Fuzzy Sets, Logic and Systems and Applications
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Lecture - 49
Fuzzy Rules and Fuzzy Reasoning

Welcome to the lecture number 49 of Fuzzy Sets, Logic and Systems and Applications. In this lecture, we will discuss Fuzzy Rules and Fuzzy Reasoning.

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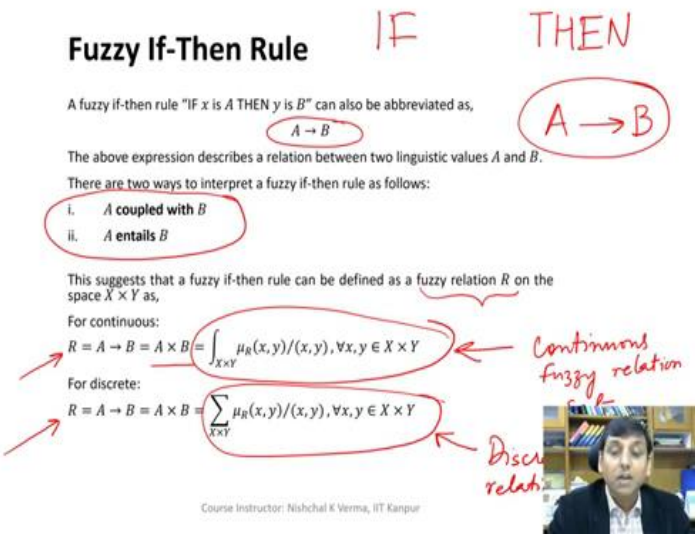

So, here when we talk of fuzzy rules, fuzzy rules are always in the form of if the linguistic variable is some linguistic value, then again the linguistic variable as the output is some linguistic value. So, the fuzzy rule has a particular syntax, the syntax is like this. Like if we have any fuzzy rule, fuzzy rule will look like this fuzzy rule will have IF part and $THEN$ part. So, any fuzzy rule will have IF and $THEN$ both the words and please understand that any fuzzy system will have a set of fuzzy rules. Without a set of fuzzy rules, there can be no fuzzy system or in other words there can never exist any fuzzy system without a set of fuzzy rules.

So, a fuzzy if-then rule like $IF\ \ x\ is\ A\ then\ y\ is\ B$ and we already know that $IF\ \ x\ is\ A$ is called as antecedent or premise and in this rule the $THEN\ \ y\ is\ B$ is called as consequence or conclusion part. So, any fuzzy rule has two parts, the first part is the premise part or the antecedent part and the second part is the conclusion part or the consequence part.

So, this fuzzy rule in this form, the fuzzy rule $IF\ \ x\ \ is\ \ A\ \ THEN\ \ y\ \ is\ \ B$ can be written as $A \rightarrow B$. So, this means we have A and then B this means that if a particular fuzzy variable or the linguistic variable belongs into a particular fuzzy reason that is the linguistic value that is represented by a fuzzy set, then what is B? What is the output? what is the linguistic value out of it? So, this how it is written. So, $A \rightarrow B$ is also written for a particular fuzzy rule.

So, there are two ways to interpret a fuzzy $IF - THEN$ rule. First is $A\ \ coupled\ \ with\ \ B$ and then the second one is $A\ \ entails\ \ B$. So, here are the two ways as I mentioned, $A\ \ coupled\ \ with\ \ B;\ \ A\ \ entails\ \ B$. So, this suggests that a fuzzy $IF - THEN$ rule can be defined as a fuzzy relation here. A fuzzy relation and we already know that how we get a fuzzy relation from fuzzy sets.

So, if we have a; if we have a set of fuzzy sets like if we have two fuzzy sets A and B and if we are interested in finding the relation fuzzy set. So, relation fuzzy set will be nothing, but the Cartesian product of these two fuzzy sets. So, and then of course, the universe of discourse of this fuzzy relation set will be the cross product of the universe of discourse of the first fuzzy set and the second fuzzy set.

