Applied Linear Algebra for Signal Processing, Data Analytics and Machine Learning

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Lecture No. 03

Inner product application: Beamforming in wireless communication systems

Hello, welcome to another module in this massive open online course. So, we are talking about the inner product and the correlation between 2 signals. Let us look at another interesting example of this inner product that is with respect to the beamforming that is forming a narrow beam in a wireless communication system with multiple antennas.

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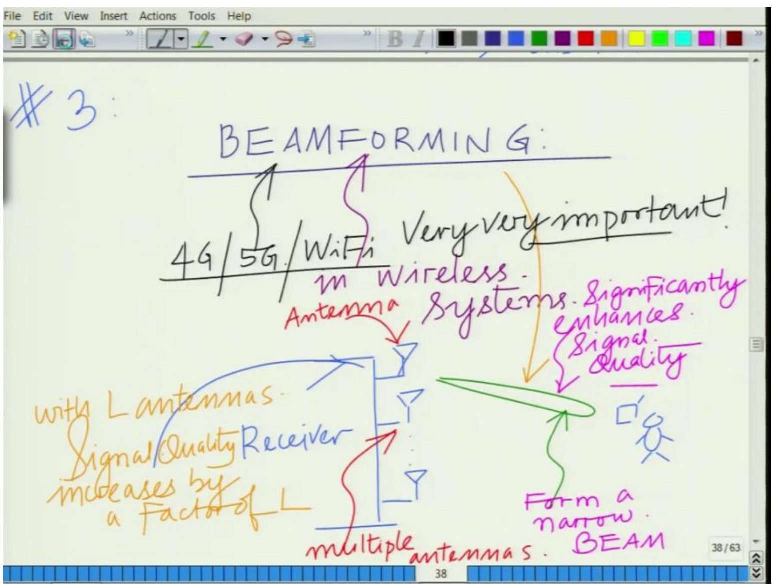

So, another interesting example of another very important, that is, essentially beamforming. So, what is beamforming? Beamforming is essentially what we mean by this is, when you have the receiver in a communication system and we have multiple antennas at the receiver and we have a user who has let us say a cell phone and then what? So, these are the antennas, so each of these is an antenna. You have several antennas, that is, you have these multiple antennas, so beamforming is only possible when you have these multiple antennas.

And you can see that you can form a narrow beam towards a particular user, that is form a narrow beam what we term as a beam and this enhances the signal to noise power ratio, enhances the signal quality and this is a very important, it significantly enhances the signal quality. So much so that if you have essentially L antennas, you can show that the signal to noise power ratio that is a signal quality can increase by a factor of L, which means your signal reception is going to be much better when you have multiple antennas. And you can do the optimal signal processing or the optimal beamforming with these multiple antennas. This is a very important and all this is important because of, what we call as, and this process is essentially what we were terming was beamforming, that is forming this narrow beam towards a particular user.

So, the narrower the beam, better the signal quality and this process of forming this narrow beam, this is essentially what we are terming as beamforming and which is a very-very important aspect in modern wireless communications especially if you look at 3G, 4G, 5G. 5G is all the important because in 5G you have a very large number of antennas, Massive MIMO systems or millimeter wave beam.

So, we will talk about some of these things, so the largest antennas which are basically hundreds of antennas, so naturally beamforming becomes very-very important. Especially as the number of antennas becomes large, so this is a technology, so this beamforming is a technology that is actively used in I would say 4G and also 5G and also, we can also say to some extent Wi-Fi, so something that is very-very important.

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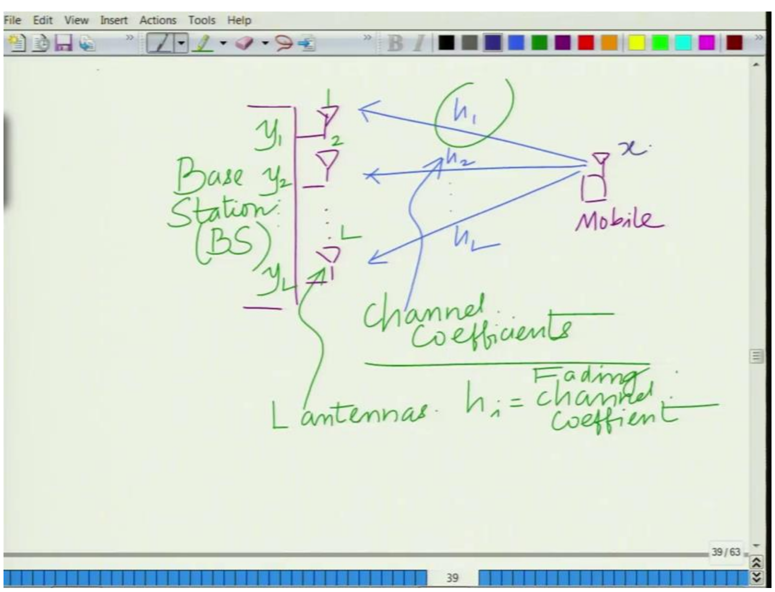

