Applied Linear Algebra for Signal Processing, Data Analytics and Machine Learning
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Lecture 45
OFDM System model: Transmitter and Receiver Processing

Hello welcome to another module in this massive open online course. So we are talking about OFDM which as I have already told you is one of the dominant waveforms that is used in 4G, 5G cellular standards as well as Wi-Fi and so on and we are trying to understand the implications of linear algebra in OFDM. How can linear algebra be used to model and analyze this OFDM system and the point at which we are in currently is the following thing; we have the OFDM system models.

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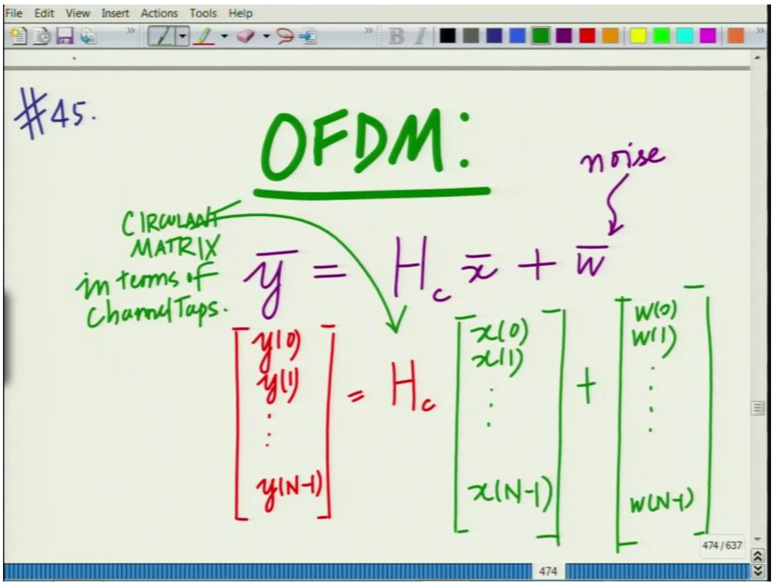

