.

So, here we see that we have a squared the values we do not have to touch we do not have to change its corresponding membership, its corresponding generic variable 1, 2, 3, 4 and 5. We do not have to touch that we do not have to change these values we only have to square the values of the membership. So, here we are squaring 1.

So, 1 will become 1 only and then 0.8 when we square it we are getting 0.64 and here is square of 0.6 will become 0.36, similarly square of 0.4 will become 0.16 and then square of 0.2 will become 0.04. So, this way we see that we are getting a new discrete fuzzy set.

So, this fuzzy set is a fuzzy set which is for $very\ bright$ I can write here the this as the discrete fuzzy set for $very\ bright$. So, what is interesting here is to note is that we had $bright$ only we were given the discrete fuzzy set for $bright$ and here we are converting the $bright$ into $very\ bright$. And as I have already discussed when we add $very$, before any linguistic value before any primary fuzzy set.

So, then this $bright$ becomes $very\ bright$ here by adding $very$ and since we are adding $very$ and we have already seen that $very$ comes with squaring of the membership values or membership function in case of continuous fuzzy set. So, in case the fuzzy set is a discrete fuzzy set then simply we have to square the respective membership values, but if it is a continuous fuzzy set, then we will have to simply square the continuous membership function that has been given for the primary set.

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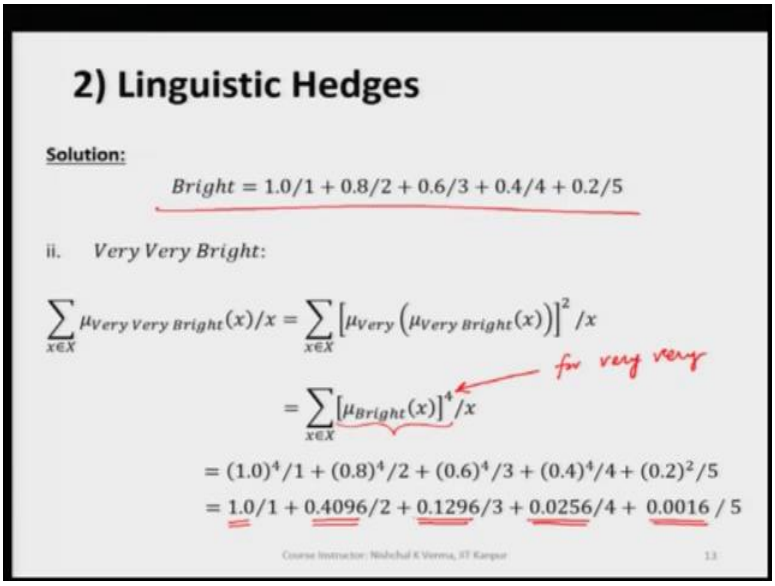

Let us now move to the second example which is for $very\ very\ bright$. So, $bright$ has been given to us now we have to convert the given $bright$ discrete fuzzy set into $very\ very\ bright$. So obviously here we have two times $very\ very$, so we have to square it two times, we have to square the respective membership values in twice. So, when we square the membership values twice this the respective membership values basically becomes the original membership values raised to the power 4 which you can see here, this is for $very\ very$.

So, when we use the $bright$ and we want to have $very\ very\ bright$ we this way by taking the powers increased by 4, the powers of the respective membership values by 4 we are getting its membership values like this. And please note that here no change will happen to its corresponding generic variable values which are 1, 2, 3, 4, and 5. So, we do not have to change these values only the change will happen to the corresponding membership values in case of the discrete fuzzy set simply we take the membership value and we use the membership value raised to the power 4 and then whatever value comes we will write. But if it is a continuous fuzzy set, then the membership function will become the twice square it means we will write the $\mu(x)^4$.

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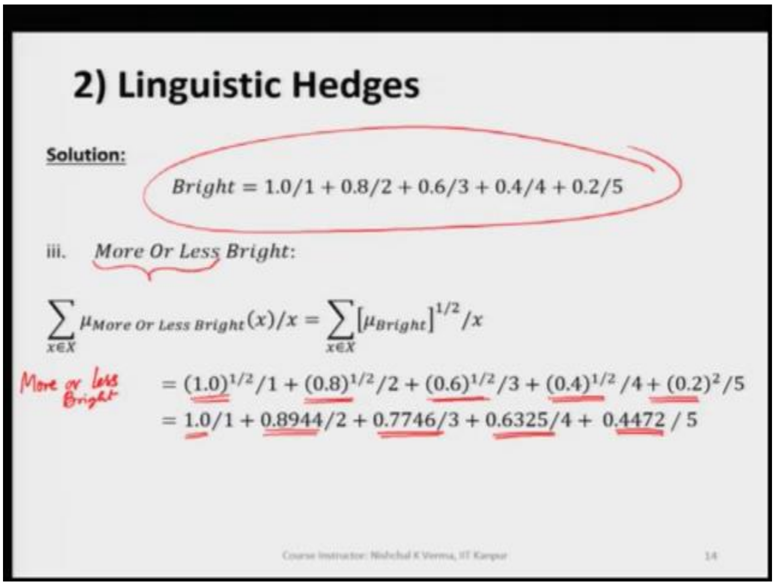

