Applied Linear Algebra for Signal Processing, Data Analytics and Machine Learning
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Lecture 46
Single-Carrier Frequency Division for Multiple Access (SC - FDMA) Technology

Hello, welcome to another module in this massive open online course. So, we are looking at OFDM that is Orthogonal Frequency Division Multiplexing and basically how it can be modelled and how linear algebra specifically can be used to model and analyse an OFDM system and of course, as I have already told you OFDM is a dominant technology that is used in 4g, 5g wireless systems.

Let us now look at another interesting technology which is again used in 4g and 5g wireless systems also, which is known as a SC - FDMA Single-Carrier Frequency Division Multiple Access and I will explain to you the difference between OFDM and SC - FDMA in a moment, but SC - FDMA is also yet another dominant waveform that is once again used in 4g and 5g wireless systems.

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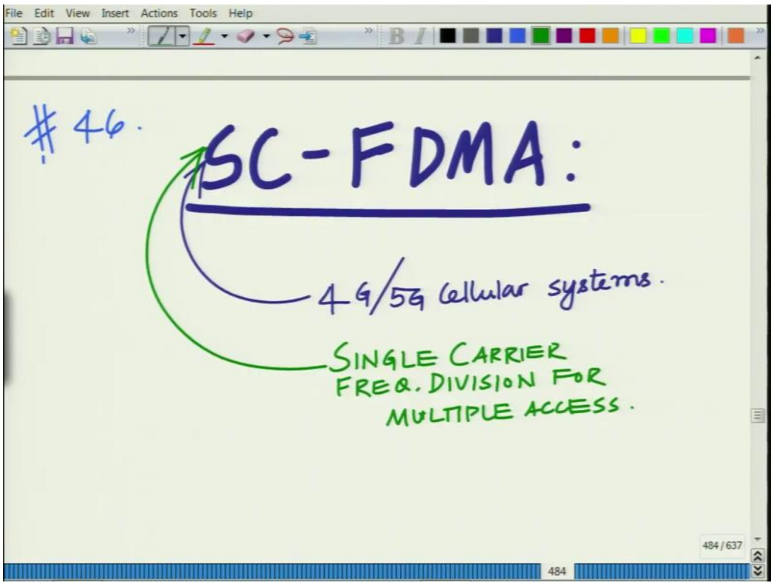

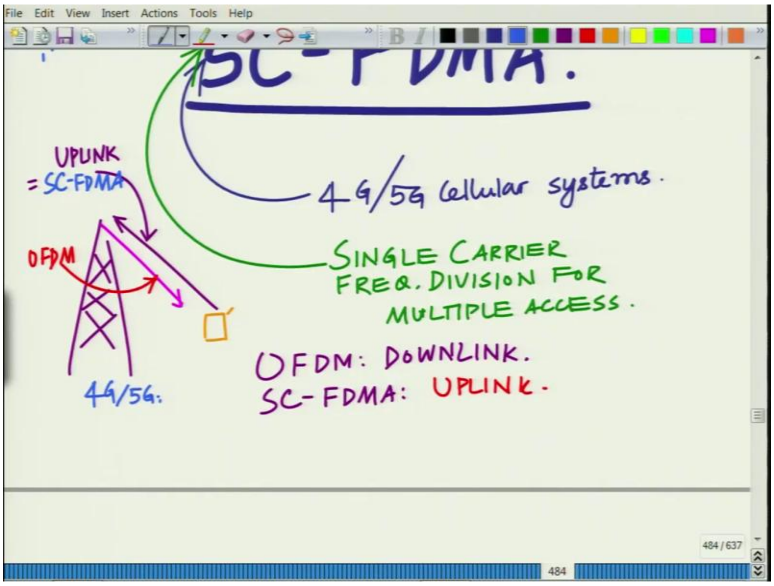

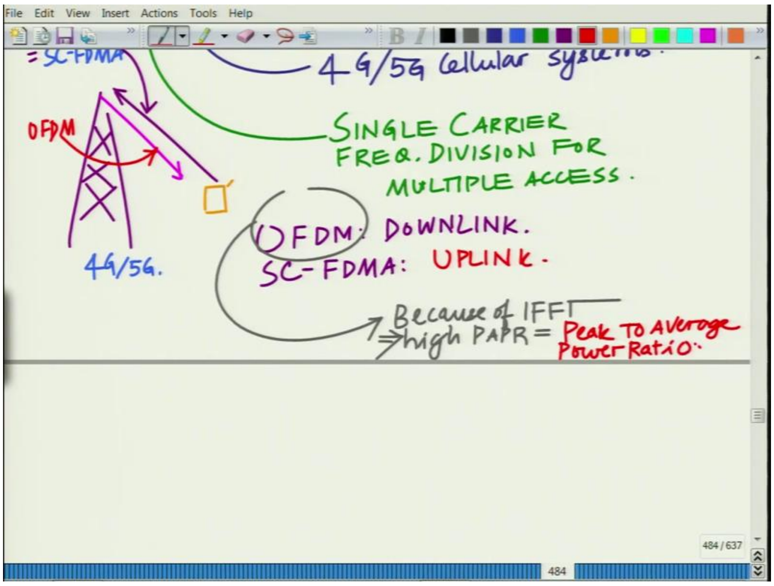

So, let us talk about SC - FDMA and of course, remember this is a course on linear algebra. So, eventually, we want to look at how linear algebra can be used to model the system, to build a model for the system. So SC - FDMA, once again this is a dominant, another dominant technology that is used in 4g and 5g cellular systems. This is used in 4g, this is also used in 4g and 5g systems and SC - FDMA stands for Single - Carrier Frequency Division for Multiple Access.

So and the difference between OFDM and SC - FDMA is the following whereas OFDM is used in the downlink that is the base station to mobile, SC - FDMA the difference is that this is used in the uplink, that is if I look at a cellular system and of course I have the mobile side I have mobile and this is the downlink which is essentially your downlink. We are saying is OFDM that is the base station to mobile.

But if you look at the uplink from the mobile to base station, this is basically the uplink and uplink is basically your SC - FDMA and this is in 4g as well as to a great extent in 5g. The reason being why do we need to use a different modulation technique or a different waveform in the uplink as compared to the downlink? The reason is because OFDM which is used in the downlink suffers from a unique problem this is known as PAPR that is Peak to Average Power Ratio.

OFDM has a high peak to average power ratio because remember in OFDM the IFFT is performed prior to transmission. So in OFDM, we have IFFT processing, OFDM because of IFFT. This implies this is high PAPR that is your Peak to Average Power Ratio.