So, for any continuous fuzzy sets A and B, if we are interested in the relation fuzzy set say R so A so, this R can be written as the fuzzy relation set can be written as a forward arrow, that means the $A\ \ coupled\ \ with\ \ B = A \times B$ and we can always write the same with this expression. We have already done in the previous lectures.

So, integral sign and then the universe of discourse of it will be X cross Y and then we have mu r of x, y oblique x comma y and then for every x comma y will be belonging into the universe of discourse of X capital X cross capital Y. So, this is for the continuous one for the continuous fuzzy relation set.

Similarly, if we have A and B both are discrete fuzzy sets then of course, the fuzzy relation set will be the discrete set, discrete fuzzy sets. So, here we have already done this in the previous lectures, so I am not going into this again. So, here this is the fuzzy relation set fuzzy, in other words I can say the discrete fuzzy relation set discrete fuzzy relation set R.

So, that is how we get fuzzy relation set out of a fuzzy set A and fuzzy set B by taking the cross product of it or I would say the Cartesian product of it.

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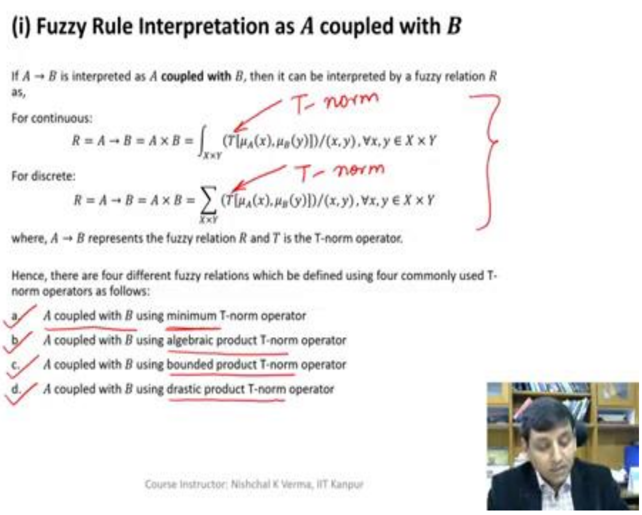

So, here let us now go one by one. So, fuzzy rule interpretation when we do with by using $A\ coupled\ with\ B$. So, when we interpret A, $A \to\ B$ arrow forward, $A \to\ B$ the interpret by $A\ coupled\ with\ B$ then it can be interpreted by a fuzzy relation R as I have mentioned here. R we know how we how did we get, see here. So, the same will involve here when we say A coupled with B will involved the T-norm, T-norm or S- co norm

So, here also we have T-norm in the discrete version of it. So, we can simply write the fuzzy relation set by R and then we use T-norm when we interpret the fuzzy rule by $A\ coupled\ with\ B$. And since we are using T-norm here and we know that we have four types of T-norms normally available. So, we can use here the same as $A\ coupled\ with\ B$ for the relation fuzzy set using minimum T-norm operator. So, we know that the first and the most basic T-norm operator is the minimum. So, we take the minimum. The first kind of T-norm is minimum, then the second kind of T-norm here is the algebraic product.

So, we can use the algebraic product here as one of the T-norms or S-co norms. We already know that T-norm is also called as S-co norm and similarly S-norm is also called as T co norms. So, the third one is when the relationship, the relation fuzzy set can be obtained by using the bounded product as the T-norm operator.

Similarly the forth one here is the drastic product as the T-norm operator. So, here we can use any of the four T-norm operators to get the fuzzy relation set for either discrete fuzzy sets or the continuous fuzzy sets and both the versions are here as mentioned.

