

**Energy Resources and Technology**  
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**Lecture - 3**  
**Complete Cycle Analysis of Fossil Fuels**

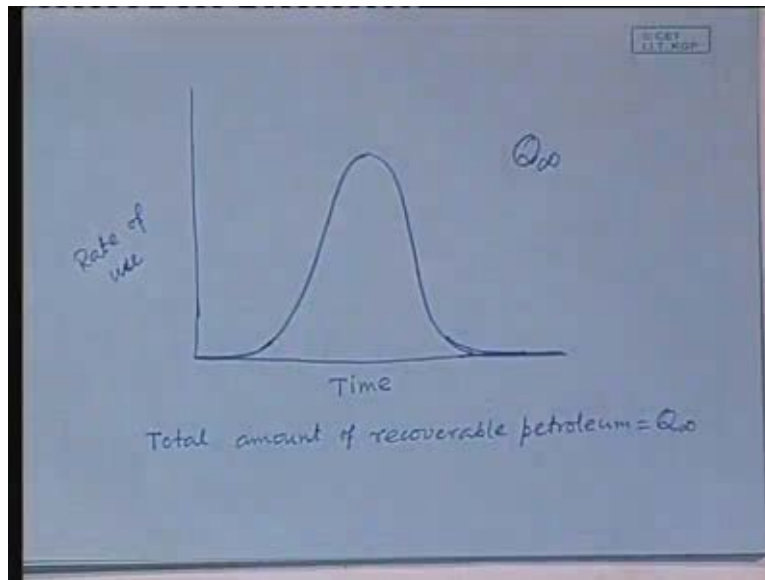
So, in the last class we had talked about the thermodynamics of, thermodynamic basis of energy engineering, but now we have come to the stage where we can talk about the actual content of the negentropy stocks and the main negentropy stock that we would be concerned with are the non-renewable resources, fossil fuels. Now, these fossil fuels namely coal, petroleum, natural gas and there are also a few somewhat uncommon fossil fuels like the shell oil and things like that. These are all accumulated, concentrated solar energy that had been accumulated for millennia, for millions of years and finally that is available to us. Notice that they are all in finite quantity; cannot be infinite, of course, right and so, the main crisis that we are facing is essentially that these finite resources are sort of close to coming to an end and in newspapers, in radio talks, in TV, in politician's statement in the parliament, you will often hear these things like we are very close to things being exhausted and stuff like that and some people will also say that is it really true that the petroleum is being exhausted, isn't it true that every year we are striking new oil fuels?

So, it is true that the the old ones are being exhausted, but the new ones are being discovered, so it is not all that a big crisis that you are making out to be. So, in this situation often the layman finds himself in a precarious situation. Who to believe? Is it really exhausting or it is only some sort of a myth that it is being really exhausted. So, in this situation, we need some kind of a scientific basis for talking and for energy engineers that is even more vital; vital in the sense that when we talk about energy, its availability, how you use, what will be the policy and what will be the politics of energy that is even more taking center space these days, we have to have a clear idea about the real status. How much, where exactly are we in terms of the exhaustion. So, that will be the matter of discussion in today's class.

Now, when we talk about any finite resource like, like say petroleum, it is not difficult to see that man did not know its use before a certain time, right, before say the advent of the internal combustion engines, what was the use of petroleum? Practically nothing; there were no oil fields. Only a few places where you find oil very close to the ground that means you had oil, right, if you dig a bucket you get oil; there are places like that, so in those places people found some use of it, but that is of course, in terms of the quantity that would be negligible. So, if you talk in terms of the rate of exhaustion, obviously that starts from a zero level and then as technology improves, the newer technologies are discovered, that uses that particular resource, the use increases, the exploitation increases.

Now, do you see that that should follow a certain type of curve, something that has zero value at minus infinity and that then, it sort of grows. To what level will it grow? If we talk about the rate of use of the resource like petroleum, then what would be the curve like?

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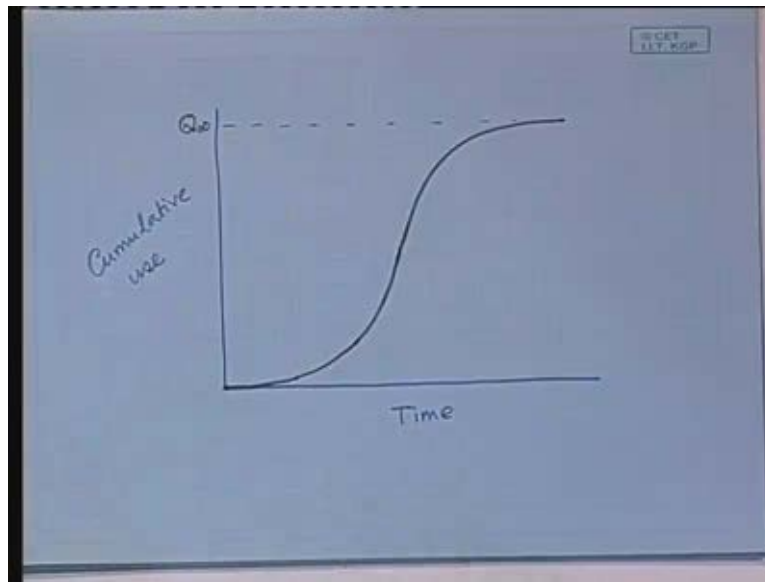
You will have the two axis; this axis representing what? Time and this axis representing the rate of use, obviously that has to start from zero value. Not only it has to start from zero value, but also the slope should be zero at that point. The slope should be zero means

at very old time the the rate at which the rate changes, it should also be zero, because people did not know its use. So, it rises, but after that, how will the, logically speaking, how will the curve grow? Exponentially; exponentially means it will go to infinity. Can the rate of exploitation, could go to infinity for finite resource?

Of course, yes; so, it should reach a maximum, it should reach a maximum and then, we expect this kind of a curve, where it reaches a maximum and then, it has to and then what will happen? Must drop, because it is finite, must drop. So, it should drop and it should come down like that and you have to have a curve that finally sort of tapers off to infinity. When it is completely exhausted, almost then you do not have any further change. So, this should be, logically speaking should be, the curve. Is that clear? Now, the area under this is the total amount of petroleum that was available for exploitation and this quantity is normally called  $Q_{\infty}$ .  $Q_{\infty}$  means the total amount of petroleum exploitable, petroleum recoverable, so that obviously is the area under this curve.