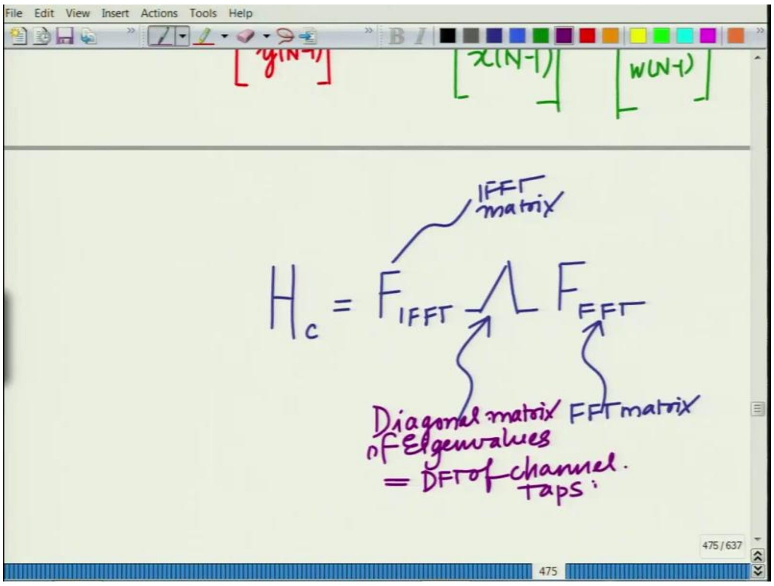

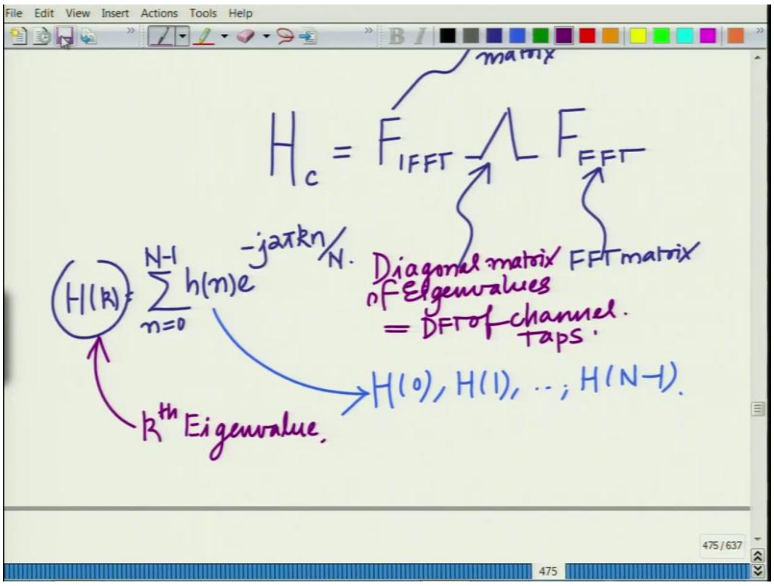

So we have the OFDM, Orthogonal Frequency Division Multiplexing, so we have the OFDM system model that is y bar equal to H c times x bar plus W bar this of course is the noise. We will not dwell upon this too much except to realize that noise is a very important aspect of any communication system. So this if you look at this vector, this is y 0, y 1, y N minus 1.

This is the output. This of course is your H c which we have talked about at length and you have your x bar which is x 0, x 1, up to x N minus 1 plus of course the noise vector W 0, W 1 up to W N minus 1 and this matrix we have seen H c which is a very interesting matrix. This is H c is a circulant matrix. In fact, we have derived the Eigen value decomposition with circulant matrix in terms of the channel taps.

This is the circulant matrix of the channel taps so we have H c and we have seen the Eigen value decomposition of H c, H c can be written as the Eigen vectors are the IFFT vectors and Eigen values are the DFT coefficients of the channel taps. So this is F IFFT times lambda times FFT so this is your IFFT matrix. This is your FFT matrix and this is the diagonal matrix of (Eigen) this is the diagonal matrix of Eigen values which are nothing but the DFT or FFT of the channel taps.

That is, we have these represented by H 0, H 1 up to H N minus 1 where H of k is nothing but the DFT, kth DFT point. So this will be n equal to 0 to N minus 1 h n e raised to minus j 2 pi k n divided by N. These are the Eigen values. This is the kth Eigen value. This is the kth Eigen value. Let us now use this Eigenvalue decomposition of this circulant matrix to analyze the OFDM system.

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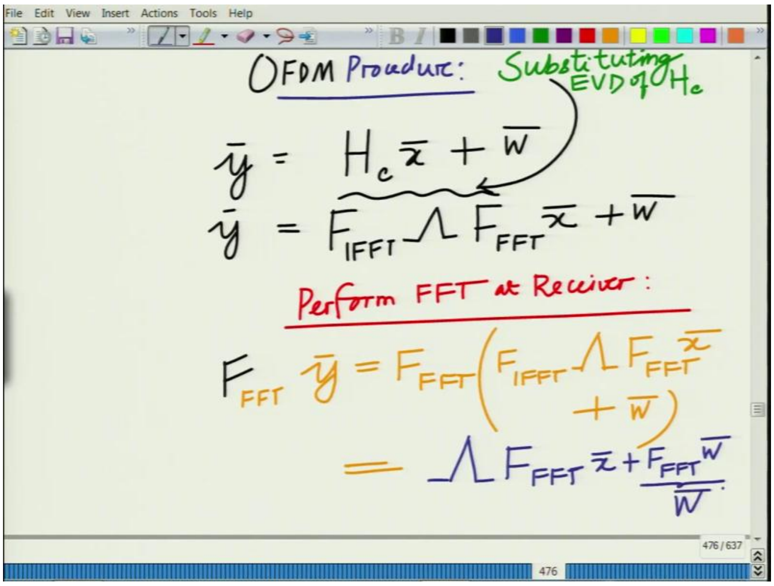

