Applied Linear Algebra for Signal Processing, Data Analytics and Machine Learning
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Lecture 24
Wireless application: Multi-antenna channel estimation

Hello, welcome to another module in this massive open online course. So, we are looking at least squares and its various applications. Let us look at yet another application of least squares that is in the context of channel estimation.

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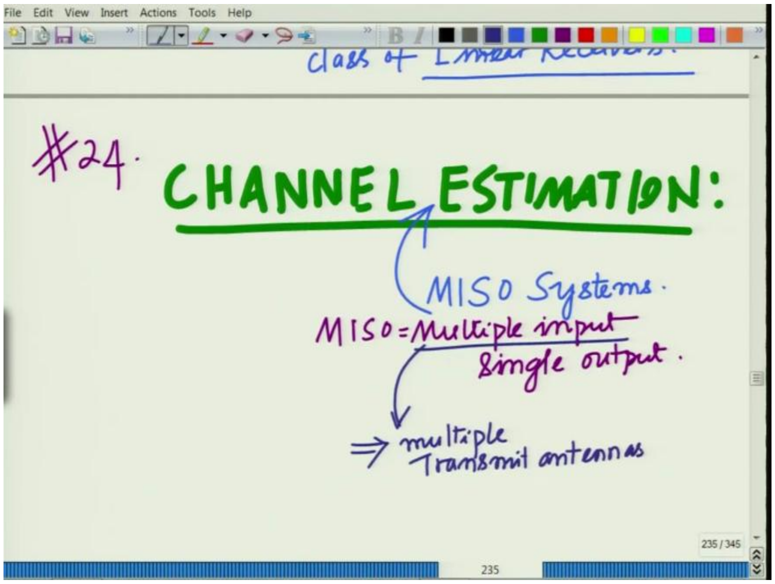

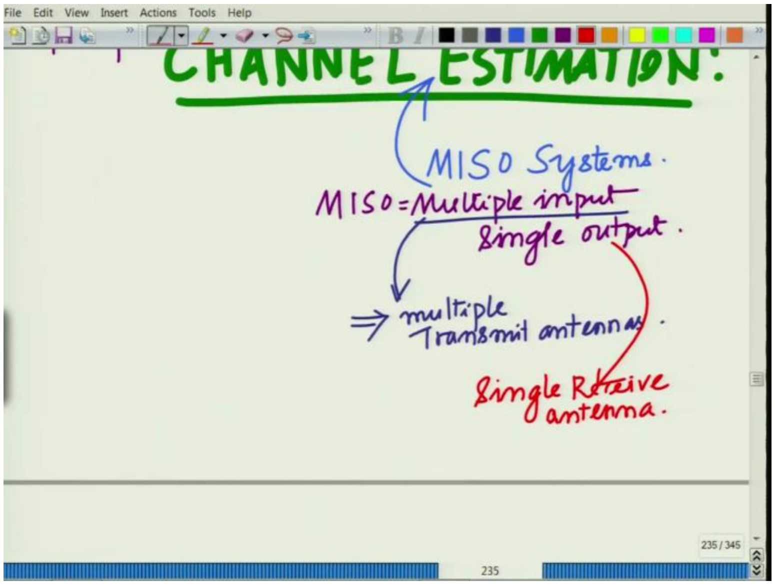

So, in a wireless system another important application of least square is what is known as channel estimation. You can also think of this as system identification. We will in particular look at channel estimation for MISO systems. Some of you might have already guessed MISO stands for Multiple Input Single Output, just like you have SIMO, you have MISO multiple input single output this means that we have multiple transmit antennas at the transmitter in a wireless communication system and a single output that is a single receive antenna. So, this is the meaning of this, so multiple input, this implies multiple transmit antennas and single output, this implies single receive antenna. So, the system looks like single receive antenna.

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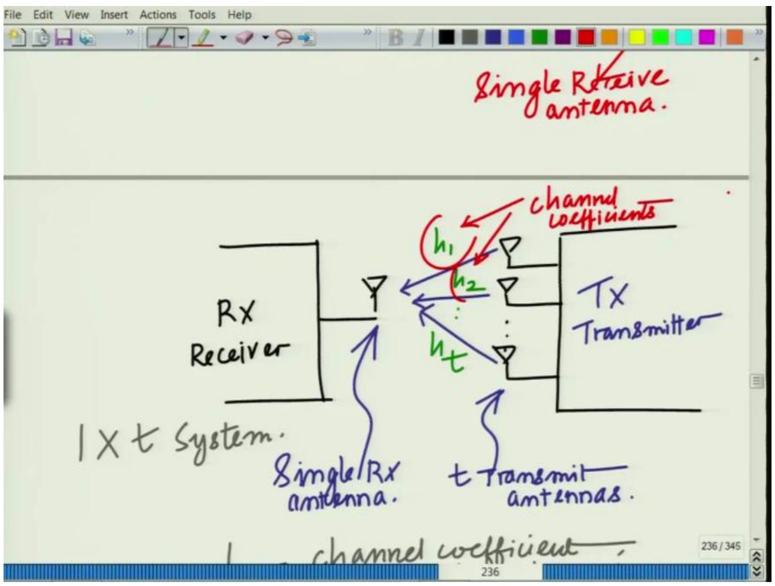

