


**Fuzzy Sets, Logic and Systems and Applications**  
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**Lecture – 50**  
**Fuzzy Inference System**


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**Fuzzy Inference System [FIS]** 

- Fuzzy inference is the process of mapping from a given input to an output using fuzzy logic. It involves all the pieces that we have discussed in the previous lectures: membership functions, fuzzy logic operators, and if-then rules, etc.

**What is a Fuzzy Inference System (FIS)?**

- A nonlinear mapping that derives its output based on fuzzy reasoning and a set of fuzzy if-then rules. The domain and range of the mapping could be fuzzy sets or points in a multidimensional spaces.
- Fuzzy inference systems have been successfully applied in various fields such as automatic control, data classification, decision analysis, expert systems, computer vision, etc.
- Because of its multidisciplinary nature, fuzzy inference system is also known as,
  - Fuzzy-rule based system
  - Fuzzy expert system
  - Fuzzy model
  - Fuzzy associate memory
  - Fuzzy logic controller
  - Fuzzy system



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Welcome, to lecture number 50 of Fuzzy Sets Logic and Systems and Applications. In this lecture we will discuss the heart of any fuzzy system that is the Fuzzy Inference System. So, fuzzy inference system basically is very important system, that means, the fuzzy inference system is having certain processes that is being carried so in nutshell I will say that the process of mapping from a given input to an output using fuzzy logic.

So, fuzzy inference system basically does what? Fuzzy inference system basically maps the given input into the output using fuzzy logic. So, if I have any fuzzy system let us say fuzzy inference engine, in short we call this fuzzy system as FIS. So, I can just write here the fuzzy systems say fuzzy inference system as FIS here.

Of any fuzzy system here if I say FIS so it takes some input and it produces some output. So, fuzzy inference system basically maps a given input to an output using fuzzy logic. So, this FIS will take the help of what, the fuzzy logic, this is based on fuzzy logic. There are so many such systems available which helps us in managing to map the input into the

output, but here the fuzzy inference system does what, it also does the same thing means it maps the input into the output, but the logic that it uses here is the fuzzy logic.

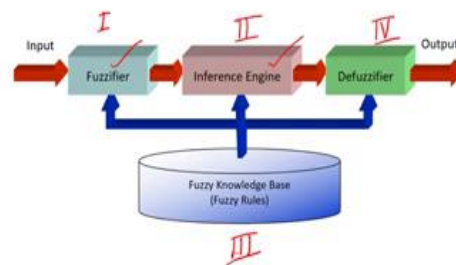
So, basically we can say that it involves all the pieces that we have discussed in our previous lectures. So, we have covered so many topics and the fuzzy inference engine almost uses everything so that we have covered so far. So, some of them some of these can be, like membership functions, fuzzy logic operators, if then rules, composition rules, etcetera etcetera so many almost all of the topics that we have covered so far in this course so far will be used in the fuzzy inference system.

So, here a non-linear mapping by the FIS basically it helps us in deriving its output based on fuzzy reasoning and a set of fuzzy if-then rules. So, the domain and range of the mapping could be the fuzzy sets are the points in a multidimensional spaces. The fuzzy inference systems have been successfully applied in multiple domains so many domains and basically it helps us in managing to give us a suitable model.

So, modeling, control, etcetera etcetera and then it has been applied into various domains for example, computer vision, control, automation, data mining, machine learning etcetera etcetera. So, here because FIS is a multidisciplinary entity, the FIS is known by its various names so the name could be fuzzy rule based system, fuzzy expert system, fuzzy model, fuzzy associative memory, fuzzy logic controller, fuzzy system. So, FIS is known by its multiple names. So, because fuzzy logic can be applied to many interdisciplinary areas and the areas using the FIS in with multiple names.

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## Fuzzy Inference System



*A Typical* Architecture of Fuzzy Inference System

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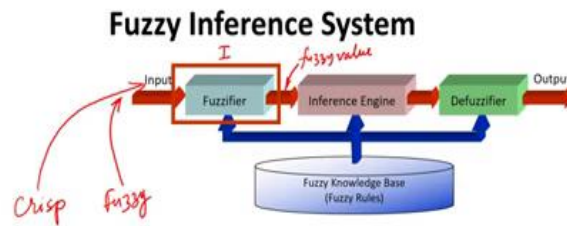


So, let us now understand first the building blocks of any fuzzy inference system. So, any fuzzy inference system will have 4 building blocks. First block here is this is the first block which is fuzzifier, the second block is the inference engine, the third block is the fuzzy rule base, fourth block is the defuzzifier.

So, as I have already mentioned that a fuzzy inference engine a typical fuzzy inference system basically takes the input and it maps suitably to a particular output, and this in between the input and output we have a block which is called the fuzzy inference system, and this fuzzy inference system has 4 blocks. First block is the fuzzifier, second block is the inference engine, third block is the fuzzy rule base. So, fuzzy rule base or fuzzy knowledge base this has a set of fuzzy rules and as I have already mentioned when we discussed fuzzy rules, fuzzy rules are always in the form of if and then rules.

So, we have set of fuzzy rules available to help the inference engine and finally, fourth block is the defuzzifier and then defuzzifier produces the suitable output or output comparison, comparative to the input that is given to the fuzzifier. So, this is a typical architecture, this is a typical architecture of a fuzzy inference system.

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**Fuzzifier:**

Converts the crisp input to a linguistic variable using the membership functions stored in the fuzzy knowledge base.



