Energy Resources and Technology
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Lecture - 38
Magnetohydrodynamic Power Generation

… power generation.

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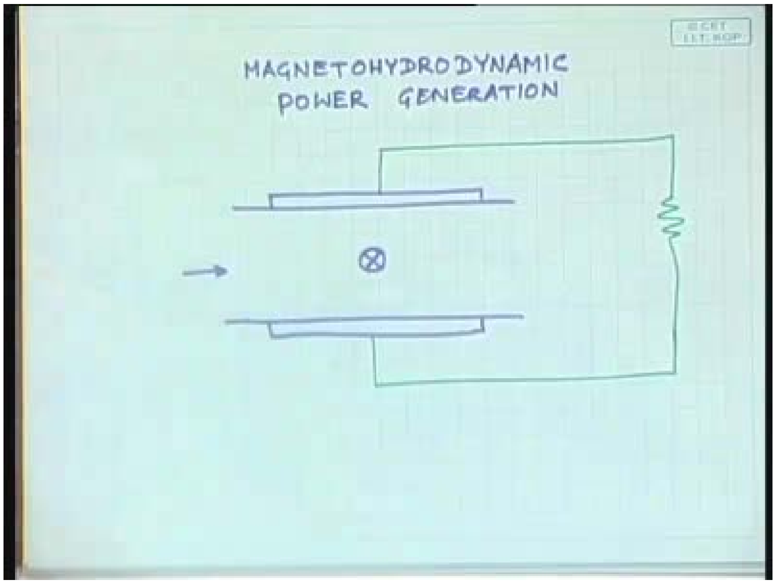

The basic idea is simple. If you have some kind of a channel and if you allow a very hot gas to flow through, hot gas in the sense plasma that means the bulk of the gas is ionized, the electrons have been stripped off, so you have essentially electrons and ions flowing and if you have a magnetic field in the direction perpendicular to the sheet of paper that means we normally denote it like this that means it is like arrow going into the paper, then the electrons will be deflected by this magnetic field. The holes, not holes, the ions will also be deflected by the magnetic field in opposite direction, so that if you put electrodes in the two sides, then some voltage will be induced in these two electrodes and if you connect it through some external circuit, through some external circuit, a current will flow. So, this is the essential idea of the magnetohydrodynamic power generation.

That means here because of the magnetic field there is a charge separation and that charge separation is collected by electrodes and that is what allows the current to flow through an external circuit.

Now, you might ask how you actually do it, what creates this plasma? So, the essential idea of the magnetohydrodynamic power generation is where you have a fuel which is either a liquid fuel or a gaseous fuel that will be burnt creating a very high temperature and that high temperature gas will be right in the beginning allowed to flow through this kind of a channel. It is like a nozzle. That means it passes at a high speed thus extracting power that is contained in that, in that gas and then after the power is generated, a part of the energy is extracted.

The rest of the energy still contains a lot of energy that goes through and that is then further extracted by a conventional thermal power plant. That means this MHD cycle becomes, MHD cycle in conjunction with the conventional steam based thermal power plant becomes what is known as a combined cycle power plant, where the MHD is the toping cycle and the Rankine cycle is the bottoming. Is that clear? Toping in the sense that where it is high temperature that is when the toping part of the energy is extracted and when it goes out of this at a relatively lower temperature that is when the bottoming part is effective.

But, there is some, something more to it. Firstly, how much can we raise the temperature simply by burning a gas? That means these kinds of systems are considered where there is a availability of natural gas. As you know, in India there are places where natural gas based power plants are coming up and those places, the MHD cycle will be very effective. It is also considered to use coal in the sense that you have already learnt that coal can also be converted into gas, gasification of coal and then the coal can be burnt. So, if you are considering that kind of a cycle, then coal can also be used as a fuel. But, essentially you have to have some kind of a clean gaseous fuel as a fuel. But still, the temperature that is attained here is not sufficient to ionize most of the gas. So, there has to be something additional put in and in order to increase the level of ionization, either caesium or potassium is injected. That means a bit of caesium and potassium is injected into the gas.

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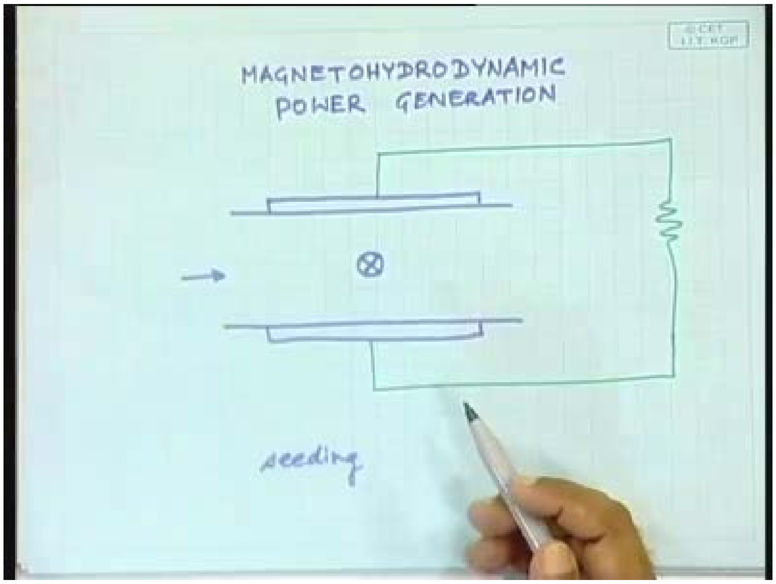

That is called seeding and that caesium or potassium and potassium whatever it is, that immediately gets ionized and that adds to the overall level of ionization of the material. That means the free electrons that are free to move in this plasma is principally contributed by the seed. So, even if the temperature is not all that high, if it is sufficient to ionize the seed that is sufficient. So, either caesium or potassium is added as a seed. What would actually contribute to the bulk of the current? As you can see, there would be the electrons and there would be the ions.