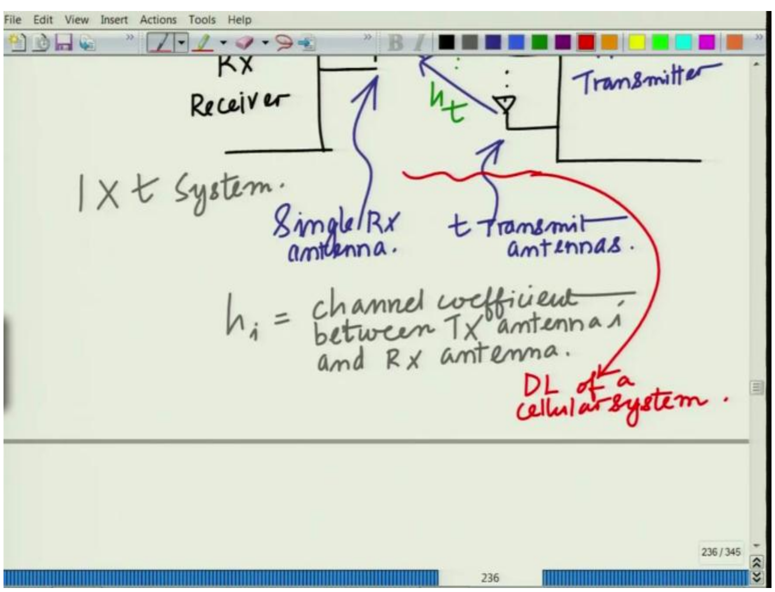

So, essentially the system looks like you have the wireless communication system, you have the receiver which has a single antenna and you have transmitter which has multiple antenna and between each transmit antenna and the receive antenna you have a channel coefficient. So, let us say you have t transmit antennas and you have single receive antenna.

You have t transmit antennas and single receive antenna, and these are the channel coefficient h1, h2, ht. So, you can think of hi as the channel coefficient. hi is the channel coefficient between transmit antenna i and the single receive antenna. So, let me write that, hi, is the channel coefficient between the transmit antenna i and the receiver antenna. So, this is essentially our system model and this is also referred to as 1 cross t system.

So, 1 is basically the single receive antenna and t is the number of transmit antennas and these are the channel coefficients. That is what we are saying, that is this hi, h2, ht. These are your, these are basically, these are the channel coefficients. So, you can think of this typically as a downlink scenario. Where the base station, for instance the base station typically which is larger can have many more antennas and its transmitting to a mobile, let us say, that has a single antenna so you have multiple transmit antennas at the base station and a single receive antenna at the mobile.

So, it becomes a MISO system. Multiple Input Single output system. So, you can think of this as the, typically you can think of this as a downlink of a or you can also think of this as a modem to laptop, Wi-Fi router to your laptop kind of a system where the router has multiple antennas and your laptop has a single antenna so on and do forth.

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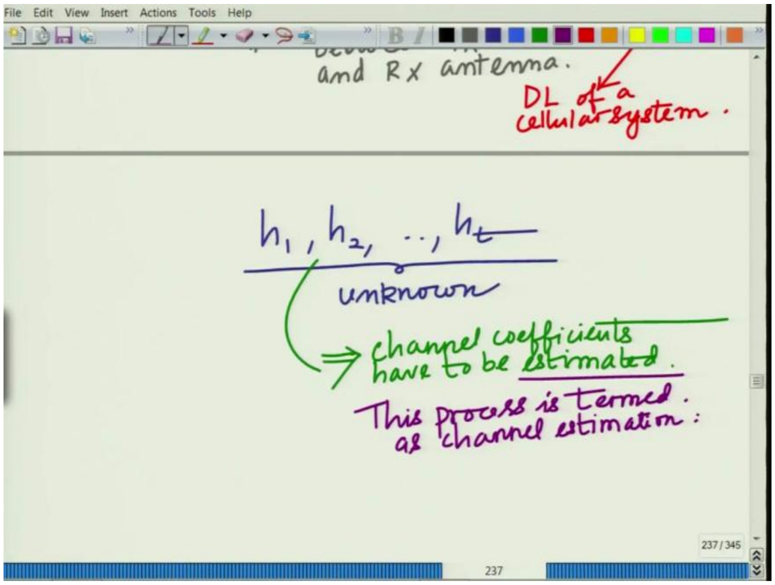

Now, what happens is these channel coefficients h1, h2, ht, these channel coefficients, these are typically unknown. Now, since these are unknown, these channel coefficients which are unknown, these have to be estimated and this process is termed as channel estimation because at the receiver one needs the knowledge of the channel coefficients for the decoding of the symbols transmitted symbols.

So, the channel coefficients have to be estimated, this is termed as channel estimation. So, these are unknown. So, this implies that the channel coefficients have to be estimated. One has to estimate the channel coefficient, determine the channel coefficients and this procedure, this process of estimating the channel coefficients is termed as, this is termed as channel estimation.