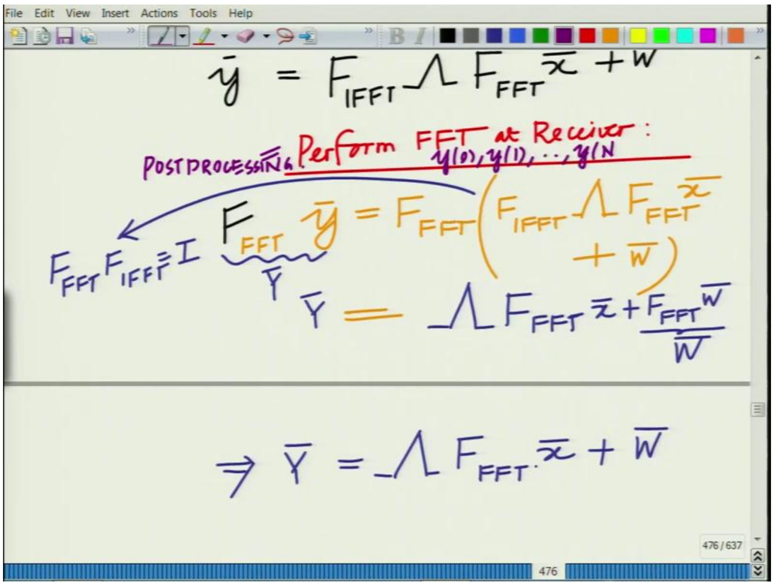

So, we start with this Eigen value decomposition y bar equals H c x bar plus W bar. Substitute the Eigen value decomposition for H c. So this is F IFFT times lambda times F FFT times x bar plus W bar. So, what we are doing here? Substituting the EVD of, so we are substituting Eigen value decomposition of H c. Now the next step, so this is we are talking about the OFDM procedure.

Now perform the FFT at the receiver, we perform, in OFDM what we do is we perform the FFT of the samples, perform the FFT at the receiver which is basically as we all know represented by F FFT times y bar. So this will be equal to F FFT times whatever is there here that is F IFFT times lambda times F IFFT times lambda times F FFT times x bar plus W bar, which is equal to FFT into F IFFT, now that is identity we know that so this, what we are left is simply lambda times F FFT times x bar plus F FFT times W bar. We will call this as the output noise of the FFT capital W bar.

So here we are using the property, we are using the property. So at F FFT small y bar we will denote by capital Y bar. So we have Y bar, capital Y bar equal to lambda times F FFT times x bar plus capital W bar which is nothing but F FFT times W bar where we are using the property F FFT into F IFFT is identity. So these are inverse of each other. So, using the property so this uses the property F FFT into F IFFT equals identity so this is the property that we are using.

So this implies we have Y bar equals lambda times F FFT times x bar plus capital W bar. So you can think of this as the post processing this performing FFT at the receiver this is basically equal to post processing or the processing at the receiver. So this is, so what are we doing at the receiver? We are performing the FFT of these output samples that is y 0, y 1 up to y N minus 1, remember these are the output time domain samples at the receiver. So we perform the FFT of these y 0, y 1 up to N minus 1.

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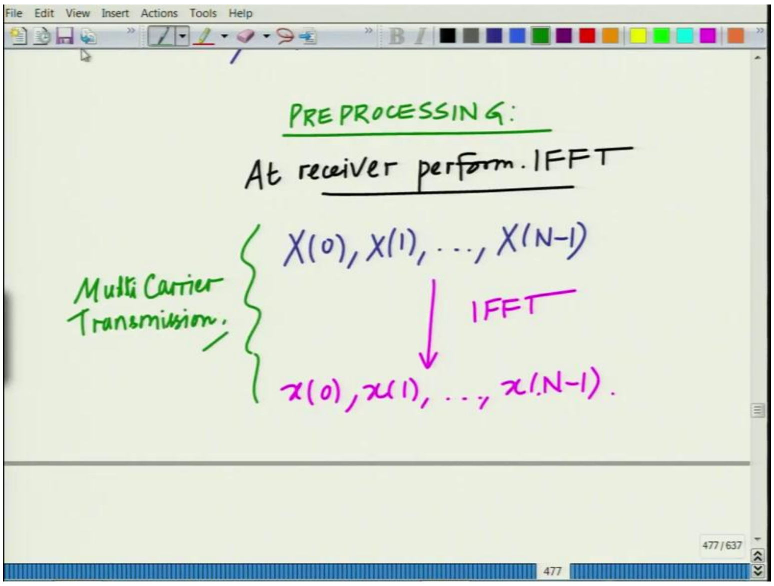