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So, let us go one by one to all the blocks and then see what these blocks are doing for us. So, the fuzzifier basically, in the fuzzy inference systems takes the input from the outside world. This input can be either, this input can be either a crisp input or the fuzzy input. So, if the input is a crisp input, if the input is a crisp input let us say, if the input is a crisp input then the fuzzifiers job is to converts this crisp input into a suitable fuzzy value.

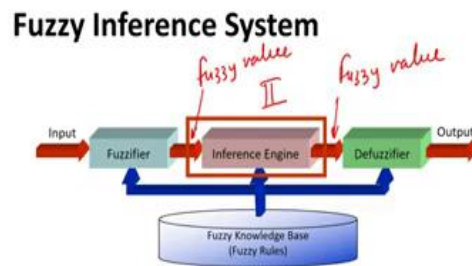
So, here we get some fuzzy value, some fuzzy value, means the fuzzy quantity comparable to the crisp input that is fed to the fuzzifier or fed to the fuzzy inference system. If the input is fuzzy input then fuzzifier normally does not do any job. So, this fuzzy quantity automatically gets transferred here as the fuzzy value.

So, finally, the output of fuzzifier is a fuzzy value which is the input to the inference engine. So, first block here as I already mentioned. So, first block is a fuzzifier and fuzzifier if the input to the fuzzifier are FIS, the fuzzy inference system is crisp it produces the fuzzy value comparable to the crisp value which is fed.

If the input is already fuzzy then fuzzifier normally doesn't do any job if this fuzzy value is directly transferred to the fuzzy inference System or inference engine as its input. Now in short we can say the fuzzifier converts the crisp input to a linguistic variable or linguistic value here, using the membership function.

So, basically the fuzzifier takes the help of membership functions and these membership functions are helping in order to convert the crisp value into the fuzzy value. So, here basically comparable to the crisp value that is fed it produces it gives it assigns some membership value which with the help of the membership functions.

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Schematic diagram of Fuzzy Inference System

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Now, next is the, next block here is the inference engine. So, now since the fuzzy, now since here the fuzzy quantity is already available this is a fuzzy quantity or fuzzy value is available as input to the fuzzy inference engine or inference engine, and this inference engine basically computes, basically inputs basically infers, based on the fuzzy value. With the help of fuzzy knowledge base fuzzy rule base and it produces the fuzzy output, normally the fuzzy value.

So, here also we have the fuzzy value available. So, the inference engine takes the fuzzy value and comparable to this fuzzy value it produces the output in terms of the fuzzy value.

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## Fuzzy Reasoning

- Fuzzy Reasoning is also known as the approximate reasoning. It is an inference procedure that derives conclusions from a set of fuzzy if-then rules and known facts.
- Let us first discuss compositional rule of inference, which plays an important role in fuzzy reasoning.

### Compositional Rule of Inference:

- This idea is proposed by Prof. Zadeh.
- Similar concept has been used for max-min composition of fuzzy relation sets.
- Moreover, the extension principle is actually a special case of the compositional rule of inference.

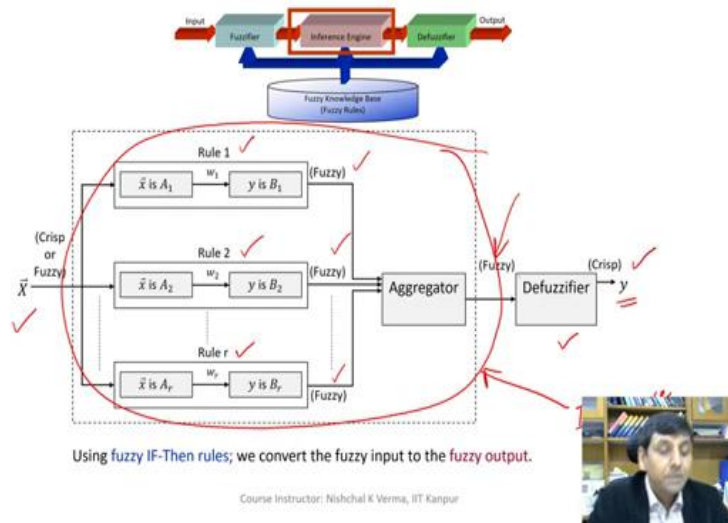


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So, this FIS, this this fuzzy inference basically does certain reasoning. So, this inference engine basically contains the fuzzy reasoning. So, fuzzy reasoning here basically as we have already discussed in the previous lecture that the this here is the approximate reasoning and it is an inference procedure that derives the conclusions from the set of fuzzy. If then rules and these if then rules that are available already in another block another basket, the fourth one the third one and then based on certain compositional rule of inference.

So, here we have the compositional rules we have already done this in previous lecture so I am not going into the detail we have certain compositional rules like max-min composition here and then based on that we have you know certain output based on the processing, inside the inference engine. And as I have already mentioned that the fuzzy set the fuzzy rule set are the rule base helps finally in managing this composition.

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So, this way we get the fuzzy output here out of the inference engine, and here you can see that as to how these rules are helping the. So, if let us say we have a set of  $r$  rules the 1st rule, 2nd rule and then the  $r^{th}$  rule. So, we have a set of  $r$  rules which are all producing us the fuzzy outputs here the, so fuzzy values here based on the input which is whether the crisp input or the fuzzy input and these fuzzy values these fuzzy outputs out of all the rules fuzzy rules these are finally aggregated together.