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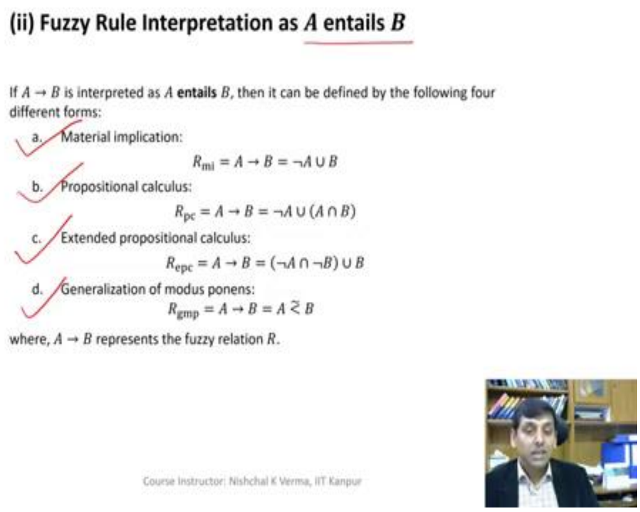

Now, fuzzy rule interpretation as $A\ entails\ B$ here. So, we have if $A \rightarrow B$ is interpreted as $A\ entails\ B$, then it can be defined by the following four different forms. So, first form here is the material implication and the second form is propositional calculus. The third form is extended propositional calculus, the fourth form is the generalization of modus ponens. So, material implication is represented by $R_{mi} = A \rightarrow B = \neg A \cup B$,propositional calculus is represented by $R_{pc} = A \rightarrow B = \neg A \cup (A \cap B)$.

Similarly, we have extended propositional calculus and here this is represented by $R_{epc} = A \rightarrow B = (\neg A \cap \neg B) \cup B$. Then fourth one is the generalization of modus ponens represented by $R_{gmp} = A \rightarrow B = A \gtrsim B$. So, $A\ entails\ B$ here represents the fuzzy relation R.

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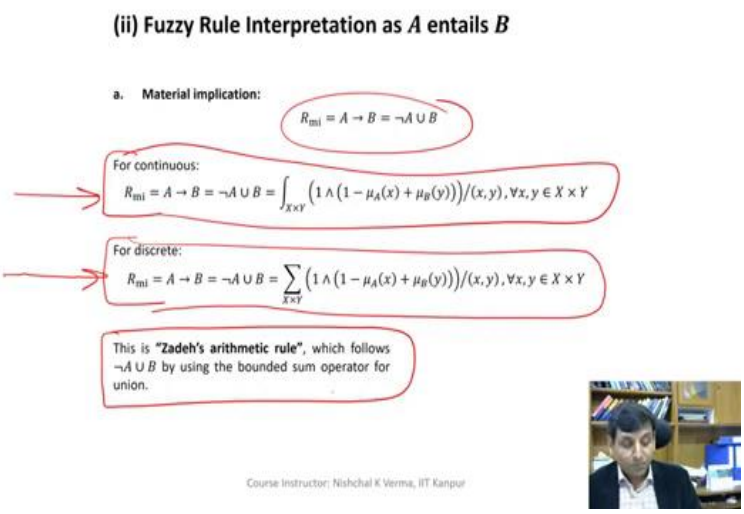

So, let us go one by one and see what is the material implication. So, material implication is defined as I have already mentioned by $R_{mi} = A \rightarrow B = \neg A \cup B$.

So, if we have a continuous fuzzy set let say the R_{mi} will be like this and if we have a discrete fuzzy set the R_{mi} be for discrete fuzzy set will be expressed by this expression. Similarly the continuous one will be this. So, this is also called the Zadeh's arithmetic rule. This is very important to note that the Zadeh's in arithmetic rule which follows the $\neg A \cup B$ by using the bounded sum operator for union.