Excellent! So, how do you model this beamforming system? Let us look at a system as I said with L receive antennas, so you have the L receive antennas and you have the mobile user with a single antenna. So, this is your mobile, let us say this your base station which we also call as the BS and then what happens is you have the channels between the mobile and the base station, these are denoted by this channel coefficients $h_1, h_2, .., h_L$, so these are basically what we are calling as the channel coefficients, these are essentially what we are calling as the channel coefficients.

So, these are L antennas, so this is antenna 1, antenna 2, so on antenna L, so you have the L antennas and h_i is the channel coefficient, in fact one can also call this as the fading channel coefficient and the output symbols on these L antennas, we denote them by $y_1, y_2, \ldots, y_L$ and the transmit symbol is x.

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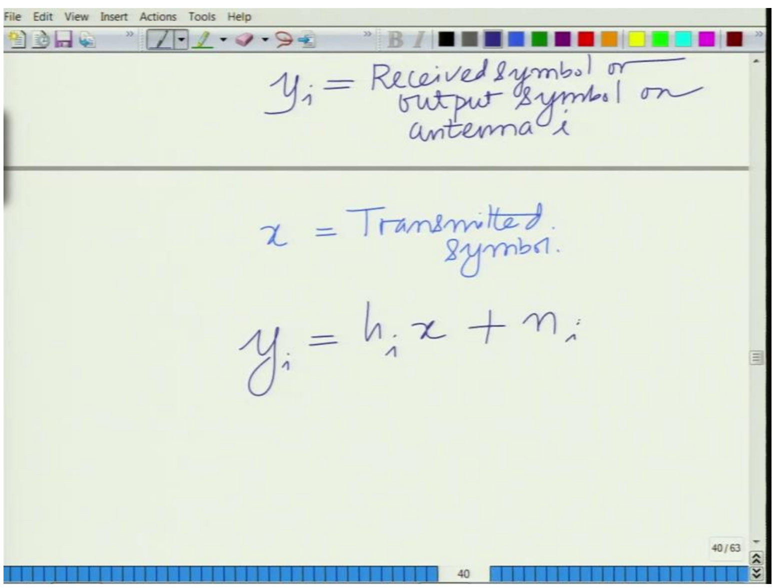

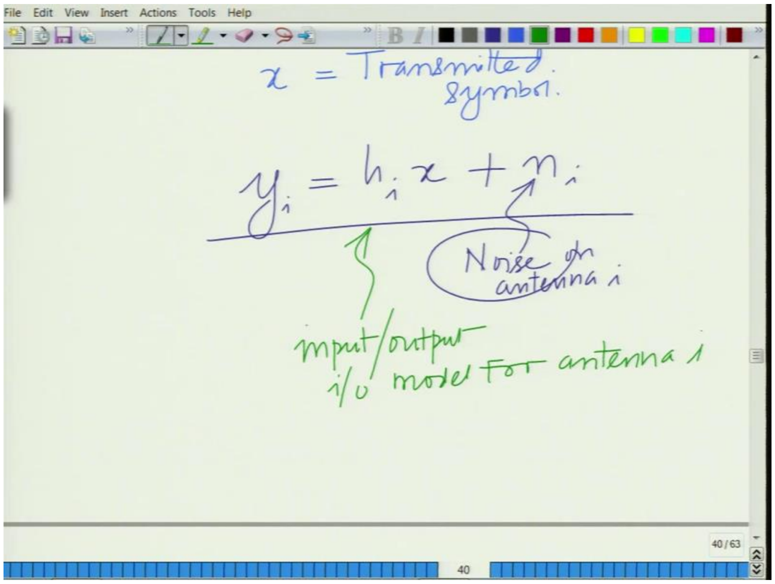

So, y_1 or y_i equals the received symbol, you can say this is the received symbol or you can say output symbol on antenna i, h_i is the corresponding fading channel quotient, x is the transmitted symbol. So y_i is related to x as, essentially $y_i = h_i x + n$, obviously you are always going to have the noise. So, this is the noise on antenna i. So, essentially what you have is you have the model for antenna i, so you have the, this is also called the system model or you call the input, output or you can call the I/O model for antenna i.