Now, it is normally convenient to draw not this rate, because the rate has quite a lot of variability. Variability, because that depends on the market place; suddenly there is Iraq war, the price goes up and stuff like that, so local temporal perturbations play a role here. So, normally it is more convenient to draw the cumulative amount of exploitation.

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So, we prefer to draw a curve, where time is here. The cumulative use is in the y-axis. Cumulative means how much has been exploited till date. That is going to plot against time. So, if you plot that, then it will be nothing but the integral of this. So, what will be the character of the curve like? It will again start from zero; let me draw with another colour, it will again start from zero and it will rise, it will rise and then, yes, the slope will come down and finally, it will saturate to ....., right. It would saturate to a magnitude which is  $Q_{\infty}$ . So, this point here is ....., clear? So, that should be the, logically speaking that should be the curve for the cumulative use of the fossil fuel.

In fact, this should be the curve for the cumulative use of any resource that is finite; that is finite. So, you have a curve like this. If I ask you what the curve is like similarly, what is the curve for the discoveries, initially we did not know any oil field, right. So, there also it started from zero and then we started discovering. Though discovery sort of progresses in steps, right, you suddenly discover a large oil field so it goes in steps, it does not go rather smoothly, but nevertheless, you can approximate that by a smooth curve, but what will be the character of the curve? If I ask you, I want to plot the cumulative discoveries as a function of time, what will be the curve like? Come on, what, what does your common sense say?

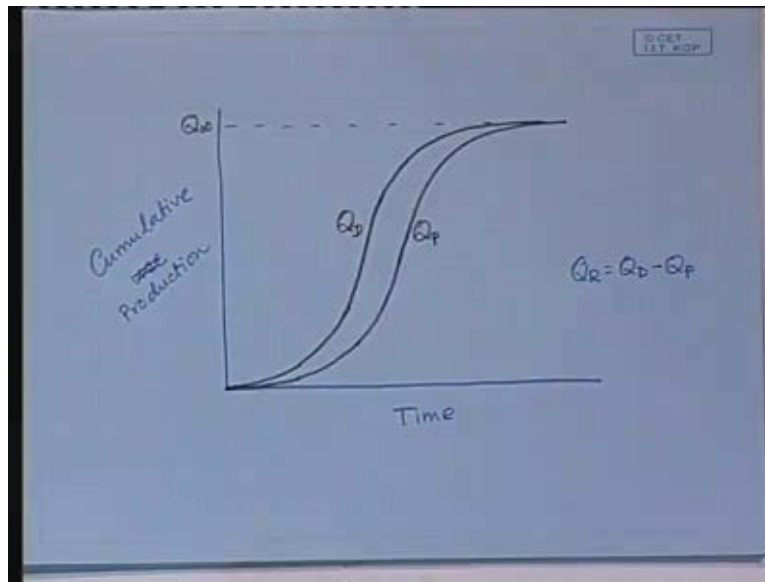
Similar, yes similar; but, what is the difference?

Student: Sir, it will saturate.

It has to saturate, because if it does not saturate, goes on, then the total amount of discovery is infinity. So, it has to saturate and it has to saturate to the same value, because that is the total amount of recoverable petroleum and you can discover only that amount, you cannot discover anymore, right. So, it has to saturate there. It has to start from the same point; it has to start with the same slope it has to end with the same slope, right. Why does it have to start at the same slope? Because, rate at which the knowledge or the rate of discovery is progressing that also starts from zero and as you go close to the exhaustion point, obviously you no longer discover anything. So, the rate at which the discovery progresses that also goes to zero. So, these two points should be identical for these two curves, but isn't it true that the cumulative discoveries should all point of time be greater than the use. Unless you discover, you ..... do not use it. So, it will be of the same character. It will have to start from the same point at the same slope, it will have to end at the same point at the same slope, but at every point of time it has to have a value greater than the use.

As yet, I have plotted cumulative production and now I am talking about what should be the character for the cumulative discoveries? Because, unless you have discovered some amount, you know that some amount is there, you do not utilize it. So, his question is why should the cumulative discoveries always be greater than the cumulative production, because even at any point of time if you know that certain Giga barrels of oil is there, you have exploited a part of it, so the cumulative production is less than the cumulative discoveries. The amount that is there known. So, the character of the discovery curve should be something like this.

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It should start from the same point, but it should be larger at every point of time, right. All the constants that I talked about are satisfied with this curve, clear. So, this is Q discovery, Q D and this is Q P, cumulative production and cumulative discoveries. It is better to use the term production here rather than use. So, you have this and then at any point of time, there should be some proven reserve, the amount that you know is there. Right now, we know that this amount of oil is there, right. So, at every point of time, there will be some proven reserve and what is the proven reserve? The subtraction from Q D to Q P; Q D minus Q P is the reserve, so Q R is Q D minus Q P. So, what will be the curve for Q R? Can you see from these two? These two, Q D and Q P have specific characteristic curves and I am asking you what will be the character for Q D minus Q P?

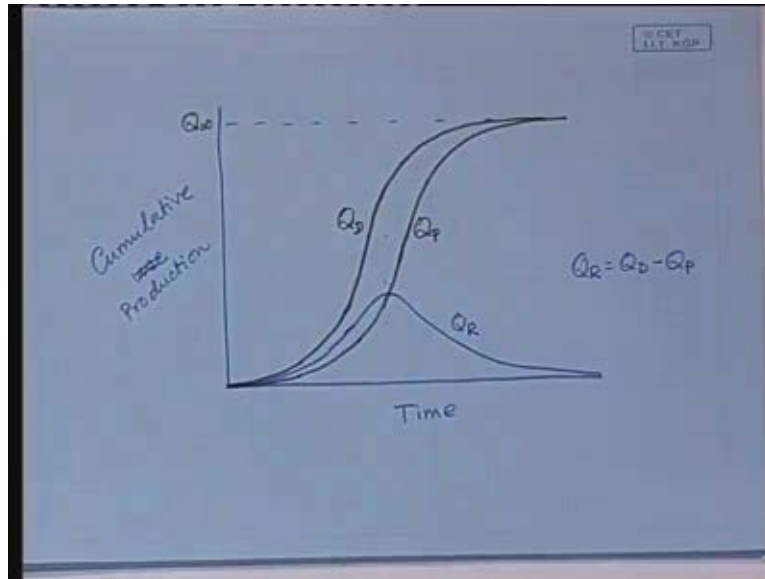
Student: Parabolic.