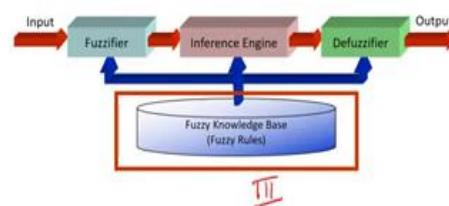
So, here you see the these values are aggregated suitably, and when these are aggregated means normally the aggregation is here as the maximum the union. So, here we use union of all these fuzzy values and then the outcome of this fuzzy, fuzzy this aggregator is the fuzzy value again you can see here. So, here we have the fuzzy value as the outcome the output so this fuzzy value is since it has to since the fuzzy system the fuzzy inference engine, inference system that we are using here it has to interact with the outer world which is working in the, working with the crisp value crisp logic.

So, the output should be the crisp. So, here the fuzzy value is further defuzzified to convert this into the crisp value that is  $y$  here you can see. So, for this we use the defuzzifier block, I will be discussing in the next slide. So, basically what is this? This is nothing, but the fuzzy inference or I can say the fuzzy, just the inference engine, inference engine this is inference engine.

And inside inference engine we use composition, compositions with the fuzzy rules or the fuzzy rule compositions and then we aggregate and finally, we get the output based on the input which is fed into it whether it is normally it is the fuzzy value, and which is which we get with the help of the fuzzifier. And finally, the output of it goes into the defuzzifier and this defuzzifier converts this fuzzy value into the crisp value.

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### Fuzzy Inference System



#### Fuzzy Knowledge Base:

The rule base referred to as the knowledge base.

- A rule base contains a number of fuzzy IF-THEN rules;
- A database which defines the membership functions of the fuzzy sets used the fuzzy rules.

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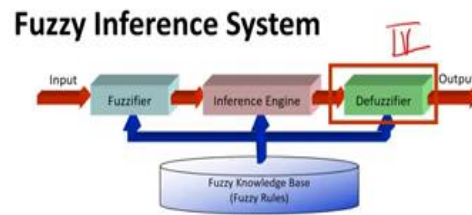
So, now let us move to the third block the third block is the fuzzy knowledge based knowledge base. So, fuzzy knowledge base here is nothing but a set of rules, for fuzzy if-then rules a set of fuzzy if then rules. So, a rule base referred to as the knowledge base, normally this is called the knowledge base also. So, either we say the fuzzy set of fuzzy then rules are fuzzy rule base or knowledge base here in fuzzy inference system all are conveying the same meaning.

So, a rule base basically contains a number of fuzzy if-then rules. So, this must be understood that, without the fuzzy rule base nothing can be done. So, all these similarly all other blocks are also very important as we say the fuzzy knowledge base is important because without the rule the inference engine cannot function.

So, inference engine takes the help of the fuzzy rule base, fuzzy if-then rules and then produces the output suitable output based on the input that it takes.



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#### Defuzzifier:

- It converts the **fuzzy output** of the inference engine to **crisp**.
- Here are some commonly used defuzzification methods are as follows:
  - Centroid of area (COA) ✓
  - Bisector of area (BOA) ✓
  - Mean of maximum (MOM) ✓
  - Smallest of maximum (SOM) ✓
  - Largest of maximum (LOM) ✓

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So, the output of it as I have already mentioned that the output is fuzzy value and then we come to the fourth block here which is defuzzifier. So, this fuzzy value which is the outcome of which is the output of inference engine which is a fuzzy value. So, this defuzzifier helps us in converting the fuzzy value into a crisp value into a suitable crisp value.

So, there here we have so many strategies available for converting fuzzy value into the crisp value, but we use some commonly used methodologies for defuzzifying the fuzzy value into crisp.

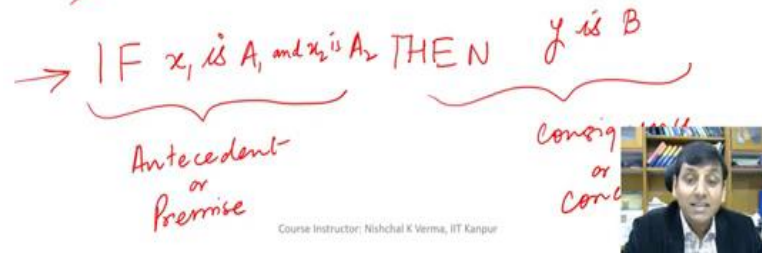
So, some of them are listed here so some commonly used defuzzification methods you can see the one is the centre of area, the other one is the bisector of area, and then mean of maximum, smallest of maximum, largest of maximum. So, these are some of the commonly defuzzification strategies that we can use to convert a fuzzy value into crisp value.

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## Fuzzy Reasoning

Now, we will discuss computational aspects of the fuzzy reasoning using the following:

- (i) Single Rule with Single Antecedent
- (ii) Single Rule with Multiple Antecedents
- (iii) Multiple Rules with Multiple Antecedents



Now, let us go a little more into the detail of managing the inferencing. So, when we go into the inferencing as I have already mentioned that we have the reasoning and this reasoning actually is managed with the help of compositions and the fuzzy rule base. So, here we can have the fuzzy reasoning like this so let us start with very basic.