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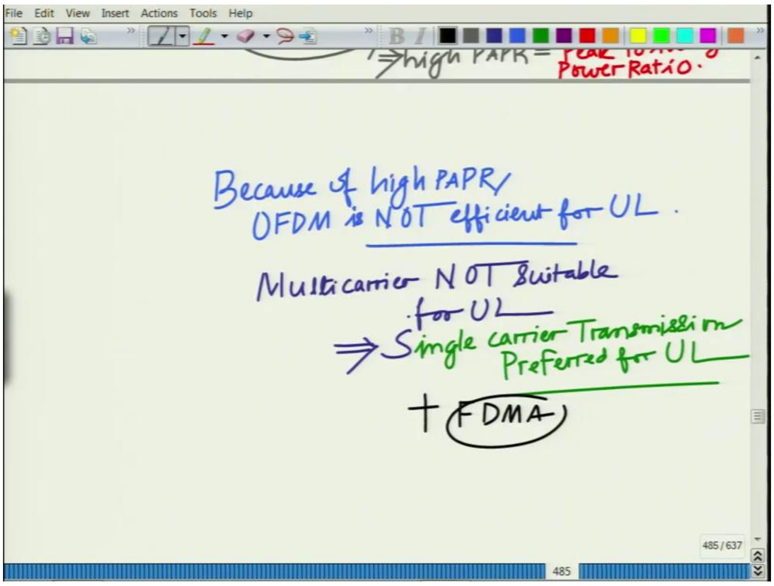

Therefore, because of high PAPR, so OFDM is not efficient for the uplink. So it can be used in the downlink, that is the base station to mobile but this is not efficient for the uplink and therefore the question is what can now be used in uplink? So uplink 1 cannot resolve to multi carrier transmission that is 1 cannot use the IFFT. So, uplink is still single carrier and that is essentially what is known as Single Carrier Frequency Division for Multiple Access.

So therefore, uplink so, basically multi carrier not suitable for uplink, which implies single carrier; which implies that single carrier transmission, which implies that single carrier transmission is preferred for the uplink and what but we would like to retain the frequency division multiple access that is there in OFDM plus FDMA.

We would like to retain the FDMA structure that is the Frequency Division for Multiple Access. We would still like to retain that aspect, but without the multi carrier transmission.

So, single carrier plus frequency division multiple access, how is that possible? And the procedure for that is as follows.

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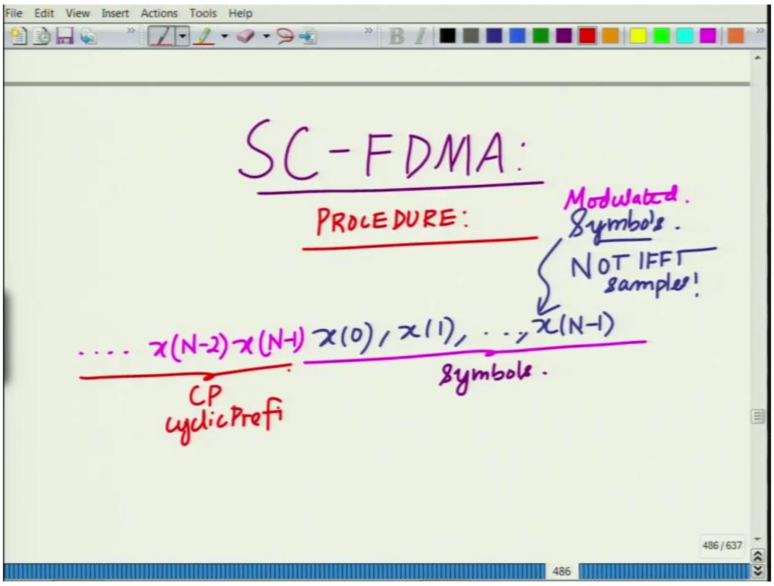