No; not a parabolic. Parabolic means it starts from zero, but the slope is not zero at that point, right. Here the slope is zero, here is the slope is zero. Therefore, for the subtraction curves also, slope must be zero.

Student: Sir, bell shaped curve.

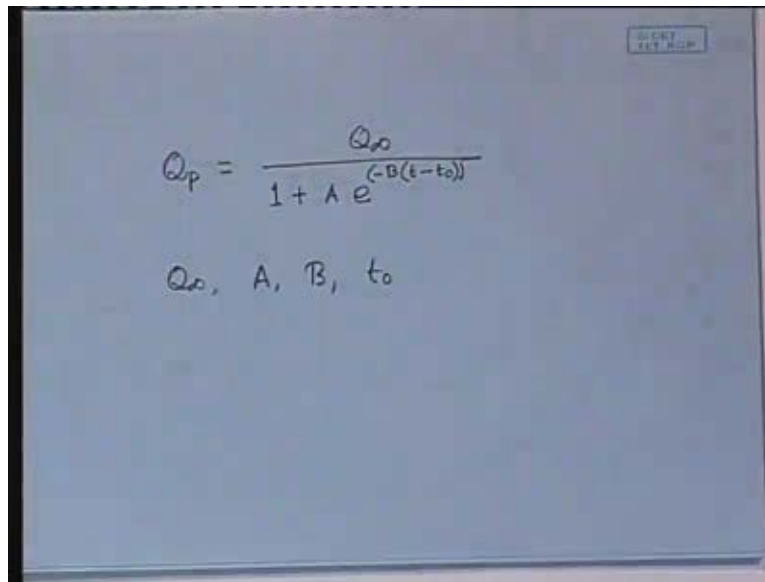
Yeah, bell shaped curve. It should be a bell shaped curve more or less centered at this point, right. So we can draw it like this.

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So, this is our  $Q_R$ ; fine. Now, the scientific way of tackling the problem of assessing where we are in terms of the availability of the fossil fuels is to determine these curves. We should actually try to determine this curve. Once we can determine this curve, ....., many of the conclusions we can draw. I will come to that, but then in order to determine these curves, we must know their equation. So, we let us write down the equation for a curve like a  $Q_P$ .

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$$Q_p = \frac{Q_\infty}{1 + A e^{-B(t-t_0)}}$$

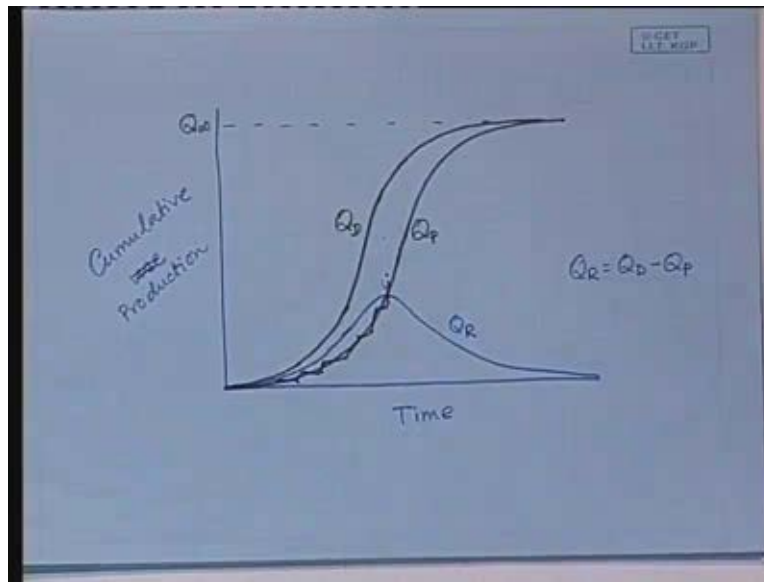
$Q_\infty, A, B, t_0$

So,  $Q_p$  that curve's equation can be, I mean that curve can be fitted into the equation  $Q$  infinity divided by 1 plus  $A e$  to the power minus  $B t$  minus  $t$  naught. So, this is the equation for the logistic and you will get, if you can estimate the values of  $Q$  infinity,  $A$ ,  $B$  and  $t$  naught you can draw the graph. So, unknowns in this equation are  $Q$  infinity,  $A$ ,  $B$  and  $t$  naught. What is  $Q$  infinity? Again, in terms of concept, it is the ultimately recoverable quantity of petroleum on Earth. What is  $A$  and  $B$ ? These are two curve fitting constants and what is  $t$  naught? Some kind of a base here, because the difference between this curve and that that is simply shifted and so, you need to have some kind of base here that will shift. So, you have this.

The problem in the curve fitting affair is that we only have partial data because we have not gone up to this point. Had we gone up to this point, there would be no point in doing all these exercise. We are somewhere midway, right.



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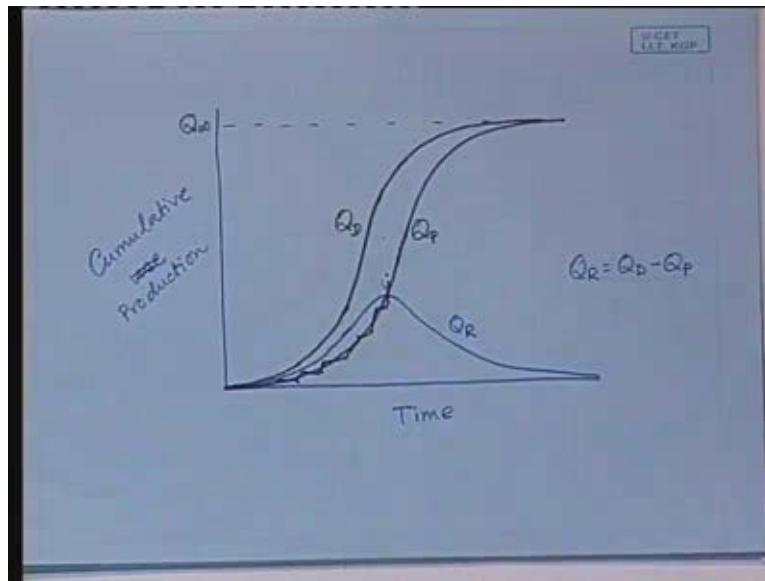


So, we have the data till the midway and supposing the production data is more available than the discovery data, suppose we have the production data and we have up to this point, so we have all these data points will be scattered something like this. So, if you really plot, they will look something like this. But nevertheless, you have, you can see a curve going through. You will need to take the mean value, so that you can approximate with the curve; but, that will not suffice in giving the equation, right, because this could be fitted into that or this, many things. So, unless you have an estimate of  $Q$  infinity, you can never fit it into the curve and if you cannot fit into the curve, you can do no prediction.

