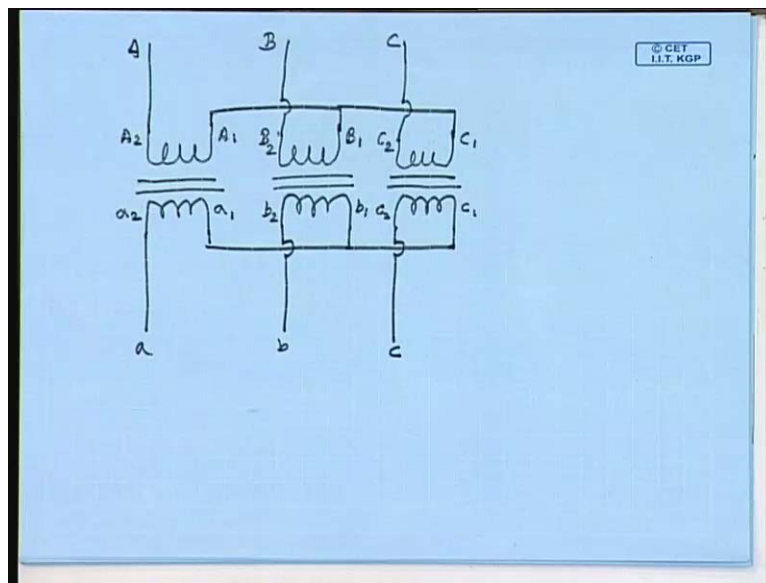


Electrical Machines - I
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Lecture - 12
Three Phase Transformers Connections

We have mentioned earlier that the simplest of the 3 phase transformer will have six windings and twelve terminals.

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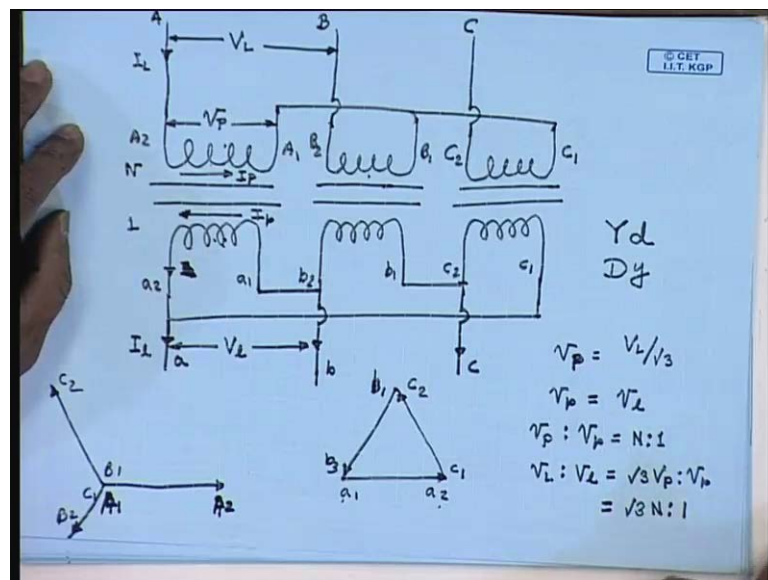
Whether they are made from three single phase transformers or built as a single 3 phase unit, there will be at least six windings and twelve terminals. And we have also decided on a terminal convention for them. The HV and LV windings of a particular phase are marked with the same letters. For example, this is called A₂, this is A₁. Then the same instantaneous polarity terminals on the L V side will be called a₂ a₁; capital letter stands for HV winding; small letter stands for LV winding. Similarly, this will be B₂ B₁, b₂ b₁, C₂ C₁, small c₂ small c₁.

Now, there are different types of connections possible with these twelve terminals. For example, the input side can be star connected by connecting the terminals with suffix 1 and supplying power to the rest of the terminal. Similarly, the low voltage type can also be star connected. This will be high voltage phase: A, B, C. This will be low voltage

phase: a, b and c. We have also seen that, in this case, the high voltage winding and line voltage phasors will be in phase with the low voltage windings and line voltage phasors respectively.