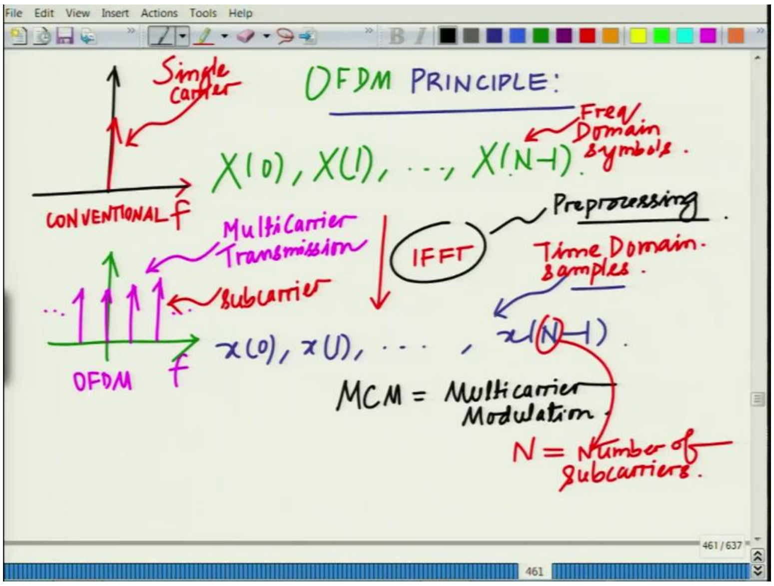

Now we also have some pre-processing. What is the pre-processing? Now the pre-processing step is the following thing; at receiver before transmission, remember we already know at receiver perform the IFFT. Remember we have already seen this step perform the IFFT, if you go all the way back here and essentially if you look at this, if you look at our discussion on OFDM at the receiver we are performing the IFFT. This is the pre-processing step.

Let me refresh your memory. We have pre-processing in the OFDM system. So what is a pre-processing that we are employing? So, we look at this what we have essentially is that we have the actual symbols x 0, x 1, x N minus 1, you perform the IFFT you get x 0, x 1, x N minus 1. Remember we said this is synonymous with multi carrier. This is synonymous with multi carrier transmission.

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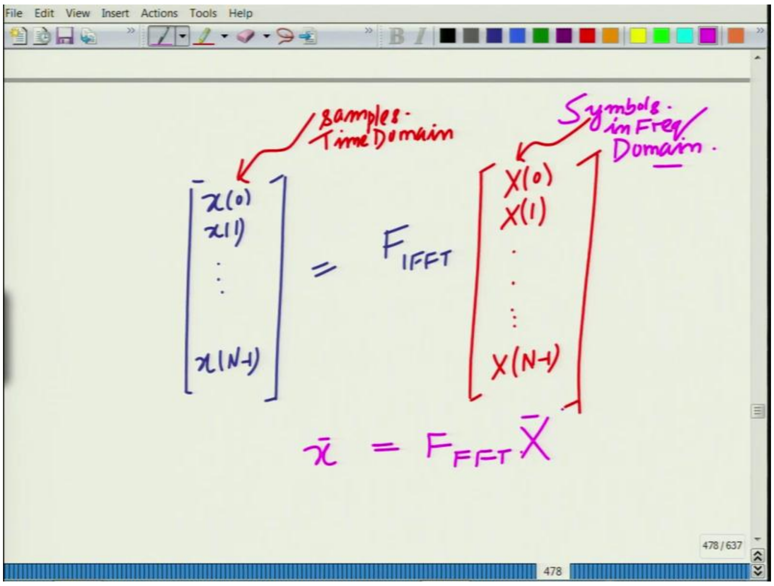