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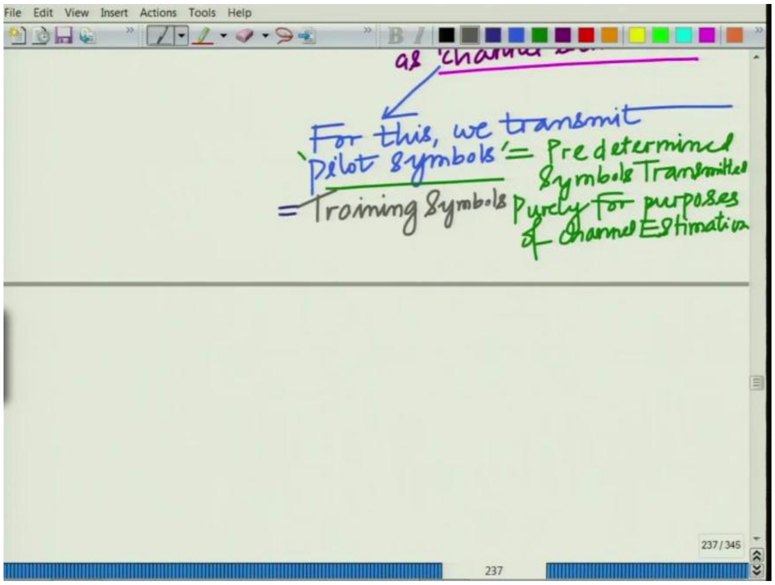

And for channel estimation we transmit pilot symbols. Pilot symbols are essentially, what are these pilot symbols, which are known symbols transmitted by the transmitted purely for the purpose of channel estimation. So, these are not information bearing symbols, these are a set of predetermined symbols that are transmitted by the transmitter which are also known at the receiver purely for the purpose of channel estimation.

So, these are, you can think of this are predetermined. So these are predetermined symbols transmitted purely for the purposes of channel estimation. So, these are the pilot symbols these are transmitted purely for the these are also known as training symbols. So, these pilot symbols frequently in wireless communication, these are also known as training symbols and in fact, the sequence that we are transmitting the set of pilot symbols or the sequence of pilot symbols, this is known as the training sequence.

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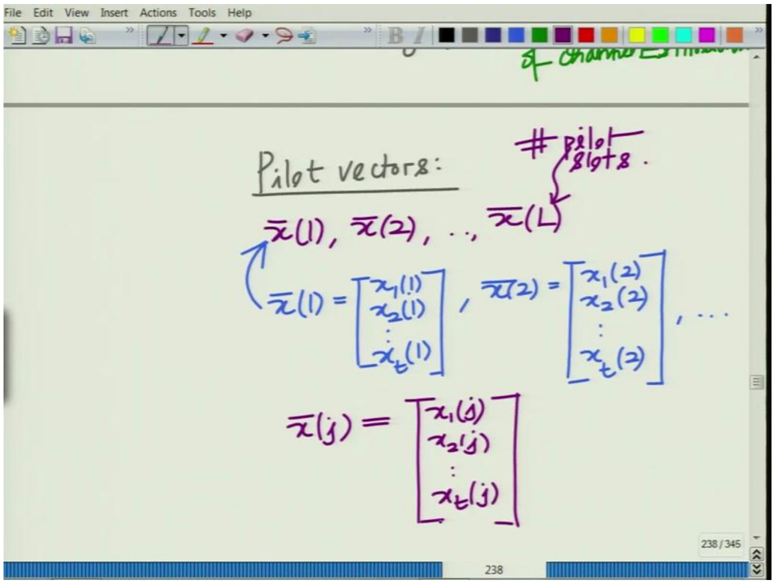

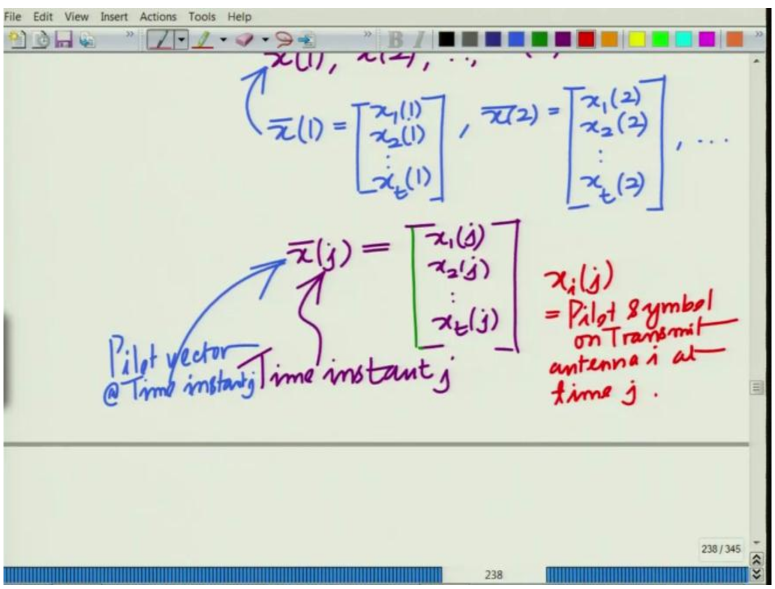

So, let us say, we transmit the pilot symbols, we transmit pilot vectors rather, x bar 1, x bar 2, x bar L where L is the number of pilot vectors or the number of pilot, you can think of or roughly this is also called as the number of pilot symbols, although each symbol is a vector and we can further write this, x bar 1 as x11, x21, xt1, similarly, x bar 2, if you just x12, x22, xt2 and so on. And therefore, what is this x bar j? This is a pilot vector transmitted at time instant j, this comprises of x1j, x2j and so on up to xtj, this is a t dimensional vector where, so this is the time instant j, this is the pilot vector at time instant jx bar j.