Now, let us quickly go to the third part of the example here and here the bright is given to us and we have to find the $more\ or\ less\ bright$. So, $more\ or\ less$ is a hedge as I have already mentioned. So, like we had $very$ and then $very, very, very$ and then extremely like that. Now, let us take $more\ or\ less$ as hedge and let us see what happens with $more\ or\ less$. $More\ or\ less$ basically we get when we change its membership value we dilate its membership value in other words I would say.

So, $more\ or\ less$ means we rather than squaring or raising the power we are decreasing the power here. So, decreasing the power means we are taking the square root of the original membership value or membership function. So, in case of the continuous fuzzy set we simply take the $\mu(x)^{1/2}$, whereas if it is a discrete fuzzy set we simply take the square root of all the respective membership values and here also we will not touch any of the generic variable values.

So, so all the generic variable values will remain unchanged. So, let us find $more\ or\ less\ bright$. So, $more\ or\ less\ bright$ is here $more\ or\ less$, $more\ or\ less\ bright$ fuzzy set is here. So, what we are doing here is we had 1. So, we are taking the square root of 1 and then we are taking the square root of 0.8, we are taking the square root of 0.6, we are taking the square root of 0.4, we are taking the square root of 0.2. And this way we are getting when we are taking square root of 1 we are getting 1, we are getting here in this case when we are taking 0.8 and then we are taking square root of 0.8 we are getting 0.8944.

So, similarly we are getting all these values when we are taking the square root and this way we are forming a new set and here we have formed a linguistic hedge. So, hedges are coming out of the modifications of the original or the primary fuzzy sets by adding the adjectives or adverbs. So, this way the bright fuzzy set is converted into the $more\ or\ less\ bright$. So, when we say $more\ or\ less\ bright$ it means we simply take the square roots of all the respective membership values which are characterizing the fuzzy set, this is discrete fuzzy set.

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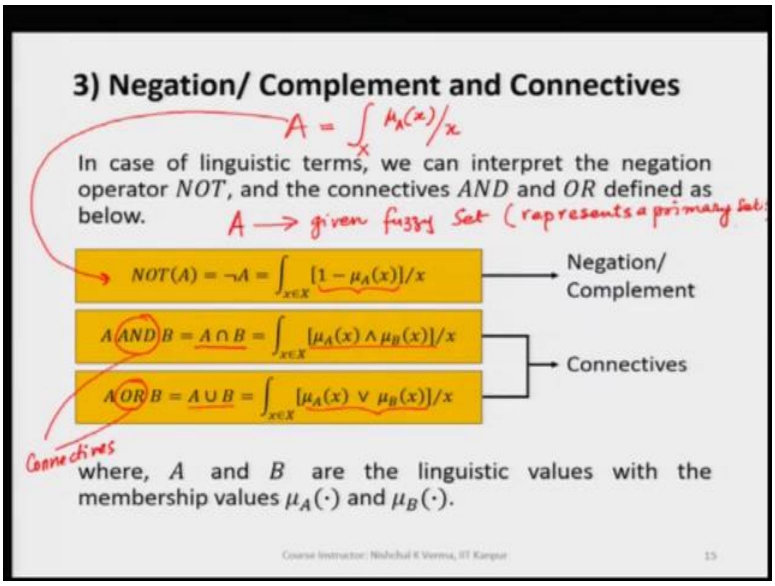

So, that way we have understood as to how we managed to get the, a primary fuzzy set converted into the linguistic hedges. So, now let us understand here move to the 3rd class and 3rd class here is the negation, complement and connectives. So, when have been given a primary fuzzy set, a primary term all these names can be interchangeably used and when we try to find a negation of it let's say we have a primary set say $middle\ aged$ and then we say $not\ middle\ ages$, then how to get the $not\ middle\ ages$ fuzzy set out of the given fuzzy set $middle\ ages$.