Now, I will come to this issue little later, but let us at this at this point of time provide the motivation. Why do we need to do this? What are the questions that really bug us? The questions are something like this. We know that the society is hopelessly dependent on oil, for everything, starting from running the cars, running the buses, running the trains, transportation, to you know, these days you go to the market without any bag and the fellow who is selling it gives the bag and the bag is made of petroleum product, so everything, almost; much of the clothes that we wear - petroleum products, so it is everywhere. A huge amount of petroleum is used all the time and we know that that is

finite and the economy very much depends on the price of the petroleum. Price of the petroleum goes up and down, but then you can easily see that there is a tendency of going up, right. For the last few years, you have been reading in the newspapers all the time that the international price is going up.

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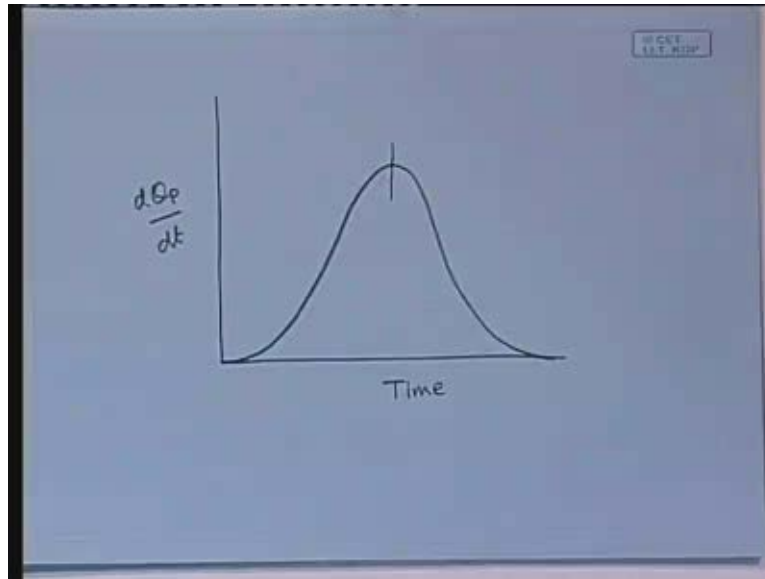


So, naturally the question is if we have a curve like this for the cumulative production, a curve like this for the cumulative production, at which point of time do we have the turning point? Do you understand what I mean by turning point? That means after which it will be, you know that the crisis will be there, you know that scarcity will be there, after which the national planning, policy, everything has to be tuned to this crisis. So, the turning point is important thing. What is the time after which you will really not have enough petroleum to run your cars or the petrol or diesel will be so expensive that the average middle class person will not be able to afford it. What is the time, can you estimate?

Normally it is assumed that when we reach like 75% of the, when you have already exhausted 75% of the resource, then we are **in**, we are there. So, these times are important, when? Now you see, in that, it will be more convenient to look at the

derivatives. So, if you look at the derivatives of that that means the rate, what will be the derivatives like?

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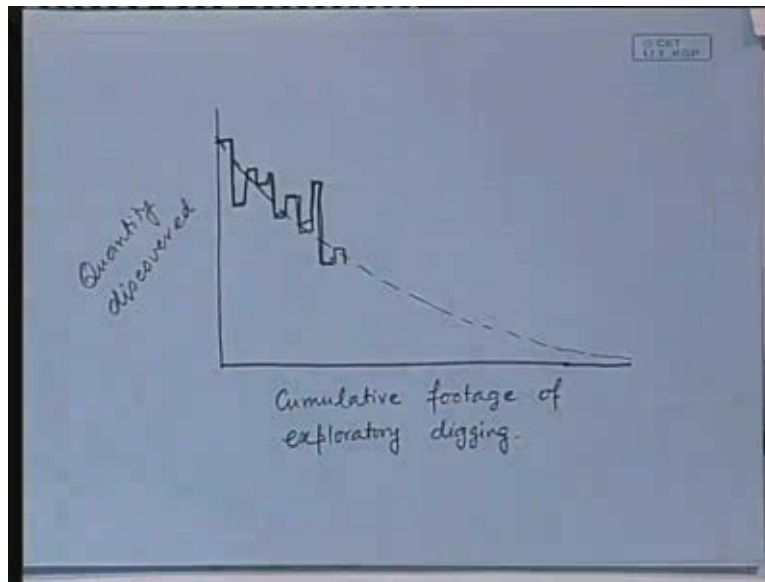
So, here is the time and here I want to draw  $dQ_p/dt$ . We have already seen that that will have a bell curve. So, it is this point that is important; it is this point that is important, because this point shows that after that the production will drop, but the production dropping does not mean the number of cars will drop. The technology is there, technology is improving; so, there would be more number of cars in the market. Yet, the production is dropping. What does it mean? It is onset of the crisis. There will be another point somewhere here, when the 75% of this, this area will be covered. So, when is that time? So, our job then is to estimate those things and in order to estimate that logically, scientifically you need to estimate these numbers, clear. Otherwise, all, everything becomes only guess, which, let me tell you all the things that were here in the newspapers or the politician's statement in the parliament, they are all guess work; very little of it is based on such sound mathematical **parameters**, but you cannot rely on that; you have to be engineers.

So, we have the problem, a parameter estimation problem at hand, in which the equation that we need to fit is this and the parameters that we need to estimate are these. How to estimate the parameters? Obviously, as I told you that you cannot even start this business unless you can estimate  $Q_{\infty}$  and it is something that cannot be estimated from the data that you have so far. Is that point clear? The data that you have so far are only this much. Based on that data can you estimate this value? Definitely not; so, you need to have an alternative yardstick of estimation of these values, not based on the production data; production data will say nothing about it. So, the problem essentially then bows down to, how to estimate  $Q_{\infty}$ .

