Fuzzy Sets, Logic and Systems and Applications
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Lecture – 59
Tsukamoto Fuzzy Model

Hi, welcome to lecture number 59 of Fuzzy Sets Logic and Systems and Applications. In this lecture today I will discuss Tsukamoto Fuzzy Model.

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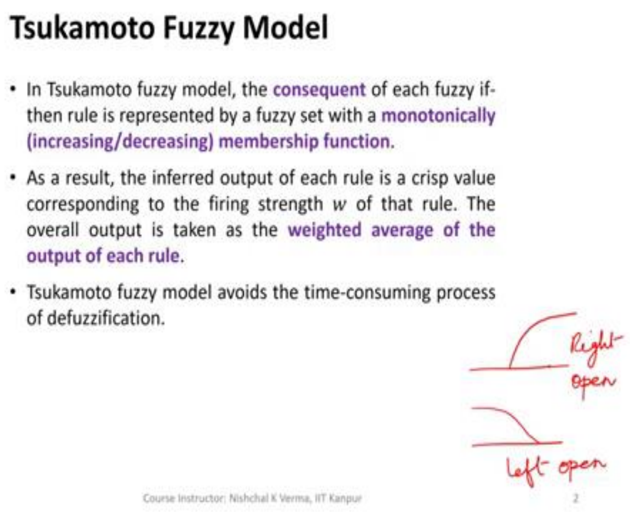

Tsukamoto fuzzy model is a very special kind of model, where we have the consequent part of the fuzzy rule, monotonically increasing or decreasing membership function. So it looks like that this fuzzy rule has a crisp function, but actually the consequent part of the fuzzy rule of this fuzzy model, that means Tsukamoto fuzzy model is always represented by a monotonically increasing or decreasing membership function. And, the highest value of this membership function can go up to 1 not beyond that.

So, in nutshell I can say that Tsukamoto fuzzy model involves the fuzzy rule a set of fuzzy rules of the form where its premise part is fuzzy and the consequent part is also fuzzy. But, the consequent part here is not exactly a fuzzy set that we have seen in the case of Mamdani model or the Larsen model. But, it is different here this fuzzy set is different here the consequent part of the fuzzy, the consequent of a part of the fuzzy set is different. In the sense that this fuzzy set of the consequent part is represented by or characterized by a membership function, which is monotonically increasing or decreasing.

So, if this is the case then, the fuzzy set of the this type which is characterized by the monotonically increasing and decreasing membership function will always be of this kind like either it is left open or right open fuzzy set. So, this is right open and this is left open.

So, this kind of thing with the consequent part happens. And here as I mentioned that the consequent part of the fuzzy rule is fuzzy, like we have in the case of Mamdani model, Mamdani fuzzy model and the Larsen fuzzy model. The Tsukamoto fuzzy model also has the fuzzy rule of the kind of the type where the premised part is fuzzy and the consequent part is also fuzzy. So, this has to be noted very clearly.

Now, the inferred output of each rule please understand carefully here that the inferred rule inferred output of each rule each fuzzy rule is a crisp value. And this is found corresponding to the firing strength, that means the weight of that particular rule. So, the overall output is taken as the weighted average of the output of each rule. So, Tsukamoto fuzzy model basically avoids the time consuming process of defuzzification because here the final output is the weighted average of output of each rule. So, that is where this makes the difference.

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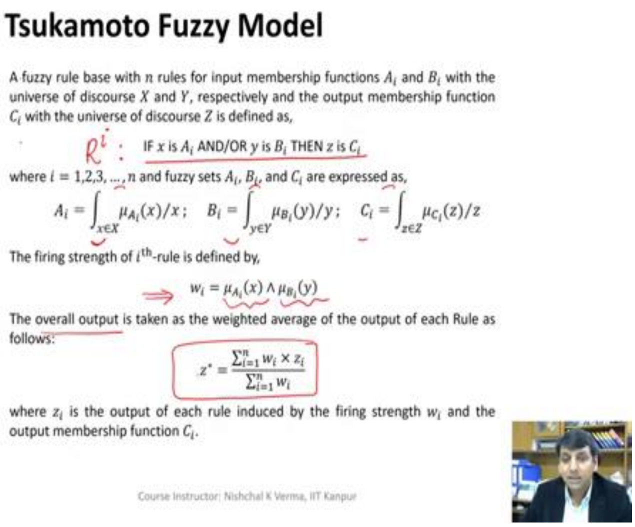

So, let us understand the Tsukamoto fuzzy model. So, here we have a rule as we have taken in case of Mamdani fuzzy model and the Larsen fuzzy model. So, the same rule is being taken. So, this does not need any explanation here. And this rule has the premise part where we have two antecedents connected by AND or OR connective and then THEN part we have the which is known as the consequent part and this is z is C_i and i here signifies the i^{th} fuzzy rule.

So, in any model any fuzzy model, we can have any number of rules say here we have n number of rules n number of rules. As so, that is why i has been used here and this is our i^{th} rule. So, I can represent this by R_i. So, i^{th} rule can be if small x is A_i AND or OR y is B_i then z is C_i, where i can go from 1,2,3 ...$\ldots n$ and fuzzy sets A_i, B_i and C_i are expressed as you can see here.

Similarly, B fuzzy set and then the C fuzzy set. The firing the strength of i^{th} fuzzy rule is defined by here the w_i this is i^{th} rule so, that is why w_i has been written. And then here, the $\mu_{A_i}(x)$ and then we have a min sign, the open triangle sign and then here we have $\mu_{B_i}(y)$. So, and this is $\mu_{B_i}(y)$.

So, what are these values basically? These are the values corresponding to the input that we feed that we applied to the model. And, corresponding membership values will give us the weights and then these weights basically after taking either min or product will give us the firing strength of the rule or weight of the rule.

So, when we have this firing a strength of the rule available, then the overall input can be found by taking the weighted average of the output of each rule, you can see here like this. So, z^* has been used. So, please understand here that, we do not use any union of the outputs of the rules, means we are not doing the we are not taking the union of the outputs of the rules that are applicable for the particular input.

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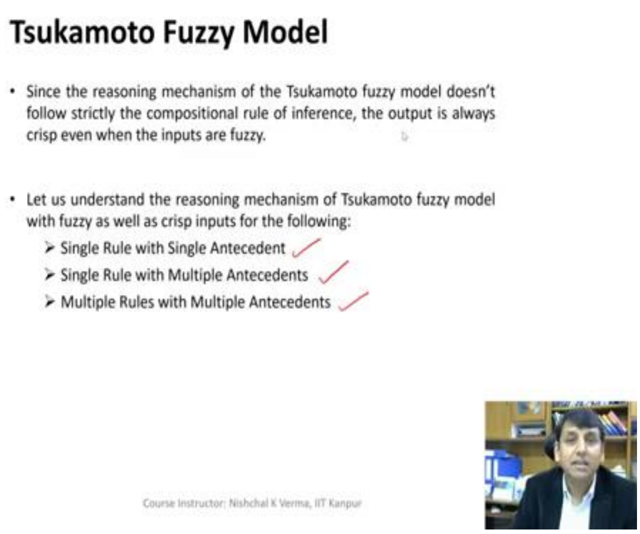

So, let us now move ahead and have the cases of single rule with single antecedent and then single rule with multiple antecedents then multiple rules with multiple antecedents and these cases we will be discussing with fuzzy inputs as well as the crisp inputs. Here as I mentioned that the Tsukamoto fuzzy model does not follow is strictly the composition compositional rule of inference what is that compositional rule of inference? This is max min or max product.