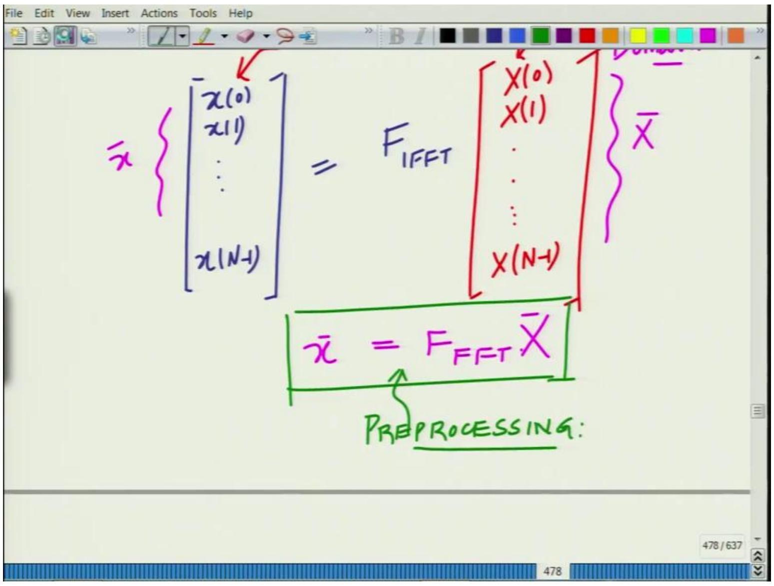

Therefore, what we essentially have is if you look at these samples, that is you have the x 0, x 1, x N minus 1. These can be expressed as the IFFT matrix, the IFFT matrix times capital X 0, capital X 1 times capital X N minus 1 and as we know these are simply the samples in the time domain and these are actually the symbols, these are actually the communications symbols in the frequency domain. So we have x bar, x bar equals F FFT times capital X bar.

So, this is your vector small x bar. This is your vector capital X bar. So this is our pre-processing step. So, we have that transmit vector, so this is essentially the pre-processing step. So this is essentially the pre-processing step that is essentially remember, I am sorry this is the IFFT. This is the IFFT, this is the FFT times x bar. So, remember before we are transmitting, we are taking the symbols, you are not transmitting the symbols directly in OFDM but performing the IFFT at the transmitter. This is an important point.

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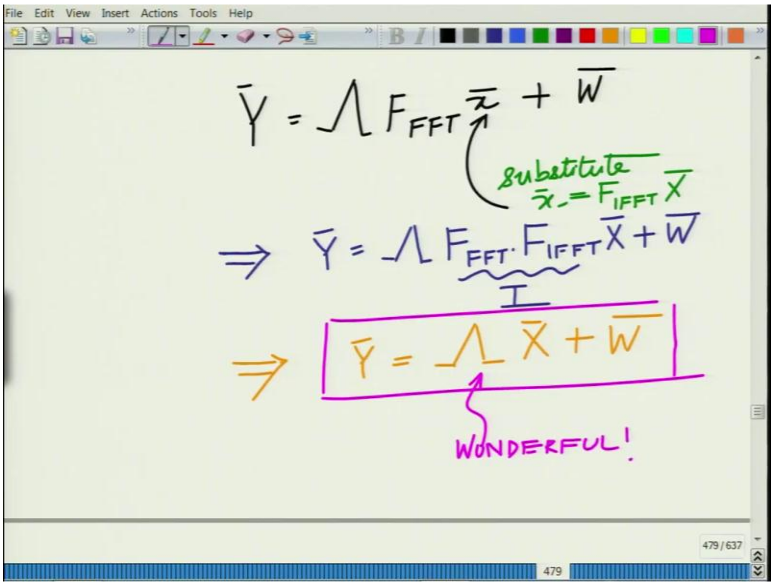

Now substitute this in the model that we have above. Remember we have Y bar equals; you go all the way back you have Y bar equals lambda times F FFT times small x bar plus W bar. Now substitute here, the fact that small x bar equals F IFFT times capital X bar and therefore we will have Y bar equals lambda times F FFT times F IFFT times capital X bar plus W bar once again you see F FFT times F IFFT.

This is identity and therefore we are left with this wonderful model Y bar equals lambda times X bar plus W bar. We are left with this wonderful model. This is the wonderful OFDM model. Why am I calling this as a wonderful model? We are left with this wonderful model. Why am I calling this wonderful? Reason is obvious, lambda is a diagonal matrix.