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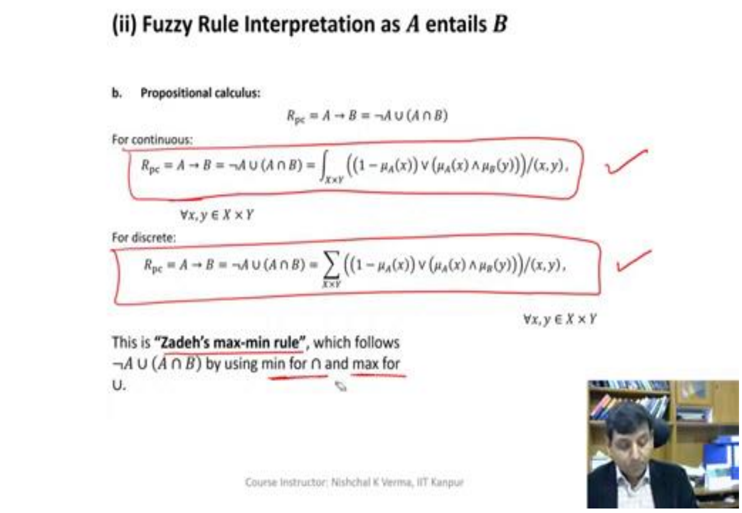

Now, the propositional calculus where the R, the fuzzy relation set $R_{pc} = A \rightarrow B = \neg A \cup (A \cap B)$. So, when we apply this to continuous fuzzy sets A and B, we are going to get this expression.

Similarly, for discrete fuzzy sets when we apply the propositional calculus, we get a relation on propositional calculus R_{pc} by this expression and this is called the Zadeh's max-min rule. This is very important to note which follows the $\neg A \cup (A \cap B)$ by using min for intersection and max for union. So, wherever we have the min we use the wherever we have the min we use intersection and for max we use union.

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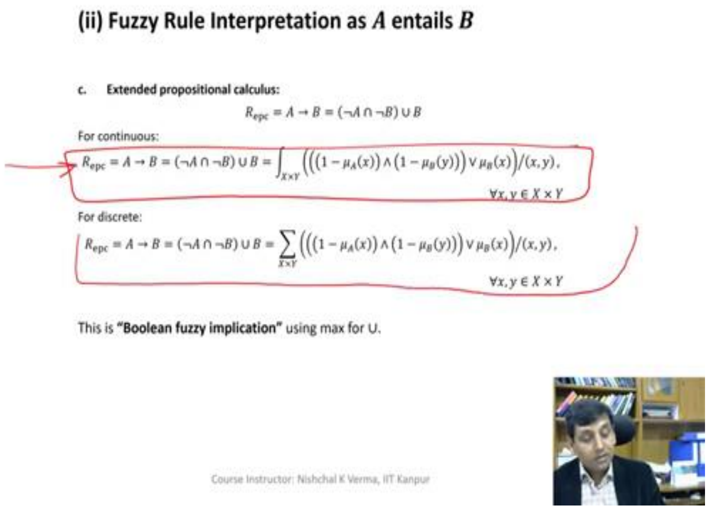

Now, the third one that is extended propositional calculus. So, here we have R_{pc} and $R_{pc} = A \rightarrow B = (\neg A \cap \neg B) \cup B$. So, when we apply this for continuous fuzzy sets A and B, the relation fuzzy set which is R_{epc}, the extended the relation fuzzy set for extended propositional calculus will become R_{epc} and this is represented by this is found by this expression for continuous fuzzy sets.

Similarly, for discrete fuzzy sets for discrete fuzzy sets, we get like this. The R_{epc} we get like this and this is called the Boolean fuzzy implication; this is called the Boolean fuzzy implication using max for $\cup$ means wherever we have max wherever we have a union, we use max.