So, the first case could be first kind of first kind of scenario could be like we have a single rule in and in FIS. So, single rule with single antecedent. So, please understand that we have a rule fuzzy rule, a fuzzy rule has two parts first part is the IF part see here and then second part is THEN part this is called antecedent or premise, here this is called consequence or conclusion.

So, here this IF part normally has, I mean it can have single antecedent like if I take a fuzzy general generic variable  $x$  so this can be like this  $x$  is  $A$ . So, if we have a single antecedent and of course, the output can be like this something like this  $y$   $B$ . So, I can have either a single antecedent or I can have multiple in antecedent by using some connective like AND or OR whatever.

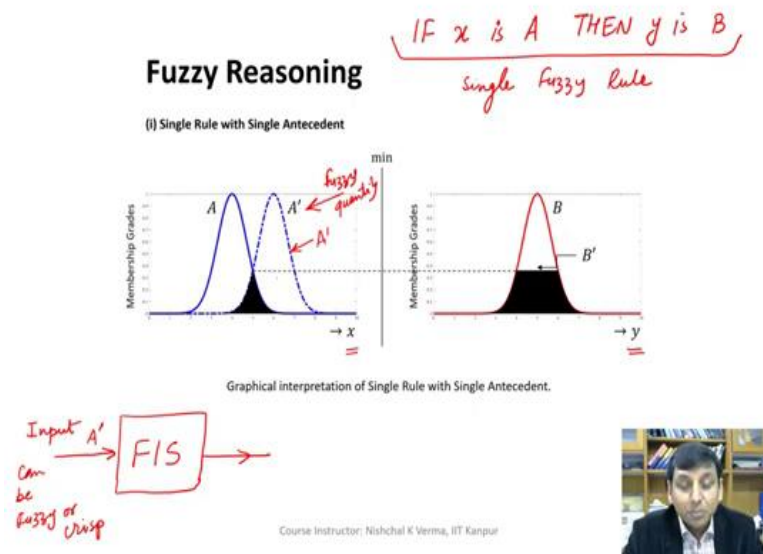
So, and then I can have here  $x_1$  is  $A_1$  and let us say  $x_2$  is  $A_2$ . So, we have two antecedent so like likewise we can have multiple kinds of scenarios. So, first scenario here is that I can have single rule with single antecedent, means the FIS has only one rule which is here and this fi this, fuzzy rule has only the single antecedent means  $x$  is equal to is  $x$  is

A and then y is B. Then the second scenario could be a single rule with multiple antecedent as I just mentioned.

So, multiple antecedent means we can have  $x_1$  is  $A_1$  and  $x_2$  is  $A_2$  or something like this. Similarly, while I can have  $x_3, x_4$  as the generic variable, generic variables. So, here in these two scenarios we have only a single rule, then we can have multiple rules in the third scenario. So, multiple rules with multiple antecedent, means we can have multiple rules like this. I mean if  $x_1$  is  $A_1$  and  $x_2$  is  $A_2$  then y is B.

Similarly, we can have another rule if  $x_1$  maybe because we have to change the value of the generic variable. So, we can say we can use another symbol here let us say 1 so  $x_{11}$  or maybe we can use  $x_1$  is let us say  $A_{11}$  or something like that to differentiate the generic variable values lying in two particular fuzzy reason.

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So, similarly we can have multiple rules with multiple antecedents so let us understand this by taking some examples here. So, we have the first scenario which is we see here the first scenario is the single rule with single antecedent. So, what is happening here is the we have a fuzzy rule you can see as I already mentioned, we have IF let us say x is A this is already given THEN y is B.

So, this is the fuzzy rule, single fuzzy rule that is available that is given, this is a first scenario single fuzzy rules so we say this as the single fuzzy rule. So, here we have the single fuzzy rule this is single fuzzy rule.

Now, when we have this as the single fuzzy rule this is already given. Now if we are interested in using this fuzzy rule in our fuzzy inference system and if an unknown input which is coming to this fuzzy inference system what will be the output comparable to the input that is coming to the fuzzy inference system.

So, here the input that is coming to the fuzzy inference system. So, let us first understand what I am saying, I am saying that if I have a fuzzy inference system and my input here is an input which is coming to this fuzzy inference system and this fuzzy inference system involves only the single rule with single antecedent. So, this is the first scenario and this input can be either fuzzy this input can be here can be fuzzy or crisp, fuzzy or crisp.

So, let us first take the fuzzy case when the input is fuzzy. So, we have the IF part mean the antecedent part the single antecedent so we have the generic variable  $x$  here and the output here is  $y$ . Single and antecedent means we have only  $x$  is equal to  $A$ , I mean which is given to us. Now a new input which is not known to us is coming to the FIS here, so the new input which is fuzzy is coming as  $A'$  here so the  $A'$  you can see here  $A'$  is coming and as I have already mentioned that this is a fuzzy input.

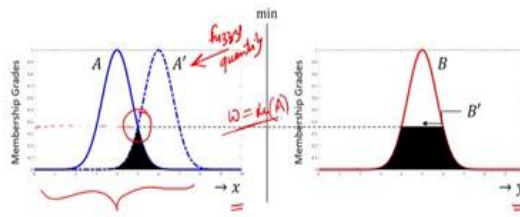
So, fuzzy input, fuzzy I will write here the fuzzy quantity so fuzzy quantity is already always in the form of a fuzzy set. So, fuzzy quantity let us say this fuzzy quantity is in the form of a fuzzy set and this fuzzy set is characterized by a Gaussian function which is this. So, here this is a fuzzy quantity, fuzzy set  $A'$ . Now when  $A'$  comes to this FIS which has the fuzzy rule and the rule has a single rule is, in the first scenario the rule is the single rule with single antecedent.