This is the pilot vector at time instant j and these are the pilot symbols. In fact, if you look at xi of j, this is the pilot symbol on transmit antenna, transmitted on transmit antenna i at time j. So, we have L pilots vector x bar 1, x bar 2 so on up to x bar L and x bar 1 in turn contains the symbols x11, x21, xt1. So, if you look at this essentially the vector x bar j, that is the pilot vector transmitted at time instant j comprises of the symbols x1j, x2j so on up to xtj where xij is the symbol transmitted on antenna i at time instant j.

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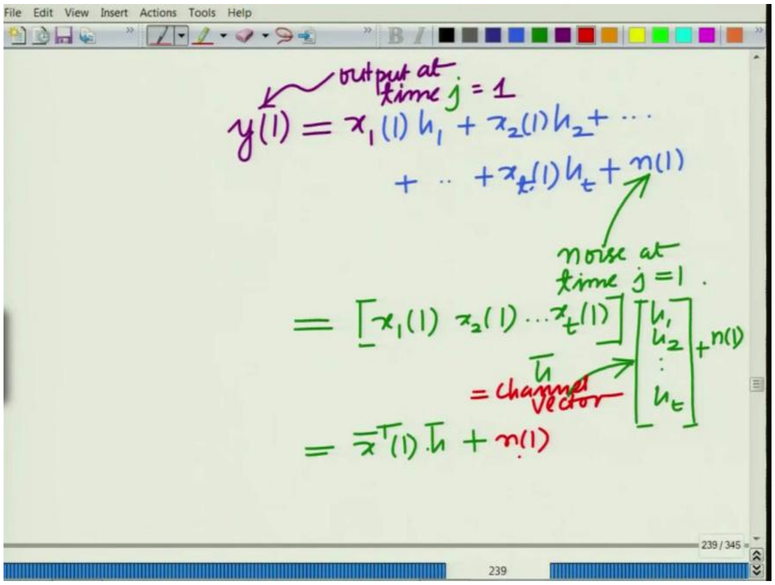

So, that is the notation and it is very simple to reasonably simple so we can model the output y1 as that is the output at time 1, this is equal to x11 times h1 plus the symbol on transmit antenna 2 at time instant 1 times h2 plus xt1 at ht plus n1. So, essentially which and this is the noise sample, this is the noise at time t equal to 1 which you write as, now this is interesting, you can write this as the vector x11, x21, I am sorry, I can call it, time j equal to 1, I think that would be better at time, because t we are using to denote the number of transmit antennas at time j equal to 1 and times, h1, h2, ht plus n1 which is nothing but x bar 1 transpose h bar, this is our channel vector h bar, this is h bar which is equal to the channel vector, this is your channel vector plus n1.

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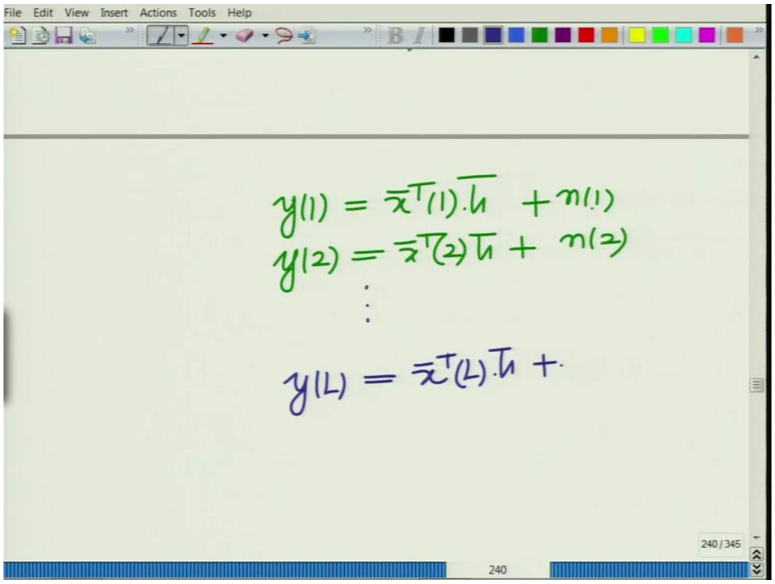