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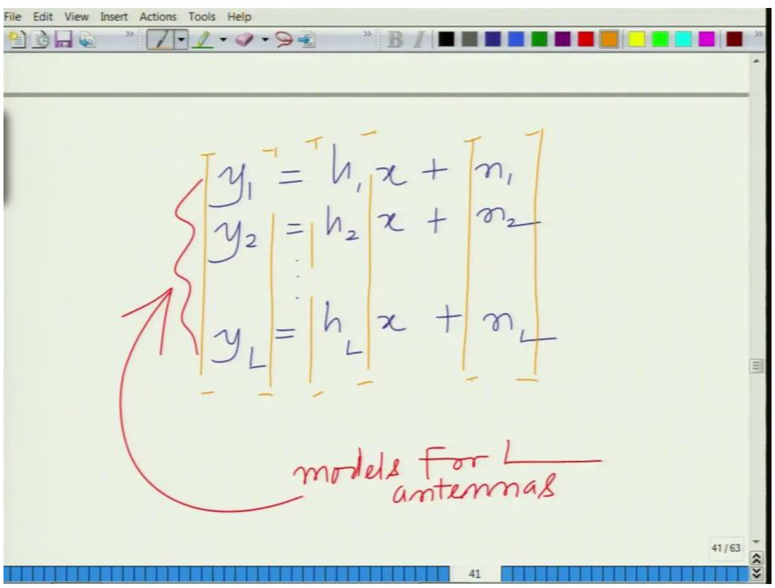

So, essentially you will have a system that is basically if you look at this you will have

$$y_1 = h_1 x + n_1,$$

$$y_2 = h_2 x + n_2,$$

$$\ldots$$

$$y_L = h_L x + n_L.$$

This is the system model for the L antennas. And now we stack this as a vector. So, now interestingly you can put these things as a vector, so this becomes your

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_L \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_L \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_L \end{bmatrix}$$

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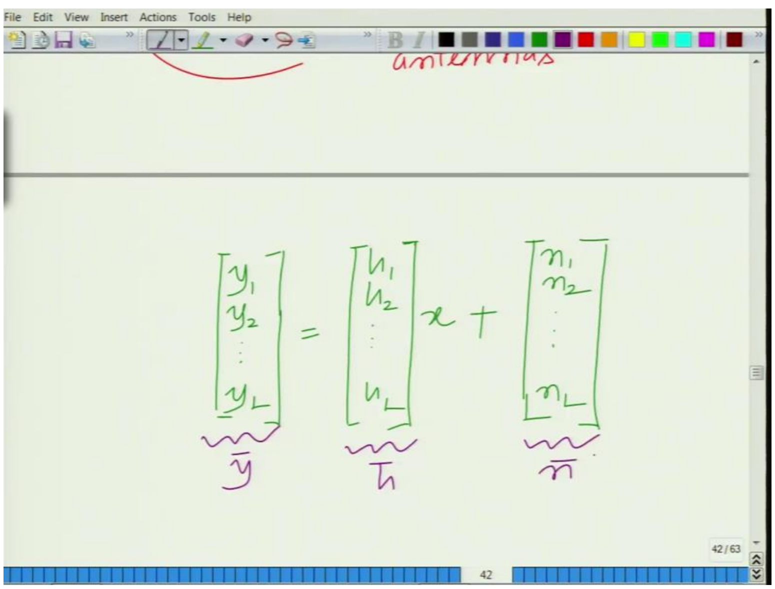
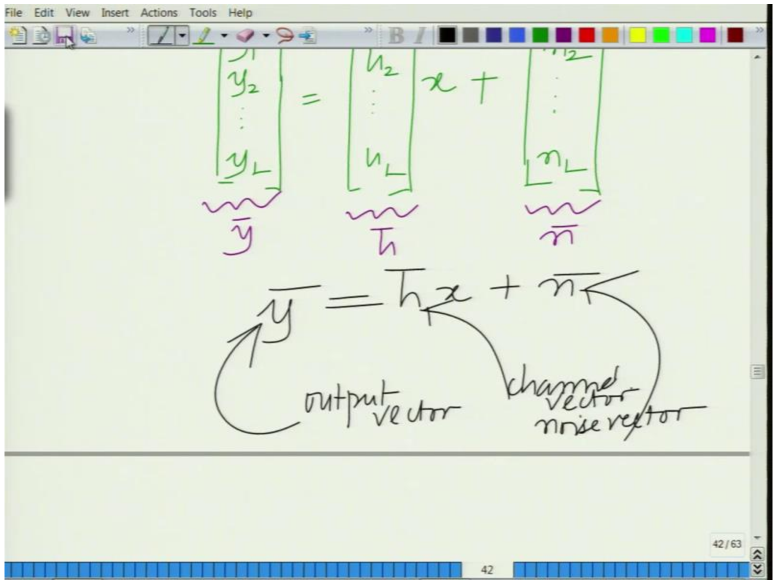