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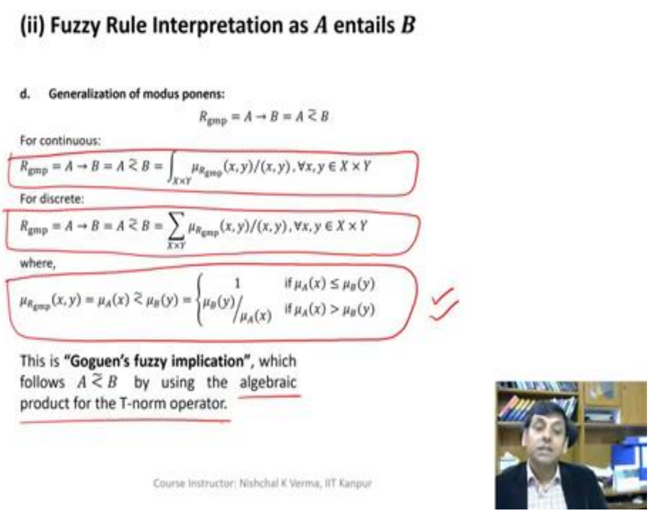

So, then comes the forth one here, the generalization of modus ponens and the fuzzy relation set that comes out of this is represented by $R_{gmp} = A \rightarrow B = A \gtrsim B$ and this is represented by R_{gmp}. And here this is for the continuous fuzzy set where we have the $R_{gmp}(x,y)/(x,y)$ here. So, R_{gmp} is computed simply by this expression here.

And if we have a discrete one discrete fuzzy sets A and B, then the fuzzy relation set that is

$$R_{gmp} = \sum_{X \times Y} \mu_{R_{gmp}}(x,y)/(x,y)$$

This is called as the Goguen's fuzzy implication which follows $A \gtrsim B$ by using the algebraic product for T norm operator.

So, this way we can very easily compute the fuzzy rule interpretation as $A\ entails\ B$ for fuzzy sets, A and B to get the fuzzy relation set.

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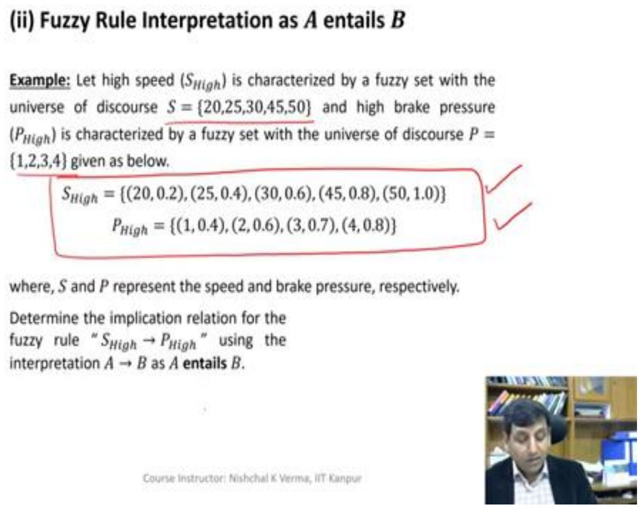

Now, let us take an example here to understand this fuzzy rule interpretation as $A\ entails\ B$. So, for this example the S_{high} and P_{high} where the universe of discourse for S_{high} is this; for S is this and for universe of discourse for P here is 1, 2, 3.

So, we have two discrete fuzzy sets here S_{high} and P_{high} is represented by the two fuzzy expressions, two fuzzy sets.

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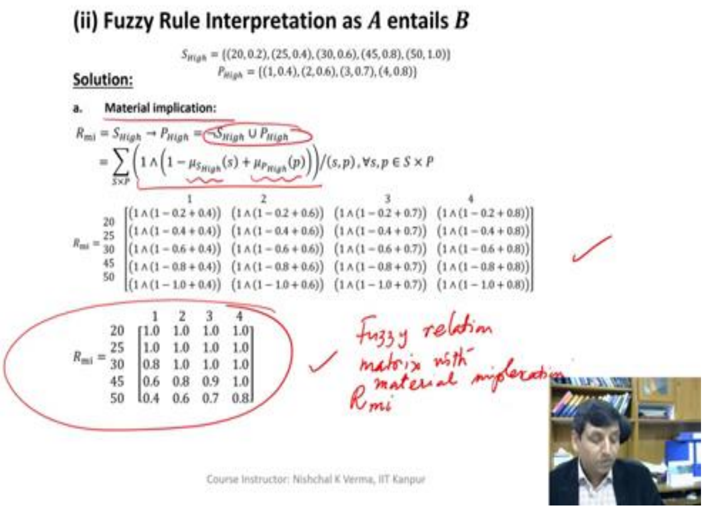

Now, since we have S_{high} and P_{high} so, we can now further go ahead and get the R_{mi}. So, this is R_{mi}, the material implication. So, this is very simple here when we use R_{mi} so, we use for R_{mi}, we use this negation of $S_{high} \cup P_{high}$ here and this we already know as to how we get here.