So, since here we are not taking the union so, max is not applicable; here the output is a weighted average of the outputs that are coming out from each rule which is which are applicable. So, let us now, discuss all these cases one by one for the fuzzy inputs and the crisp inputs. So, let us discuss first the single rule with single antecedent here.

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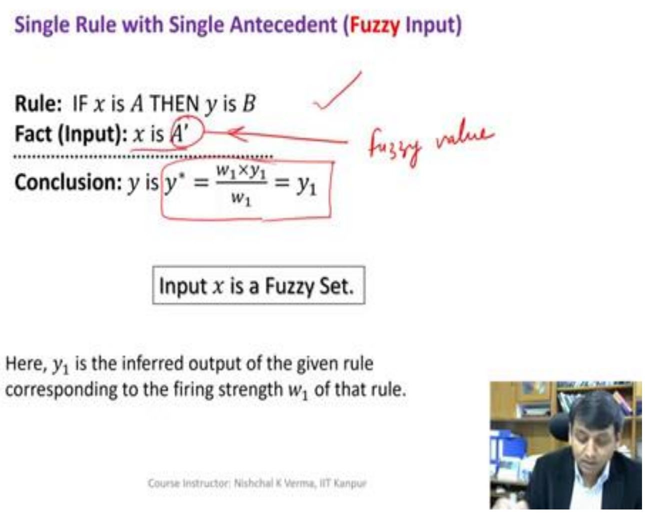

So, as we already know that, if we have a model fuzzy model let us say called Tsukamoto model. So, Tsukamoto model is defined by is characterized by only one rule that a single rule and which is of this type, where we have single antecedents like if x is A then y is B. So, if this is the rule that is there for the model and this rule is known.

Now, if a new input comes to the input comes to the model as the input here x is A', A' is nothing but what is A'? A dash is fuzzy set some fuzzy value fuzzy set or fuzzy value fuzzy set is used for describing quantifying the fuzzy value. So, I can write here the fuzzy value. So, if this is the case, then our output corresponding to the x input that is fuzzy here will be this. So, we will discuss this here.

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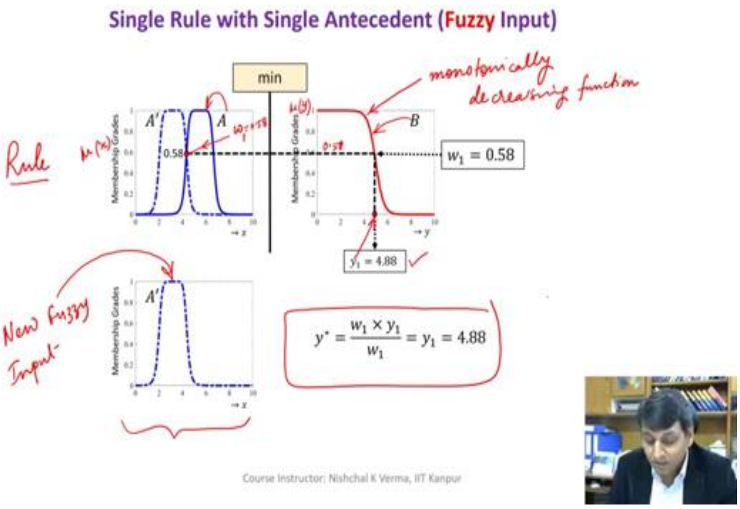

So, if we have a single rule with single antecedent which is here. So, x is A is already there, then corresponding to this x is A we have the output which is B and this is the B fuzzy set. And please note that here the premise part has a fuzzy set which is bell shaped fuzzy set, whereas the consequent part you see the fuzzy set that we have here is the left open fuzzy set. And the highest value of this fuzzy set is going up to only 1 and this is monotonically decreasing fuzzy set.

So, let us I can write here the monotonically decreasing fuzzy set or monotonically decreasing function the fuzzy set function. So, and this is actually the fuzzy set. So, here this is the rule that is given to us for the Tsukamoto fuzzy model. So, here a new input this is a new input which is coming, new input I can write here new fuzzy input that is coming here. And if this input is coming to the model as input to the Tsukamoto model as input, so what we do? We try to superimpose this fuzzy value this fuzzy set on the fuzzy set A which was already given which is already given in the rule so, A is already there.

So, A we already have. So, we try to superimpose A' with A and then we find the intersection point if any. So, here we have the intersection point, which is called weight and I can represent this by w. Since this is the for the first rule, I mean here you can have only you have only one rule. So, I am not writing w_1 or w_2 like that, but I can simply write w. So, w represents the points point of intersection and since we have only one antecedent here.

So, we are not going to use any compositional rule here, like min or product. So, this w is our strength of this rule. So, w is here basically 0.58 this w is equal to 0.58. Now, when we have this w available, this is also known as the firing strength of the rule. So, when we have this now, corresponding to this point here we try to find the y, we see here that this is nothing but the $\mu(x)$, that means the membership function membership value corresponding to the x and here this is $\mu(y)$.

So, corresponding to 5, 0.58 we try to find the y_1 which is here. So, y_1 is you can see the y_1 here y_1 is 4.88. So, this 4.88 is the value the crisp value which we are corresponding to the fuzzy input that we are feeding to the model to the Tsukamoto model, which is which has a single rule which is characterized by the single rule with single antecedent and the input is fuzzy here that is A'.

So, the interesting point here is that we always get a crisp output of the rule means even if we provide the fuzzy input, we are getting the crisp output which is y_1 here. So, y_1 is for this case 4.88 now. Since we have only one rule. So, the weighted average is going to be the same here it is w_1, which need not be there. So, I can write here w_1 and this w corresponding to this this rule the output is 0.58 we have y^* is equal to w_1 into y_1 over 1 and so, we are getting 4.88. So, here the weighted average is going to remain the same.