So, essentially you can write this as the vector model, so you have $\bar{\boldsymbol{y}} = \bar{\boldsymbol{h}}x + \bar{\boldsymbol{n}}$. Now, to perform beamforming, remember we started with this idea that you can form a narrow beam and maximize the signal quality or the signal to noise power.



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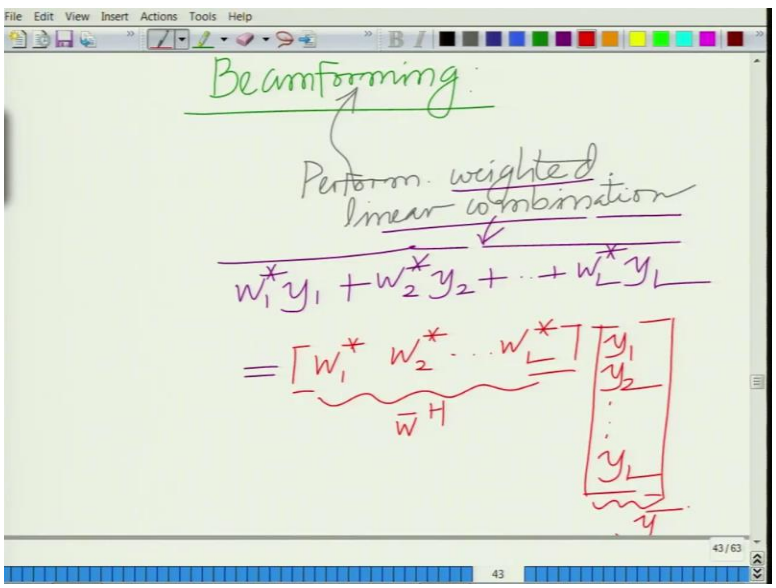

So, to perform beamforming, what we do is essentially we perform what we term as weighted linear combination, that is essentially what we write is? We have

$$w_1^* y_1 + w_2^* y_2 + \cdots + w_L^* y_L$$

So, essentially this is your weighted linear combination of the outputs. Which you can write as the row vector that is your $[w_1^*,\ \ w_2^*,\ \ \ldots, w_L^*] = \bar{\boldsymbol{w}}^H$ times $[y_1, y_2,\ \ \ldots, y_L]^T$, so this is essentially what you call as, write as, $\bar{\boldsymbol{w}}^H \bar{\boldsymbol{y}}$, this is of course your output vector or signal vector $\bar{\boldsymbol{y}}$.

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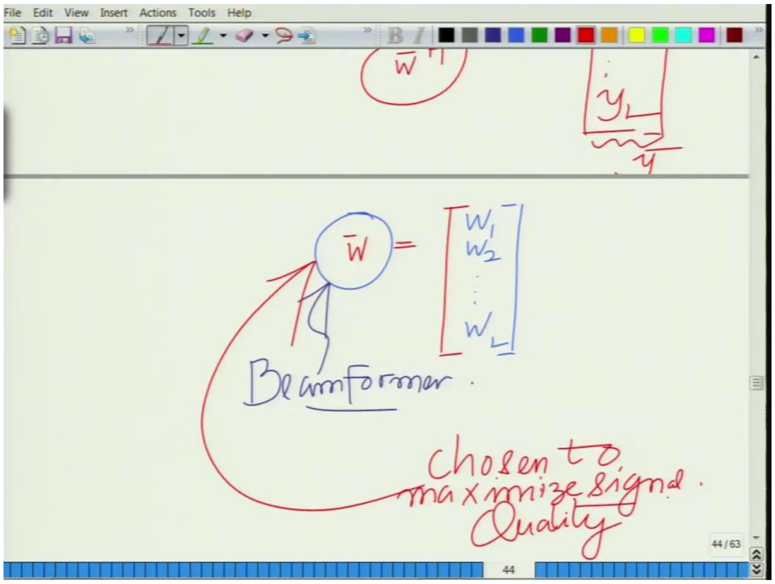