From a matrix that is completely filled, a circulant matrix we have now come to a matrix is diagonal and we like diagonal matrices because diagonal matrices are easy to invert, easy to process, easy to apply that is what makes OFDM really efficient. That decoupled nature with the diagonal, which the diagonal matrix uses. So, this is a diagonal matrix. That is what makes this wonderful.

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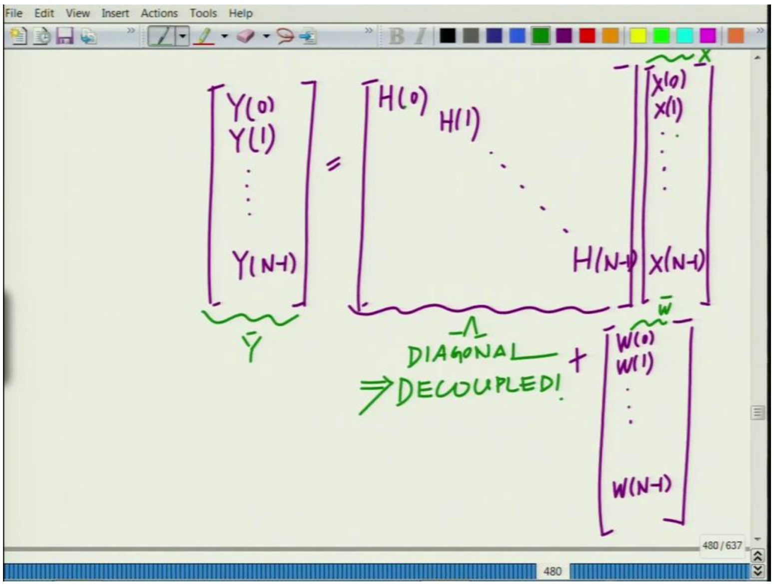

So, if you write this you will have then, if you write this you will have then Y 0, Y 1 up to Y N minus 1. This is equal to the diagonal matrix H 0, H 1 up to H N minus 1 times X 0, X 1 up to X N minus 1 plus your noise which is W 0, W 1 up to W N minus 1 and because this is a diagonal matrix, remember this is nothing but your diagonal matrix, this is your lambda.

This is your Y bar, this is your X bar, this is your W bar and this is a diagonal matrix. This is diagonal implies that different symbols are decoupled, that is the big advantage that you have. So, if you look at Y 0, Y 0 equal to X 0, X 0 plus W 0, no interference from X 1 or any symbol. Y 1 equal to H 1 into X 1 plus W 1 so on and so forth.

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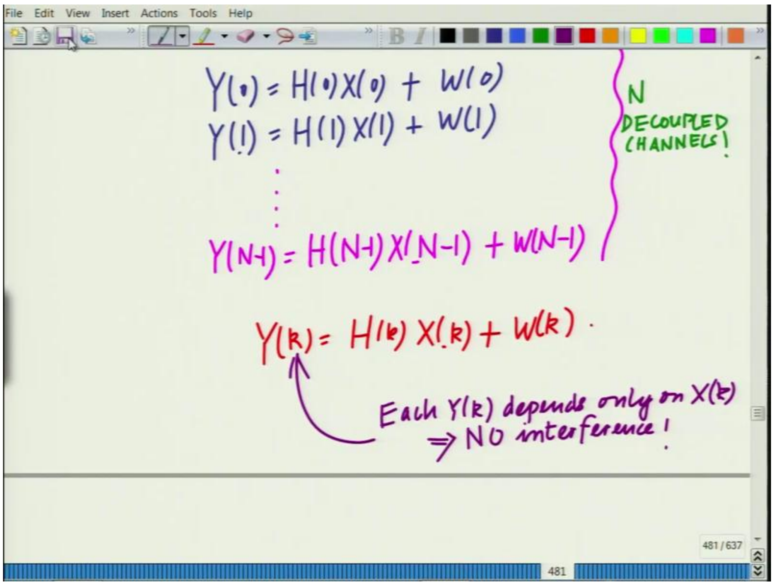

So, if you are right this thing you will realize you will have Y 0 equal to H 0, X 0 plus W 0. Y 1 equal to H 1, X 1 plus W 1 so on Y N minus 1 equals H N minus 1, X N minus 1 plus W N minus 1 and if you look at this these are N decoupled channels. So, with an ISI we started with an ISI system interface, inter symbol interference and we realized that, that system is intractable because of the inter symbol interference that leads to this degradation, distortion and then we applied OFDM on that and what has resulted is that this results in a very nice convenient framework where you have these N decoupled channels in which each output Y 0, look at this each Y 0 depends only on X 0, Y 1 depends on X 1 so on.