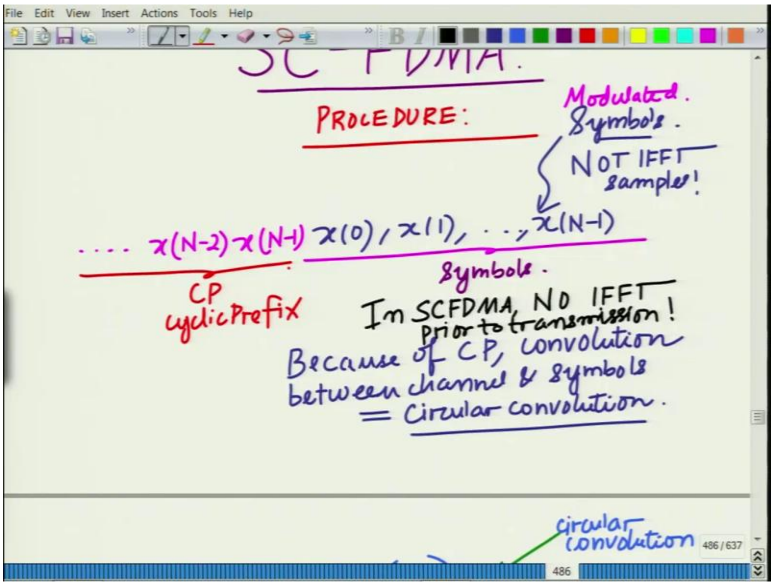

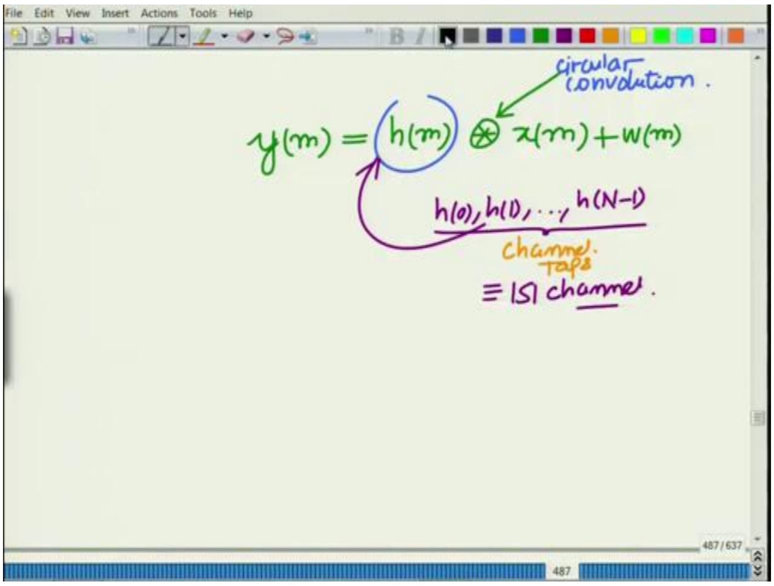

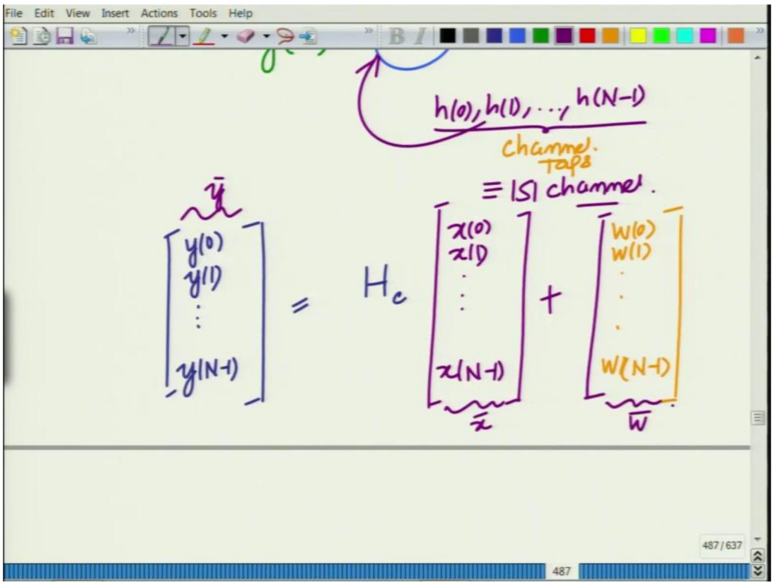

So, SC – FDMA, what is the procedure so, we begin by asking the question SC FDMA what is the procedure for, what is the procedure for SC – FDMA? So, SC - FDMA what we do is we take the symbols as I already told you we do not do the IFFT, we directly transmit the symbol. So, in this case you have the time domain quantities x 0, x 1 up to x N minus 1. These are the symbols not IFFT samples.

I think this is important to remember, in SC - FDMA you are directly transmitting the symbols that is the modulated information symbols that is your BPSK are KPSK symbols, not the IFFT samples, samples at the output of IFFT. In fact, there is no IFFT at the transmitter. So, these are the directly, so these are basically what you are transmitting, these are basically the modulated symbols not the IFFT samples, but nevertheless we add the cyclic prefix.

So, nevertheless we add the so, these are your symbols. So, this is essentially, this is your cyclic prefix. So, remember this is making the whole symbol look cyclic. It makes it look as if it is a periodic signal. Therefore, what happens is the channel, the convolution between the channel and the symbols that becomes a circular convolution. So, I can express this as a result of this so, adding CP because of CP remember similar, this is similar to OFDM because of CP convolution channel and symbols becomes a circular convolution and remember that is why you have the circulant matrix.

Remember that circulant matrix that represents the circular convolution. Nevertheless, if you write this, you will have the output y m equals h m again you will have the same this thing y m equal to h m circularly convolved with x m plus w m. This is essentially your circular convolution.

This is essentially your circular convolution and therefore, now if you look at the outputs if you write these hmm you might well recollect, these are your h 0, h 1 up to h N minus 1, these are the channel taps. So, these are your N channel taps and these represent the ISI channel. Remember, we went through this discussion as the bandwidth increases symbol time decreases, symbol time decreases so, essentially what happens is the symbols are smeared out by the channel and therefore, this results in inter symbol interference so on and so forth.

Such a channel can be represented as the using the multiple channel taps and now essentially therefore, if you look at this output therefore, now look at this so, we do not perform the IFFT prior to transmission. So, now here the of course, the thing you have to remember in SC - FDMA is, in SC – FDMA, no IFFT prior to transmission.

This is the major difference with respect to OFDM. There is no IFFT performed that is prior to transmission, you are simply taking the symbols, block of N symbols adding the cyclic prefix and transmitting it over the channel, which of course, it is a inter symbol interference channel and therefore, you will have the output, will be if you look at y 0, y 1, y N minus 1 this will be H c times a x 0, x 1, x N minus 1 plus, you will have the W 0, W 1, W N minus 1.

So, this is you can call this as your y bar, this is your x bar and this is your W bar and remember your x bar this is directly your symbol vector. So, you are transmitting the symbols not the samples.

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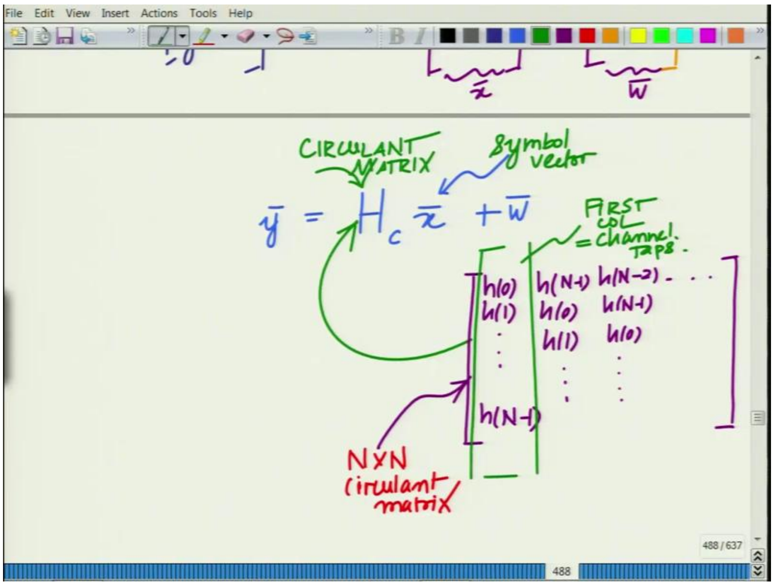