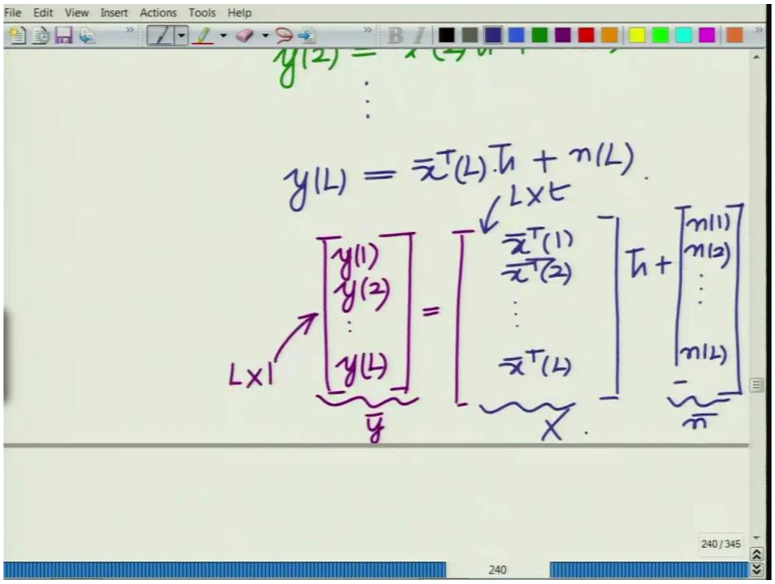

So, the output at time instant 1 equal to x bar transpose 1h bar plus n1, similarly the output at time instant 2 equals x bar transpose 2 h bar plus n2 and so on and so forth, the output at time instant L, this is equal to x bar transpose L into h bar plus nl and therefore, putting these things now as a vector, you have y1, y2, yl. This is an L cross 1 vector you can call this as y bar this is equal to x bar transpose 1, x bar transpose 2, x bar transpose L. You can see this is L cross t matrix times h bar plus we have n1, n2 up to nL. This is the matrix n bar and this is the matrix X and this matrix X.

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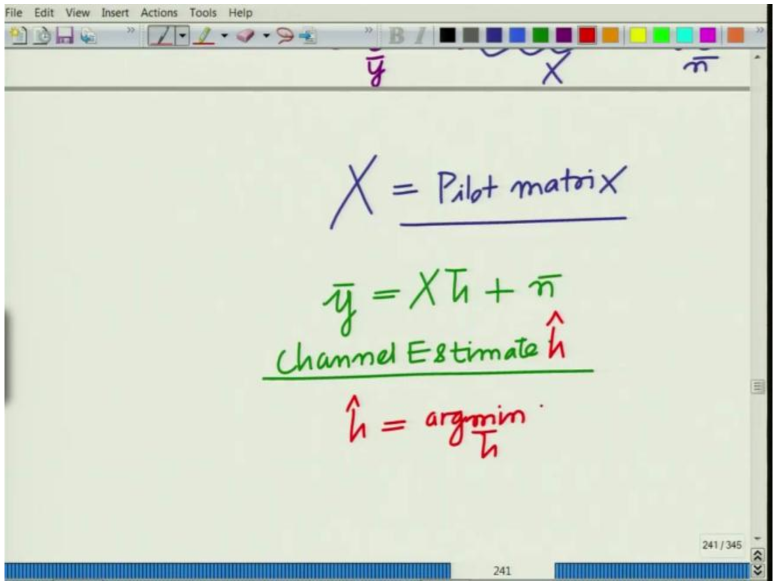

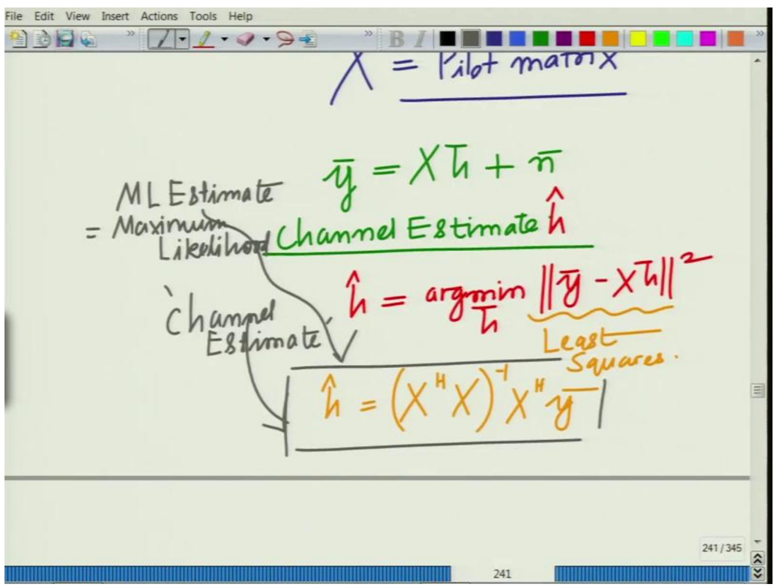

So, this matrix x which contains the pilot vectors, this is termed as the pilot matrix. So, you have the output vector y bar which is equal to x which is your pilot matrix that is of size L cross t times h bar which is the channel vector which contains the channel coefficient h1, h2, ht plus the noise vector. And, therefore, once again you have this interesting model, y bar equal to x h bar plus n bar where x is the pilot matrix.

Now, the channel estimate for this situation can be determined as, let us denote this by h hat, now h hat can be denoted once again defined as the h bar that minimizes the least squares cost function, you can see once again, this is your essentially least squares. This is your least squares cost function and therefore, your h hat is once again given by x Hermitian x inverse x Hermitian y bar. So, this is essentially your channel estimate.

And this is how the least squares can be also used for channel estimation in wireless communications systems so as we are saying least squares has a wide variety of applications, we have seen previously the equalizer or basically the 0 forcing receiver in a MIMO system and now you are seeing. Now you are seeing, now we will look at another important application which is to determine the unknown channel coefficient.

In fact, this is also can be derived much more rigorously, this is termed as the noise samples are independent Gaussian, it can be shown that this is the estimate which has the maximum likelihood. So, this is termed as the ML estimate that is this least square estimate, this is termed as the ML estimate, this is equal to the maximum, this is basically your maximum likelihood estimate this is termed as the maximum likelihood estimate.