So, here if I have been given any fuzzy set A, let us say A, the given fuzzy set, given fuzzy set and this fuzzy set represents; this represents basically a primary set which is part of the term set. So, if we are interested in finding $NOT(A)$, means as I mentioned if A is a $middle\ aged$, then if you are interested in $NOT(middle\ aged)$ then we simply take the NOT of it and NOT of it is represented by this sign here if NOT is represented by this sign.

So, NOT is here and $NOT(A)$ can be symbolically written like this and what is done here to get $NOT(A)$ we have already learned this when we have studied, when we have discussed in the one of the previous lectures, the negation the complement. So, what we do here is we take we subtract the corresponding membership values from 1 which you can see here.

So, simply when we have been given a fuzzy set let us say if I write the given fuzzy set A like this, if it is a continuous fuzzy set we will be representing it like this the integration sign and then the universe of discourse just below it and then $\mu(x)$ and then x here. And this is here what is done for $NOT(A)$ is simply we take the complement of it and when we take compliment of it the membership function is subtracted from 1 which you can see here rest other things remain the same.

So, this way we can get NOT is as the negation or the complement and here in other case we can have the connectives like $A\ and\ B$ when we have let us say a fuzzy set A and fuzzy set B both the fuzzy set have been given to you, then how to connect both the fuzzy sets together? For example, I can say a $middle\ aged$ and $young$. So, we have two fuzzy sets and both the primary terms are getting connected by AND.

So, AND is here a connective. So, the connective can be either AND, OR, so these two are the connectives. So, here there could be $A\ AND\ B$ or $A\ OR\ B$. So, like $middle\ aged$ and $young$ or $middle\ aged$ or $young$. So, whatever way any two fuzzy sets can be connected. So, this and can also be replaced by but, so if we use, but or and both are same.

So, let us first take $A\ AND\ B$. So, when we take $A\ AND\ B$, so this means what? When we have already done this exercise in one of the lectures previous lectures. So, when we have two fuzzy sets let us say and they are being connected by AND connective. So, we simply use intersection. So, $A\ AND\ B$ will become A intersection B means both the fuzzy sets are being intersected means we have we will have to take the intersection of the primary term set A and the primary term set B.

So, this way when we do what is happening with the corresponding membership function is this we take the min of $\min[\mu_A(x), \mu_B(x)]$. Similarly, when we connect $A\ AND\ B$ by and we take the union of A and B and similarly we use the max sign in place of the min sign. So, here we connect both the membership function, functions of $A\ AND\ B$ like this like $\mu_A(x)$ and then we take max $\mu_B(x)$, means we take $\max[\mu_A(x), \mu_B(x)]$ which you can see here.

And please note again that the generic variable values x will remain the same, will remain unchanged. Here these connectives are very interesting and even negation also. So, negation and connectives both are basic connectives shown here, but since we have already done in the previous lectures that we have multiple kinds of negations, multiple kinds of connectives. So, like we could use for connectives various kinds of t-norms and s-norms.

So, as an when it is required we can use that also, but here if nothing is mentioned then we simply use the basic s-norm and t-norms. So, s-norm is used for OR and t-norm is used for AND. So, this way the negation complement and connectives can be managed and this is quite interesting to note here that any primary fuzzy set any primary linguistic value, when we say linguistic value linguistic value is nothing but a fuzzy set, linguistic value is represented by a primary fuzzy set.

So, any primary term set, any primary fuzzy set, any linguistic value can be converted into its hedges or in other fuzzy sets with the negation or you know with connectives you can manage to get a new fuzzy set.

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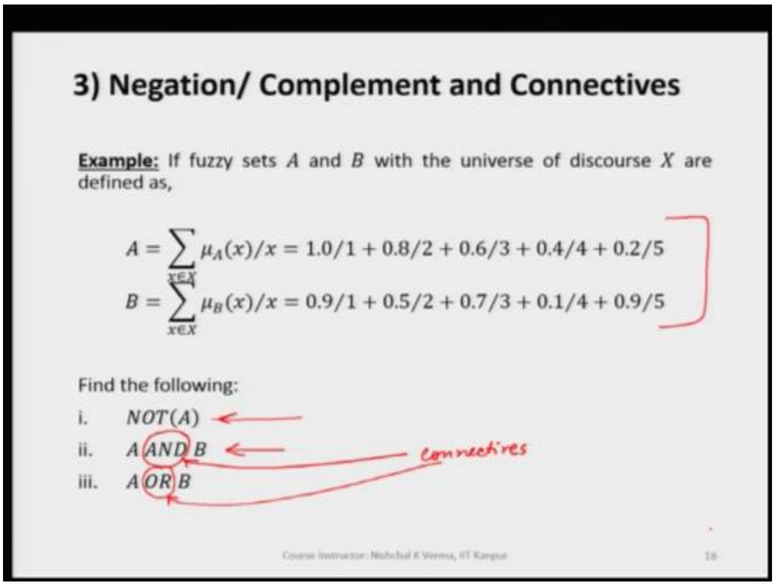