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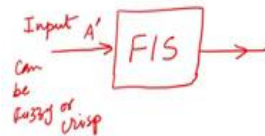
# Fuzzy Reasoning

IF  $x$  is  $A$  THEN  $y$  is  $B$   
 Single Fuzzy Rule

(i) Single Rule with Single Antecedent



Graphical interpretation of Single Rule with Single Antecedent.



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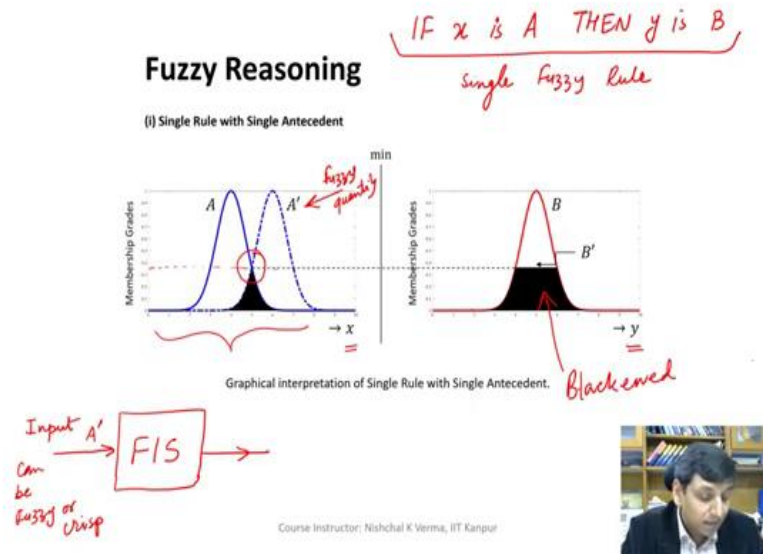


So, what is happening? So, now we will first try to superimpose  $A$  and  $A'$ . So, when we do that see here  $A$  and  $A'$  both are superimposed. So, when we superimpose since both are, both are the  $A$  and  $A'$  both are fuzzy quantities. So, when these are superimpose you see here is that cross section.

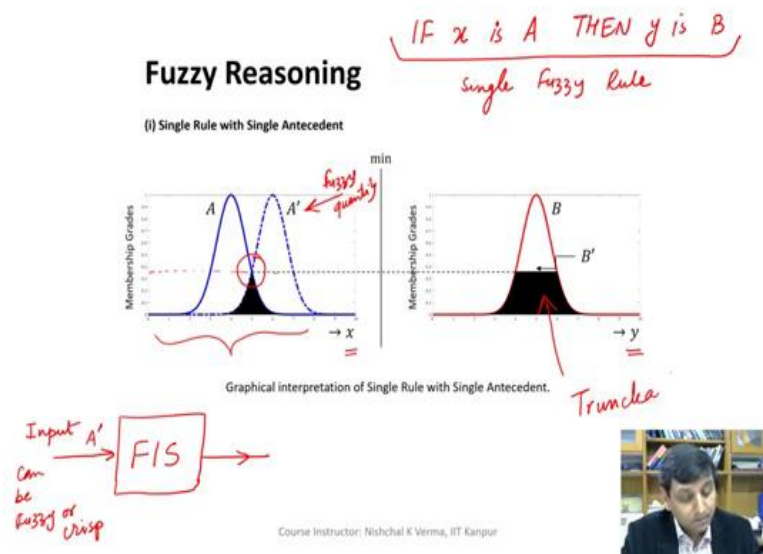
So, you see here this is the cross section. I am just encircling here. So, this point is found this cross section is found and we normally say this is  $w$ . So, this  $w$  is taken and. So, this  $w$  is nothing but the cross section point and then, then corresponding to this cross section point we have the  $\mu$ ,  $\mu$  is the membership value so this also called the degree. So, let us say this has certain  $\mu$  and then for, then the output here you will see the  $y$ ,  $y$  is already a fuzzy quantity.

So, what we do here is we with the same degree we truncate the fuzzy set. So, you see when we truncate the fuzzy set here, we are truncating the fuzzy set given fuzzy set  $B$  so what we are getting is the shaded portion the blackened area you see that some  $\mu$  we are getting see here. So, we get some  $w$ , I will write the  $w$  and this  $w$  is along this line we are truncating here so, you see here we are truncating like this and this is what is the outcome this is what is the outcome.

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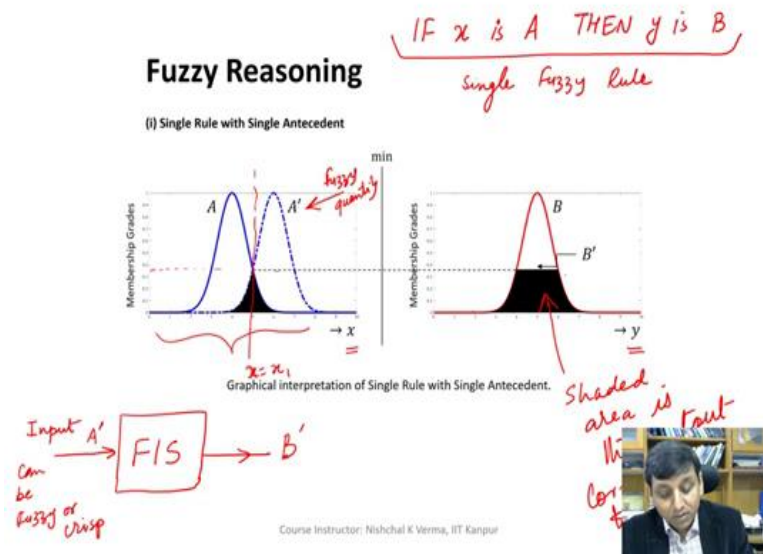


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So, this blackened area or the truncated area, I will write truncated area, truncated I would say blackened area or the shaded area the shaded area is the output corresponding to the input, that is  $A'$ .

