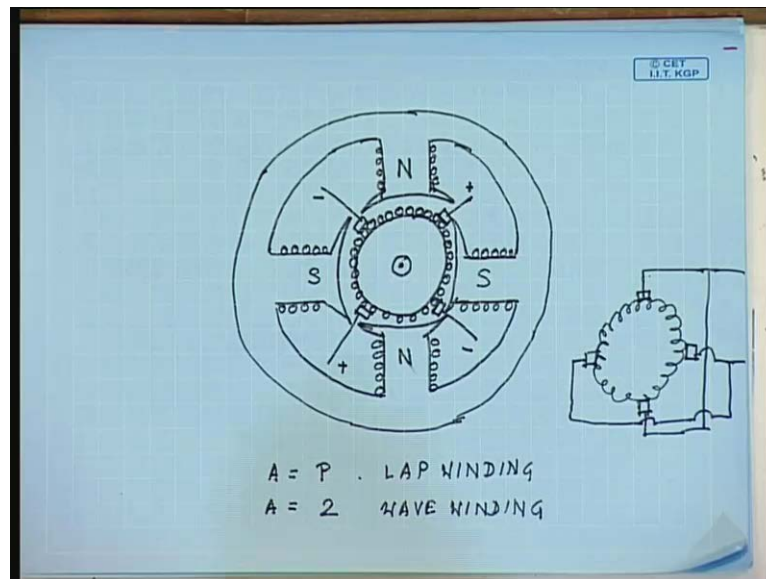


Electrical Machines- I
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Lecture - 23
General EMF and Torque in DC Machines

In the last class, we have seen that the cross section of a direct current machine.

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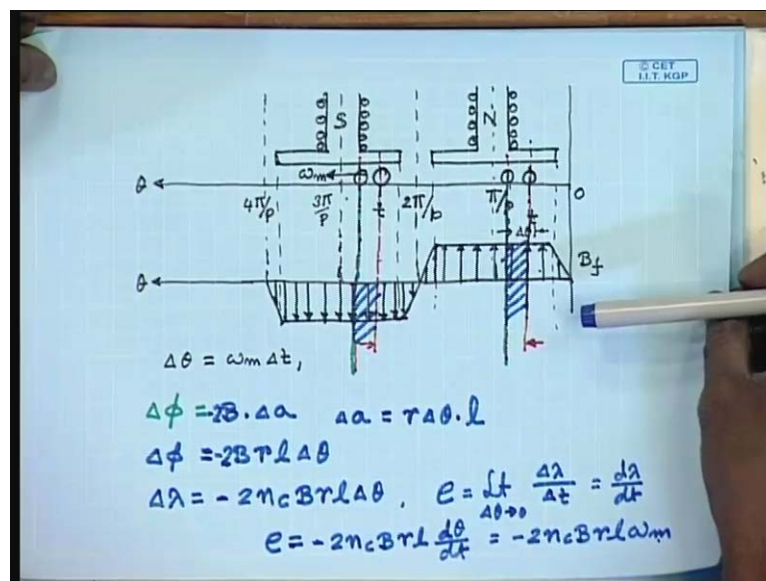
Can be drawn somewhat like this, it has a stator core to which holes are connected stationary poles field poles. In general a dc machine can have more than two poles. The poles are fitted with pole shoes, which cover the most of the air gap part and carries winding. These windings are electrically excited with dc current, such that alternate north and south poles are generated. The armature the rotor carries the armature. There is mounted on a shaft axial shaft the connection to the rotor are obtained by means of commutators and brush segments. The brushes are normally pressed what we call the inter-polar region?

That is it connects to the conductor that are at the inter-polar region. This is plus plus minus minus the brushes divide the armature is a closed winding the brush divide the armature in several parallel path. The closed winding of the armature can be schematically represented like this, and the brushes here the positive creases are connected together, the negative are also connected together and two terminals are

finally brought out. However, the brushes divide the whole armature winding into a number of parallel paths number of parallel paths a is equal to the number of poles of the dc machine for lap winding and the number of parallel paths a equal to 2 for wave winding.