So, if we want to compute this, the R_{mi} simply for computing the membership values of R_{mi}; we use the $\mu_{S_{high}}$ here; $\mu_{S_{high}}$ this one and $\mu_{P_{high}}$. So, both the membership values, we take and then we compute the membership value of R_{mi}. So, R is the relation fuzzy set for material implication.

So, this way we get this relation matrix this relation; this fuzzy relation matrix and finally when we compute, we are going to get the R_{mi} here like this. So, we can write here if fuzzy relation matrix R_{mi} and mi is nothing but the material implication with we can write here with material implication.

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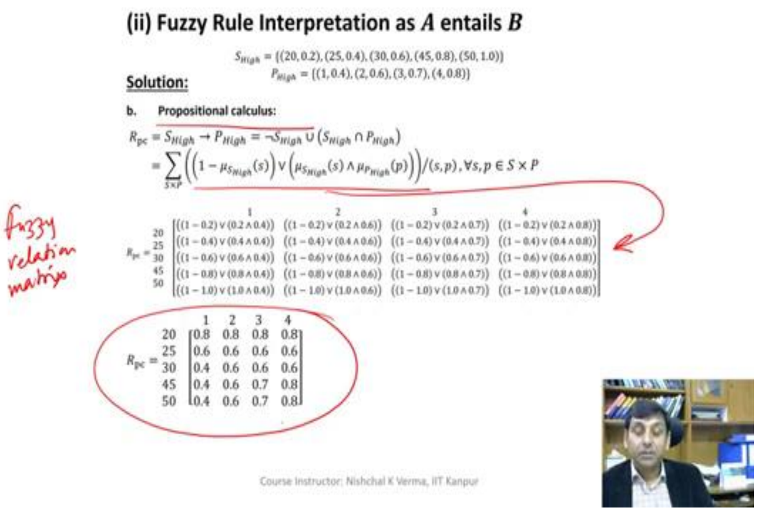

Similarly, we use the same two matrices, we use same fuzzy sets S_{high} and P_{high}, both are discrete and when we use propositional calculus, so for propositional calculus we use the $\neg S_{high} \cup (S_{high} \cap P_{high})$. So, when we do that here, we get the a we get the fuzzy relation matrix fuzzy relation matrix R_{pc}. So, this is what we are going to get after using the propositional calculus, the expression is here. So, we know has to how we are going to compute. And then finally, we are going to get R_{pc} which is the fuzzy relation matrix based on the propositional calculus.

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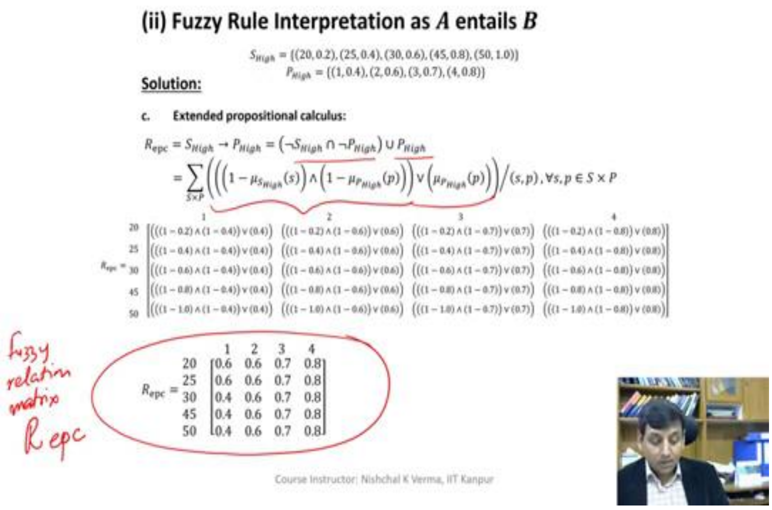