Ions would be, for example the caesium ion or a potassium ion that will contain its mass. Each of this ion’s mass will be far exceeding the mass of an electron, right, many times about 25 to 1000 times. As a result, the bulk of the transport, they are in equal numbers, the number of electrons assuming that one is stripped off from one, would be in equal numbers or other words in some cases, electrons will be larger in number provided larger number is stripped off from each ion. So, we are considering the actual bending of the path and then finally reaching that electrode, right and the bending of the path will obviously depend on the mass of that, the ion or the electron taking part in the activity.

If it is heavy, then it will have a larger probability of just going through. If it is light, it will bend and take part in this activity. So, then this lead us to conclude that bulk of that current that is generated is due to electrons, though theoretically both would take part. Ultimately, the activity that happens here is due to electrons. So, let us consider the motion of electrons in a bit more detail. Have you understood the essential structure?

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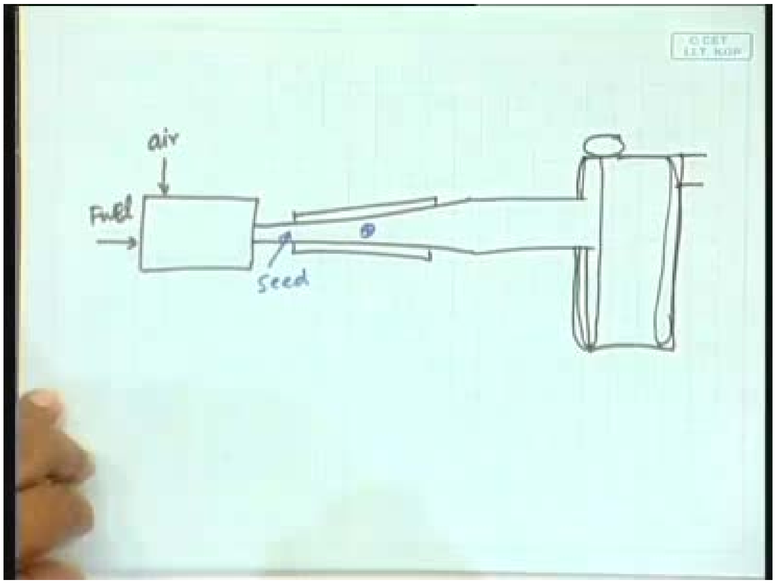

The structure of the MHD generator would be something like this that you will have a chamber in which you will have the injection of air and injection of fuel. So, this is the combustion chamber. From the chamber it will be sort of going out like a nozzle and the electrodes will be in the two sides of the nozzle and the magnetic field will be perpendicular to it and when it goes out of the nozzle, then it is that only in this part the energy is generated and after that it goes to a normal, what is this? This is the normal boiler, where in the normal way you will have, you already learnt that there will be a boiler drum, there would be the water walls like this, there will be water walls like this and finally there will be, this will go out, there will be the super heater section, the economizer section and all that. So, it is just before that takes place, a part of the energy is extracted at a higher efficiency through this MHD process and at this point, there will be the seed injection. So, if you have understood this part, then let us go into what actually happens inside this, what happens to each of the electrons?

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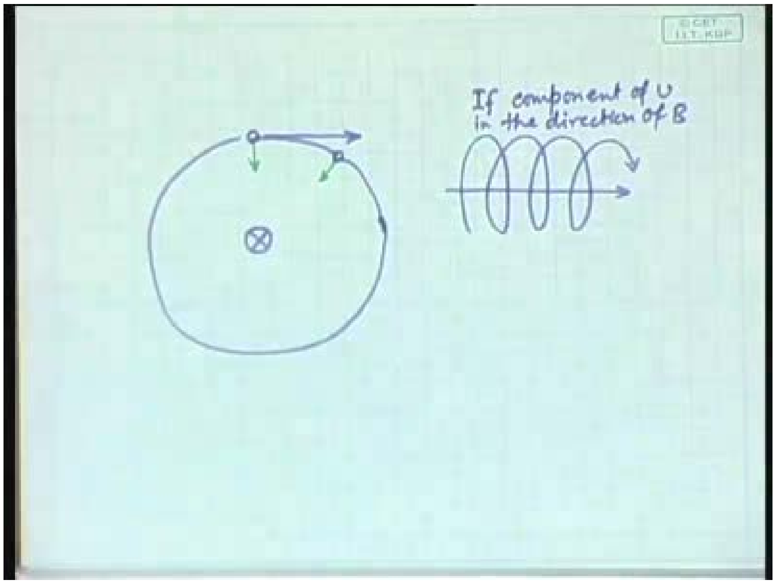

Now consider one electron, say here and this fellow is moving in this direction and you have a magnetic field that is going into the board like this. What will be the motion of the electron like? That is obtained from the left hand rule. So, apply the left hand rule and tell me what will be the force felt by this electron?

Student: Downwards.

Downwards; so, it will feel a force downwards and as a result, its motion will be deflected away from this linear path to a path like this. So, it will move like this and say it has come here. At this point when it has come here, then the motion is no longer like that. The motion is now like this and what will be the direction of the force? Again towards the center, so it will be like this. It will further move like this. So, do you see ultimately what will its behavior be like? It will be circular. So, it will be a circular motion like this if nothing happens to it. That means it is not colliding with anything or something like that, then it will be a circular motion and logically speaking it will go on moving in the circle. But no, that cannot happen, because we had assumed that there is a, there is a, well, this fellow has a velocity in the direction perpendicular to the magnetic field, but there is no reason to assume that there could be a component in the direction of the magnetic field, which will be completely unaffected by the magnetic field.