Now, when such an arrangement is rotated inside the stator, this armature is rotated inside the stator an EMF is induced in the armature coils, and across the brushes as the coils are series connected between brushes as well as if a current is allowed to flow through these conductors by either connecting an external resistance and rotating the machine in which case it access as generator or by supplying the armature from an external voltage source in which case it access a motor. A torque is also produced. In this class we will try to find put the expressions of this generated emf and the generated torque as functions of different machine parameters for this purpose, let us take what we call a developed view of this cross section? Developed view is nothing but cutting the cross sectioned diametrically and unfolding it in a straight line.

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If we do that exercise along these axes and fold it open then it will look somewhat like this. Thus this is my position 0. There are p number of poles if this is the inter polar region then the axis of the first north pole will be at a position π by p. the next inter polar region will come at 2π by p. Next pole will come at 3π by p. And, then you will come back to the original position and the next polar region inter-polar region will be at

4π by p . Since, if you had a pole pair n pair of poles the action due to a pair of poles is identical for all pole pairs will concentrate only on the behaviour of the machine under one set of pole pairs.

So, our diagram see here let us say this is the north pole, and this is the south pole. The poles are fitted with pole shoes which cover most of the armature surface therefore. you can draw the pole shoes somewhat like this. So, this is north pole, this is south pole the flux density waveform produced by the field winding can be approximated by a flat top for which goes to 0 in the at the inter polar region. In this region flux is going from the pole to the armature, which we call positive flux and denote by these arrows. And, this is a negative flux which flows from the armature to the south pole and denoted by these negative arrows.

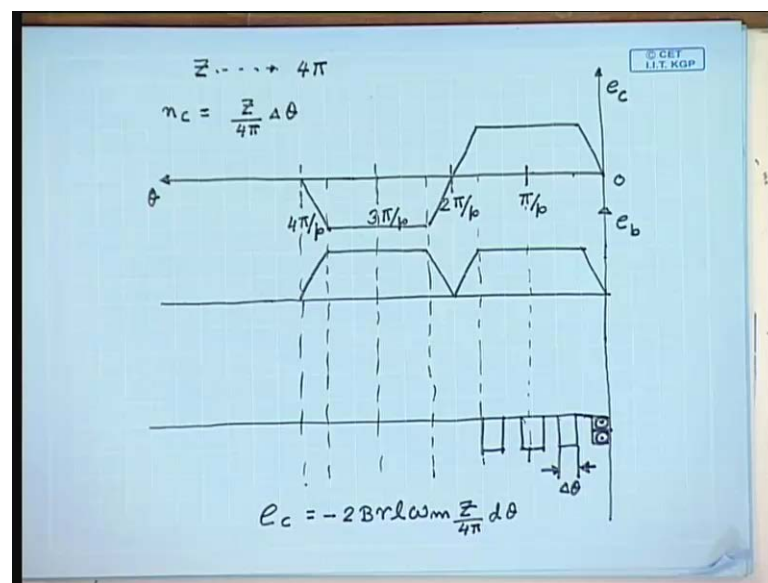
So, this is the flux density waveform due to the field winding alone. Now, consider at any time a coil on the armature full pitch coil by that we mean that if one end of that coil is under north pole the other end will be under south pole. Let this be the position of the coil at time T and let the armature along with this coil be moving in this direction with an angular velocity of ω , then after a short time $T + \Delta T$, this coil will move by an angle $\Delta\theta$. This angle will be $\Delta\theta$, what will be $\Delta\theta$? $\Delta\theta$ equal to ω into ΔT . Now, let us look at the change in flux linkage of this coil $\Delta\lambda$.

At time T the flux passing through the coil can be found by integrating the flux density waveform between these two limits, and at time $T + \Delta T$ the flux density waveform flux, then flux linkage can be found by integrating the flux density open between these two lines. However, let us find out the change in flux linkage or rather change in flux first $\Delta\phi$. The change in flux $\Delta\phi$ if we try to find out we see that the flux passing through this area. Positive flux passing through this area has gone out of the coil as well as some amount of negative flux has been included and from the symmetry of the flux density waveform they are equal.