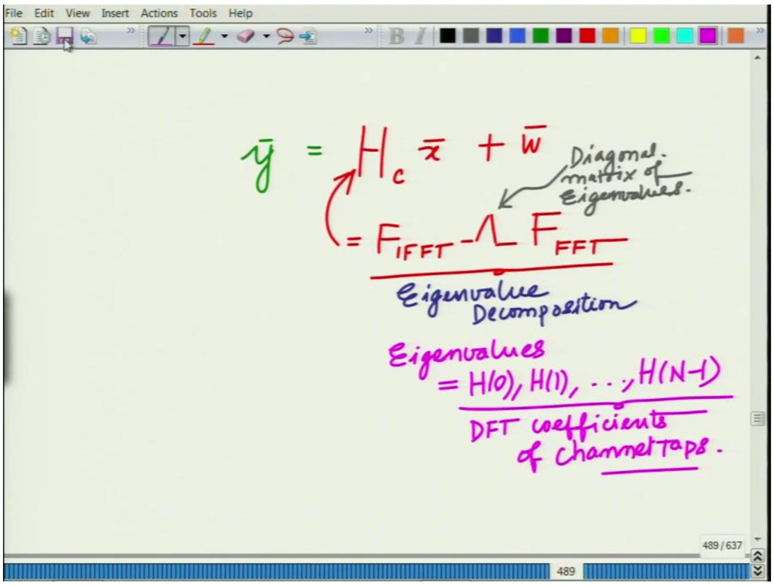

And therefore, I can represent this as y bar equal to H c, x bar plus W bar this is your symbol vector and this is your circulant matrix. Hopefully all of you remember that this circulant matrix is basically one in which each successive column or each successive row is obtained by circularly shifting the previous row or column.

In fact, the first column simply comprises of the channel types h 0, h 1 h N minus 1 each subsequent column is obtained by downshifting, circularly downshifting the previous column. So, this is your circulant matrix and the interesting circulant matrix comprising of the channel in terms of the channel taps, not a problem. You can write it down that is your h 0, h 1, h N minus 1, h N minus 1, h 0, h 1 so on h N minus 2, h N minus 1, h 0 so on and this is essentially what will be your this is your N cross N circulant matrix.

And if you look at the first column in this, this is nothing but basically your channel taps, that is your channel tap vector and each column will be basically obtained by circularly downshifting the previous column and therefore, now we have y bar equal to H c times x bar where H c is a circulant matrix Now, recall I can write the Eigen value decomposition of H c as F of the IFFT matrix which are the Eigen vectors lambda, diagonal matrix of Eigen values which are nothing but the FFT coefficient or DFT coefficients of the channel taps times IFFT matrix inverse which is nothing what the FFT.

So, this is basically the Eigen value decomposition. This is the Eigen value decomposition and this is the I of course, IFFT, FFT matrix and this is the diagonal matrix of Eigen values, diagonal matrix of Eigen values and these Eigen values are nothing but your DFT coefficient. This is the DFT coefficients of the channel taps. These are the DFT coefficients of the channel taps.

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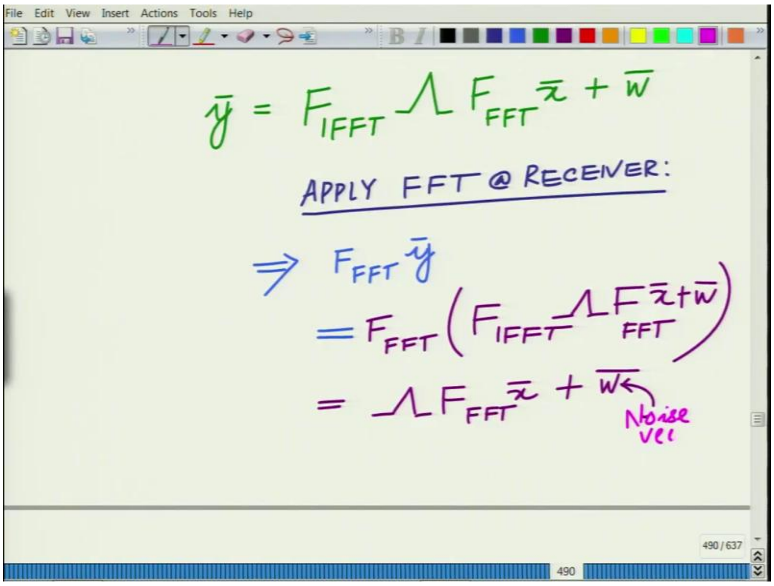

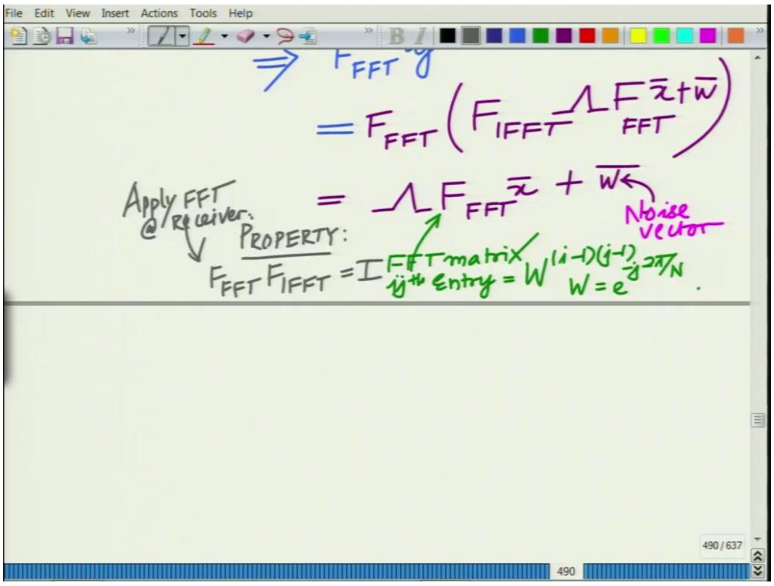

And therefore, now if you substitute this what you will have is that you will have y bar equals F IFFT lambda F FFT times x bar plus W bar. Now, apply again similar to OFDM now, this part is similar to OFDM apply FFT at receiver or perform FFT at receiver. This implies you will have F of FFT times y bar, this will be equal to F of FFT times F of IFFT times lambda F of FFT into x bar plus W bar.

Now F FFT into F IFFT this is equal to identity. So this will simply be lambda times F of FFT times x bar plus W bar which is basically your noise vector. This is basically x bar is your symbol vector, this is your FFT matrix, this is basically your FFT matrix. Remember I jth entry equal to W raised to the power of i minus 1, W raised to the power of, W raised to i minus 1, j minus 1 where W equal to e raised to j 2 pi or e raised to minus j 2 pi over N and of course, lambda is a diagonal matrix of Eigen values which are nothing but the DFT coefficients of the channel taps.