So, what will happen to that then? You have, then you will have there is a magnetic field and here is the electron. If there is no component in that direction it will move like this, but if there is a component which will be unaffected, so it will move like a helical path. It will move in a helical path. So far so good, it will move like, just consider it will move in helical path. Let me draw. So, here is the magnetic field. So, if component of the velocity in the direction of B is present, then it will be like this.

Well, there need not be only the magnetic field, there could also be electric field.

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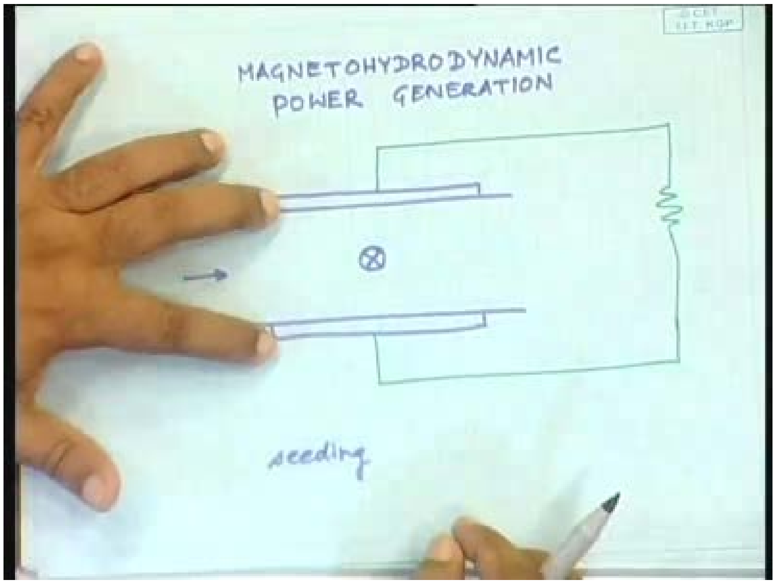

Because as you can see, there would be a pair of electrodes and the pair of electrodes being in two different voltage means each of these electrons will face electric field also.

So, what will be the motion like if there is an electric field also? It will drift in the direction of the electric field. So, while supposing I am not considering this that means I am not presently considering the component in the direction of B but just this and the electric field. In that case it will move like, move like what? It would have been a circle if there is no electric field. If it is a circle there will be a drift.

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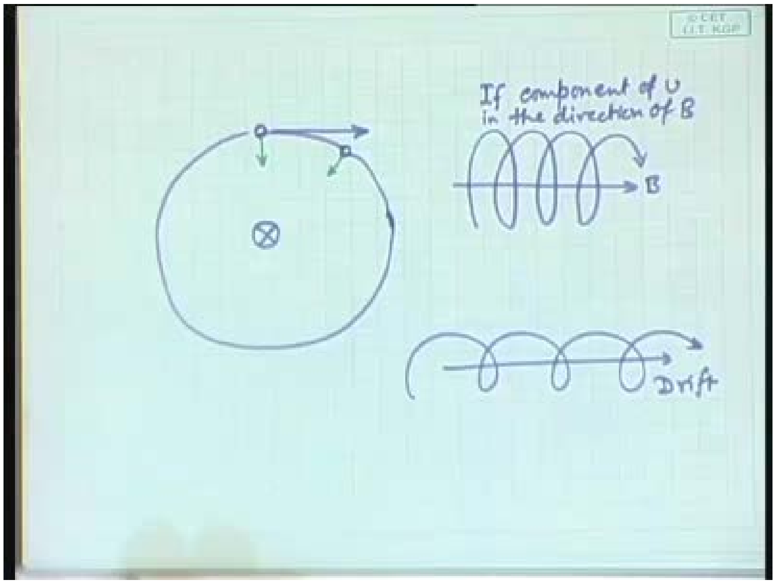

So, it will be, it is not really helical motion, remember, but it will move like this. So, there is a drift. So, this is, this was the direction of B earlier and in the presence of both, it will be a combination of all these motions. So, first we considered what? If there is only B and the component of the velocity is only in the direction orthogonal to B. Second, we considered if there is a component of the velocity in the direction of B what will be the motion and third we considered what will happen if there is an additional electric field. So, it will be like this. But not really, because there will be a large number of electrons and ions in that gas. So, you cannot really say that an electron will go just like that, because it will collide. It will automatically collide. So, ultimately the electron will traverse a path that will go through various collisions.

But before we go into that, let us just figure out what will be the radius of this? You can easily figure that out from Newton’s law. So, let us write that.

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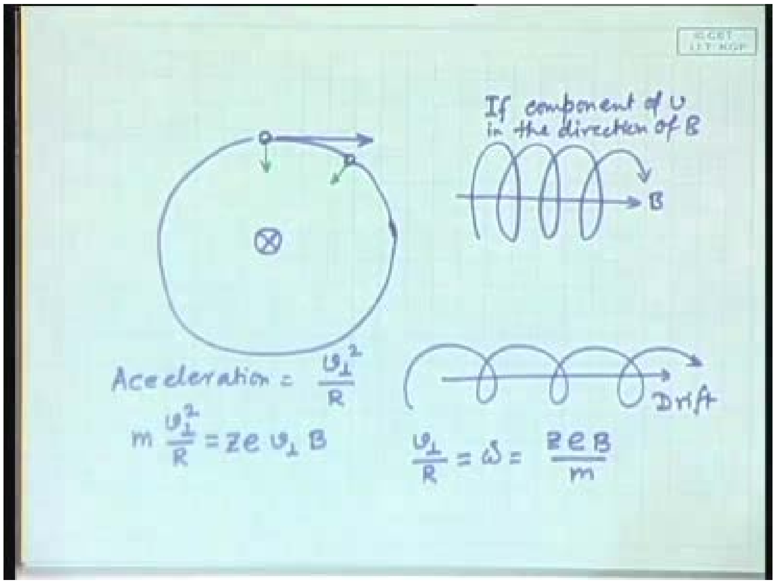

$$\text{Acceleration} = \frac{v_\perp^2}{R}$$

$$m\frac{v_\perp^2}{R} = Ze\,v_\perp B$$

$$\frac{v_\perp}{R} = \omega = \frac{ZeB}{m}$$

The acceleration will be, will be v perpendicular square by R. I am denoting that perpendicular because it is, it is related with the perpendicular direction of that velocity. Then, Newton’s law will say that the force is mass into acceleration, so mv perpendicular square by R, mass into the acceleration is the force and what will be the force? Force will be electronic charge times v perpendicular times B, provided I am considering only one electron. But, if you write it in a general sense that means something that is applicable both to electrons as well as the ions, then you will have to multiply with the number Z representing how many of the electronic charge is there in that ion?