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So, what is that which the FIS is producing, corresponding to the input the fuzzy input  $A'$ . So,  $A'$  let us say this the truncated fuzzy set that we have is  $B'$ . So, here we are getting  $B'$  produced. So, what I have said here what I have mentioned here is that the input to FIS is fuzzy. Now on the same lines we can have the output when we take the crisp input. So, if we take the crisp input for example, if I have a crisp input instead of  $A'$  we have some crisp input like this here.

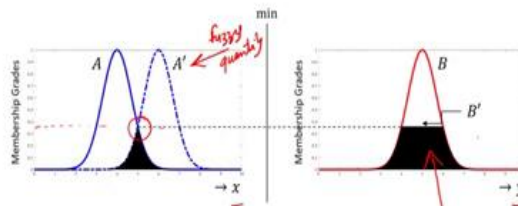
So, here also let's say  $x$  is some value  $x$  is  $x'$ ,  $x_1$ . So, this is the crisp value and if we use this crisp value and then again when we superimpose this,  $x$  or on  $A$  so then if we get any intersection point and along this intersection point we truncate the output fuzzy set.

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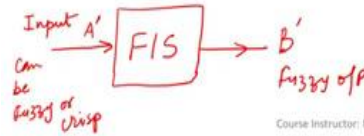
# Fuzzy Reasoning

IF  $x$  is A THEN  $y$  is B  
Single Fuzzy Rule

(i) Single Rule with Single Antecedent



Graphical interpretation of Single Rule with Single Antecedent.



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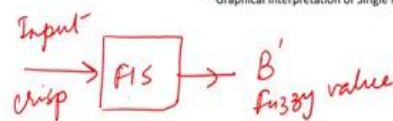
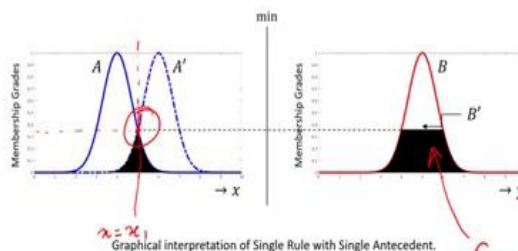
So, here also on the similar lines we can get the fuzzy output. So, what is interesting here is that the output in both the cases we are getting as the fuzzy. So, here we are getting a fuzzy output, here we are getting as a fuzzy output.

So, what we have seen here is that if we have any fuzzy inference system, which is having a single rule with single antecedent we are going to get the output which is the fuzzy output whether the input is crisp or fuzzy and we have seen as to how we are getting the output. Now when we, as I have, as I have already mentioned that when we take the crisp we can say it like this, like we have crisp output.

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# Fuzzy Reasoning

(i) Single Rule with Single Antecedent



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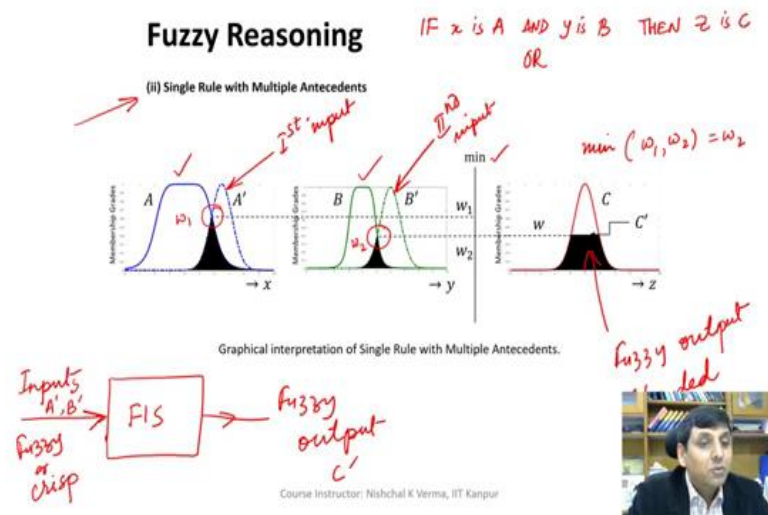




I can just draw it here let us say this is  $x_1$   $x$  is equal to  $x_1$  so here we have the intersection point and then we get the degree which is basically some membership grade. So, wherever it cuts wherever it intersects and then when it intersects and the same value is used for truncating the output fuzzy set as we have seen in the case of the fuzzy input.

So, here what we are getting is again the fuzzy value, fuzzy value. So, this FIS is giving us the input in this case the input is crisp and the output that we are getting is what? The  $B'$  which is fuzzy value. So, this is how we can manage to get the output through the FIS having single rule with single antecedent.