And essentially, while here we have used the property, the property that we have used is basically that F of FFT times F of IFFT equals identity, because we are applying the FFT at, apply FFT at receiver, that is applying the FFT at the receiver is nothing but you can think of it as processing with the FFT matrix that is F FFT that is what we have seen before that is the FFT operation can be represented as multiplying with this FFT matrix whose i jth entries is W raised to the power i minus 1 into j minus 1 where W is e raised to j 2 pi over N, N is the number of sub carrier. So, please understand each and every step in each and every quantity that is involved thoroughly. If you understand it, it is very simple to follow.

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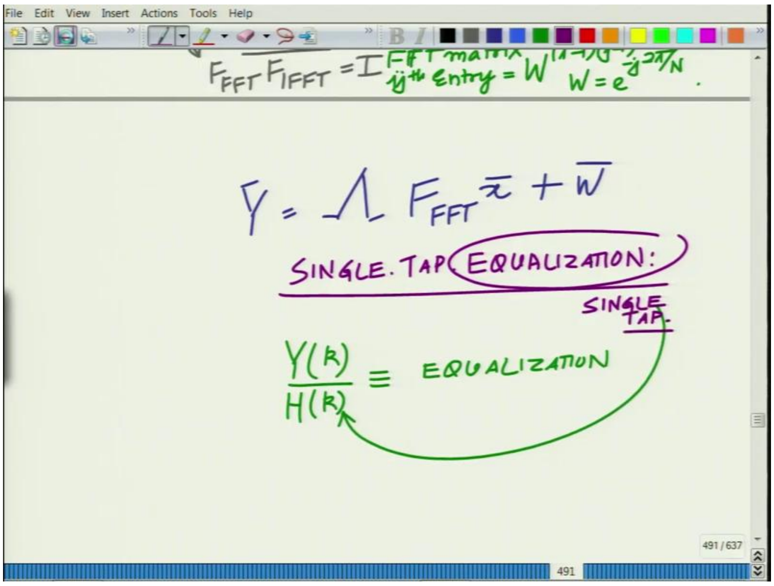

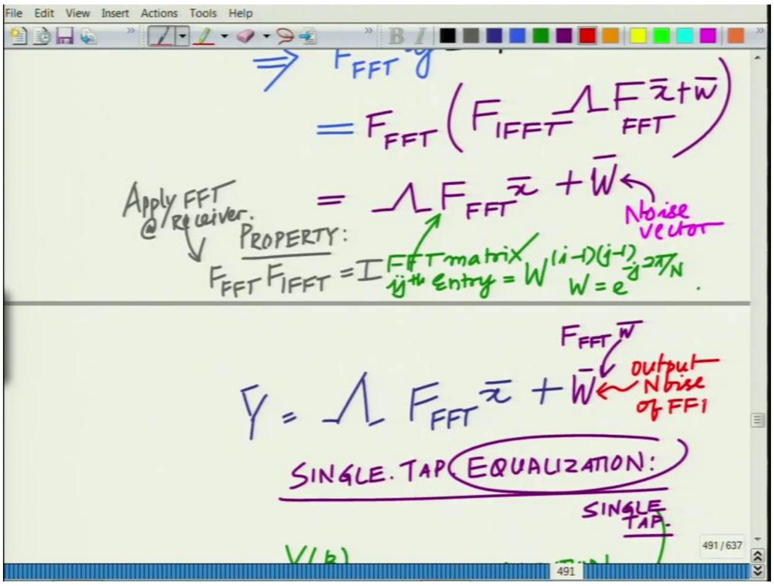

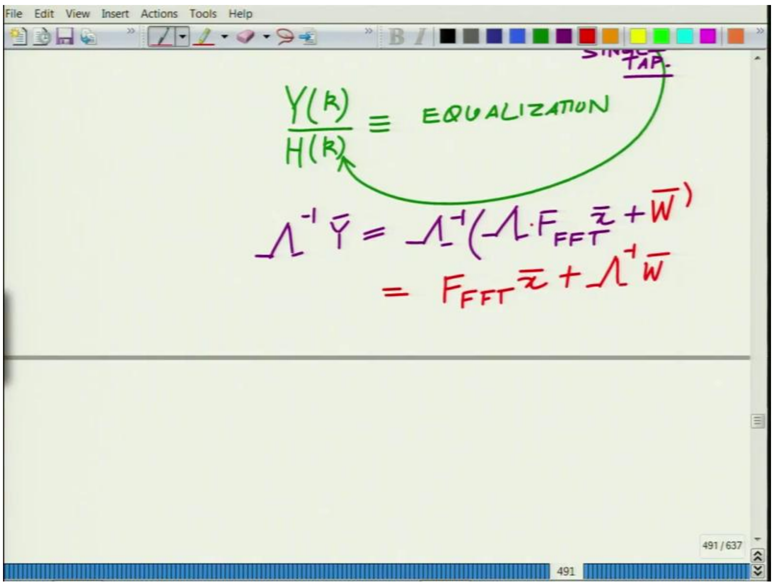

Now, let us what we do is we obtain this, let us call this as y bar this is similar to what we obtained in OFDM, capital Y bar. So, you have the capital Y bar equals lambda F FFT times x bar plus W bar now, we do the single tap equalization that is on each sub carrier divided by the channel coefficient. Overall we perform lambda inverse, where lambda is a diagonal, so this equalization; this is an interesting term. This essentially means flattening the channel response.

So, that is essentially where you are equalizing the channel response and that is why you are dividing by H k. The channel is H k dividing by H k to flatten the response that becomes your equalization. So, the equalization in this case is simple that is your single tap equalizer. So, remember Y k divided by H k, this is essentially your equalization and basically this is your single tap, since you are dividing only one quantity that is your H k, this is your single tap equalizer.

This is your single tap equalizer and therefore, what we are doing is, we are performing lambda inverse. Overall if you look at it Y bar which becomes lambda inverse times F or lambda F FFT x bar plus, this is your, we call this as the capital W bar. So, this is your F FFT times. So, this is essentially your capital W bar. What is this capital W bar? It is FFT at the output of the noise at the output of the FFT.

So, this is F FFT times the small w bar. So, this is your noise, output noise; output of what? Output of the FFT, that is your capital W. So, this is your W bar which now gives you lambda inverse and this lambda is identity. So, that gives you your FFT, F FFT x bar plus lambda inverse W bar that is because your lambda inverse lambda is identity and remember lambda is a diagonal matrix.