How much is the flux contained in this part it can be found from the flux density waveform and the area the area of that the armature area due to this angle $\Delta\theta$ Δa equal to the arc length r into $\Delta\theta$ into the length of the armature. So, $\Delta\phi$ equal to minus two B into Δa , that is $\Delta\phi$ equal to $Br l \Delta\theta$. So, and 2

times this and that is in the negative direction. So, this is minus 2 times $b r l \Delta \theta$ and the change in flux linkage $\Delta \lambda$ equal to number of turns in this coil let us denote it by n_c . So, $\Delta \lambda$ equal to minus 2 $n_c b r l \Delta \theta$. So, the magnitude of the emf induced in this coil at the instant t is given by e equal to limit $\Delta \theta$ tending to 0 $\Delta \lambda$ by Δt . That is equal to $d \lambda / d t$. So, e equal to minus 2 $n_c b r l d \theta / d t$ that is minus 2 $n_c b r l \omega m$. So, we see that the voltage induced a coil at a given position θ is proportional to the flux density at that position and the rotational speed of the machine.

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So, we can draw the waveform of the induced voltage in that particular coil as it moves along the armature surface. Since, 0 this is π by p this is 2π by p this is 3π by p this is 4π by p , as the coil moves along this direction the emf induced has a similar waveform as that of the flux density waveform. That is where θ is the position of the first end of the coil. Now, this is to be understood that we do not see this waveform on the brush. Since, the coils are connected to the brush through commutators, let us try to visualize the situation the brushes are placed at the inter-polar region. Therefore, this coil will connect to the brush first when it is at position 0 and the induced voltage in the coil is 0, and if this is the only coil then the commutator segment spends half the armature periphery, therefore for half the cycle it will remain connected to one commutator segment and then for the rest half it will connect to the other commutator segment, hence

in effect the voltage induced in the brush voltage in at the brush will be the rectified waveform of the voltage induced in the coil.

This is the voltage induced in the coil this is the voltage in the brush, across the brush. This will be would have been the voltage across the brush had there been only one coil on the armature, but now consider a practical dc machine where there will be large number of such coils. For example, if we look at the armature surface, this contains slots. and dc machines usually have double layer. So, you each slot there will be two coil lengths. So, and if we assume that one of this slot width of denote the width of the slot by $\delta\theta$, then the number of turns n_c in a coil can be represented in terms of $\delta\theta$ as follows.

Let there be total conductor in the armature these are uniformly distributed over the armature periphery. So, and the armature is double layer. So, the z numbers of conductor are distributed over a mechanical angle of 4π . So, the number there is the number of turns in a coil that is conductor in a coil is n_c , So n_c can be written as z divided by 4π into $\delta\theta$ hence the voltage induced in a coil e_c equal to $\text{minus } 2 B r l \omega m z$ by $4\pi \delta\theta$, that is the voltage induced in a single coil. Now, it is to be understood that between two brushes. There are number of coils connected in parallel. In fact, all the coils that are in the span 2π by a brushes are set apart by an angle of the number of conductors between two opposite polarity brushes is given by 2π by a . The number of coils connected in series between two brushes is given by 2π by a .

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$$e_b = \int_0^{2\pi/a} e_c d\theta$$