Now, there are two possible ways of estimating  $Q_{\infty}$  and people apply both and only when they lead to the same value, they talk with some confidence that fine, this is the value of infinity, because  $Q_{\infty}$  is futuristic. Notice the essential meaning of it. I have not yet discovered this amount. I have discovered as yet today somewhere here. I have discovered only this amount, but on some logical scientific basis I am trying to predict that this is the total discoverable amount. Obviously, that cannot be done on the basis of what you have discovered so far. It is a tricky problem though. So, what people do is there are some methods available to geologists, by which it is possible to estimate the total quantity of petroleum in a certain reservoir. You know ..... there are reservoirs and they have some geophysical means by which it is possible to estimate the size of the cavity, size of the area, volume in which it is stored. So, as you go on discovering, even the ones that you do not know how to ..... all those things, the geologist have some kind of estimate of what it could be. So, one way is to rely on the geologists estimate, but there is also a statistical estimate.

Let us show how it comes. Initially when there was no discovery, people did not know. When people did not know, then if you, if you started digging or when there was no oil field around, people started digging at places they found. So, by digging small amounts they found some reservoirs. But, as time went by, you will have to spend more and more effort, more and more money, more and more digging, in order to discover even a small amount. So, what does it mean? It means that initially discovery was easy.

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As time progresses, discoveries become difficult and that can be fitted into another curve, a curve something like this, where you have in the x-axis how much you have dug that means cumulative footage of exploratory digging. That means you are digging, suppose you keep track of how much you have dug, then the total amount of digging that means here we have dug some amount, another place you have dug some amount; here you have dug 100 meters, there you have dug 300 meters, so total is 400 meters. So, keep on adding them and as time progresses, this will go on right this will go on the cumulative, will go on increasing and in the y-axis, we plot the quantity, the quantity discovered.

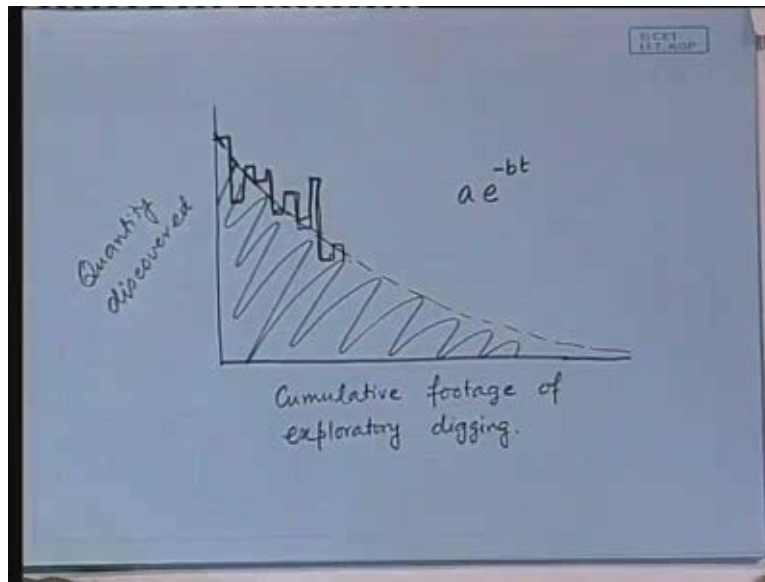
So, how much you have discovered? What should be logically the shape of this curve? S-curve; wait, let us logically work it out. Initially the cumulative footage of exploratory digging was very small. We have not dug much. Only with a spade you have dug something. So, it should start from zero. It should start from zero in this side, but for small amount of digging, then how much have you discovered? That is large, so it does not really start as a bell curve, because at this point, when you have really dug very small amount, you have discovered a lot. Initially, I told you that the discovery was easy. What does it physically mean in terms of numbers? You have dug a small amount, but you have discovered a lot. So, here there would be a large number. So, here there would be

somewhat a large number. It does not really talk about the zero point. You have not dug anything, you are sitting ideal and you have discovered; no, it is not like that. So, small amount of digging, but you have discovered a large amount; that was the initial days and then as time progresses, it becomes more and more difficult. How does it translate in terms of this curve?

You are going on digging; cumulative is going on increasing, but the amount that you discover that should go down. But, it is not, you know, monotonic going down, because suddenly you discover some field, suddenly you discover the Iraq field, so suddenly it goes up. Suddenly you discover Bombay high, it goes up. So, they should be something like this, you know, curve should be something like this. But, in the main it should go down. All this going up should be there, because suddenly you discover. But, on an average it should go down, so that you can plot a curve going through. How should it end? Will it simply go to this point? No; obviously it cannot. It should sort of extend to infinity, right. .... that ultimately, even though initially the curve is like this, ultimately it should be a curve something like this.

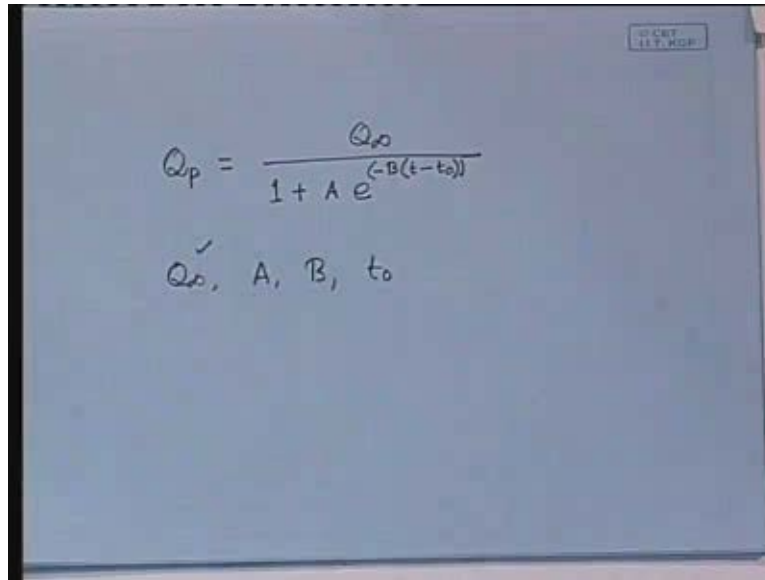
What kind of curve is this? Exponential curve and you have the data up to some point, right; you have the data up to some point. So, can you then fit into this curve? That essentially becomes a question. You have these data, right, say up to, we have, we have come up to this point and the rest is still there. But, you know the way this curve is progressing and up to this point at least you can fit this curve and then what is the equation? Logically the equation of this curve  $e$  to the over ....., yes.