Line currents on the HV and LV side will also be in phase. In fact, the conversion ratio assuming ideal transformer will be same between the phase voltage of the two sides and the line voltage of the two sides will be same. And that will be equal to the turns ratio of the windings. However, this situation will change. This is also true if both sides are delta connected. However, this situation will change if for example, one side is star connected, the other side is delta connected. Let us see what will be the corresponding conversion ratios.

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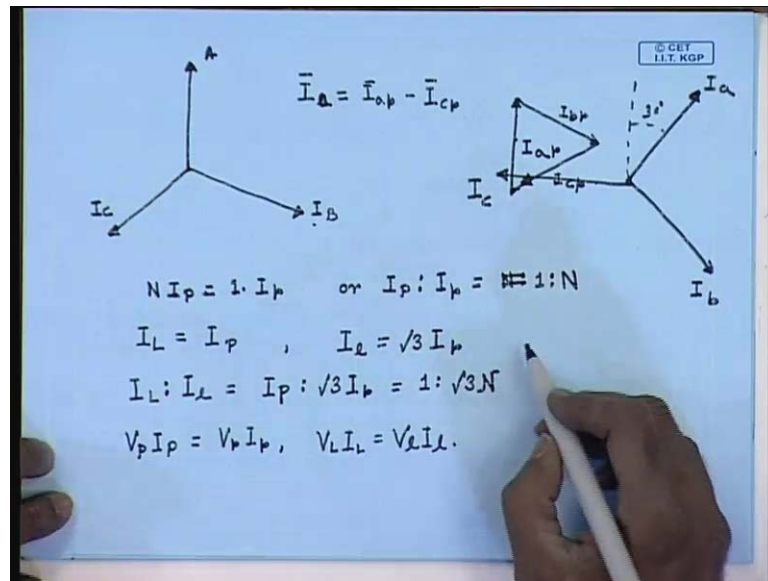
So, this is the high voltage side winding – phase A; and the low voltage winding. This is A 2 A 1; this is B 2 B 1; this is C 2 C 1. Similarly, this terminal is a 2, this is a 1; this is b 2, this is b 1; this is c 2, this is c 1. So, let us say the high voltage side is star connected. So, we form the star connection by shorting the terminals A 1, B 1 and C 1 and giving supply to the remaining terminals. So, this is phase A of the high voltage side; this is phase B of the high voltage side; this is phase C. This is the line current I_L flowing; and this is the line to line voltage V_L . And this is the phase voltage V_p . This is the winding current or phase current I_p . Similarly, this is the phase current on the LV side I_p ; and this is the line current on the LV side I_L .

Now, the low voltage side let us say is delta connected. How do I get this delta connection? Let us say for that, we connect a 1 to b 2, b 1 to c 2 and c 1 to a 2; still get supply from the terminals a 2 b 2 c 2, which constitutes the lines – low voltage lines a, b, c. This is the low voltage line current $I_{small\ l}$. And this is the low voltage side line voltage $V_{small\ l}$. This is called star-delta connection; that is, the high voltage side star represented by a capital Y and low voltage side is delta represented by a small letter d. Had the high voltage side been delta connected, it would have been written as capital D small y delta-star connection.

Now, let us first try to look at the phasor diagram – the voltage phasor diagram first. Since the basic transformer relation must hold; so this is the phasor let us say B – B 2 B 1; that is, this is the terminal B 2; this is B 1. This is... Let us call it A 1 A 2. This is the phasor C 1 C 2 and this side is the phasor B 1 B 2. So, the corresponding winding voltages will be in phase. So, the voltage $V_{a\ 1\ a\ 2}$; this is a 1, this is a 2. Now, on the secondary side, a 1 is connected to terminal b 2. So, this will be the phasor b 1 b 2; and b 1 is connected to c 2. So, this will be the phasor c 2 c 1. So, we see that, voltage small a 2 a 1 is parallel to the voltage V capital A 2 A 1. So, since these two are coupled to each other, similarly for other phasors. So, this is the phasor diagram of a star-delta connected 3 phase transformer.