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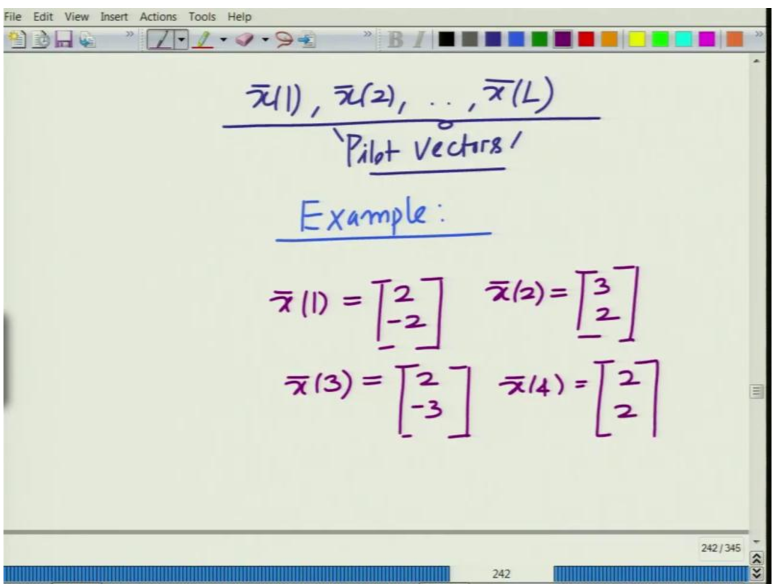

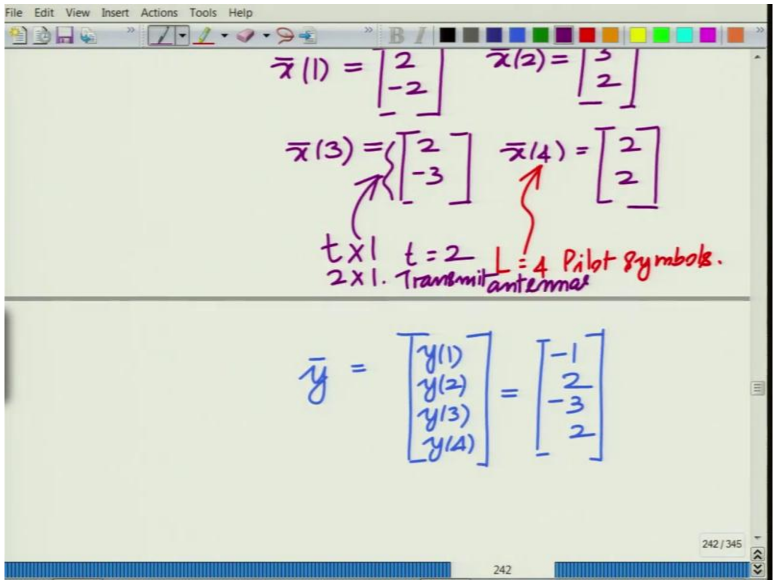

And we have already seen that these vectors x bar 1, x bar 2, x bar L, these are the pilot vectors. So, this is the maximum likelihood estimate. What we are essentially doing in this is, transmitting a set of pilot vectors, pilot containing the pilot symbols and using this knowledge of this predetermined or known pilot vectors at the receiver so the inputs are known that is the pilot vectors, the outputs are the observations, that is observation vector y bar using the outputs and the known inputs one can estimate the channel. The h bar which comprises of the channel coefficient and this is essentially the process of channel estimation.

Let us look at a simple example to understand this better, consider a simple example to understand this channel estimation process better, for instance x bar 1, this is equal to 2 minus 2, pilot vector at time instant 2, this is 3 comma 2, pilot vector at time instant 3, this is 2 comma minus 3 and pilot vector at time instant 4, this is 2 comma minus 3 and pilot vector at time instant 4, this is 2 comma 2.

And the observation vector y bar, let us say which is equal to y1, y2, y3 and y4, this is let us say, equal to minus 1, 2, minus 3, 2. So, we can see the number pilot symbols L equal to 4, L equal to 4 pilot symbols and this is basically your t cross 1 vector, you can clearly see t equal to 2 transmit antennas. So, we have 2 equal to 2 transmit antennas, L equal to 4 pilot vectors. Let us now form the pilot matrix.

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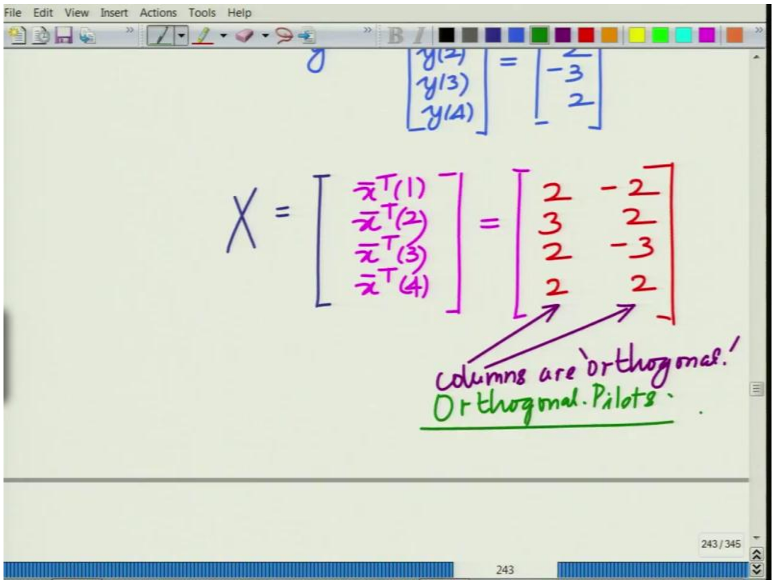

We know that the pilot matrix is given as x bar transpose 1, x bar transpose 2, x bar transpose 3, x bar transpose 4 which is, if you write this down 2, minus 2, 3, 2, 2, minus 3, 2, 2 and you can see this satisfies an interesting property columns are orthogonal, this is not always true but columns are orthogonal, that is if you take the dot product of the columns, it is 0.