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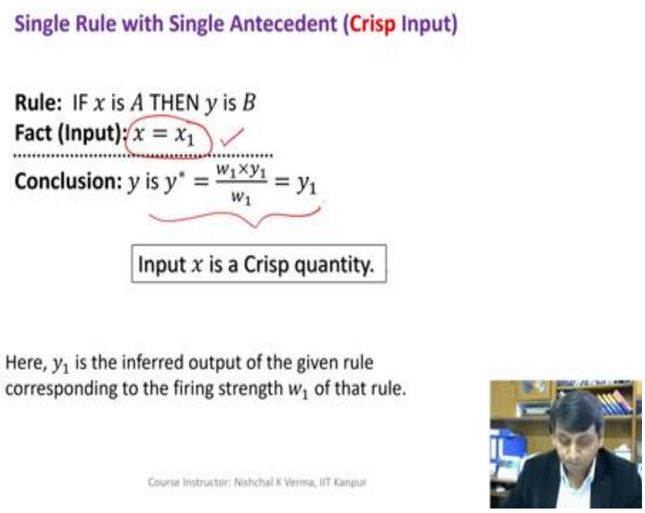

Now, what if we have the crisp input? So, if we have crisp input x is equal to x_1 the output here is going to be y_1 and let us see how do we find the corresponding output here for the crisp input.

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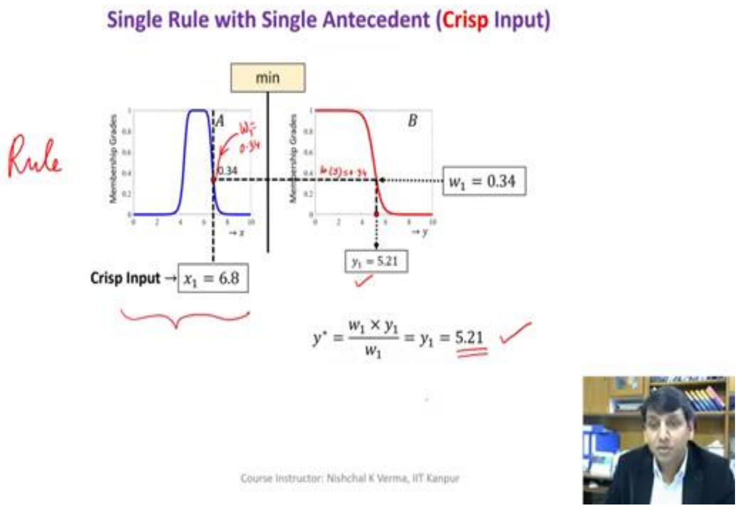

So, here we have the rule and corresponding to this rule, we have a fuzzy set A that is already given. Now, we have the input some input let us say x_1 is equal to 6.8. So, we have taken x_1 is equal to 6.8 which is lying within the region of universe of discourse and also it is lying within the region of the fuzzy set A.

If it is not lying within the region of fuzzy set A, this rule will not be applicable. So, this input has to intersect the fuzzy set before the rule can be applicable. So, here we are taking x_1 is equal to 6.8 and corresponding to this input we are getting the intersection point which is, let us say w_1 here and this w_1 is equal to 0.34 and this 0.34. We have got this called the rule strength fuzzy rule strength or strength of the fuzzy rule they also called the weight of the rule.

So, corresponding to this now, coming to the consequent part fuzzy set which is here and corresponding to this we are getting the corresponding to this $\mu(y)$ this becomes $\mu(y)$ is equal to 0.34. So, corresponding to this we are getting our y_1 here as 5.21. And, again since we have only one rule so, we have only single rule. So, the weighted average is going to remain the same.

So, this way even if we have the fuzzy input or the crisp input for this case, for this model the Tsukamoto model, where we have only one rule with single antecedent we are going to get the crisp output. And the output calculation is very simple we are not going to apply here the compositional rule of these are not actually applicable. Because the min if we have let us say apply the max-min composition. So, min will not be applicable or product even will not be applicable because we have only one antecedent.

Similarly, the max is not applicable because here first of all we have only one rule, but even if we have more number of rules then also the output is the weighted average of the outputs of the rules.

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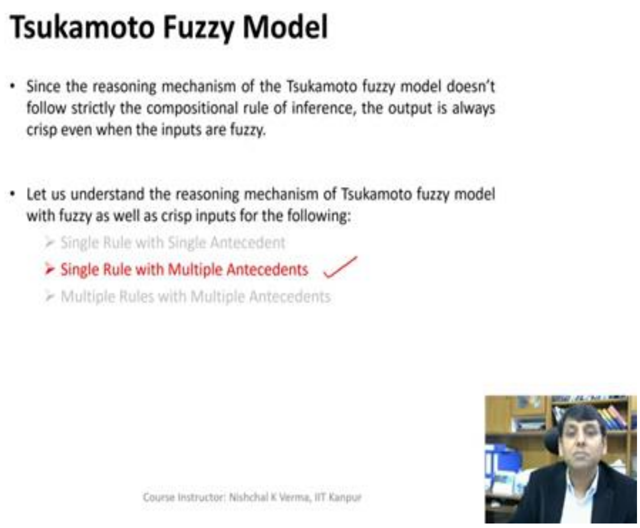

Now, let us come to the other case the second case second case here is that we have a Tsukamoto model and let us say it is characterized by the single rule but with multiple antecedents. So, we have single rule let us say and then this rule has multiple antecedents.

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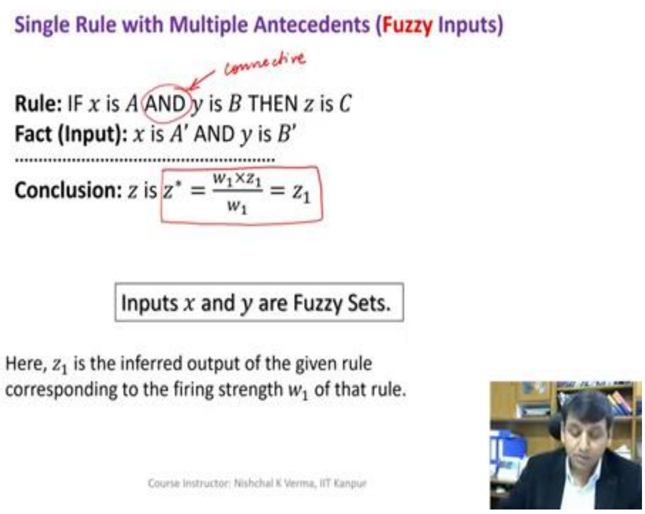

So, let us now, look at this rule and see how does it look like. So, we have here this rule where we have only one rule the model is characterized by model is represented by only one rule, where we have here in this case we have two antecedents. So, we see that we have first antecedent as x is A AND the second antecedent in this case is y is B and both of these antecedents are connected by the connective AND. So, instead of AND it could be any other connective like OR, BUT etcetera.

So, when we say single rule with multiple antecedents. So, multiple antecedents can be any number, here we have only two we can have any number like 3, 4, 5, 2, 3, 4, 5 and so on. So, here for simplicity we have taken only two antecedents and these are joined by connected by the connective AND. So, this is connective as we already discuss alright. So, next is to find the output final output corresponding to some input. So, we will go ahead and first apply the fuzzy input.