Now, let us look at the phase voltage, line voltage, phase current and line current transformation ratio. For that, let us assume that, the high voltage side has N number of turns and the low voltage side 1. So, this is the N is to 1 transformer; in which case, we see that, on the high voltage side, the phase voltage V_p equal to the line voltage V_L by root 3. On the other hand, on the low voltage side, the phase voltage V_p equal to the line voltage V_l . But, from the transformer relation, V capital P is to V small p is equal to N is to 1. Therefore, V capital L is to V small l equal to root 3 V capital P is to V small p. This will be equal to root 3 N is to 1. So, the phase voltage ratio is same as the turns ratio; whereas, the line voltage ratio is root 3 times the turns ratio.

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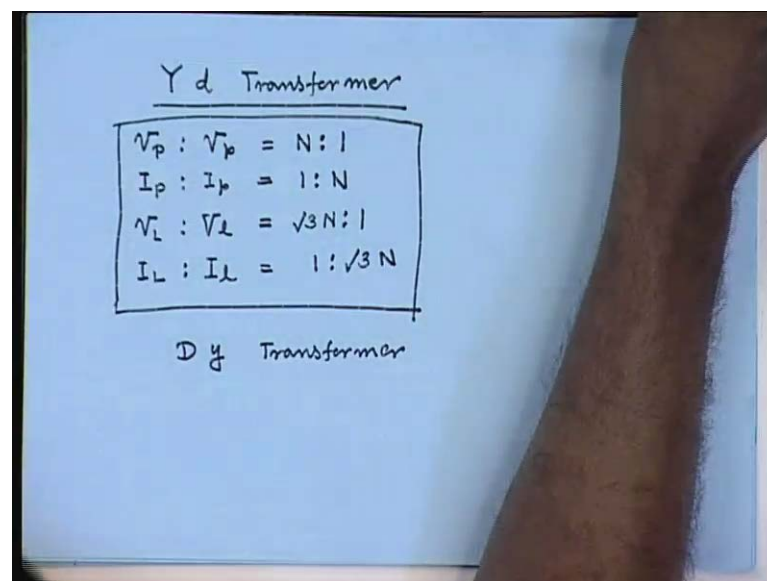
Let us look at the currents. So, what will be the current phasor diagram? Again the currents I capital P and I small p will be in phase in this diagram. So, the current flowing from A 2 to A 1 will be in phase with the current flowing from A 1 to... In the LV side in the same phase. So, let us first draw the current from the primary side from the h V side. So, this is let us say the phase current I A, which is same as the line current; this is I B and this is I C. Now, this I A is same as the current flowing through from A 2 to A 1. So, this will be in phase with the current flowing from A 1 to A 2. So, the secondary current flowing through phase A 1 to A 2 will be in phase with this current. So, this is the current I A 1 to A 2, is the phase current I a, which will be in phase with this. Similarly, the phase current I b will be in phase with the high voltage current I B. So, this will be the phase current I b and this will be the phase current I c.

What will be the line currents? Let us say ((Refer Time: 15:36)) I a phase, I b phase, I c phase. What will be the line current? Let us say line current I small a. This is line current I l of the a phase will be phase current I a p minus this current, which is the phase current I c – minus I c p. So, this is I a minus I c p. What will be the I a? This is I c p. So, the phase current I c line phase – I mean the line current I a on the secondary side will be the vector sum of this phasor and the negative of this phasor, which will be somewhat like this. And this angle be 30 degree. This is I small a. Similarly, it can be shown that, the I small b will be 120 degree with respect to this. And I small c will be 120 degree with respect to this. What will be the ratios? Again, if we assume the individual transformers

to be ideal; then, we know N times $I_{\text{capital P}}$ equal to 1 times $I_{\text{small p}}$ or $I_{\text{capital p}}$ is to $I_{\text{small p}}$ equal to 1 is to N .