$$= 2\omega_m \frac{Z}{4\pi} \int_0^{2\pi/a} B r l d\theta$$

$$\phi = \frac{a}{p} \theta, \quad d\phi = \frac{a}{p} d\theta$$

$$e_b = 2\omega_m \frac{Z}{4\pi} \cdot \frac{p}{a} \int_0^{2\pi/p} B r l d\phi$$

$$E_b = e_b = 2\omega_m \frac{Z}{4\pi} \frac{p}{a} \phi_p$$

$$E_b = \frac{Z}{2\pi} \frac{p}{a} \phi_p \omega_m = k \phi_p \omega_m$$

$$k = \frac{Z}{2\pi} \frac{p}{a}$$

$a = p$ for Lap winding
 $a = 2$ for Wave winding

$$\omega_m = 2\pi n = \frac{2\pi N}{60}, \quad E_b = \frac{\phi_p Z N p}{60 a}$$

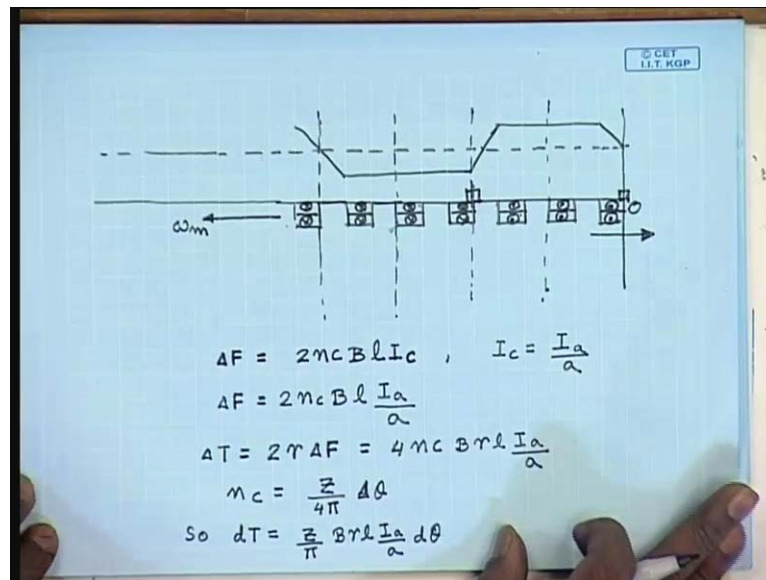
Therefore the induced voltage between the brushes can be written as integral e_b in a practical d c machine can be written as integral 0 to because one of the brush will be at the inter polar region 0 the other will be at 2π by p , but the number of coils connected in series between these two brushes will be 2π by a , where a is the number of parallel paths. So, 2π by a $e_c d\theta$, e_c which is equal to $2\omega_m n z$ by 4π integral 0 to 2π by a $B r l d\theta$. Now, let us do a change of variable as follows it has defined ϕ equal to a by p θ .

Therefore, $d\phi$ equal to a by p $d\theta$. So, e_b can be written as $2\omega_m n z$ by 4π into 2 by a integral 0 to 2π by p $B r l d\phi$. This term under integration can be easily identified as the total flux passing through the north pole, hence this is termed the flux per pole of ϕ_p therefore, the simple formula of induced voltage across two brushes is given by. So, this is also the terminal voltage of the dc machine or the generated back emf in the dc machine because the voltage is measured across the brushes. So, the back emf expression is given by e_b equal to z divided by 2π p by a $\phi_p \omega_m$ this is sometimes written as $k \phi_p \omega_m$, where k is the back e m f constant as given by k equal to z by 2π p by a .

Now, a equal to p for lap winding, and a equal to 2 for wave winding. There is a slightly more slightly different formula for this e_b also if we write ω_m in terms of rpm then ω_m equal to 2π into n , where n is the rotational speed of the dc machine rotor in

rps or equal to $2\pi n$ by 60, where the capital n is the rotational speed in rpm. So, e_b can be written as $\phi p z$ by $\phi p z n$ by 60 into p by a , this formula is also very much in use. So, we have seen that how to find out the induced emf in a dc machine from the first principle. Now, let us find out the torque for that.

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We again go back to the original diagram of the developed diagram, the flux density waveform due to field winding. We have mentioned is trapezoidal that is of this shape and the armature contains slots and coils in the slots in double layer. We have seen that if this armature is moving at a speed ω_m in this direction. Then, there will be a induced voltage in this coils, what will be the direction of the induced voltage from faradays principle? We can find say that the induced emf will be in such a direction that if a current is allowed to flow in the coils it will oppose the change of flux linkage.