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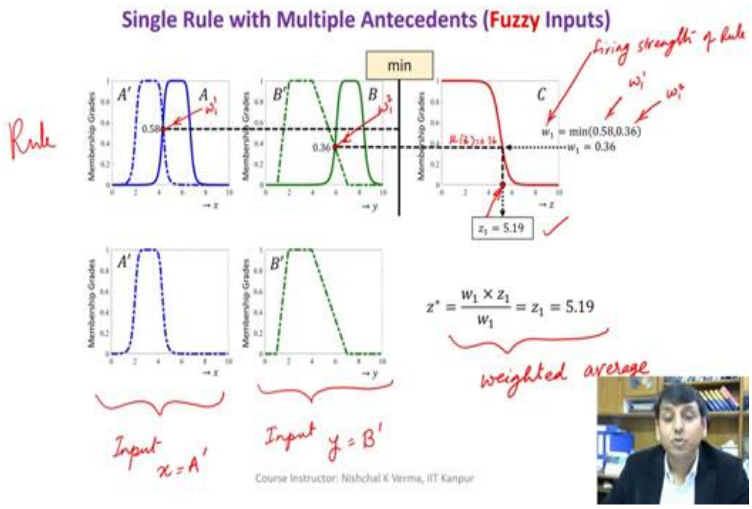

So, here we have the single rule is here, but here we have two inputs the input 1 which is x is equal to which is input x is equal to A', this is this means the fuzzy input is provided and then the another input simultaneously fed is y is equal to B'. So, these two inputs are together supplied to the model. Now, let us see how will this rule help us in finding the corresponding output. So, like we have done in the past, what we do here? We superimpose the inputs the fuzzy inputs on A and B respectively. A and B are already there in the fuzzy set in the fuzzy rule.

So, here we see that when we superimpose we find the intersection, here when we here when we superimpose we get w_1 and then we get here another intersection here which is w_1. So, this is let us say w_1^1 and this is let us say w_2. Now, since we have here two antecedents. Now, if we use min composition from max-min. So, the w_1 becomes min of the w_1^1 and then here w_1^2. So, the firing strength of the rule firing strengths of rule becomes w_1 is equal to 0.36 and corresponding to this 0.36 the z_1 which is the output fuzzy set we get z_1 is equal to 5.19 here and it is very clear from this picture here.

So, our z_1 is z_1 corresponding to x is equal to A' and y, y is equal to B' here and since we have only one rule. So, the weighted average becomes the same. So, this is weighted average. And since we have only one rule as I said, the 5.19 will become the final output.

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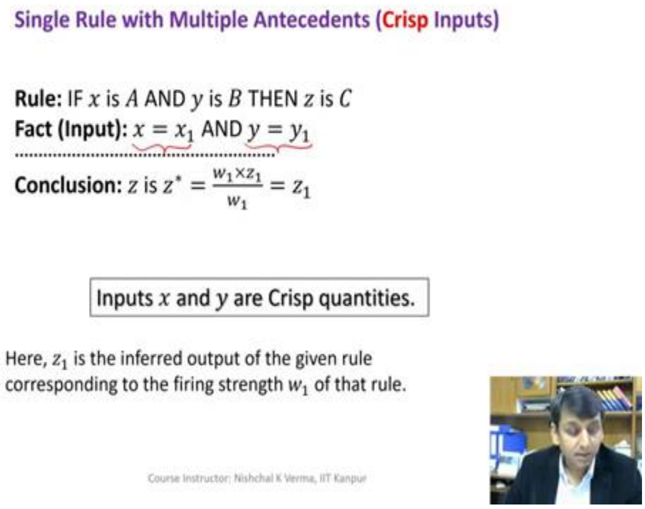

Now, what if we supply crisp input to the Tsukamoto model characterized by single rule with multiple antecedents. So, let us see that here we are supplying x is equal to x_1 some crisp value and y is equal to y_1 here x_1, y_1 are the crisp values that are being supplied.

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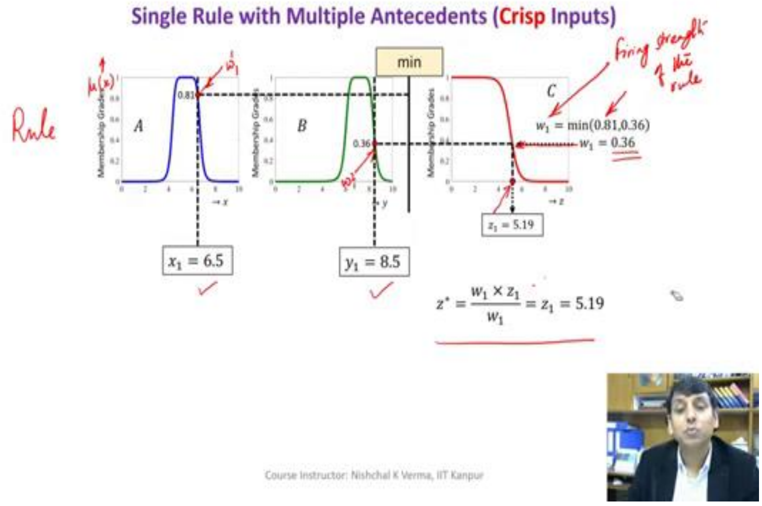

So, let us take x_1 is equal to 6.5, y_1 is equal to 8.5 and when we apply this, when the here we use this x_1 value and y_1 value we see that the rule that was given and that is characterizing the Tsukamoto fuzzy model the rule is here is here. So, this is actually known.

So, this rule is known. So, when the rule is known we can say the model is known. So, all the parameters of this rule is known means A is known B is known C is known. So, here corresponding to this x_1 we have the membership value is 0.81 which is called the weight here. So, w_1 let us say here w_1^1, yeah this is the first rules of first rule is w_1 and then for the first antecedents. So, w_1^1 and then here corresponding to y_1 similarly we get let us say w_1^2 and if we take the min composition here.

So, we take the min criteria when we take the min criteria. So, firing strength of the rule comes out to be this is firing strengths of the rule which is coming out to be w_1 and w_1 here is 0.36. And corresponding to this 0.36 you see here this is 0.36 and corresponding to this the through the output fuzzy set which is monotonically decreasing fuzzy set, we are getting z_1 here 5.19. So, now since we have only one rule again here in this case because, this is single rule with multiple antecedent model Tsukamoto model.

So, the final output is a weighted average and weighted average is going to be the same. So, 5.19 is the final output corresponding to x_1 is equal to 6.5, y_1 is equal to 8.5. Now, let us go to the third case, where our Tsukamoto model is characterized by the multiple rules with multiple antecedents.