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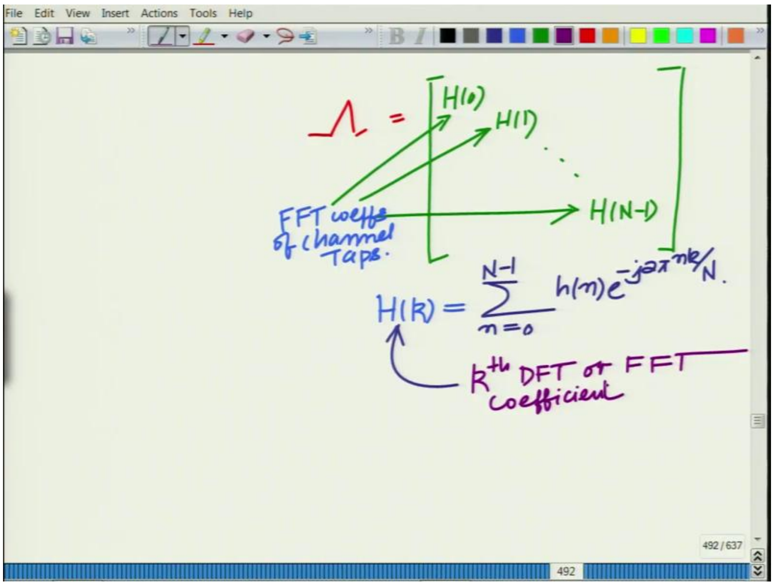

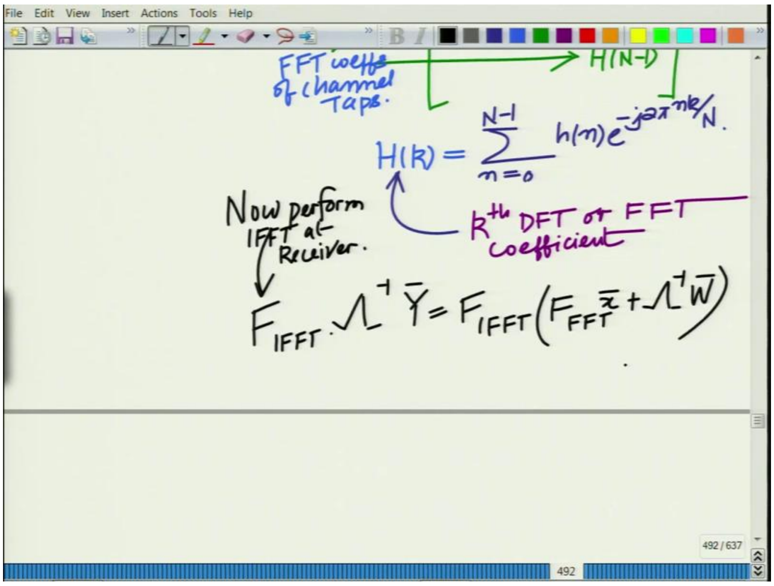

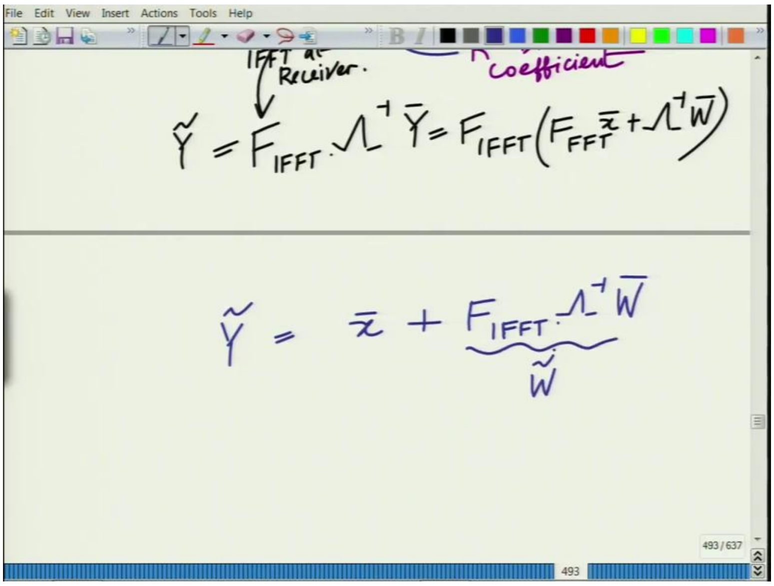

Therefore, it is easy to invert. Remember lambda, if you look at, if you recall the structure of lambda; this basically contains simply your DFT coefficients of the channel taps. So, these are basically the DFT coefficients or these are your FFT coefficients of the channel taps and we also saw what we mean by H k. H k equals summation n equal to 0 to capital N minus 1, h n e raised to minus j 2 pi n k over N.

This is essentially the kth DFT coefficient or FFT, kth DFT coefficient or FFT coefficient and now, what we perform after the single tap equalization, remember an OFDM the estimation process ends with single tap equalization because they are pre-processing using the FFT matrix, but here that is not the case. We still have to recover the symbols x bar, the small x bar.

Therefore, we now have to perform F IFFT that is F IFFT times lambda inverse Y bar; that is now perform IFFT. So, at receiver in SC - FDMA we are performing both FFT and IFFT. Contrast this with OFDM where you are performing IFFT at transmitter, FFT at receiver. In OFDM nothing at the transmitter both IFFT and FFT; FFT followed by IFFT at the receiver.

So, this will be F IFFT lambda inverse Y bar, which is F IFFT times F FFT x bar plus lambda inverse capital W bar which if I call this as Y Tilda. Now, if I call this as Y Tilda, what you will see is this Y Tilda I called F IFFT to F FFT is identity. So, that will be your x bar plus F IFFT times lambda inverse capital W bar, let us call this as W Tilda.