Now, we have seen that due to the motion the flux linkage change is in the negative direction that is flux link positive flux linkage tends to decrease, so the current will flow in this coil in a such a fashion that this decrease in the flux linkage is compensated that is the currents in these coils will produce positive flux in this direction. So, what will be the direction of current flow? it has to be understood that the positive flux means flux flowing from the field to the armature definitely, then the current flowing in this coils will be of dot polarity and current the other side of the brush will have cross polarity due to the induced emf. Now, we also know that if there is a current carrying conductor

under a magnetic field, then the conductors experience a force and on the Fleming's left hand rule. We can find out the direction of the force in this case the direction of the force will be opposite to motion.

We can also find out the magnitude of this force acting on some conductor, let us say one coil delta f will be equal to the number of conductors at any given position which due to double layer winding is 2 n c. So, this will be 2 n c and then B into l and into I c that is the current flowing through this conductors. Now if the armature has a number of parallel paths, then I c equal to armature current I a by a there put delta f equal to 2 n c B l I a by a, now let us look at the original diagram of the dc machine the conductors here are experience a force of delta f in the direction opposite to the motion.

Similarly the conductors diametrically opposite are also experiencing a force and again in the direction opposite to the motion, so the torque produced by this conductor sets that these two sets of conductor is given by delta T equal to 2 into r into delta f equal to four n c B r l I a by a. However, we have found out n c equal to z by 4 pi into d theta. So, d T equal to z by pi b r l I a by a into d theta. Now, we know that the total torque in order to find the total torque will have to add the torque produced by individual coils between two brushes, because the rest of the coils will, now be taken care of by other sets of.

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$$T = \frac{Z}{\pi} \cdot \frac{I_a}{a} \cdot \frac{P}{2} \int_0^{2\pi/P} B r l d\theta$$

$$= \frac{Z \phi_p P}{2\pi a} I_a = \frac{Z}{2\pi} \cdot \frac{P}{a} \cdot \phi_p \cdot I_a = K \phi_p I_a$$

$$P_e = E_b I_a = K \phi_p \omega_m I_a$$

$$T_e \omega_m = P_e = K \phi_p \omega_m I_a$$

$$T_e = K \phi_p I_a$$

$$V = E_b = K \phi_p \omega_m$$

$$V I_a = K \phi_p \omega_m I_a$$

$$T_e \omega_m = K \phi_p \omega_m I_a$$

$$T_e = K \phi_p I_a$$

So, there are between two brushes you will have to integrate them. So, torque T equal to by pi into I by I a by a you have to integrate over 1 pole pitch that is integral 0 to 2 pi by

p brush pairs. However, there are $p/2$ such pole pairs which is generating the torque this is the torque produced by 1 pole pair. So, the total torque or in order to get total torque this has to be multiplied by $p/2$. So, the total torque expression will this we again identify as flux per pole. So, this will be $z \phi p / 2 \pi a I_a$ equal to $z / 2 \pi$ into p by a into ϕp into I_a , this is again can be written as $k \phi p I_a$, where k is the torque constant and we found that a torque constant is same as the back emf constant found during calculation of the induced emf.

This can also be shown from the power balance. Let us say this is the armature with the brushes with an external resistance connected and being rotated in the direction with a speed of ω_m let the current here be I_a if we assume no loss in the machine, that is if we assume armature resistance of the machine to be 0, then the power produced by the generator equal to e_b into I_a , the e_b equal E_b equal to $k \phi p \omega_m$ I_a . The electrical power produced must be equal to the mechanical power input, so in order to rotate this armature some amount of mechanical work has to be done that is it has to be rotated against a generated torque T_e .