So, Z is the how many of the electrons charge is there in that ion. So, in general, it will be this expression. So, v cancels off and you get, so this gives you v by R which is nothing but the omega as Z e B by m, so that is the omega, fine. So, that is the angular velocity of the motion.

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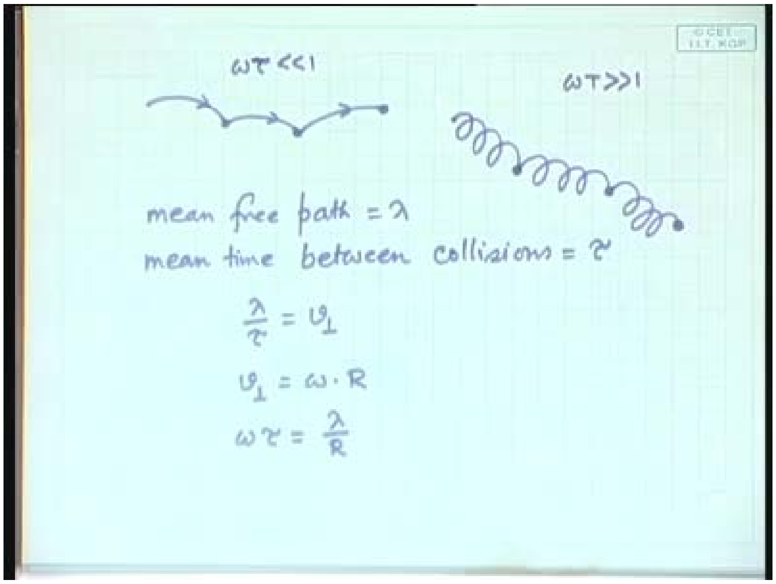

mean free path $= \lambda$

mean time between collisions $= \tau$

$$\frac{\lambda}{\tau} = v_\perp$$

$$v_\perp = \omega \cdot R$$

$$\omega\tau = \frac{\lambda}{R}$$

But now, as I told you, that it will actually cover paths like this and then another collision, another collision, another collision and so on and so forth, right, it will go through many collisions either this type or you might say that if the angular frequency is much higher, then it will be a collision like that. There are two possibilities. What is the difference between these possibilities? Here, the mean free path was smaller and here the mean free path was larger. In comparison to what? In comparison to what?

Suppose your, the mean free path is lambda and the mean time is tau, then you can say that lambda by tau is … Now, we know that v is omega times R, the radius. So, we have, if you substitute, we have omega tau is equal to …, fine. Now, you would notice that this, this term omega tau is what actually makes the distinction between these, this type and that type. If omega tau is large that means this is large, this is the mean free path is larger than the radius, far larger than the radius, then it will be this type. If it is small, then it will be this type. So, the omega tau is the quantity that we normally use in order to distinguish between the types of the average. These are all average; you will not be able to say that each one will traverse paths like that, but on an average if omega tau is small, then the mean free path is small, then it will be like this, sorry, in that case it will be like that. So, this will be where omega tau is less than less than 1 and this will be where omega tau is greater than greater than1. So, these are the two possibilities.

Now, what will actually be the result of all these? Consider this.

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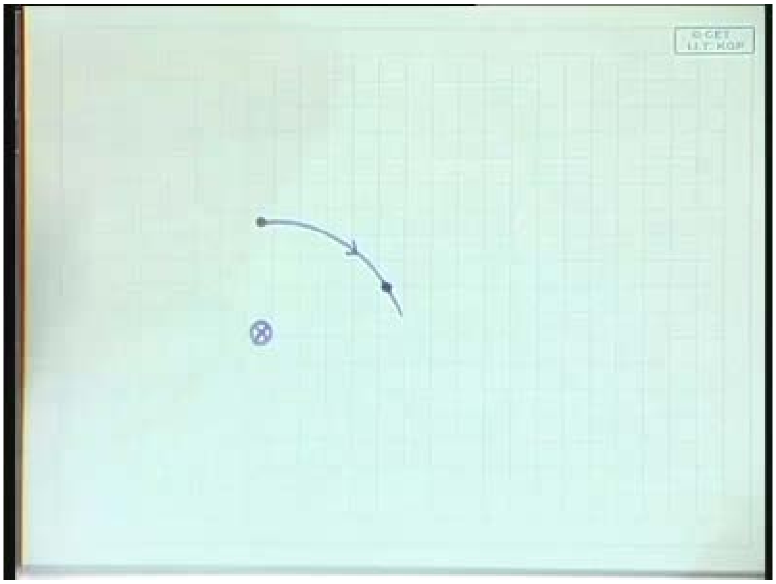

You have, consider again one electron and the magnetic field and this fellow as you understood would be moving like this and suppose it has come here.