Y capital N minus 1 depends only on small x n minus 1 which is not there in the OFDM system because of the, which is not in the conventional single carrier system because of the inter symbol interference. So, if you look at this system the great advantage of this system the beauty is that you have Y k equals H k, X k plus W k, each Y k depends only on X k. This implies there is no interference, no inter symbol interference, no cross channel interference and so on and so forth. So, this is a nice decoupled system and therefore, the decoding becomes extremely easy.

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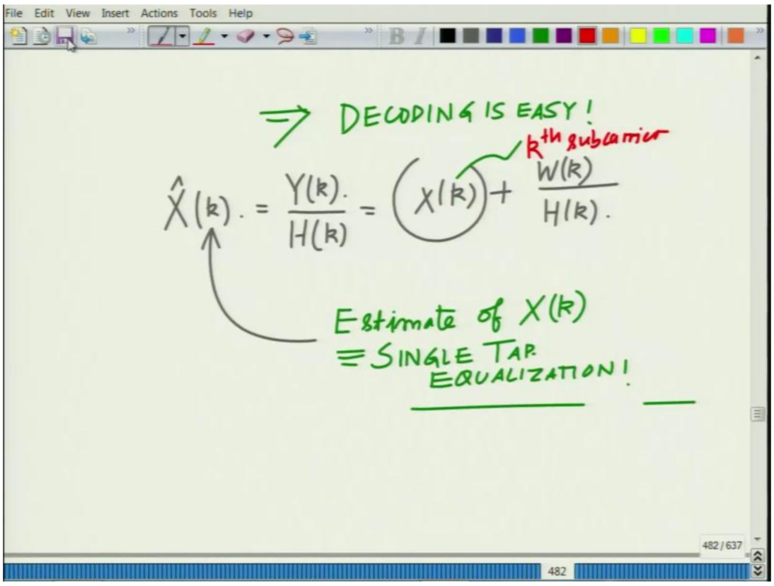

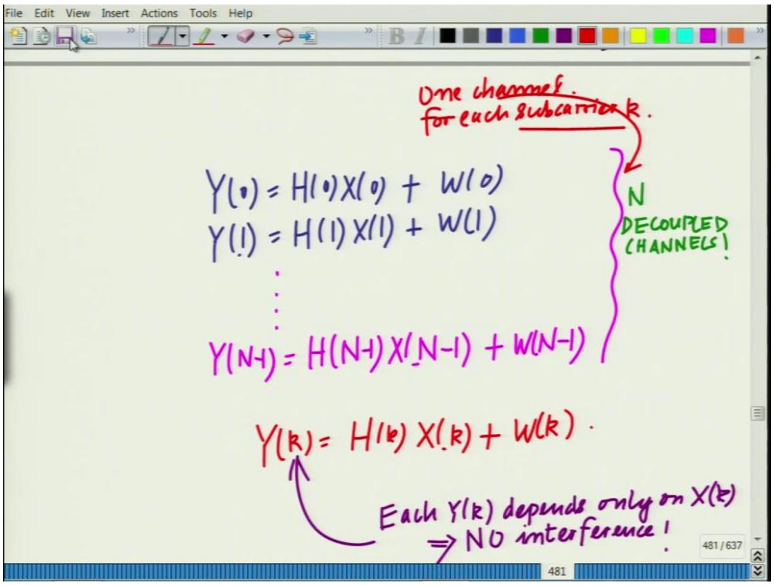

This implies that the decoding, remember that is where the problem was, implies decoding, I should not say easy maybe that simply taking it too far. It is relatively easy let us say that, low complexity. How are we doing the decoding, you take Y k, you divide it with H k that simply gives you X k plus W k divided by H k which is of course, the noise but you have a estimate of X k. So, this is your X hat k. This is the estimate of the symbol X hat k.