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So, it should be some  $a e^{-bt}$ . All you need to do is to estimate these two values  $a$  and  $b$  from this curve. Once you have done so, if you draw this curve, you are essentially extending to the future and the area under this curve is  $Q_{\infty}$ , right. So, that is the other way of estimating  $Q_{\infty}$ . Notice that we have not discovered the fuel and we cannot base ourselves on the amount that we have discovered. There will be future discoveries, but then logically you can extend this curve. It is possible to extend this curve and it does follow that route. So, you have this equation which you need to estimate, draw this and obtain  $Q_{\infty}$ .

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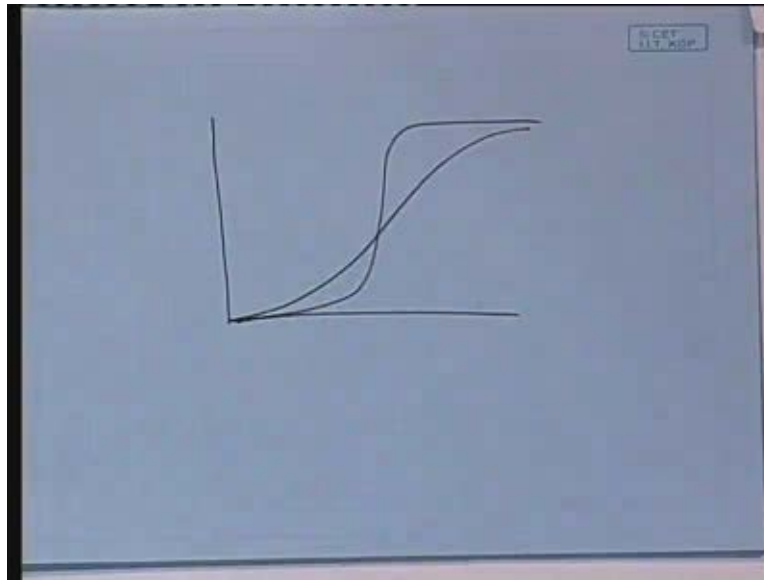
A photograph of a whiteboard with a blue border. In the top right corner, there is a small rectangular stamp that reads "SECRET" above "117 WOP". The whiteboard contains handwritten mathematical equations. The first equation is 
$$Q_p = \frac{Q_\infty}{1 + A e^{(-B(t-t_0))}}$$
 The second equation, written below the first, is 
$$Q_\infty, A, B, t_0$$

The whiteboard is otherwise blank, with some faint, illegible markings in the bottom left corner.

Thereby, you have obtained the  $Q$  infinity and now if you know the value of  $Q$  infinity, then it becomes a problem of estimating three numbers; it becomes a problem of estimating three numbers. Now, this three numbers, estimating the magnitude of three numbers based on certain data is also not a trivial affair. In what sense? In order to do that estimation, because that will be a problem that you have to solve yourself; not that somebody does it, you have to solve. I will give you the problem, I will give you the data and you have to solve it. So, let me clarify what you need to understand before you go about doing it. You need to understand what is the role of  $A$ , what is the role of  $B$  and what is the role of  $t$  naught.



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Without knowing that you will not be able to estimate that, because here, okay, let me, the curve could be like this and the curve could also be like this, same equation, same equation. So, what makes, what is the effect of A, what is the effect of B and what is the effect t naught, unless you understand that you will not be able to solve this problem.

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$$Q_p = \frac{Q_0}{1 + A e^{(-B(t-t_0))}}$$

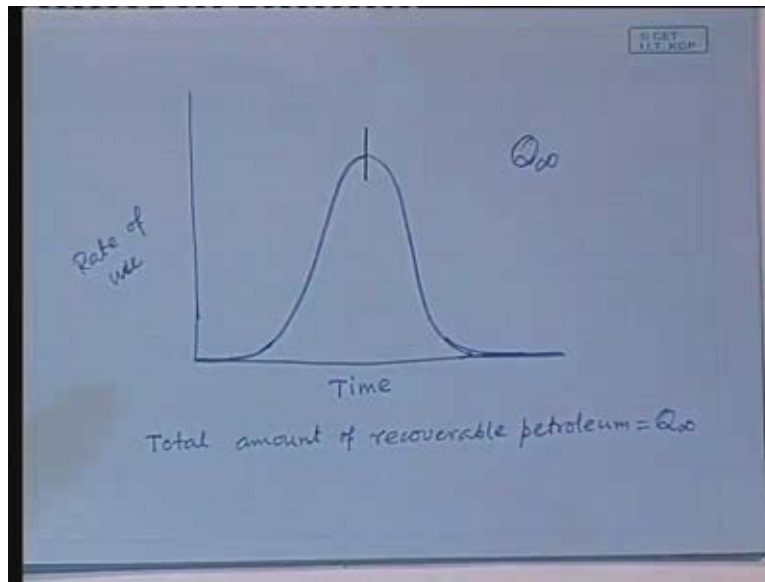
✓  
 $Q_0, A, B, t_0$

$$Q_0 = 3 \times 10^9$$
$$A = 10$$
$$B = 0.125$$
$$t_0 = 1985$$

So, take it as an assignment; take it as an assignment that in this equation, put the values.  $Q$  infinity is 3 into 10 to the power 9. These are generally done in terms of barrels. So, that is, that will be in the unit of barrels, but here will we are noting putting units.  $A$  is say 10,  $B$  is say 0.125 and  $t$  naught is say, a year it should be, 1985. I will give you some values. In fact, some years back, a student of energy engineering of this institute estimated these values from the data that are obtained for Indian reserve, the reserves that are in India, with data taken from the oil, in the oil, that is Oil India Limited, Assam Oil Corporation and stuff like that. So, the Indian companies and ONGC and he estimated values that are close to these; I only rounded off.

So, a good estimate of the amount of oil that is there in India is this, so many barrels. Anyway, so put these values into this equation and draw this curve. Let that be your datum curve. Now, vary  $A$  up and down and draw two more curves keeping everything else fixed. So, that will tell you the effect of  $A$ . Then keep the  $A$  there and vary  $B$  and again draw curves; that will tell you the effect of  $B$ . Similarly, keep  $A$  and  $B$  constant and vary  $t$  naught that will tell you the effect of  $t$  naught. Next day, when we assemble again next week, you will have to come with those results and report it here, so that on that basis we will be able to go ahead further and then, I will give you a problem in which you will have to estimate the values. So, the question is once you have estimated these values, once you have estimated these values can you then find out the midpoint of this curve, this point?