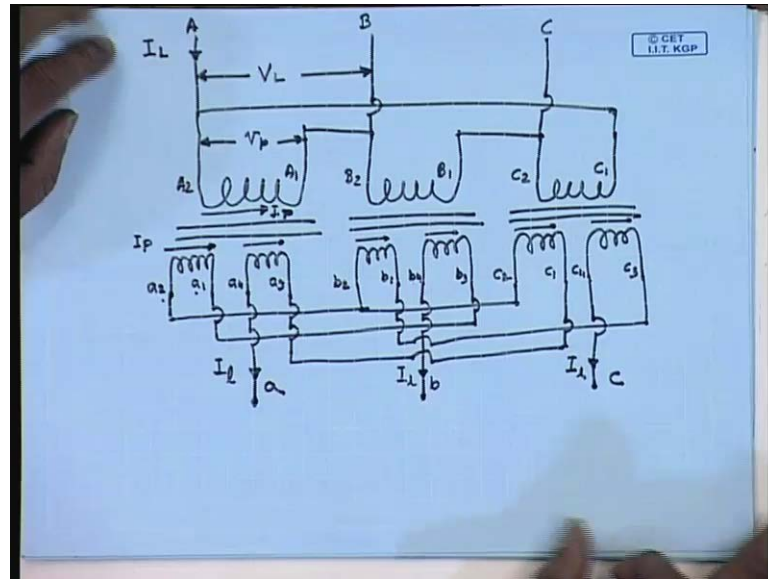
Similarly, for star-delta connection, we know the line current $I_{\text{capital L}}$ is same as $I_{\text{capital P}}$. On the other hand, line current on the delta side – $I_{\text{small l}}$ equal to root 3 times $I_{\text{small p}}$. Therefore, I_{L} is to I_{l} equal to I_{P} is to root 3 $I_{\text{small p}}$. And that is equal to 1 is to root 3 N . Again, it should be noted $V_{\text{P I P}}$ equal to $V_{\text{small p I small p}}$ as well as $V_{\text{L I L}}$ equal to $V_{\text{small l I small l}}$. This relation holds irrespective of the transformer connection. This is because we have assumed the transformer to be ideal. Hence, we have seen that, if one of the sides of the 3 phase transformer is star connected and the other – the high voltage side is star connected, and the low voltage side is delta connected; then, in a nutshell, we can write the relations for the Y d transformers.

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Transformation relation for the Y d transformer is V_{phase} is to on the high voltage side to V_{phase} on the low voltage side. This is same as N is to 1. I_{phase} on the high voltage side is to I_{phase} on the low voltage; this is equal to 1 is to N . V_{line} on the high voltage side is to V_{line} on the low voltage side equal to root 3 N is to 1. I_{line} on the high voltage side is to I_{line} on the low voltage side is 1 is to root 3 N . The same similar relation can be written for D y connected transformer. Although star-star, delta-delta, star-delta or delta-star are common connections for 3 phase transformers; there is another interesting connection for the 3 phase transformer. This is called the zig-zag connection.

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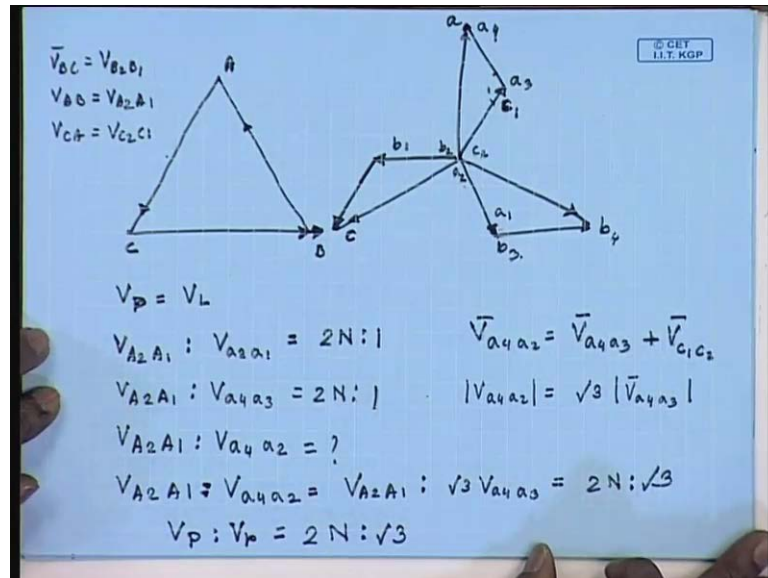
For the zig-zag connection, one of the sides, this is let us say the high voltage side of the transformer – A 2 A 1, B 2 B 1, C 2 C 1. And the low voltage side is zig-zag connected. For zig-zag connection, the secondary winding is split into two parts: each low voltage winding is split into two parts. This is small a 2 a 1; this is called a 4 a 3; this is b 2 b 1; this is b 4 b 3; this is c 2 c 1; this is c 4 c 3; indicating all the even number terminals have same instantaneous polarity, and all the odd number terminals have the same instantaneous polarity.