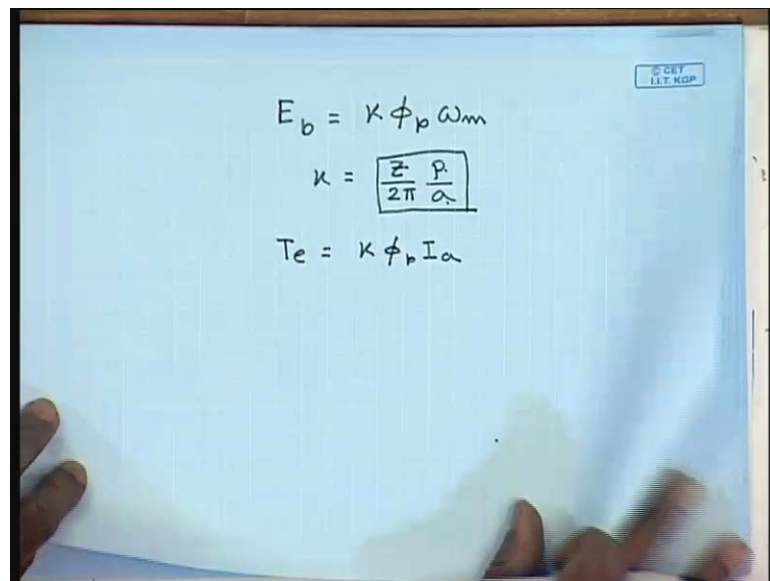
Therefore $T_e \omega_m$ should be equal to P_e equal to $k \phi p \omega_m I_a$ or T_e equal to $k \phi p I_a$, which is same as what we have obtained for as the torque expression from the first principle. This we have shown when the machine works as a generator, however the same conclusion will be arrived if we consider a motor mode of operation. In the motor mode of operation the only change will be the direction the armature terminals will be connected to an external source of voltage v and if we neglect the armature resistance then in steady state v equal to E_b equal to $k \phi p \omega_m$ therefore, electrical power input to the machine is $v I_a$ equal to $k \phi p \omega_m I_a$.

This must be same as the mechanical work done where direction of torque will also now change because the direction of current has changed, so $T_e \omega_m$ should be equal to $k \phi p \omega_m I_a$ or torque in the motor mode will also be given by $k \phi p I_a$. Therefore, we have seen that the dc machine, when the armature rotates under the influence of field poles field flux a emf is induced across the brushes the magnitude at the e m f is proportional to the flux produced by the field poles that is flux per pole ϕ is the and the rotational speed of the armature in the same manner.

We have shown that if a current is allowed to flow in the armature conductors either by connecting a load in the generating mode or by connecting the armature to a separate voltage source to make it run as a motor then an electromagnetic torque is generated the magnitude of which is again proportional to the flux per pole and the armature current in the armature circuit. For the motoring mode the current is injected into the armature by an external circuit, so that electrical power is input to the machine, and hence the generated electromagnetic torque will be in a direction which will assist motion that is mechanical work will be done.

So, the direction of electromagnetic torque will be in the same direction as the that of rotation that it will assist rotation, and hence electrical power input will be converted to mechanical power output. The reverse will happen in the generating mode that is if a load is connected then the current flowing through the armature winding would try to stop the motor that is the electromagnetic torque produced would be in a direction which is opposite to the direction of rotation, however if the flux per pole and the magnitude of the armature currents are same in the motoring, and in the generating mode, then the magnitude of electrical torque produced will be given by the same formula that is $k \phi_p I_a$ both in motoring and in generating mode, similarly the induced voltage either in the motoring mode or in the generating mode will be given by the formula $k \phi_p \omega_m$.

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A photograph of a whiteboard with handwritten mathematical formulas. The formulas are:

$$E_b = k \phi_p \omega_m$$
$$k = \frac{Z P}{2\pi a}$$
$$T_e = k \phi_p I_a$$

The whiteboard has a small logo in the top right corner that reads "SCET IIT KGP".

So, whether the machine operates as a motor or as a generator the formula for induced voltage in a dc machine is given by $E_b = k \phi \omega_m$, where $k = \frac{z}{2\pi p a}$, z is the total number of armature conductor p is the number of poles and a is the parallel path and electromagnetic torque is given by $T = k \phi I_a$ where k has the same expression .

Thank you.