Typically, such a pilot is usually employed in wireless communication system because it makes channel estimation easier that is the pilot symbols transmitted, the sequence of pilot symbols transmitted from the 2 transmit antennas are orthogonal and this is termed as an orthogonal pilot sequence, it is very efficient and convenient for channel estimation, we are going to see why very soon. So, this is termed as an orthogonal pilot sequence or this is basically termed as orthogonal pilots because the columns are orthogonal.

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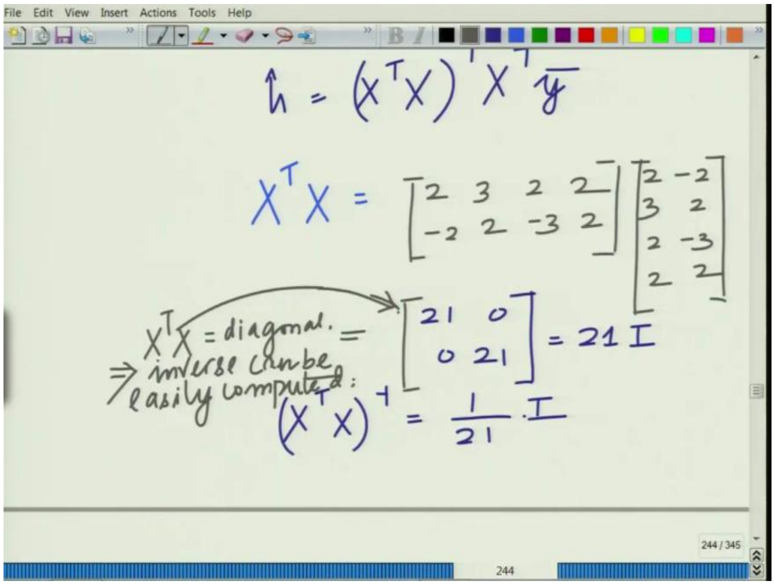

Now as we have seen the channel estimate h hat is given by X transpose X inverse x transpose y bar, let us now evaluate X transpose X, this is equal to 2, 3, 2, 2, minus 2, 2, minus 3, 2 times 2, 3, 2, 2, minus 2, minus 3. Now, if you look at this, you will notice something interesting, this is equal to, this will be 21, 21 diagonal matrix, this is 21 times identity which implies that X transpose X inverse equals 1 over 21 times identity. Now since the pilot sequences are orthogonal X transpose X is a diagonal matrix.

Therefore, X transpose X can be easily inverted, this makes it very computationally efficient or computationally much more easier to evaluate the channel estimate, therefore, the orthogonal pilot sequence is usually preferred for many advantages, one is because one can show that general estimation error is lower and more importantly, computationally much easier to evaluate the channel estimate, when the pilot sequence is orthogonal. So, this X transpose X. X transpose X equals its diagonal implies the inverse can be easily computed. So, one can easily compute the inverse.

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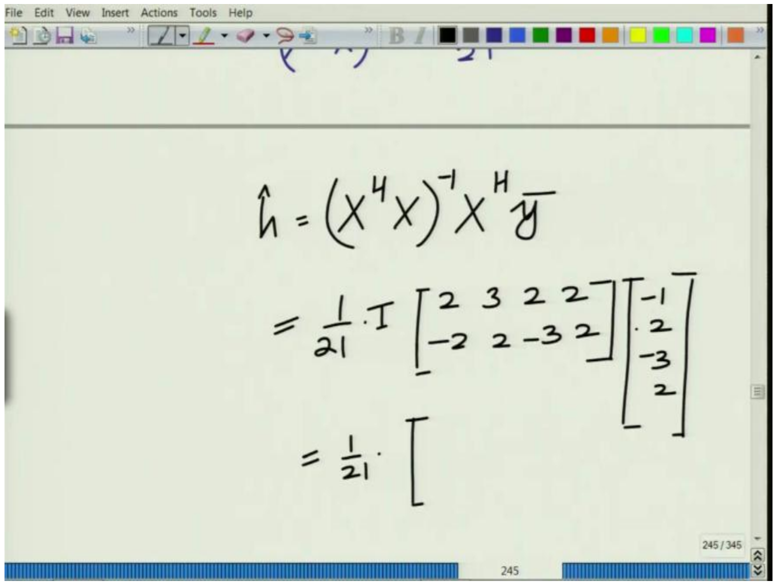