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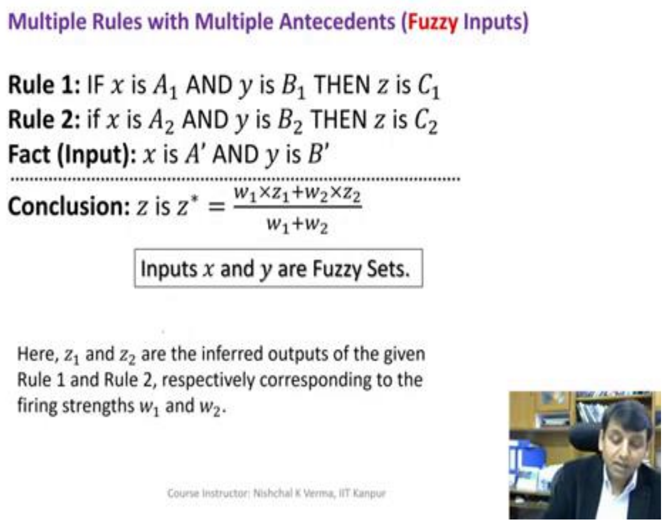

So, we have here two rules for simplicity we have taken only two rules, but it can be any number of rules say up to n. And similarly, we can have antecedents here any number of antecedents here we have for simplicity only two antecedents.

So, two rules and two antecedents for simplicity we have taken. Now, if we supply the inputs to this model to this Tsukamoto model which is characterized by the multiple rules with multiple antecedents. Let us see how we get the output final output computed.

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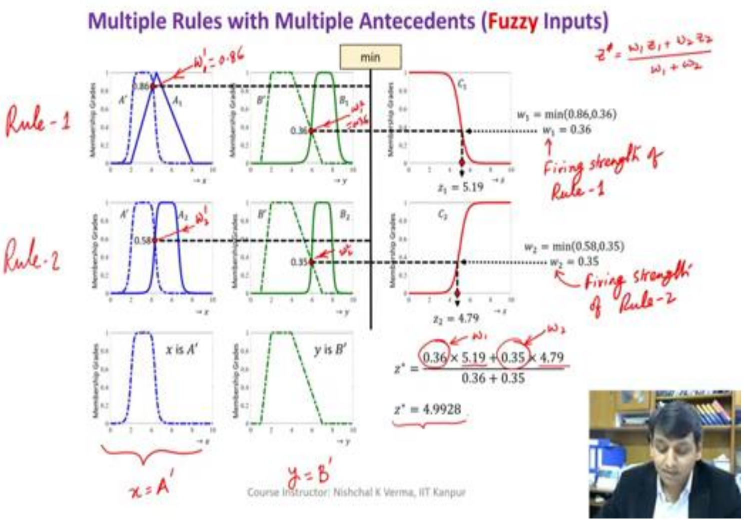

So, here we have the rule number 1, I write here as rule 1. So, rule 1 basically has A_1, B_1, C_1 fuzzy sets which are known to us the all the parameters of this rule are known. Similarly we have rule 2 here and here also A_2, B_2, C_2 are known.

So, when we feed the input x is A', A' is a fuzzy value, similarly y is let us say B' which is again a fuzzy value. So, when these two fuzzy valued values are supplied to this model, let us say let us see the how the output final output we can compute. So, x is equal to A' y is equal to B', when it goes to the when it applies to the first rule. So, as we have already done, we superimpose this fuzzy value these two fuzzy values to the fuzzy sets that are already present in the antecedents A_1 and B_1 respectively.

So, here this is the antecedent for x this is the antecedent for y. So, we superimpose A' with A_1 and we superimpose B' with B_1, B_1 is known A' is known A_1 is known B' is known A_1 is know all our the fuzzy values are known. So, now, we look for the intersection points. So, here since this is the first rule I am writing w_1 and then first antecedent we write 1 here. So, w_1^1 here is 0.86, similarly for this antecedent the intersection point is w_1^2 is equal to 0.36 this is 0.36 alright.

So, now, if we apply the min of these two. So, the min is going to be here the rule strength is going to be 0.36 and corresponding z_1 here, you see that the corresponding z_1 here is 5.19 for the first rule. Similarly, when we apply the same input to the rule 2 these points are coming out to be the weights are coming out to be here w_2^1 and then this is w_2^2.

So, w_2^1 is 0.58 w_2^2 is 0.35 and if we take min of this we are getting here as w_2 is equal to 0.35. So, this is rule strength of 1 and this is the rule is strength of 2 or I can say here the firing strength it is more appropriate to write firing strength of rule 1. So, this can be written as firing strength of rule 2 and here corresponding to w_2 you can see here corresponding to w_2 we are getting z_2, that means the corresponding output 4.79.

So, please note that here we are getting two outputs z_1 and z_2. So, we are getting z_1 and z_2 output, z_1 is corresponding to the first rule z_2 is corresponding to the second rule. And both these outputs are the rule outputs are corresponding to the input fuzzy input $A'(x)$ is equal to A' y is equal to B'.

Now, the next step is to since we have two outputs two outputs from the rules two rules. So, we go for the weighted average of it. So, weights we already know here, this is our w_1 this is our w_2 and here this is our z_1, 5.19 is z_1 this and 4.79 is z_2. So, when we take weighted average, we already know the formula of the weighted average I can once again write it here.

So, $z^* = \frac{w_1 z_1 + w_2 z_2}{w_1 + w_2}$. So, this way we compute the final output as 4.9928 corresponding to the fuzzy input x is equal to A' and y is equal to B'. And please note that this model this the outcome of this is the outcome of the model Tsukamoto model with multiple rows with multiple antecedents.

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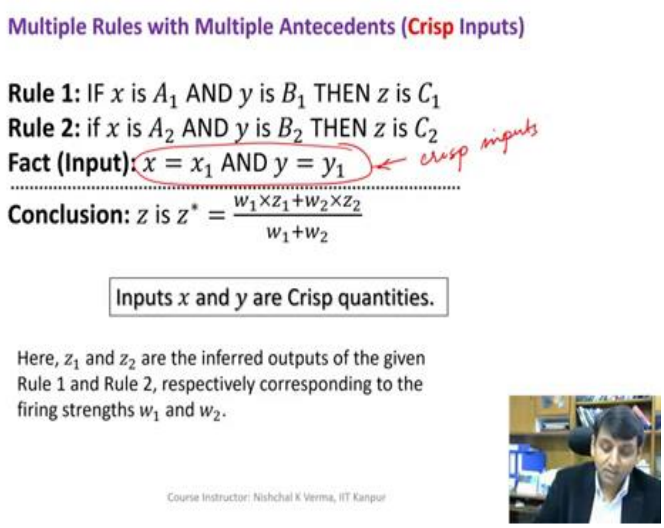

Now, what happens when we supply the crisp inputs? So, here everything remains, the same the input becomes crisp inputs x is equal to x_1 and y is equal to y_1.