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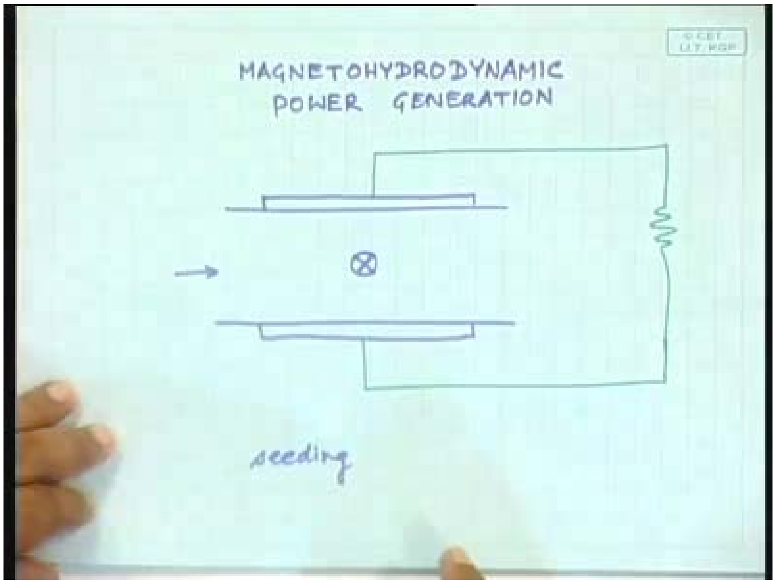

Remember, we had initially considered that this will be the direction of flow, so the electrons will be flowing like this and you have produced a B, due to which there will be a voltage generated here. That means the electrons will be actually moving like this. That is producing what is the, what you know as the Faraday effect. So, let us quantify this Faraday effect.

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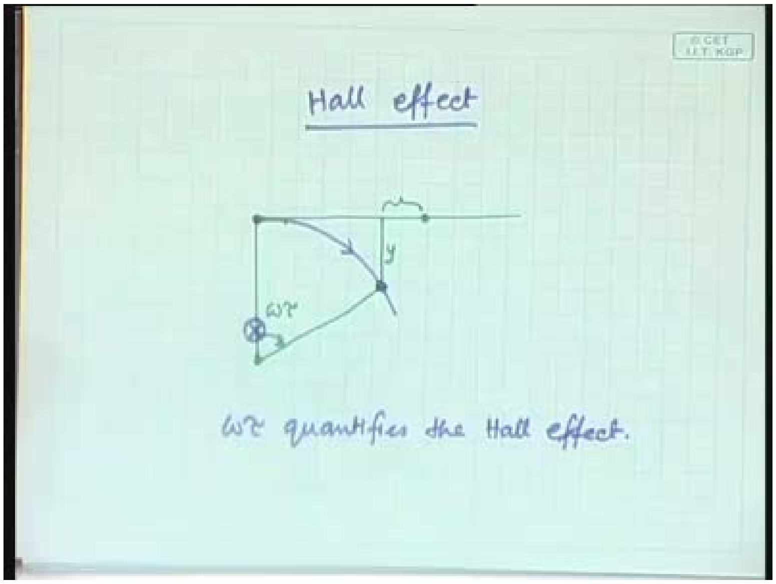

It has come here. So, it was actually moving like this. In this time it would have gone here, but instead it has come here, right. So, it has moved by an extent this in the y direction. So, there would have been, otherwise if this is not there, no motion in this direction; now, it has the, it has motion y. That is the Faraday effect, right. But, notice that there is something in addition to that. That is it would have traveled up to this point in the absence of the magnetic field, but now it has traveled up to this in the x direction. So, here is something that is representing what? The bulk of the gas is moving with the velocity, v say, what does this represent?

It will represent the fact that the electrons are falling back, electrons are failing to travel with the same velocity in the x direction, right. If electrons are falling back from that direction what its effective result is that the system will see a voltage in the x direction. If the electrons are travelling along with the rest of the mass, there will be no voltage. But if they are falling back, yes, there will be a voltage, there will be current, which you normally did not anticipate. From the idea of the Faraday motion, you would not anticipate that there will also be a voltage in the x direction, but there will be. If you allow current to flow, there will be a current in the x direction also. So, that is called the Hall effect and that voltage is the Hall voltage, the current is Hall current. So, that is the Hall effect.

How much will that be? So, you had this, this path and it has, I will, I will draw it correctly. This center would be somewhere here, here. So, if it travels up to this point before the next collision then it has traveled by this extent and what is this related to? That is related to the omega tau; so, that is related to omega tau, fine. So, the Hall effect, the amount of Hall effect will be large if the omega tau is large. For example, if the next collision happens here, obviously it has not fallen back much from the rest of the gas, but it has, it has been allowed to travel up to this. It has fallen back much from the rest of the gas. So, the extent of Hall effect will depend on the mean free path, the mean time between the collisions, effectively on this omega tau, fine.

So, it depends on omega tau which is expressed in the unit of radiance. So, you have omega tau quantifying the Hall effect. So, let us us write down omega tau. So, now for example if the omega tau is something like 0.1, you would anticipate it to move on an average only up to this extent. If it is of the order of say 1, omega tau is equal to 1, then you would on an average anticipate it to go up to this extent. If it is say 5, you would allow it to traverse almost the full cycle before the next collision, like that and as a result there will be the Hall effect.

So, now on this basis, let us try to understand the actual working.

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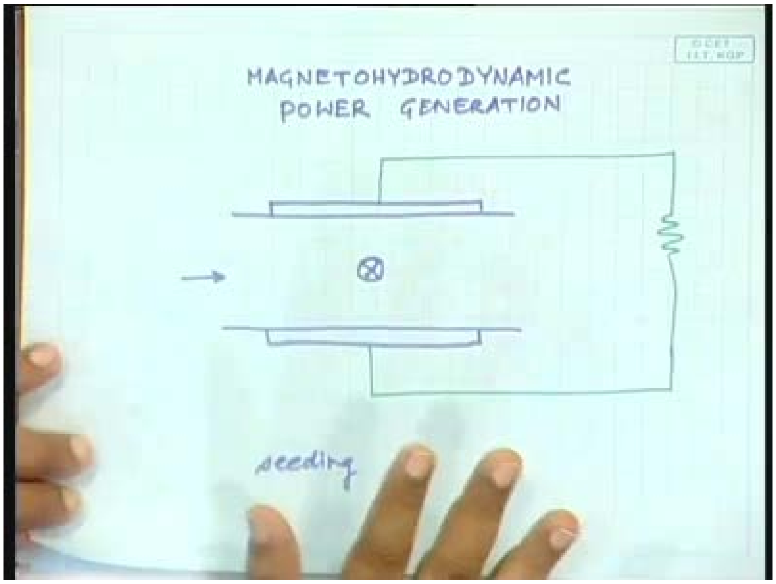

So, you see, initially the kind of picture that we had of the functioning of the MHD generator, now it is somewhat changing, right. There is some more complication due to very fundamental physical effects. So, we need to understand and we need to do something about that. How much will be the voltage induced?