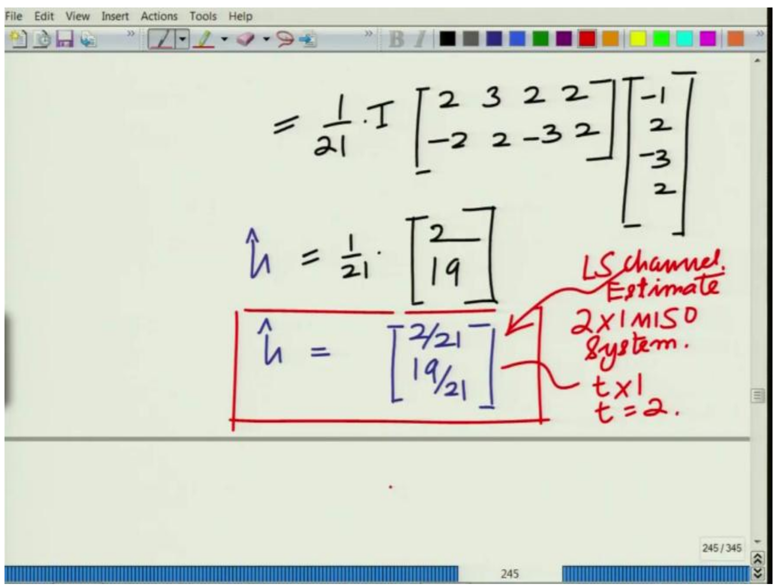

And therefore, now you have your h hat equals X transpose X inverse. In fact, I am replacing the Hermitian by transpose as you can see because all the quantities are real. Otherwise you can also write this as Hermitian, it does not really matter in this case, because the quantities are real so this one by 21 times identity into x Hermitian or in this case x transpose y bar 2, 3, 2, 2, minus 2, 2, minus 3, 2 times y bar which is minus 1, 2, minus 3, 2 and you can see this is 1 over 21 times minus 2 plus 6 that is 4, 4 minus 6 minus 2, minus 2 plus 4 so that is 2, 2 plus 4, 6, 9 plus 6, 16, 15 plus 4 which is 19 so essentially your channel estimate is, this is your channel estimate.

So channel estimate is once you evaluate the least square channel estimate this is 2 by 21, 19 by 21. This is your LS channel estimate and we are considering, we that you are reminded, we are considering a 2 cross 1 MISO system which is also obvious in this because the channel vector is of dimension 2 cross 1, this is t cross 1 so t equal to 2 transmit antennas and you have a single output or a single AC antenna.

So, this is a simple example that illustrates a practical, yet another practical application of the least squares principle which is essentially for channel estimation and you can think of this also in general as system identification. So channel is basically an unknown system. So, now you can generalize it to any unknown system, in fact, we can think of this as linear time invariant system which is like a black box, you treat it like a black box.

We do not know what is this so to estimate this, one can transmit a probing signal, so the channel is, the pilot vectors in that context will be the probing signals and then you will have the observed outputs and using the knowledge of the probing signals and the output one can identify the system which is the black box that is sitting in between.

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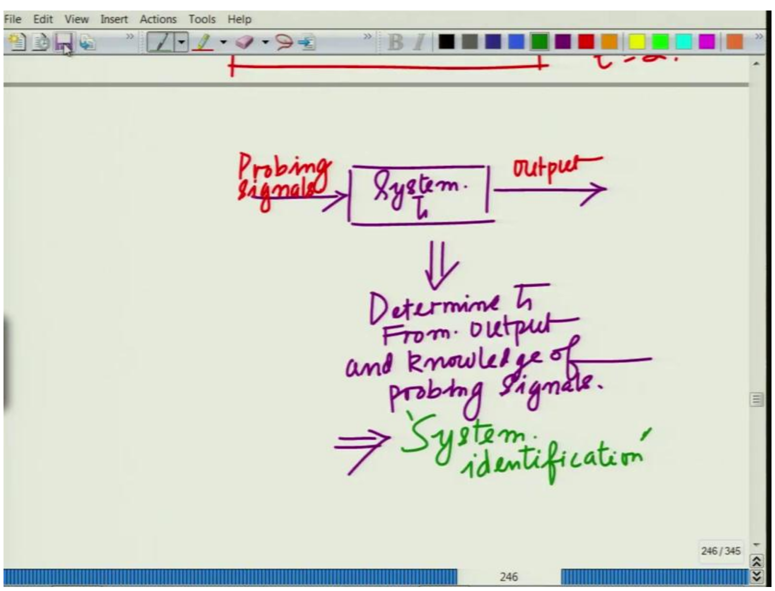

So, just to illustrate this point you have, so if you generalize this the system which we are characterizing by xh bar which is unknown and you have the output and you have the probing signals similar to the pilots, you have the probing signals and from knowledge of output and the probing signals one can determine the system h bar, so determine h bar from the knowledge of output or you can say from output and knowledge of probing signals, from output is the observation output and knowledge of probing signals.

And this is basically termed as your system identification. So, what we have seen is basically a channel estimation which can broadly think of this as a linear system. So, it basically corresponds to a broad class of problems which one characterized as linear system identification. In fact, this can be very easily extended to non-linear system identities.

So, the principle is same, transmit the probing signals, observe the output, from the output, knowledge of the probing signals, one can identify the systems so it belongs to a, and least squares can always be used in such applications and it is extensively used in such applications that is for channel identification, system identification and so on and so forth. So, this is yet another interesting applications of least squares, so we will stop here and we will continue in the subsequent modules with looking at other such aspects. Thank you very much.