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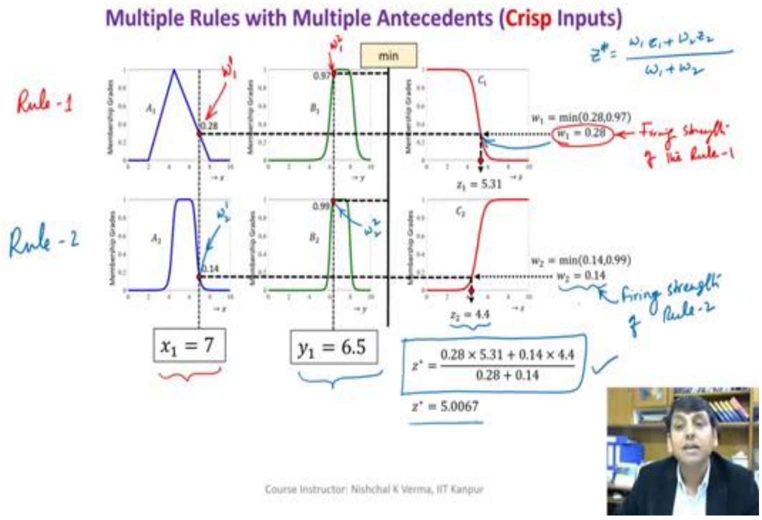

So, now, let us go ahead and see here we have the rule 1, here we have the rule 1 and in this rule 1 we have the crisp input. So, when we apply this input crisp input x_1 is equal to 7 and y_1 is equal to 6.5. So, corresponding to this x_1 is equal to 7 and y_1 is equal to 6.5 we get two points of intersection here.

So, for the first antecedent where we have A_1, for A_1 we are getting this value this x_1 is equal to 7 is giving us the intersection point at 0.28. So, this I can write as w_1^1 similarly here why w_1 one because this is for the first incident and antecedent and first rule, similarly here this is first rule and first antecedent.

So, sorry this is second antecedent; first rule second antecedent. So, w_1^2 is 0.97. Now, if we use min so, min of these two is going to give us w_1 which is as I mentioned the firing strength here, firing strength of the rule of the rule 1. So, corresponding to this w here, corresponding to this w_1 rather corresponding to this w_1 here we are getting the output z_1 and z_1 is here point z_1 is 5.31.

So, the outcome of the rule 1 is z_1 which is 5.31. Similarly now, we go to rule 2. So, here we have rule 2. Now, the rule 2 here gives us the intersection points like this the w_1, since this is the second rule, so we have we will write w_2^1 and then here we will write the w_2^2 the second rule second antecedent. So, corresponding to the y_1 here which is crisp 6.5 we are getting two values two intersection points. And when we take min we get the firing min of these two will give us w_2 and w_2 is nothing but the firing strength of the rule 2 firing strength of rule 2.

Now, we have corresponding to w_1 z_2 here and z_2 is 4.4. So, now, these two inputs we already have z_1 and z_2. Now, let us go ahead and find the weighted average. So, weighted average is giving us here z^* which is a final output and z^* is equal to 5.0067 and we already know that six we have our w_1 0.28 and w_2 0.14 and 5.31 as z_1, 4.4 as z_2.

So, if we substitute all these values here $z^* = \frac{w_1 z_1 + w_2 z_2}{w_1 + w_2}$. So, this is giving us z^* is equal to the final output corresponding to x_1 is equal to 7 , y_1 is equal to 6.5 then getting z^* is equal to 5.0067. So, this is how corresponding to the crisp inputs with multiple rules and multiple antecedents which is describing the Tsukamoto model we are getting the output.

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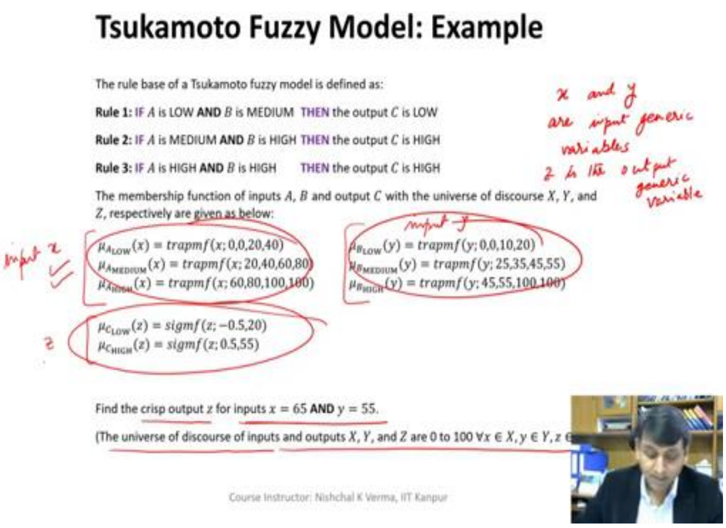

Now, let us take a very simple example here of Tsukamoto fuzzy model, where we have three rules. So, this fuzzy rule is defined by this fuzzy model is defined by three fuzzy rules and this rule as we see here these three rules are of the type of multiple antecedents. So, we see the previous part we have two antecedents in all these three rules. So, this is of the kind of model fuzzy model Tsukamoto fuzzy model which is the multiple antecedents and multiple rules. And the membership functions here are already known as since the model is known.

So, membership functions have been given here $\mu_{A_{low}}$, $\mu_{A_{medium}}, \mu_{A_{high}}, \mu_{B_{low}}, \mu_{B_{medium}}, \mu_{B_{high}}$. So, here the we have two this model has two generic variables as inputs x and y are input generic variables. z here is the output generic variable output generic variable. And please understand that, the x the complete x is divided into three fuzzy regions low, medium, high. So, this means that these fuzzy regions have three fuzzy these fuzzy regions are characterized by three fuzzy sets.

And the membership functions corresponding membership functions here are mentioned you can see here for the input and similarly for the input y. So, for input x here input x and then here we have input y generic variable. So, we see that in both the inputs input generic variables we have 3-3 fuzzy regions and in the output here z we have only two fuzzy regions and both the fuzzy regions are represented by fuzzy sets the here the sigmoid fuzzy set low and high.

The input x region is has three regions represented by three fuzzy sets all the fuzzy sets are trapezoidal type. Similarly, here input y also all the fuzzy sets all the fuzzy sets of the three fuzzy regions are trapezoidal type. So, now, when we give the input the crisp input here crisp input. So, for this crisp input what will be the corresponding output and that too in crisp form. So let us now, see as to how we can find it and the universe of discourses of the inputs and outputs are mentioned here.