And this $\bar{\boldsymbol{w}}^H$, so what is $\bar{\boldsymbol{w}}^H$? You have this $\bar{\boldsymbol{w}}$ of the weights, so you have the $w_1, w_2, \dots, w_L$ of the weights, you take the transpose and the conjugate of each of them that is column vector becomes a row vector and then it becomes, each element, you take the conjugate of each element. So, this $\bar{\boldsymbol{w}}$ this is called as the beamformer or the beamforming vector. So, this $\bar{\boldsymbol{w}}$ is called as the beam former or the beamforming vector and this can be chosen to maximize the signal quality.

And what you can see over here, it's very easy to see is that this is of course your $\bar{\boldsymbol{w}}^H \bar{\boldsymbol{y}}$, so you are forming the inner product of the beamformer and the output vectors, so this is essentially inner product of the beamformer and the output vector. So, we can clearly see the inner product or the correlation has a very important role to play in beamforming. So, we have the beamform vector or the beamformer $\bar{\boldsymbol{w}}$ and you are forming its inner product with output vector.

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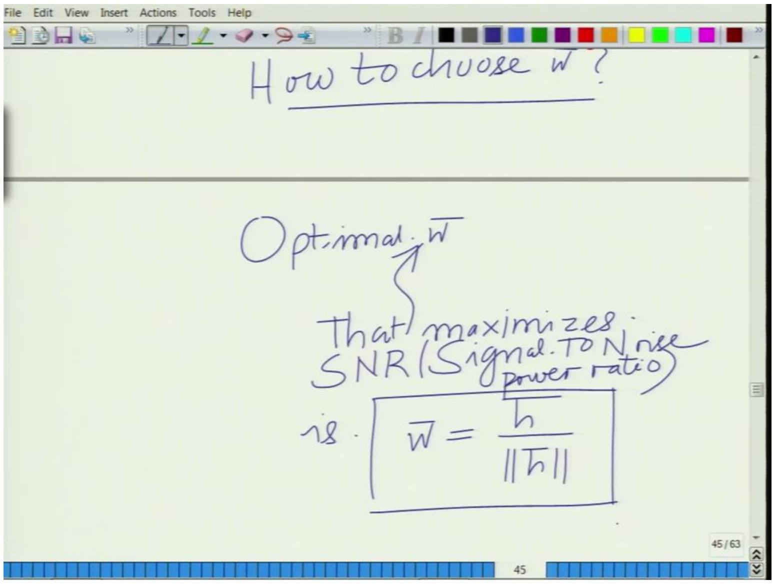

Now, the question is how to choose this beamformer? $\bar{\boldsymbol{w}}$ is termed as the beamformer, now choose the beamformer. How to choose the $\bar{\boldsymbol{w}}$? Well, the optimal $\bar{\boldsymbol{w}}$, what do you mean by optimal, which means the beamformer that maximize the SNR or signal-to-noise power ratio is $\bar{\boldsymbol{w}} = \frac{\bar{\boldsymbol{h}}}{||\bar{\boldsymbol{h}}||}$, you see these are the very interesting properties. This is basically similar to the channel vector but normalized, that is unit norm, remember $\frac{\bar{\boldsymbol{h}}}{||\bar{\boldsymbol{h}}||}$, this is the unit norm vector. And this is similar to the channel and that is matched to actual channel vector that is $\bar{\boldsymbol{h}}$, so this is also known as a matched filter.