Now, the third one is based on extended composition. So, for computing the extended propositional calculus, we use the, we take basically the $(\neg S_{high} \cap \neg P_{high}) \cup P_{high}$ here. So, when we do that, we use this expression and finally, we are getting R_{epc} here. Again this is nothing, but the fuzzy relation matrix R_{epc} is nothing but the extended propositional calculus.

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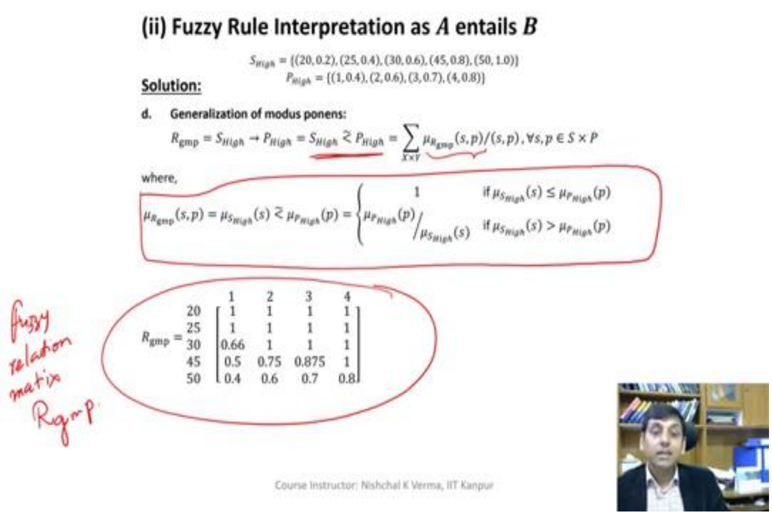

Now, the forth one is generalize generalization of modus ponens. When we use this, so $R_{gmp} = S_{High} \overset{\approx}{<} P_{High}$, you can see here. So, since we have S_{High} and P_{High}. So, when we find this $\mu_{R_{gmp}}(s, p)$, please understand this s, s here is nothing, but it is the generic variable and p also is the generic variable here. So, this way $\mu_{R_{gmp}}$, we can compute here with this expression with this condition here. And when we use this, we are going to get the fuzzy relation matrix, fuzzy relation matrix R_{gmp}. So, this way we get the fuzzy rule interpretation as $A \ \ entails \ \ B$.

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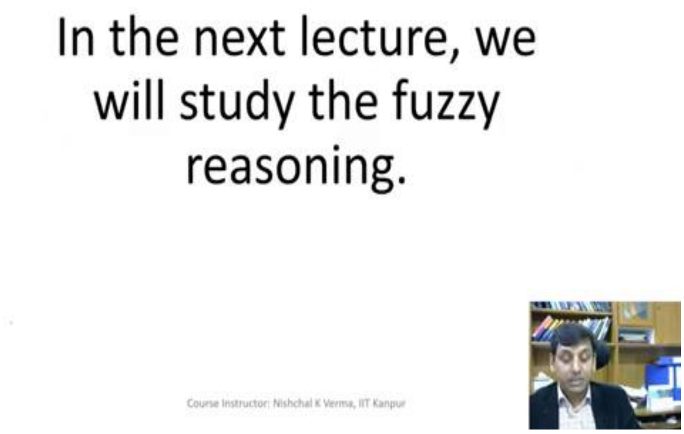

So, that is how we have been able to manage to get various kinds of relation fuzzy relation set in the form of fuzzy relation matrix and with this I would like to stop here. And in the next lecture, we will study the fuzzy reasoning

Thank you.