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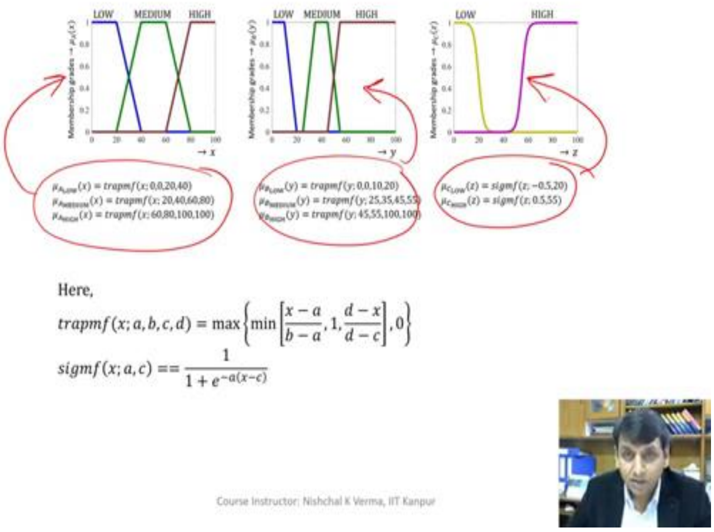

So, the fuzzy regions the membership functions that are given, if we plot the fuzzy sets will look like this low medium high, similarly here also this will look like this. So, all of these are given these are known. So, we do not have to worry for the parameters of these fuzzy set these are already given here you can see. These are the membership functions which are characterizing the corresponding fuzzy sets.

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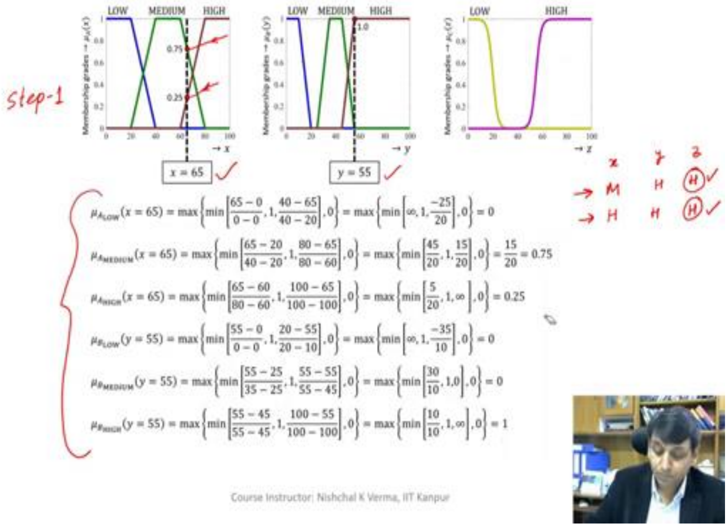

So, let us apply the input and what is the input here is x is equal to 65, y is equal to 55. So, this two this input has two parameters x is equal to 65, y is equal to 55 both these values are now, applied and since we have the three fuzzy regions.

So, the first step here is to find where it is intersecting where this x 65 is intersecting. So, this line when we see is intersecting at here two point, 0.25 and then 0.75 you can see. So, this is the first step, first step is to find the corresponding to the input that is here 65. So, corresponding to 65 what are the fuzzy regions which are applicable what are the fuzzy sets that applicable?

So, please understand that the x is equal to 65 is intersecting medium and high only not low in this case. So, low is not relevant only medium and high are relevant for x is equal to 65, this is the first step that we checked while we are computing the output. So, step number 1 I can write here. Next is similarly we check for the other antecedent like y is equal to 55.

So, y is equal to 55 we see that this corresponding to y is equal to 55 we are getting high and medium, medium is not intersected; because it is when we see where y is equal to 55 is cutting at medium, medium is at 0. So, we see that y we see that medium is not relevant only high is relevant high fuzzy set fuzzy set for high is relevant.

So, y is equal to 55 only one fuzzy set is applicable. Now, or in other words we can say that low and medium are not at all applicable. So, we have for x is equal to 65 two intersection points, whereas for y we have only one intersection points y is equal to 55. Now, we multiply these two means two intersection points and then the one intersection point for the other antecedent. So, 2 into 1 means we have 2 into 1 is 2 here. So, only two combinations are happening here what is that combination? The combination is when my x is medium and high or what is the other combination when my x is high and high.

So, we have two combinations I can write here the medium, I can write x here and y here; I can write medium by M and I can write high by H. So, first combination could be the medium high and then other combination the next one is high because the x is 65 is cutting high fuzzy set also so, high and high. So, two combinations we are getting. Now, corresponding to these two combinations what will be my output that is z. So, this we can find by this we can find by the rules that are given.

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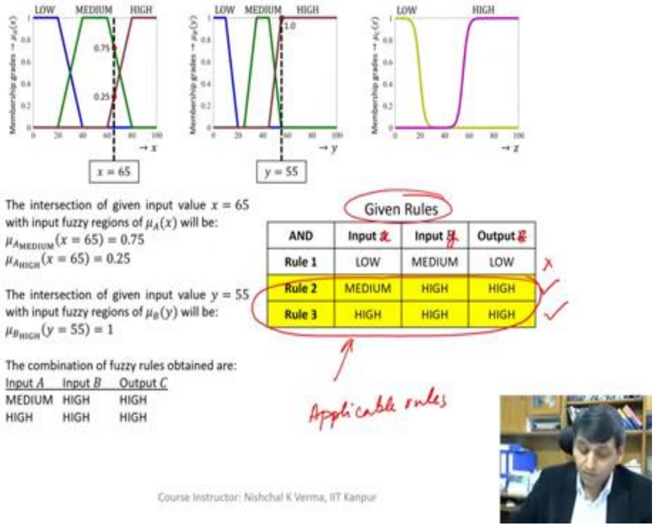

So, let us now, check here the given rules are here. So, first rule here is the rule number 1 here is when my input is A when my input is low input A is low or I can say input 1 is low and then my input 2 is medium then the output or I can write here x, y. So, input x is input y and output z here. So, my input x is low, the output y is medium then the output z is going to be low this is z.

So, this rule 1 is not applicable because we see that we have first combination medium and high the second combination is high and high, here we see that low and medium is not applicable. So, we are not seeing any combination which is low and medium so, this rule will not apply. Now, next we look for medium and high which is here. So, rule 2 is applicable where we have medium high x is equal to medium, y is equal to high I mean when x falls in the medium, y falls in high then the output also falls in high we can see here this one.

So, this way from this rule we find we can very easily find the output is going to be the z is going to fall in high region. So, I can write here high similarly what happens when x falls in H and y falls in H. So, when x falls in H here and then y falls in H output also falls in H. So, this rule is already there. So, rule number 3 also applies and with this combination we can write here high.

So, it means both the outputs are from high fuzzy region in the output. And these are the computations that we do for getting the corresponding membership values which are nothing but the intersection points here for x is equal to 65 here. For low, for medium, for high and similarly for the other antecedent the second antecedent here y is equal to 55, y is equal to 55 here.

So, for low medium and high. So, this way we get the points of intersection. Alright so now, when we know these rules which are applicable these two rules are applicable here for the input that we are supplying. Now, let us compute the corresponding outputs.