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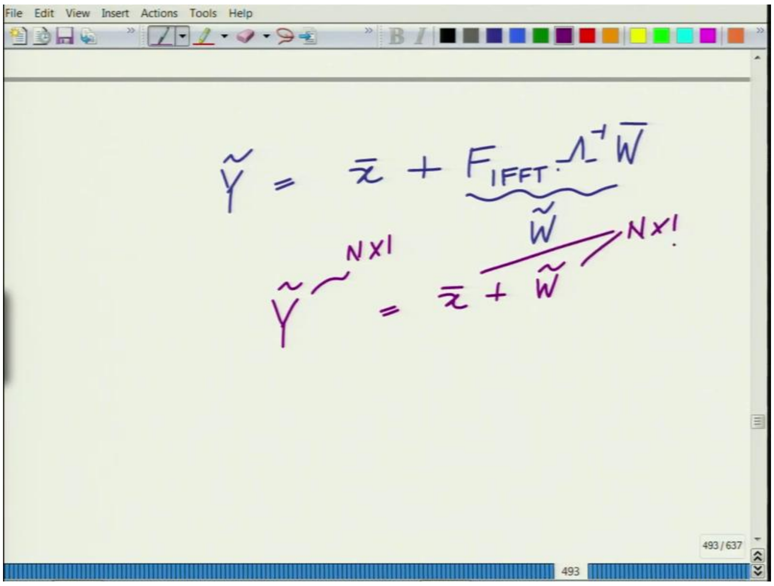

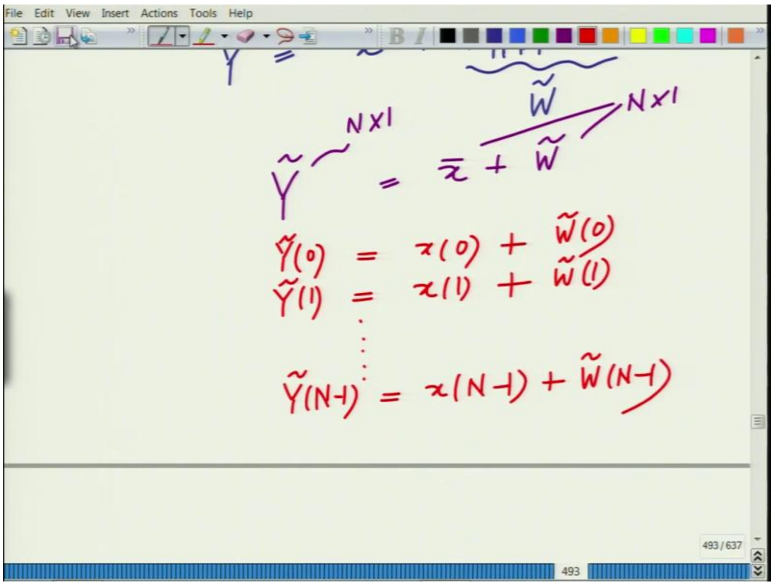

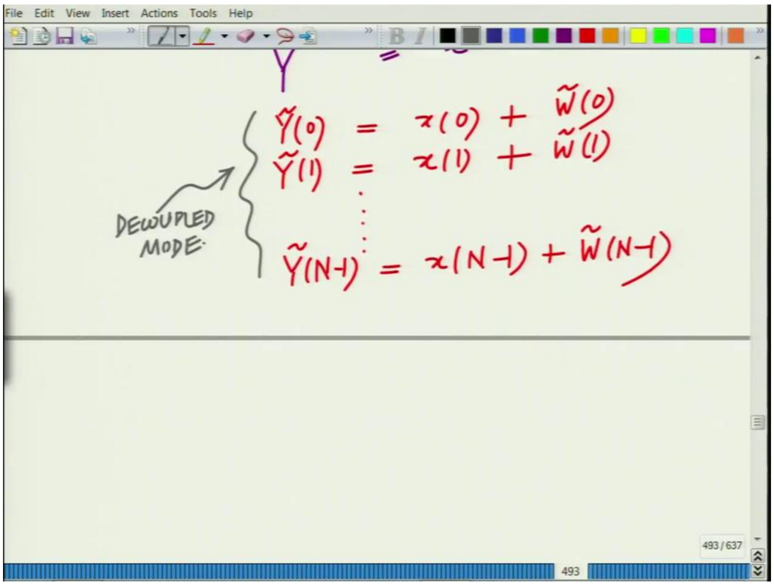

So, essentially what you will have is now after the IFFT, you will have Y Tilda which is basically your N cross 1 vector, equals x bar plus W Tilda where both of these are also naturally N cross 1 vectors and therefore, what this essentially means is now, you have a very simple model Y Tilda 0 equals x of 0 plus W Tilda 0, Y Tilda 1 equals x of 1 plus W Tilda 1 Y Tilda N minus 1 equals x of N minus 1 plus W Tilda N minus 1.

Now, why go through all this frequency division multiplexing when you wanted to simply transmit using a single carrier. The advantage is you still have a decoupled system and more importantly the equalization can be performed using the FFT and IFFT operations. This cannot, is not possible in a conventional single carrier system.

So, remember one thing, although this is a single carrier system, but the processing becomes much easier because it is being done using the FFT and IFFT which can be done in a very fast fashion. You know that from your knowledge of signal processing FFT and IFFT are very fast, rapid. Complexity is N log 2 to the base log N to the base 2.

In comparison to that, if you have to do conventional equalization which you have to invert N cross N matrices that will be N cube, much higher complexity. So, the single carrier FDMA complexity equalization complexity is still much lower because of FFT, IFFT processing. So, you will understand this aspect from this model.

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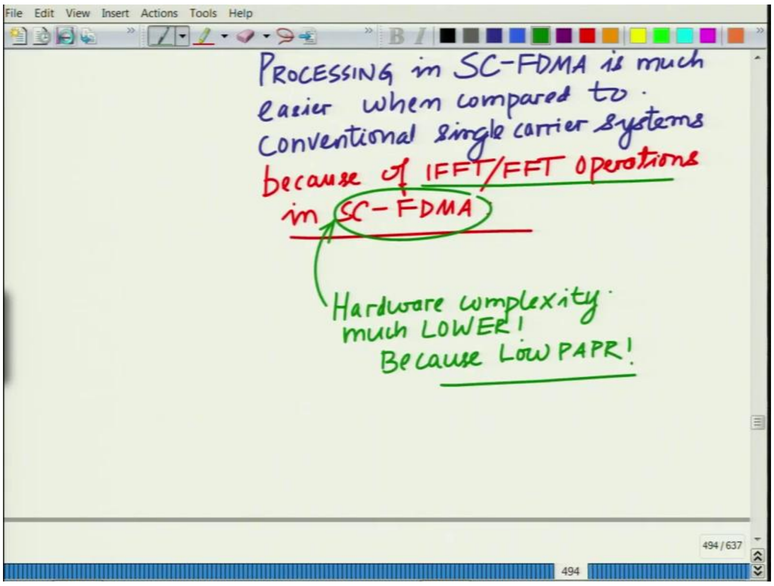

So, what is the difference here? You still get a decoupled model. You will still get a decoupled model but processing is easier, processing in SC - FDMA is much easier in comparison to conventional single carrier systems. Processing in SC - FDMA is much easier in comparison or when compared when compared to conventional because of IFFT, FFT operations.