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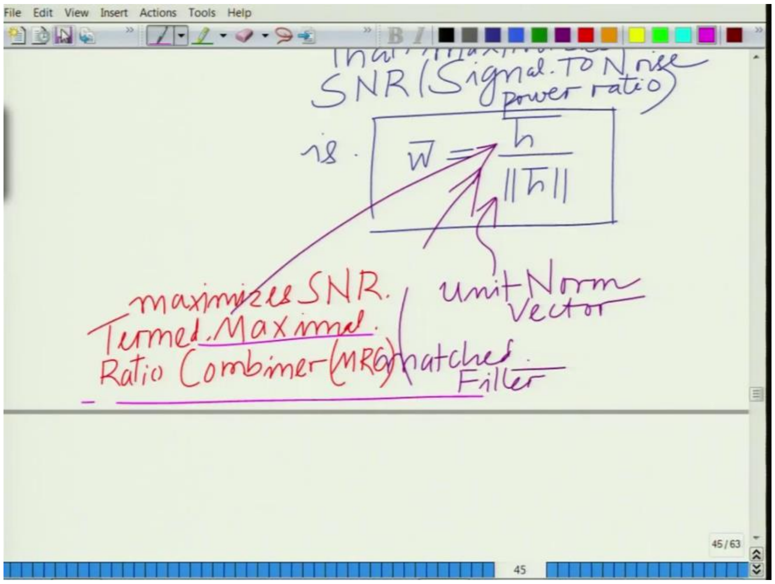

So, essentially what you will see is, this is a unit normal vector and this is matched to the channel. So, this is also a matched filter and other thing interesting aspect is this it maximizes the signal to noise power ratio. Hence, this is termed as it is in literary, and more popularly it is known in wireless communication technology as the maximal ratio combiner (MRC). So, this is simply known as the maximal ratio combiner that is the optimal beamformer that maximizes the signal to noise power ratio at the output of multiple antenna wireless communication system.

We also called that as a SIMO that is Single Input Multiple Output because we have multiple antennas at the receiver or multiple outputs at the receiver and only a single input. Of course, we are later going to see MIMO Multiple-Input Multiple-Output, but here the context of this beamforming can easily understand it from the SIMO system Single Input Multiple Output.

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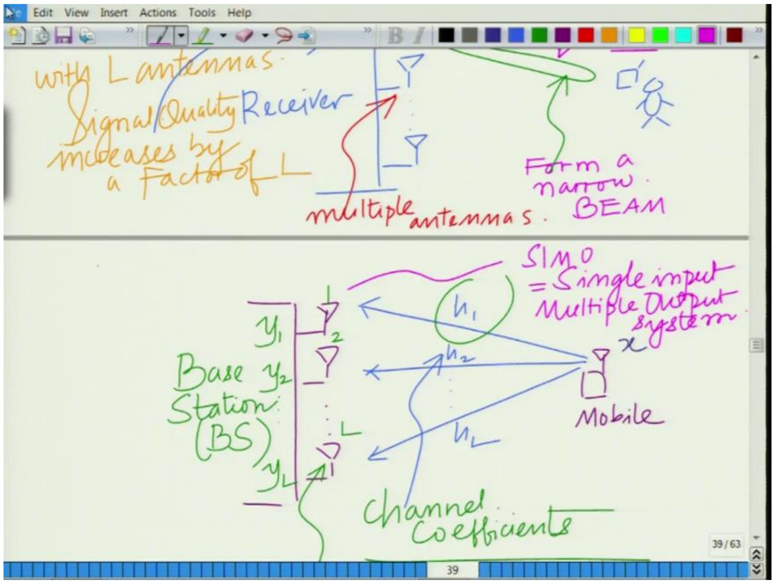

Let me also just mention that because you might find that useful, we go all the way back over here, this is what is called as a SIMO that is single input multiple output, it is very-very important class of wireless communication systems. So, multiple antennas at the receiver.

So, what we have shown is that the optimal beamformer, right? optimal beamformer which maximize this signal-to-noise power ratio is given by the maximal ratio combiner, so this is a very important result a very interesting result. And this has interesting applications in modern wireless communications, especially I told you 3G, 4G, 5G wherever you have multiple antennas at the base station an single antenna handset, I mean multiple antennas at the base station here what we are considering but you are also extend it to a scenario where you have multiple antennas at both the sides.

So, these are the interesting applications and there are a lot more interesting applications of linear algebra and in particular this concept that is the inner product or the correlation between these 2 signals. So, let us conclude this module here and let us continue in the next module. Thank you very much.