This is known as single tap equalization, this is termed as a single tap equalizer because you have only one tap that is H k in each channel, kth channel you are dividing it by H k which is the channel coefficient, by the way this is known as the kth sub carrier. So, this is the k remember this is the so, you have one channel for each sub carrier, you go back and take a look at this end decoupled channels.

You have one channel for each sub carrier k. That is essentially what you have and therefore, what I have is now N such decoupled channels. So, what is OFDM doing? OFDM is essentially taking this intractable unwieldy very, the broad band, broad band, wideband, this what you call as a fat data pipe, this huge high bandwidth wireless channel and dividing it into several N decoupled channels, each of which is albeit a low data rate, make no mistake if you look at each of these channels, each of these channels is narrowband.

So, the data rate naturally will be smaller, but the point is you have N such channels and this system is much more tractable, it is much more easier to implement the system, much more easier to implement transmission, much more easier to implement the decoding in this system compared to your conventional single carrier system and therefore, the distortion will be lower, the effective data rate or what we call as the throughput will be higher.

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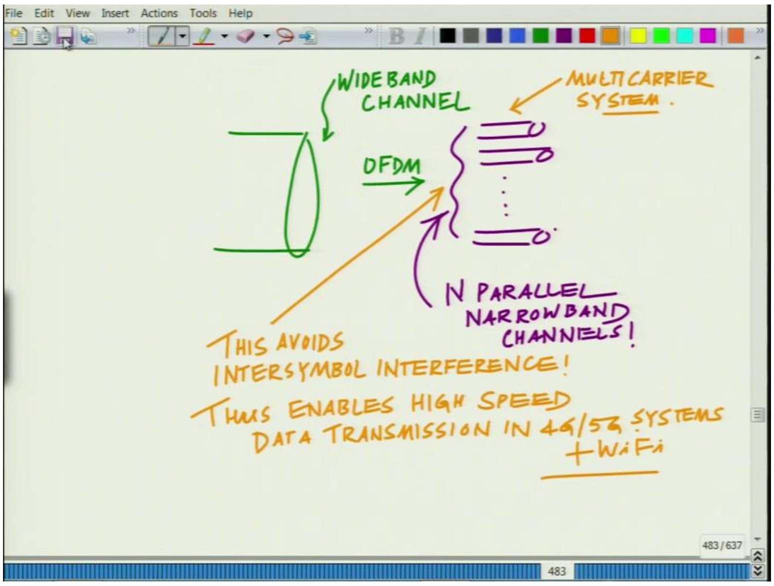

So essentially if you look at this schematically, what is happening is you have this OFDM you have this wideband channel and what OFDM does is takes this wideband channel, converts it into N parallel, splits this into N parallel, OFDM splits this into N parallel narrowband channels that is essentially so the data rate, effective data rate is going to be the same, but the second system is much more tractable.

This is essentially a multi carrier system. So you have the multiple sub carriers, one narrowband channel over each sub carrier. So this is essentially your multi carrier system. This is essentially your multi carrier system and therefore this avoids interference, enables high speed transmission of data. This avoids inter symbol, thus enables high speed data transmission in 4G, 5G systems plus Wi-Fi. It enables high speed data transmission in 4G, 5G systems and Wi-Fi.

That in essence a summary of OFDM; it is you see in linear algebra in particular, the FFT, IFFT, the matrix presentation of these FFT, IFFT matrices, the application of the FFT at the receiver, IFFT at the transmitter, these have a very important role to play and overall you can see the powerful tool, the powerful impact out of the (()) (26:35) the compact fashion in which linear algebra that is matrix analysis helps in the modelling and analysis of this OFDM system.

So, that in turn essentially shows us how variegated, how varied, how diverse and how powerful these various applications of linear algebra matrix analysis, matrix transformations properties related to Eigen values, Eigen vectors etcetera and so on can be. In fact, you can see what is currently driving your 4G phone and what is going to drive your 5G phone in the future is based on these very interesting and powerful applications of the IFFT, FFT and all of the operations all of which can be represented in fact as linear transformations and in fact, very much a part of linear algebra. Alright. So, let us stop this discussion here and continue with other interesting aspects in the subsequent modules. Thank you very much.