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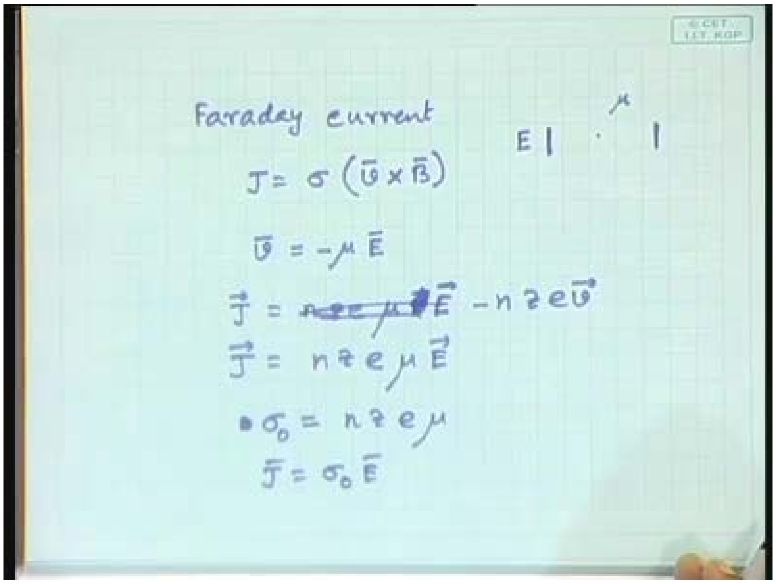

Faraday current

$J = \sigma(\bar{v} \times \bar{B})$

$\bar{v} = -\mu \bar{E}$

$\vec{J} = \cancel{nze\mu}\vec{E} - nze\vec{v}$

$\vec{J} = nze\mu\vec{E}$

$\sigma_0 = nze\mu$

$\bar{J} = \sigma_0 \bar{E}$

The voltage induced will be the or that will produce the current, so there will be two currents, Faraday current. Faraday current will be, is the conductivity times … So, that will be the current. So, the motion of these electrons or the ions will depend on, suppose you have got, you have applied a field E; there are two electrodes and you have applied a field E and there is electron or an ion here. How much will it move? That will depend on its mobility. Obviously, the mobility of the electrons is far larger than that of the ions, but nevertheless it will depend on mobility. So, that mobility is represented by mu.

So, you say that this v is equal to the mobility times E, the electric field induced. But, for the electrons it will have a negative sign, because the electrons move in the direction opposite to the electric field. Now, if this is the velocity, then the current will be J, as a vector, will be if each of the electrons have this velocity, then what will be the J? What will be the current induced? The current will be how many electrons are flowing, n, how many of these charged particles are flowing and each charged particle has a charge of z e, for the electron z is 1, but for the ions it will be something else, times the mobility times oh sorry, times the V or times the mobility times E. So, n z e v and v is this, so it will be this.

Now, let me, let me write it clearly; is equal to n z e v in the negative direction, because for the electrons flow it is, the current is opposite, is equal to n z e mu. Now, this has the appearance of the ohm's law. Here is the voltage, here is the current and here is then the effective resistance. So, effective resistance will express it in terms of the conductivity. Then, sigma zero will be our notation for the conductivity on that condition, that will be n z e mu. So, that will be the conductivity. Effectively, that will be the conductivity of the flow of charge and we will write J is equal to …, fine. So, that defines how much would be the motion.

Now, you see, let us consider both of them.

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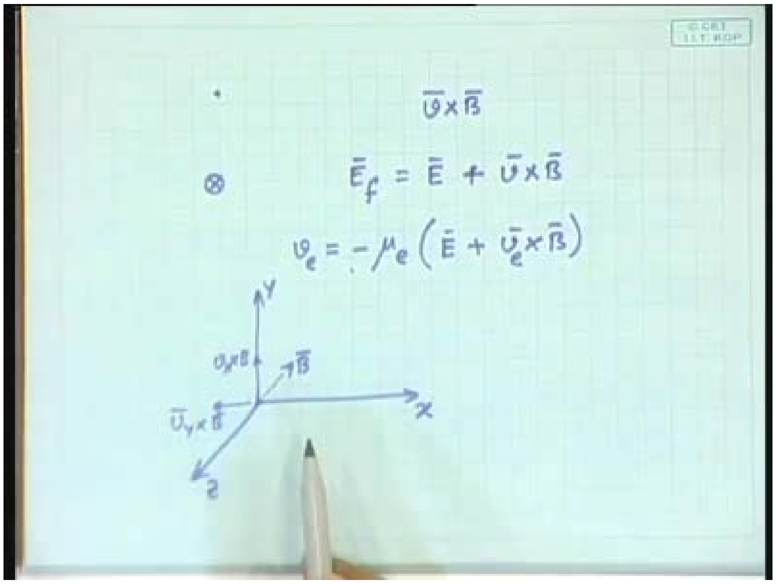

$$\bar{v} \times \bar{B}$$

$$\bar{E}_f = \bar{E} + \bar{v} \times \bar{B}$$

$$v_e = -\mu_e (\bar{E} + \bar{v}_e \times \bar{B})$$

Here there is a magnetic field, here there is an electron. This fellow is moving, but now we are considering not only the motion, now we are considering the, we are trying to consider the voltage induced and the current which are palpably evident quantities. So, the v cross B will give you the electric field that is produced by, because of this, fine and in addition to that, there will be an existing electric field. So, the effective electric field will be the combination of these two, right. So, we will say that E f is equal to the electric field existing plus v. So, the electrons drift velocity will be v is equal to minus mu, I am not talking about electrons, so let us put this subscript e times this. So, you see, v appears in both the sides. Is that clear?