Now, let us assume that, the primary is delta-connected with A 1 connected to B 2, B 1 connected to C 2, and C 1 connected to A 2. And the connections are coming from... External connections are given to A 2, B 2 and C 2. So, this is phase A; this is phase B; this is phase C of the high voltage side. This is the line to line voltage V_L – V capital L; this is the phase voltage V_p ; this is the line current – high voltage side line current I_L . Let this be the high voltage side phase current I_p .

Similarly, the low voltage side phase current is $I_{small p}$. This current I will call I a 2 a 1; this is I a 4 a 3; this is I b 2 b 1; that is, from B 2 to B 1. In order to make a zig-zag connection, what is one of these sets of windings are star connected. So, one of them... Let us say a 2, b 2, c 2 is star connected forming the neutral. The terminal a 1 is connected to b 3; and. phase b comes out from b 4. Similarly, the terminal b 1 is

connected to c 3; and the line c comes from c 4. Terminal c 1 is connected to a 3 and the a phase connection comes out from a 4. So, these are the line currents I l.

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First, let us see how the voltage and current phasor diagrams will look like for this kind of a connection. Taking the voltage V B C to be the reference phasor, this is B terminal this is C terminal. So, this is basically the voltage of B 2 with respect to B 1 because B is connected to B 2 and C is connected to B 1. So, this is the voltage V B; this is CA. So, this is BC; this AB; this is CA. So, here we have actually V B C equal to V B 2 B 1; V A B equal to V A 2 A 1; and V C A equal to V C 2 C 1.

Then, on the low voltage side, we see that, the star point is formed by connecting terminals a 2, b 2 and c 2 together. So, the potential V a 1 a 2 will be in phase opposition to voltage V a 2 a 1. That is voltage V A B. So, this will be the phasor V A 2 A 1. Similarly, the phasor B 1, B 2 will be in phasor position to V B 2 B 1 which is V B C. So, this will be the phasor B 1, B 2 and this will be the phasor C 1 C 2. So, this is terminal a 1; this is c 1 and this is b 1.

Now, the phase a obtained by adding the phasor V a 4 a 3 with that of a 3 is connected to c 1. So, phase a obtained by adding phase a 4 a 3, which is parallel to V a b - a 2 a 1 with this phasor. So, this is a 3 a 4. And this is the voltage V a; this is the terminal a. Similarly, phase c is obtained by connecting c 4 c 3 with b 1 b 2, which is parallel to this. Hence, this will be the voltage phasor V c or this is the c phase. And phase b is obtained

by connecting b 4 b 3 with a 1 a 2; this is phase b. This is what the voltage phasor diagram will look like. Again we will assume that, the turns ratio is N on the high voltage side is to 0.5 is to 0.5. So, since the primary side is delta connected, V_p equal to V_L . Also, on the secondary side, let us say, $V_A 2 A 1$ is to $V_{small a 2 small a 1}$ equal to $2 N$ is to 1. From this phasor diagram, since a 2 a 1 – the magnitudes of the a 2 a 1 and b 3 b 4... Also, $V_A 2 A 1$ is to $V_{a 4 a 3}$ is also $2 N$ is to 1.