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Now, let us go to the second scenario, second scenario that we have here is, the second scenario is single rule with multiple antecedent. So, when we say multiple antecedent it means that we have IF and then we have say, THEN part like this. So, here let us say we have THEN part is  $z$  is  $C$ .

Then, IF we have  $x$  is  $A$  and this can be this is connective and it can be either AND or OR whatever. So, this can have any connective so I am just writing with the capital letter here AND or it can be OR OR and then the second antecedent here is  $y$  is  $B$ .

So, here we have only two antecedents, but we can have similarly multiple antecedents like multiple linguistic variables  $x, y$  and so on and so forth. So, here we have this kind of situation. So, let us understand that, if we have FIS here we have the FIS and in the FIS

we send some input, which can be either crisp or fuzzy. So, I can write here the input either the crisp, I can write can fuzzy first and then fuzzy or crisp.

So, comparable to this output or with respect to this input we get certain output which is here the fuzzy output. And let's see how we get it. So, if the input is fuzzy and this fuzzy value is let us say, since there are two antecedents. So, we have I am writing here as inputs so here we have multiple inputs multiple antecedent means multiple inputs.

So, we have here  $A'$  and  $B'$ . So, since  $A$  is already there in the rule  $B$  is already there in the rule these are known and here we are applying 2 fuzzy inputs this is fuzzy input, 2 fuzzy input, I can write here the first fuzzy input the first input which is  $A'$ . Then the we have this second input so and here the second input is  $B'$  first input is  $A'$  and second input is  $B'$ .

So, let's see how we can manage to get the output, how we can, how the map how we compute the output when we have this kind of situation, what does the inference engine do? So, we see here that we have only a single rule, single rule means we have only one rule which has multiple antecedents and this in our case we have 2 antecedents  $x$  is  $A$ ,  $y$  is  $B$  and these are known. So, if  $x$  is  $A$  and  $y$  is  $B$  then  $z$  is  $C$  this is known this is the rule, this is already existing in our rule base in the FIS.

So, then we make use of this rule and we apply this rule to the input, inputs that are fetched into the FIS. So, the inputs that are fetched here in the FIS  $A'$ ,  $B'$ . So, here also we superimpose  $A'$  with  $A$  and  $B'$  with  $B$  and we see that  $A$  is  $A$  and  $A'$  both are intersecting here and  $B'$  and  $B$  are intersecting here. So, since both of these are intersecting at certain points let us say this is  $w_1$  and this is  $w_2$ .

So, these two points are taken and then we take min of this. So, we take the minimum of these two points here in this case we see here min is applied. So, the minimum of the  $w_1$  and  $w_2$ , and then since the minimum of  $w_1$   $w_2$  is  $w_2$  and with this  $w_2$  we truncate the output value the output  $C$  the fuzzy set. So, when we truncate this  $C$  we are getting  $C'$  as the output this is what is the fuzzy output.

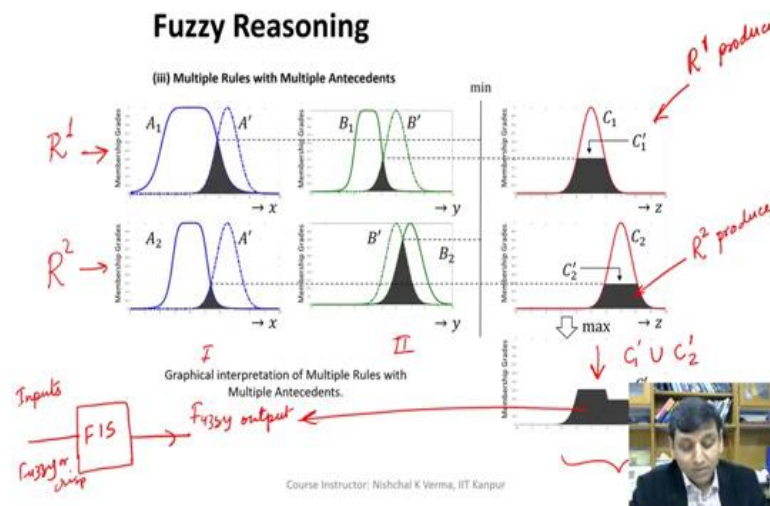
So, this is very interesting to note that as to how we are getting the output mapped corresponding to the input that we are, inputs that we are feeding to our fuzzy inference system so this is the fuzzy output. Now since this is a fuzzy output that we are getting now we can use further the defuzzifier to get the crisp output and this in the previous case also

on the similar lines we can manage to get the crisp output comparable to the fuzzy output, that we are getting here.

Now, the question comes what would have happened when we have used the crisp inputs? So, the crisp inputs here, if we would have, we would have used crisp input. So, on the same lines as we have discussed in the first scenario we here also would have gotten first the intersection points and then we would have taken the minimum of again the  $w_1 w_2$  and on the same way, same lines we got this truncated the output fuzzy set truncated and whatever comes as the shaded area here, shaded truncated fuzzy set the shaded area shaded fuzzy set this is the output.

So, fuzzy output the shaded area. So, there is the fuzzy output, now this fuzzy value can be converted into the suitable crisp value if needed we use as I have already mentioned in the in this lecture previous slide you can refer as to how we can move forward here from here to defuzzify this  $C'$  by using either center of area or there are so many other ways.