So, let us take an example here to understand the negation complement and connectives. Here we have a two fuzzy sets both the fuzzy sets are discrete fuzzy sets A and B and let us using A and B let us find $NOT\ A$ and then let us find $A\ AND\ B$. Here this is a connective. And in the third case also OR is connective, so AND, OR both are connectives. So, let us now go one by one and try to find $NOT\ A$ first.

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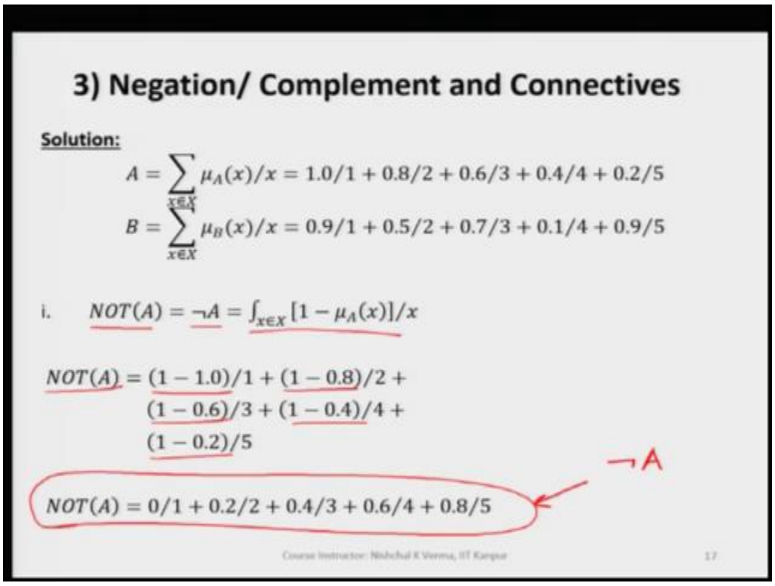

So, A has already been given. So, we represent $NOT(A)$ by a negation A and as I have already mentioned that simply when we have μ_A the membership function given. And then when it comes to negation of it we subtract the membership values from 1 or if it is a membership function we subtract this also from 1.

So, when we do that we get $NOT(A)$ like this means here we are negating all these corresponding membership values and this comes out to be this. So, $NOT(A)$ is this, this is represented by $NOT(A)$. So, a new fuzzy set $NOT(A)$ given A is this. So, this way we find $NOT(A)$ very quickly very easily.

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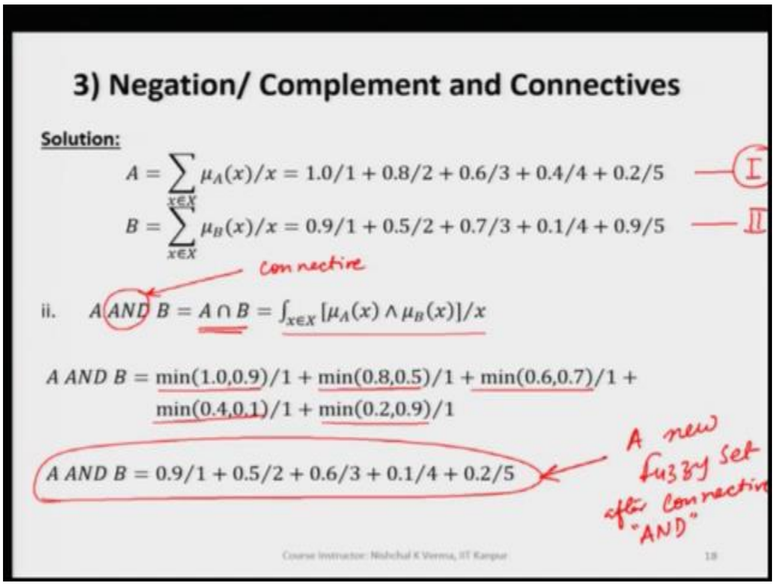

Now, let us find $A\ AND\ B$. So, $A\ AND\ B$ as I have already mentioned that AND is a connective; AND is a connective. We have been given fuzzy set A this is fuzzy set A first fuzzy set and this is second fuzzy set. So, both the fuzzy sets have been given now we have to connect these two fuzzy sets together and as I have already mentioned that when we have AND we have to take the intersection of it.