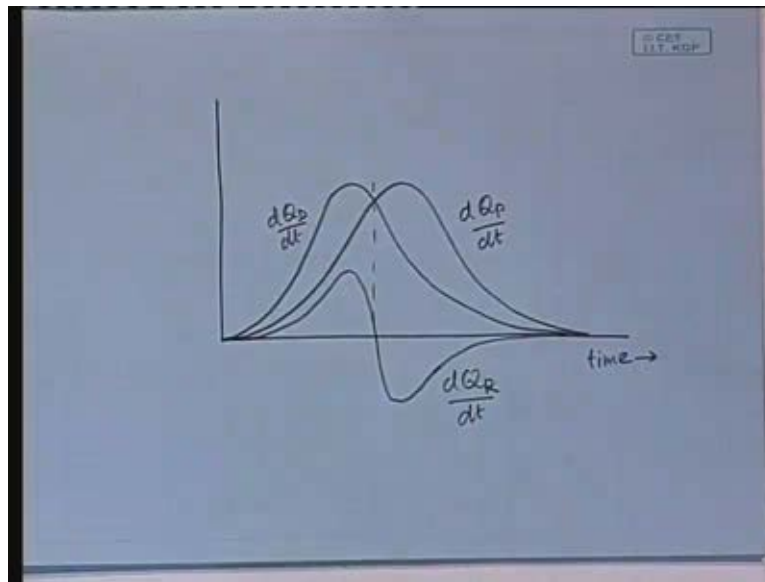
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Yes, you can; pretty simple. Can you find out at which time 75% will be exhausted? You can do that. So, the kind of problem which you will encounter may be in exams, would be of that type. Given these equations, you will be able to estimate the onset points of various types of crisis and to your surprise you will notice that if you even make 20% error in the estimation of the  $Q$  infinity, the time by years by which this time shift will be observed are 2 to 3 years only. So, presently, the estimate of  $Q$  infinity is  $3 \times 10^9$  for India. Suppose if it goes up to 4, the time it will take to exhaust 75% will only extend by 2 years, because at the same rate the exploration will also go up. These are all given by these equations, fine. So, the next day you have to come with that.

Now, a few more things; look at these curves and let us work out the derivative of all of them. What will be the derivative **shifts**?

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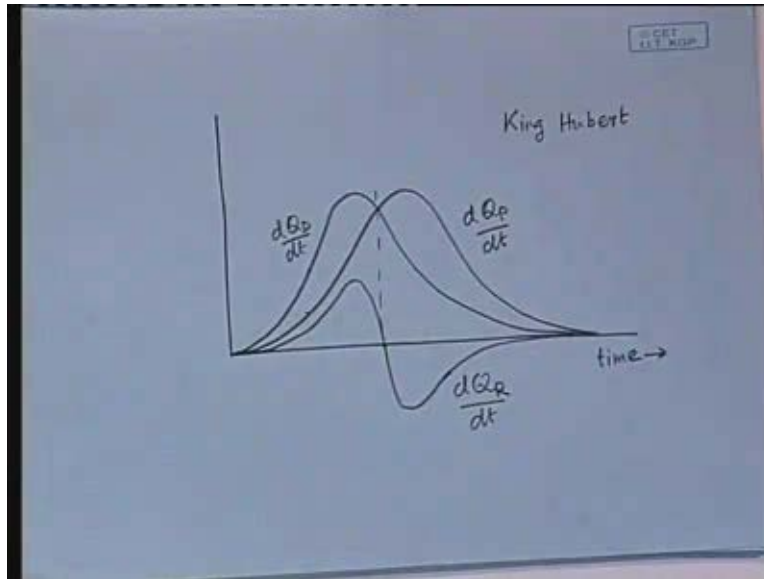


We have already drawn the derivative of the Q P, right. So, Q P's derivative was like this. What will be the derivative of Q D? It will be another bell curve, because it is given by the same equation. No, it will, it will be shifted, right; so, it will be shifted and then what will be the derivative of the reserve? So, this is  $dQ P dt$  and this is  $dQ D dt$  and what will be the derivative of the reserve? Well, this point will mark the change. Before that, the derivative of the discovery is greater than the derivative of the production. After that, the derivative of the production is, production rate is greater than the discovery rate. So, before this point, the discovery rate is exceeding the production rate, after this point the production rate is exceeding the discovery rate. So, the reserve will switch at that time. So, its curve will be something like this, alright. Here is your time.

So, you will also be able to draw these curves once you have estimated those values  $A B t$  naught and  $Q$  infinity. You work out, work out on these terms. See, if  $Q R$  is  $Q D$  minus  $Q P$ ,  $dQ R dt$  is  $dQ D dt$  minus  $dQP dt$ . So, it will be subtraction of these two. At this point, these two values are equal; the subtraction should be zero. So, here the  $dQ D dt$  is greater than this. Therefore this term will be positive. Here,  $dQ P dt$  is greater than  $dQ D dt$  and therefore, this term will be negative; the rate at which the reserve changes will

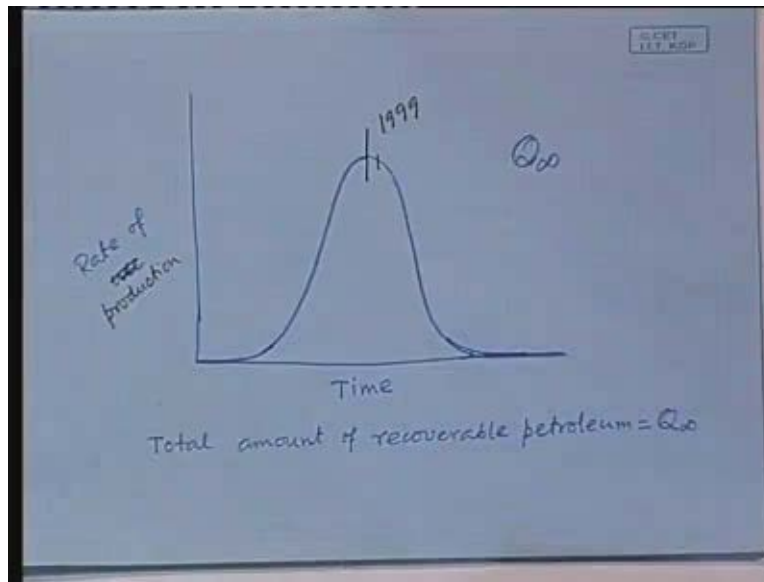
have a negative value, clear. So, you might still ask where are we, right now, in the year 2006.