So, it is a single carrier system, but the frequency division multiplexing aspect that is introduced by these IFFT, FFT operations that make the equalization the overall equalization much simpler in the SC - FDMA in comparison to conventional single. Now, comparison to OFDM, SC - FDMA complexity is relatively safe, because in OFDM, you are doing IFFT at transmitter FFT at receiver.

SC - FDMA you are doing both IFFT and FFT at the receiver. So, complexity is more or less same in comparison to OFDM. But hardware complexity once again realize this in SC - FDMA is much lower because we, I already told you in OFDM you have the problem PAPR because of PAPR, there are several other problems.

Once there is power becomes very high, the peak or the peak in comparison to average that is what we call is the dynamic range of that signal because very high designing hardware is very difficult in particular, because the bias point cannot be effectively designed and so on and the non-linearity effects start to kick in. SC - FDMA, because it is single carrier system PAPR is low, it means hardware complexity is much lower.

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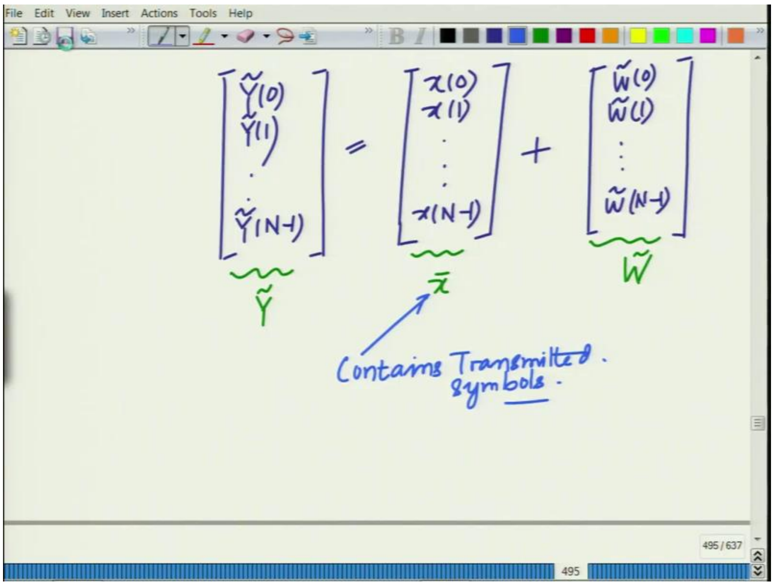

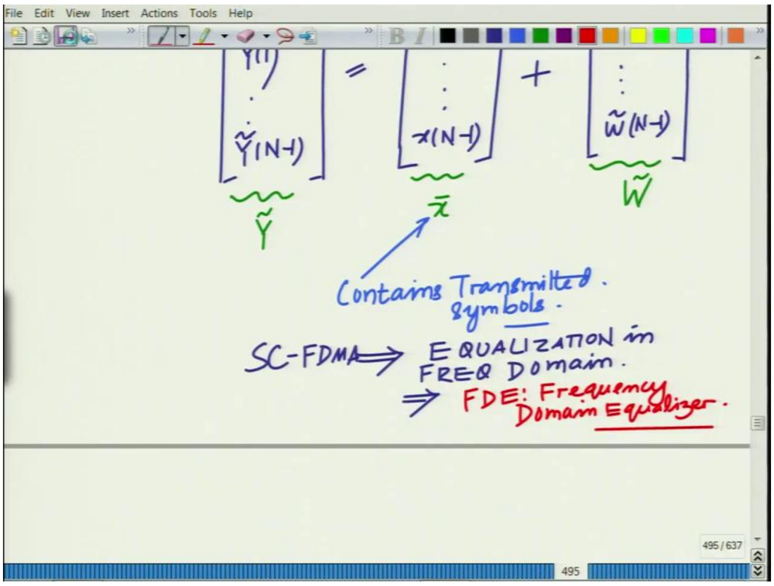

So SC – FDMA, hardware complexity is much lower because low PAPR, because of the low PAPR. So, hardware complexity is much lower and as we have seen the overall system model is basically after the IFFT, you will have Y Tilda 0, Y Tilda 1, Y Tilda N minus 1. This is equal to your x of 0, x of 1, x of N minus 1 plus you will have your W Tilda 0, W Tilda 1 up to W Tilda N minus 1.

So, this is your Y Tilda, this is your x bar. This is the noise W Tilda and you know that this x bar contains the transmitted symbols. So, essentially what you are doing is you are transmitting the symbols, time domain symbols itself and because of the IFFT, FFT operations, you are pure sort of performing the equalization in the frequency domain. So look at this at the receiver you are doing the FFT, followed by the equalization followed by the IFFT.

So equalization is done in the frequency domain. This is also roughly known as frequency domain equalization. So although it is not exactly the same thing, implies in SC - FDMA equalization is being done in the frequency domain. So this is also often referred to as FDE, Frequency Domain Equalizer. So essentially (your say) what you are saying is, it is much easier to do equalization in the frequency domain, especially after you are transmitting, adding a cyclic prefix because everything is circular.

So you can operate using the FFT and IFFT operations. So, the equalization is implementing in the, implemented in the frequency domain where you have single type equalizer remember. That is the advantage; you have the diagonal matrix, lambda multiplying by lambda inverse. That is a single type equalizer, and therefore you have frequency domain equalization and then you are reconstructing the symbols back using the IFFT.

So FFT equalization, IFFT get back the (())(37:31). So that essentially is the principal of SC - FDMA, which is once again, used in 4g, used in 5g again, one of the most important technologies and waveforms that we have currently and once again, you can see how all of this can be neatly modelled in a very compact, succinct fashion using linear algebra matrices and so on and you do not need a lot of how do you put it complicated mathematical manipulations to describe this.

Once you have the framework in terms of the FFT and IFFT matrices and Eigen value decomposition of the circulant matrix, you can see it falls, it comes out very neatly, simply using the Eigen value decomposition of the circulant matrix and followed by these FFT, IFFT operations, which are once again all represented using matrices.

So, that is the advantage of linear algebra matrix analysis, it gives you a powerful tool to represent these operations in a succinct fashion and gain valuable insights. Alright, so let us stop this discussion here. We will continue in the subsequent modules. Thank you very much.