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Now, the third scenario here is that we have the FIS, we have the FIS and this FIS takes the multiple inputs, the inputs again it can be fuzzy or crisp whatever. And then it produces the fuzzy output, and remember this fuzzy inference system has, fuzzy inference system has, this fuzzy inference system has multiple rules like the one which you have seen in the just the previous slides the in second scenario that likewise we have multiple rules available.

So, first rule is like this, this is the first rule I am just writing this as  $R_1$  and then the second rule here so we are here we have 2 rules and the multiple in antecedent here would mean that we have multiple linguistic variables. So, here we have 2 antecedents like in the second scenario we have.

So, we have this same scenario as we have in the second scenario here but apart from that we have the multiple rules. So, instead of multiple rules we have taken 2 rules or instead of more rules like 3, 4, 5 or  $n$  number of rules, we have only 2 rules just to make you understand very simple.

So, we have 2 rules 1st rule  $R_1$  and 2nd rule is  $R_2$  and we have multiple antecedents here we have 2 antecedents. So, we already have discussed as to how we proceed to get the output here when we apply  $A'$  and  $B'$  as the input to the FIS. So,  $A'$   $B'$  we already know and we have already seen that we are getting  $C'$  so, let us say the 1st rule produces the 1st rule or I would write here as the  $R_1$ ,  $R_1$  produces or generates produces  $C'_1$ .

Similarly,  $R_2$  produces  $C'_2$ . So, this is produced by the rule number 2. So, here we have 2 outputs first output is from the rule 1 and second output is from rule 2. Similarly, if we would have more rules more number of rules we would have more number of outputs. So, every rule here is producing the output and these outputs are fuzzy outputs and the input also here is fuzzy, but again if we let us say take the crisp input then accordingly as we have discussed in the previous slides in this lecture that the crisp or input also produces here the fuzzy output.

So, now every rule is contributing to the output. So, please understand that, every rule is not always contributing to the output, means every rule corresponding to the input is not producing the output. Why? Because there may be cases where some of the rules may not be applicable may not be fired so those rules normally not produces, those rules normally do not produce any output. So, these rules are simply we say that these rules are not fired so we simply exclude these rules.

So, we take only those rules which are applicable. So, here let us say these 2 rules are applicable for  $x$  is equal to  $A'$  and  $B$  and  $y$  is equal to  $B'$  and these 2 antecedents are here being used as the inputs to the FIS and these 2 are producing the corresponding output  $C'_1$

and  $C'_2$ . So, we see that we have 2 outputs the first output is the  $C'_1$  and the second output is  $C'_2$ .

So, now we need to aggregate these outputs 2 fuzzy output, because these both the rules are producing the separate fuzzy outputs,  $C'_1$   $C'_2$ . Now we have to convert this into the crisp value with respect to the inputs that are fed to the system FIS the fuzzy inference system. So, first of all what we do here in the FIS the fuzzy inference what we do we aggregate so here we aggregate.

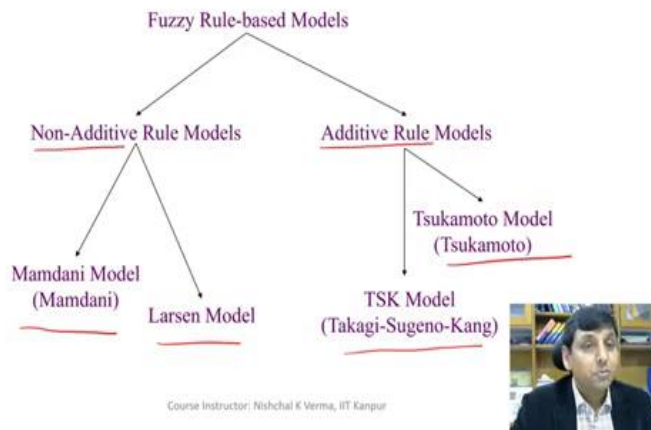
So, how do we aggregate is, we take the union of. So, what we do here is we take the union of the  $C'_1$  which is the outcome of the first rule and then we take the union of  $C'_1$  with  $C'_2$ . So, union of  $C'_1$  and  $C'_2$  is going to give this as the output. So, on the same scale on the same axis if we draw this so graphically we can get this as the output the aggregated.

So, when we aggregate we get this aggregated area and this is what is the fuzzy output. So, since this is a fuzzy output, we may be interested in the crisp output compare comparable to this. So, when we are interested in that then as I already mentioned that we use defuzzifier to get the suitable crisp value.

So, we can write here as the fuzzy output. So, here it is already written so I can simply this is the fuzzy output. So, when we use multiple rules with multiple antecedents this is the way as to how we get the output comparable to the input. And here we have multiple antecedent that means we have multiple inputs.

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### Classification of Fuzzy Rule-based Models



So, this is how the FIS work. So, we have the FIS that is the fuzzy rule base fuzzy inference system. So, various kinds of fuzzy inference systems are available and if we classify these. So, the first class is the non-additive here and then the second class is additive and then under non-additive we have Mamdani and Larsen additive we have the Tsukamoto and TSK. So, this is how the crude classification of fuzzy inference system available.

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In the next lecture, we will study the Mamdani Fuzzy Model.

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And we will discuss the FIS, these kind of FIS in more detail and we'll stop here and the in the next lecture we will go further and discuss the non additive fuzzy inference system

and the first model that will come in that discussion will be Mamdani model Mamdani fuzzy model. So, with this I would like to stop here.

Thank you.