So, effectively the electric field experienced by this fellow is this much and due to that there will be the, because this is the mobility, this will be the velocity. So, the velocity appears in both the sides. We need to separate them out to obtain the actual effect. So, we will consider this later. But, let us now figure out these are the vectors and the vectors will be somewhat difficult for us to handle. So, what we will do is we will break them out into three components - x component, y component and z component and try to find out this in all the different components and in order to do that, we will draw x y z and B will be in the direction of z, right, into the paper. x is the direction in which the gas is flowing and y is the direction perpendicular to it. So, there will be a component of the motion in the x direction and that when coupled with B, v cross B, there is a component in the x direction v x. Cross B will be in which direction?

Up, so the v x cross B will be in this direction. Similarly, there will be a component of the velocity in the v y direction and that when coupled with B will be giving you is the minus x direction, clear. So, this is v y cross B. So, now we can, on the basis of this we can write down the equations in each of the coordinates. Let us see. Let us clearly draw the vector diagrams.

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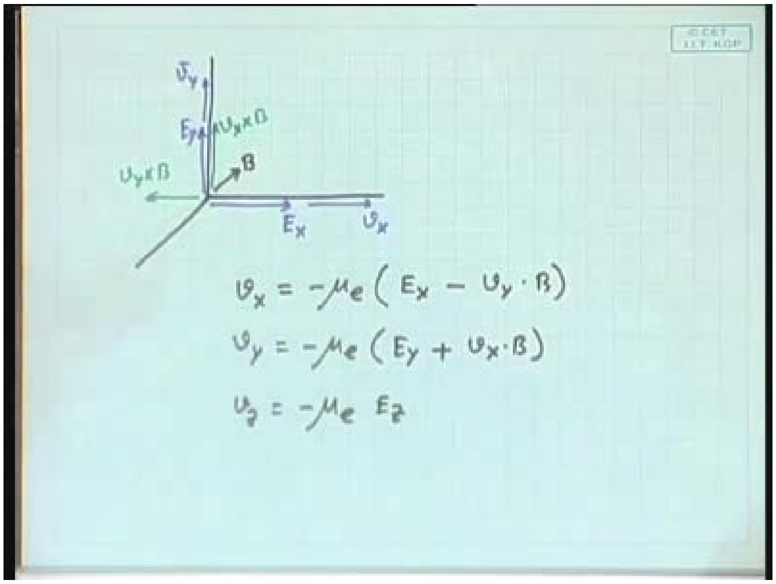

$$v_x = -\mu_e \left(E_x - v_y \cdot B \right)$$

$$v_y = -\mu_e \left(E_y + v_x \cdot B \right)$$

$$v_z = -\mu_e \, E_z$$

So, here you have x y z and you have B in that direction. E x is in this direction, v x is in this direction, E y the voltage in the y direction here and v y is in this direction and here whatever it is, I do not really care, because that does not interact with the B. These are the two things that are of our interest and we had just computed that v x cross B will be in this direction and v y cross B will be in this direction, fine. Based on that we can write the v x is equal to, the way we have just written, mu e times E x minus v y, we have times B because now we are not writing vectors, we are writing just the product of the two quantities. v y is minus mu e, v y will be E y plus v x B and v z is simply, as I told you, we need not consider this, these are the two things of our interest and meaning.

Now notice, very importantly that v x appears here and here, v y appears here and here, so you could solve these two equations to obtain them individually, clear. So, do that. Can you do that? Do it. To cut a long story short, let me give you the results.

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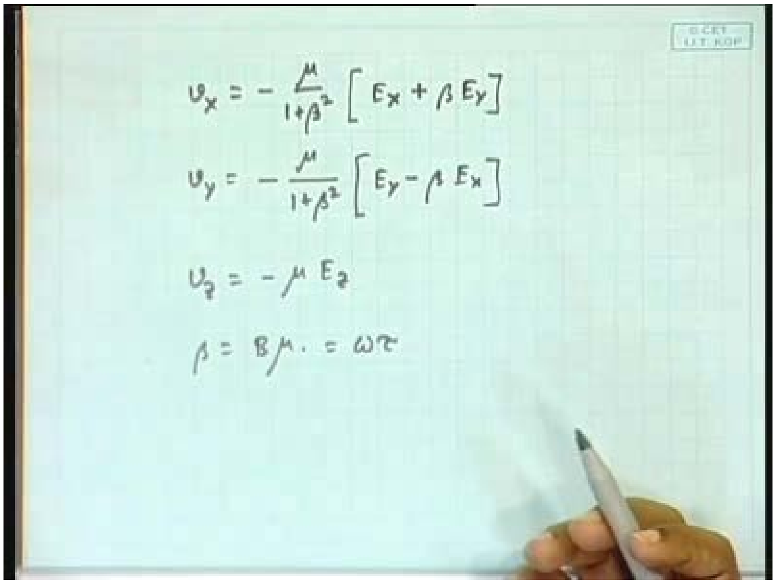

$$v_x = -\frac{\mu}{1+\beta^2}\left[E_x + \beta E_y\right]$$

$$v_y = -\frac{\mu}{1+\beta^2}\left[E_y - \beta E_x\right]$$

$$v_z = -\mu E_z$$

$$\beta = B\mu. = \omega\tau$$

You will have v x is equal to minus mu by 1 plus beta square E x plus beta E y and v y is equal to minus mu by 1 plus beta square E y minus beta E x. This beta is B mu. So, let me write on and then I will write that. v z is minus mu E z, where beta is … This is equal to omega tau. So, this is an important quantity, as I told you, quantifying the Hall effect. That is why we have put it in terms of the beta. Now, if we know the velocities we also know the currents.