Therefore, what is the ratio of $V_A 2 A 1$ is to $V_{a 4 a 2}$; $V_{a 4 a 2}$ equal to what? From this phasor diagram, $V_{a 4 a 2}$ equal to $V_{a 4 a 3}$ plus $V_{c 1 c 2}$. But, the magnitude of $V_{c 1 c 2}$ and $V_{c 3 c 4}$ are same. And this angle is 120 degree. So, this amplitude is equal... This is equal to... So, mod $V_{a 4 a 2}$ equal to root 3 times mod $V_{a 4 a 3}$. Therefore, this will be equal to $V_{a 4 a 2}$ equal to $V_A 2 A 1$ is to root 3 $V_{a 4 a 3}$; this will be equal to $2 N$ is to root 3 or V phase is to V phase equal to $2 N$ is to root 3. What about the line?

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$$V_L = V_p, \quad V_L = \sqrt{3} V_p$$

$$V_L : V_L = V_p : \sqrt{3} V_p = 2N : 3$$

$$\frac{V_L}{V_L} = \frac{2N}{3} = \frac{N}{1.5}$$

$$V_p = V_L, \quad V_p = \frac{V_L}{\sqrt{3}}, \quad V_p : V_p = N : 1$$

$$V_L : V_L = V_p : \sqrt{3} V_p = N : \sqrt{3}$$

$$\frac{V_L}{V_L} = \frac{N_z}{1.5} \quad \frac{N_z}{1.5} = \frac{N_y}{\sqrt{3}}$$

$$\frac{V_L}{V_L} = \frac{N_y}{\sqrt{3}} \quad \frac{N_z}{N_y} = \frac{1.5}{\sqrt{3}} \quad \text{or } N_z = \frac{1.5}{\sqrt{3}} N_y$$

$$N_z < N_y$$

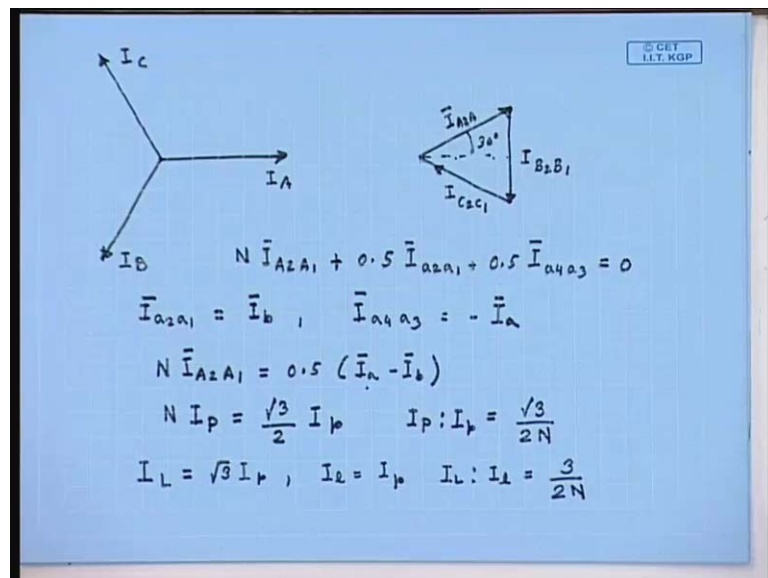
The line is simply obtained, because on the high voltage side, V_L equal to V_P . On the low voltage side, since after all this is star connected, this is incidentally called the zig-zag star connection. So, on the low voltage side, V_L equal to root 3 V_p . Therefore, we have V_L is to V_L equal to V_P is to root 3 V_p will be equal to $2 N$ is to 3. One interesting thing to note here is V_L is to V_L . In this case, equal to $2 N$ by 3 or equal to N divided by 1.5. For a star connected system, both side star – there we knew that... Or, if

the side was star connected, input side delta connected; output side star connected; in which case, we would have seen it differently. For example, for a delta-star connection, the V_p equal to V_L .