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Now let me tell you that people do keep track of these curves and they do the estimate. In fact, oh, I need to tell you the person who proposed the analysis this way. His name is King Hubert. He was a geologist in United States, King Hubert. There is an energy encyclopedia in our library; you will find a chapter by him, so that gives a good reference to the things that I am teaching today and this peak in the  $dQ P dt$  curve, rate of production, this peak is called Hubert peak.

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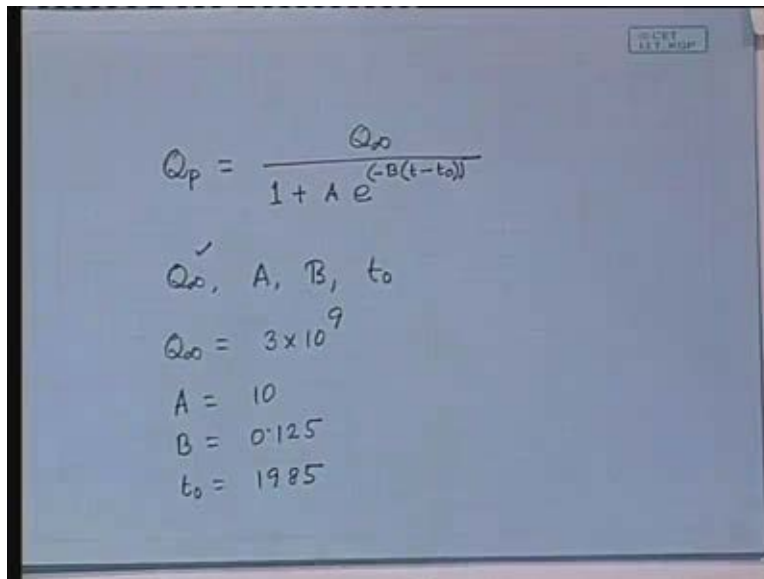
This peak is called the Hubert peak. Now, this Hubert peak, now it is known occurred in the year 1999; has already happened 7 years back. So, we are out there, so we have already crossed one crisis point and we are in the middle of the crisis. We are right now in the middle of the crisis and that is exactly what you are seeing - the daily increase of the petroleum price. No; I must tell you that that is greatly inflated, because the international petroleum price, if it is  $x$ , then twice  $x$  is the domestic taxation, so that we actually have to pay thrice  $x$ . So, it is greatly inflated, but nevertheless if it is  $x$  and  $x$  is increasing, the whole thing, thrice  $x$  also increases. So, that is how you see the petroleum price. Actually, the petroleum price in the international market is about a third of what you really have to pay, but the point is that the peak has crossed and therefore, the worldwide production is going down and since the whole is finite, you cannot really keep on increasing it.

What will happen if you artificially increase it? It is after all determined by the world market. How much will the OPEC countries produce? OPEC means oil producing and exporting countries. How much will they produce, what is the, what does determine that? If they produce too much, then what will happen? The oil price will go down and therefore, they will not be able to make money. If they produce too little, the amount that

they will sell is low, is low and therefore, they will not be able to make money. In between, there is an optimal set by the market and that is where the economy actually operates, but that is often offset by other considerations like, some country immediately may need some money and therefore, they may immediately withdraw a large amount of oil and may sell it, so that this curve will not be exactly the nice way that you see here. It will very, very vary, with some peaks, but supposing at this point, in order to reduce the world market price, some country pumps up a lot of oil, what will happen? What will be the result of that? Sometime later, it will not be available.

So, if it is a boost at some point of time, another point of time there has to be a burst; there has to be a reduction, reduction below the average line. So, what you have, what you see here is the average line. You cannot change it over a long period of time whatever you do and because it has been the onset of this crisis, we see all those ... at its worse all over the world. Necessarily, people are trying to get hold of the oil resource and that is what is resulting in these conflicts.

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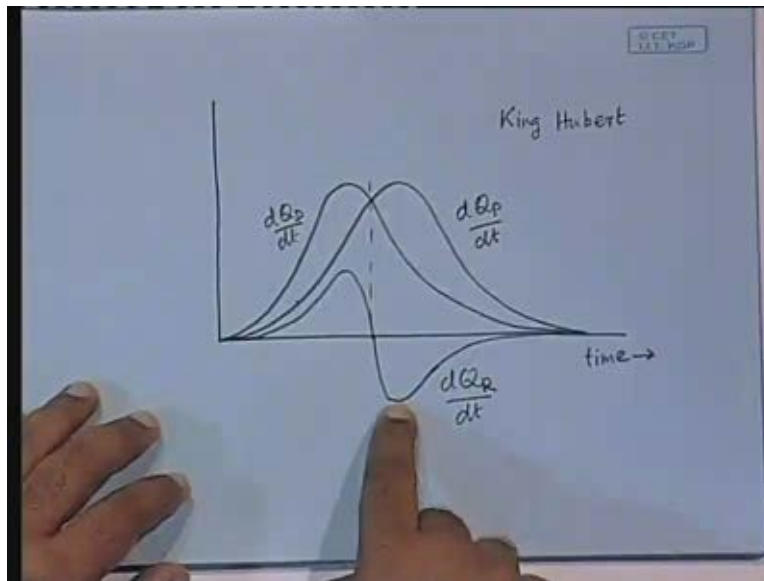


The image shows a whiteboard with handwritten mathematical equations. The first equation is  $Q_p = \frac{Q_0}{1 + A e^{(-B(t-t_0))}}$ . Below it, the variables are listed as  $Q_0, A, B, t_0$ . The values are given as  $Q_0 = 3 \times 10^9$ ,  $A = 10$ ,  $B = 0.125$ , and  $t_0 = 1985$ .

Anyway, we will come to that in a little more detail later, but presently what I want to tell you is that at the end of the day, you on the basis of the equations should be able to,

where is the equations, yes, should be able to predict the onset of crisis. I have told you that it has happened in '99, but you should be able to, on the basis of the data you should be able to, find out where it is and you should be able to find out those critical junctures, total amount, that 75% being exhausted or this point being crossed.

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So, all these things you should be able to estimate, clear. So, that is the end of this lecture; we will, do not go right now.

Thank you!