So, when we take intersection, the basic intersection uses the min. So, when we use this we find the new fuzzy set like this. So, in the new fuzzy set is with min of both the membership values. So, when we use this the new fuzzy set is coming out to be this. So, we can say this fuzzy set is a new fuzzy set; a new fuzzy set after connective AND so this is for AND, now if we use OR.

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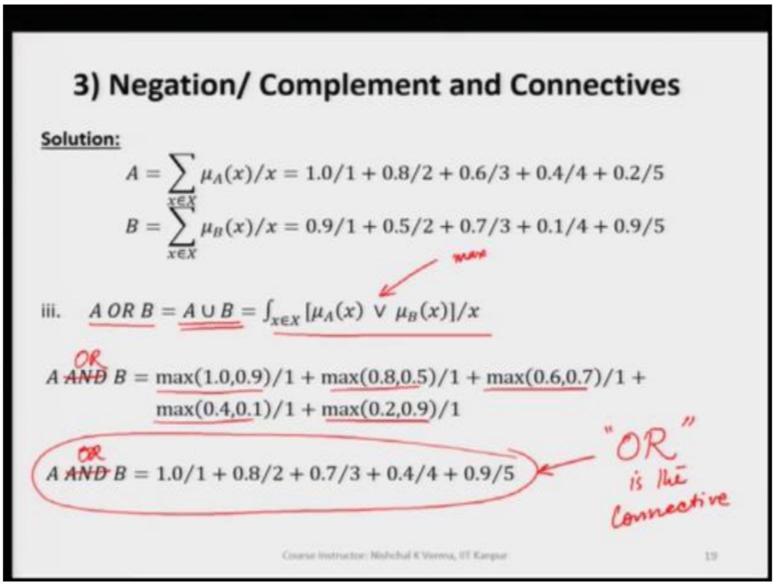

So, OR can also be very quickly managed to get we see that here when it comes to OR means when we have two fuzzy sets and when we have to make a new fuzzy set by ORing both the sets. So, $A \cup B$ gives us $A \; OR \; B$ and here we use the max sign as a basic union. So, you see here the max sign the inverted open triangle and this way we have $A \; OR \; B$ and this $A \; OR \; B = \int_{x \in X} [\mu_A(x) \vee \mu_B(x)]/x$.

So, the new fuzzy set, the new fuzzy set which is coming after connecting $A \; AND \; B$ as OR by OR we are getting a new fuzzy set here the by OR. So, OR is the connective. So, this way we see that we have been able to manage to get new fuzzy sets. So, either taking the negation of the primary set primary fuzzy set or by connecting the two primary sets or maybe even further we can connect two or more fuzzy sets by AND or OR or any other connectives and we can get the expression for the fuzzy set. So, this way we are able to manage to get the, a new fuzzy set. So, with this I would like to stop here in this lecture.

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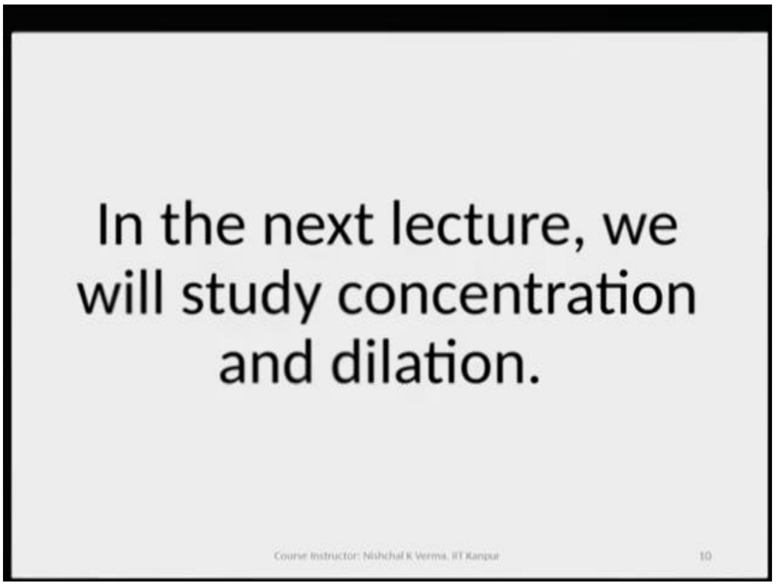

And in the next lecture we will discuss the concentration and dilation of linguistic values.

Thank you.