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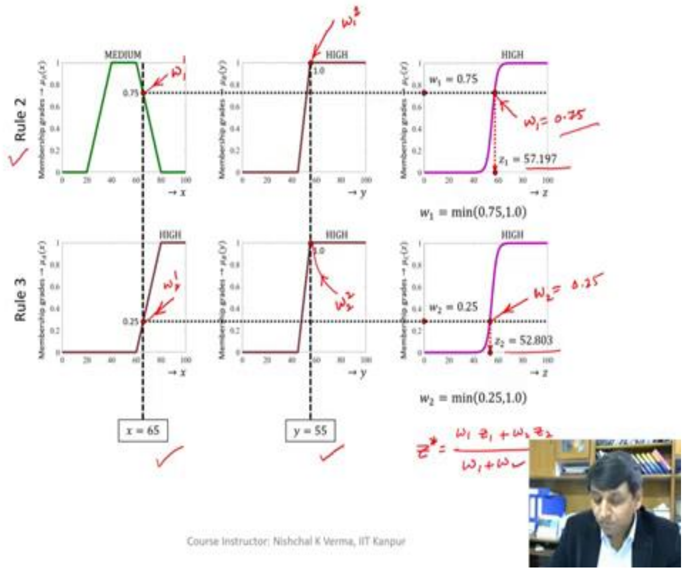

Now, this is very simple we have already done this rule number 2 we have medium high and high. So, when we have the input 65 we see the points of intersection see here. So, this I can write w_1^1 and similarly corresponding to y input we get the w_1^1 w_1^2.

So, our first step was let me just recap the first step was to get the points of intersection in the fuzzy regions, means we need to first find all the combinations corresponding to the input that we are supplying. So, we need to know what are the fuzzy sets that we are getting intersected that are applicable and that is how we make the combinations and these combinations we check with the given rules. So, then from these rules we find that what are the corresponding outputs.

And, so here in the given rules we find we check what are the applicable rules. So, these are the applicable rules here. So, the given rules are normally more than the applicable rules alright.

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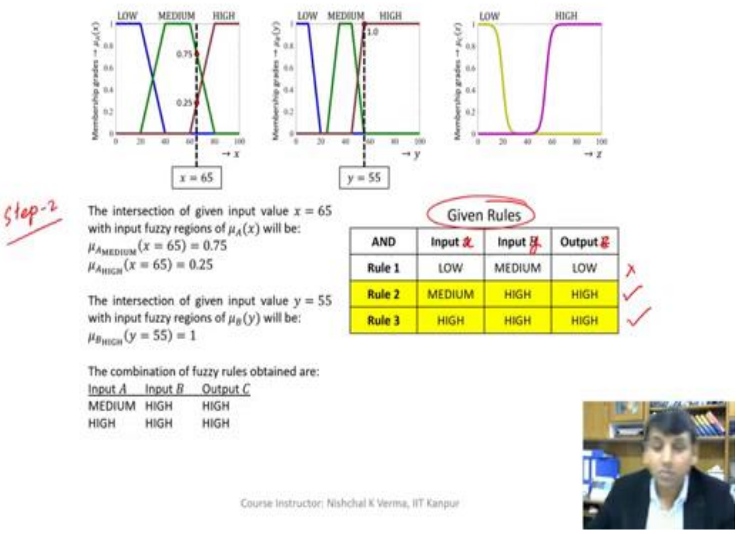

So, this way when we have come to know the what are the rules that are going to be applicable. So, this I can write as the step number 2, next is same as what we have done in lecture like for particular inputs how do we get the output for the case multiple antecedents and multiple rules of Tsukamoto fuzzy model.

So, now, when we apply x is equal to 65, y is equal to 55. So, since rule 1 is not applicable. So, we have not written here only rule 2 is written here. So, we are applying this in these two inputs in rule 2 and then we find w_1^1, w_1^2 and corresponding to this we have here the firing strength of the rule 1, which is nothing but the w_1 is equal to 0.75 will be take min.

Similarly, here this is w_2^1 this intersection is w_2^2 and if we take min of these two we are getting here w_2 which is the firing strength of the rule 2. Now, corresponding to w_1 here the z_1 that is coming out to be 57.197 corresponding to w_2 the firing strength of the rule 2 we are getting z_2 is equal to 52.803.

Now, when we have gotten these two values, these two outputs which are coming from the rule number 2, rule number 3 respectively; now, we take the weighted average of this. So, weighted average will be z^* let us say we are interested in final output. So, final output will be $\frac{w_1 z_1 + w_2 z_2}{w_1 + w_2}$; let us now, quickly here we are here.

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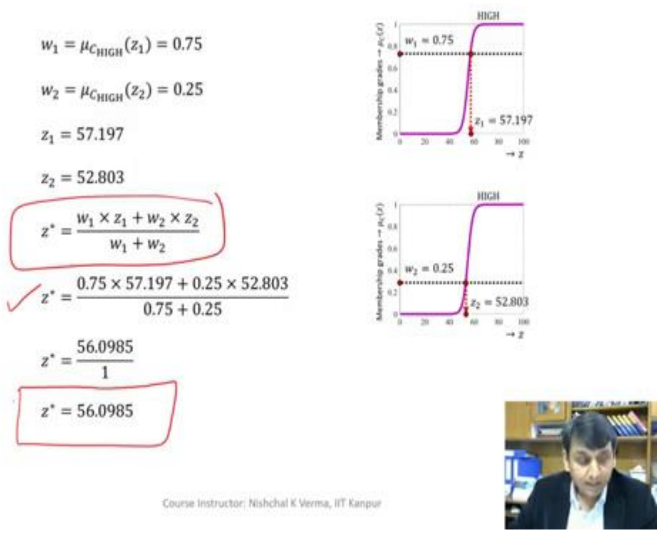

So, let us now, quickly substitute these values and our z star becomes here finally, z^* is 56.0985. So, this is the output corresponding to the input that we have supplied. Here one thing that needs to be noted here is that even if we apply even if we use rule number 1 our inputs that are here x is equal to 65, y is equal to 55 it is not cutting it is not giving any intersection point. So, that is why we have through that step 2 through step 1 in step 2 we have already filtered that. So, we are not going to get any point of intersection even when we use rule number 1.

So, in this process right from the beginning we have found out what are the rules that are applicable, where we are getting the points of intersection. So, this way using rule number 2 rule number 3, we are getting our z star is equal to the final output is equal to 56.0985 corresponding to the crisp inputs x is equal to here 65 and 55.

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So, this way we have seen that as to how we can use Tsukamoto fuzzy model to find out the output corresponding to the crisp input or fuzzy input. So, with this I would like to stop here. And in the next lecture I will discuss the TSK fuzzy model, TSK fuzzy model is nothing but the, Takagi-Sugeno and Kang fuzzy model.

Thank you.