On the other hand, on the star side, V_p equal to V_l by root 3. But, for a delta-star connection, V_p is to $V_{small p}$ would have been equal to N is to 1. Therefore, V_L is to V_l would have been V_p is to root 3 V_p equal to N is to root 3. So, this is the difference. So, suppose I have two transformers: one is delta zig-zag star with turns ratio N_z . So, V_L is to V_l . There is N_z is to 1.5. In the other case, I have the same line voltages, but with delta normal star. This is N star by root 3.

Now, if in both cases, the line voltage is on the high voltage and low voltage side are same, then we must have N_z by 1.5 equal to N_y by root 3 or N_z by N_y equal to 1.5 by root 3 or N_z equal to 1.5 by root 3 into N_y . In other words, N_z will be less than N_y ; which shows that, for the same line voltage ratio, if we use a zig-zag star connection on the secondary side, then the number of turns on the HV side can be reduced. So, that gives a cheaper transformer. That is why sometimes it is preferred.

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Now, let us look at the current phasor diagram. The input side is delta connected. So, let us say this is the line current on the high voltage side – I_A ; this is I_B ; and this is I_C ; I_A , I_B , I_C . From the circuit connection, we have seen that, we can write from here that, line current I_A equal to... This current equal to this current plus this current, which is

minus of I_C . So, it is I_{A2A1} minus I_{C2C1} . Similarly, line current I_B equal to I_{B2B1} minus I_{A2A1} ; and I_C equal to I_{C2C1} minus I_{B2B1} . If we keep this relations in mind, then the phasors: I_{A2A1} , I_{B2B1} , I_{C2C1} will look like this. This is I_{A2A1} ; this is I_{B2B1} ; this is I_{C2C1} . This angle be 30 degree. Now that we have got the winding currents, we can now draw the winding currents on the low voltage side.

Now, the currents on the low voltage side; from this phasor diagram, we find the current $I_{small a}$ equal to... This is the $I_{small a}$. The current $I_{small a}$ is basically I_{a3a4} , which is same as current $a3a4$, same as current I_{c2c1} . Now, it has to be seen that, in order to balance mmf, we must have N times I_{A2A1} plus 0.5 times I_{a2a1} plus 0.5 times I_{a4a3} . This must be equal to 0. Now, from the phasor diagram, we have seen that, I_{a2a1} . What is I_{a2a1} ? I_{a2a1} is this current flowing through this terminal through this and coming out. So, this is equal to I_b .

Similarly, I_{a4a3} – this we have already seen; this is equal to minus I_a . So, we can write N times I_{A2A1} equal to $0.5 I_a$ minus I_b or N times $I_{capital P}$ equal to $\sqrt{3}$ by 2 times $I_{small p}$, because I_a and I_b are the phase currents of the low voltage sides, which are 120 degree apart. So, I_a minus I_b is $\sqrt{3}$ times the phase current. Hence, we have I_P is to I_p equal to $\sqrt{3}$ by $2N$. But, on the high voltage side, I_L equal to $\sqrt{3}$ I_p ; and on the low voltage side, I_l equal to I_p . Therefore, we have I_L is to small I_l equal to 3 by $2N$. So, this is for the delta zig-zag star.

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Handwritten equations on a blue board:

$$V_p : V_p = 2N : \sqrt{3}$$

$$V_L : V_L = 2N : 3$$

$$I_p : I_p = \frac{\sqrt{3}}{2N}$$

$$I_L : I_L = \frac{3}{2N}$$

$$V_p I_p = V_p I_p \quad \& \quad V_L I_L = V_L I_L.$$

Therefore, we can summarize it as V_P is to V_{phase} equal to $2 N$ is to $\sqrt{3}$; V_L is to V_l equal to $2 N$ is to 3 ((Refer Slide Time: 53:52)). I_P is to I_p equal to $\sqrt{3}$ by $2 N$; and I_L is to I_l equal to 3 by $2 N$. Again here also, we see $V_P I_P$ equal to $V_p I_p$; and $V_L I_L$ equal to $V_l I_l$; which is due to the fact that, we have assumed all transformers to be ideal.