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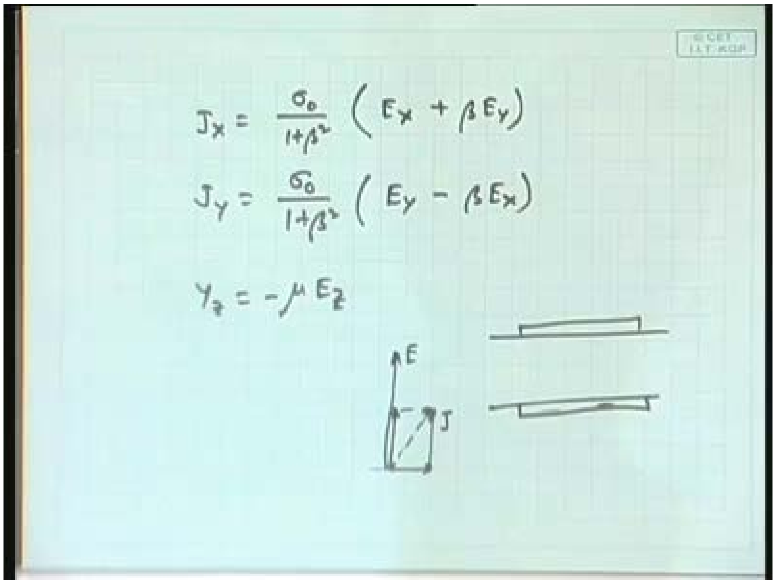

$$J_x = \frac{\sigma_0}{1+\beta^2}\left(E_x + \beta E_y\right)$$

$$J_y = \frac{\sigma_0}{1+\beta^2}\left(E_y - \beta E_x\right)$$

$$v_z = -\mu E_z$$

So, we can write the currents as J x is equal to, J is the current density in the x direction, it will be sigma by 1 plus beta square, notice the expression, so we are writing in terms of this, beta square, E x plus beta E y and here it is, J y is sigma naught by 1 plus beta square E y minus beta E x and J z is equal to minus mu E z, simple. So, we have noticed that this is the thing like the ohm's law. This is how the current vector is related to the voltage vector. But, you see that these are somewhat involved and complicated, because the current in the x direction depends on the voltage in the, both the direction. Current in the y direction depends on voltage on both the directions and z direction is independent. So, what is the, what is the meaning on the effect of all this? Let us try to figure that out.

Now, it is, it is not difficult to see that supposing the electrodes are placed such that there is no voltage in the x direction, E x is zero. As we have seen already, here is the channel and here is the electrode. That means there is no voltage in the x direction, because the electrodes will short it. If E x is zero, then what will be the J x? It will be this times beta E y and how much will be J y? J y will remain, it will be, this is zero, this, which means that even if there is no voltage in the x direction, there will be the current in x direction. So, you can now draw sort of a vector diagram.

E will be in this direction here. There will be a, there will be a current in this direction, there will be also a current in this direction. So, the current resultant will be like this, here with J here with E. They will be moved from each other; this way or that way. Probably it will be the other side, because of the, we will see that, see to that. But, the point is that the voltage and the current will not be in the same direction. Is that point understood? Effectively, all these rotation of this things and things like that we are trying to get that extracted in terms of the voltages and the currents and we have come to the conclusion that the voltage and the current will not be vectorially in the same direction, they will be in different directions.

Now, let us consider what is happening inside this fellow?

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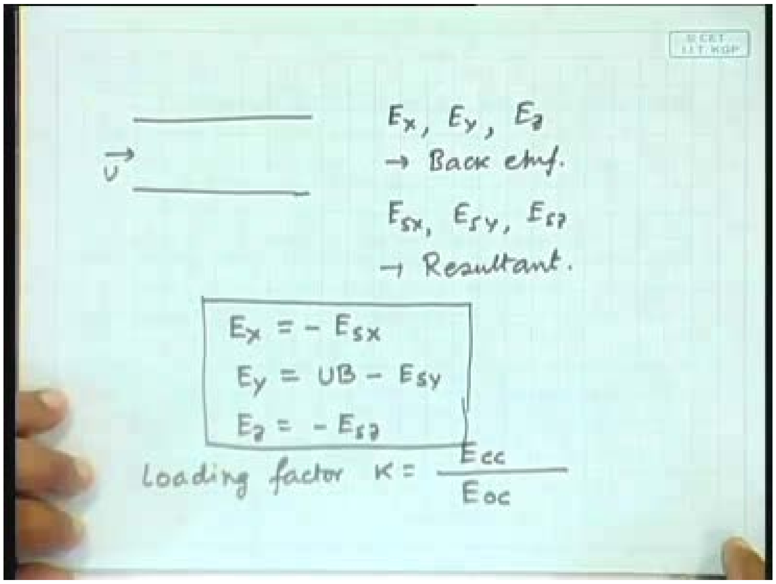

E_x, E_y, E_z

→ Back emf.

E_{sx}, E_{sy}, E_{sz}

→ Resultant.

$$\boxed{\begin{aligned} E_x &= -E_{sx} \\ E_y &= UB - E_{sy} \\ E_z &= -E_{sz} \end{aligned}}$$

Loading factor $K = \frac{E_{cc}}{E_{oc}}$

You have the plasma flowing with some velocity and you have got some kind of an electrode structure. There can be different types of electrodes that is why I am not drawing it right now. But, what happens is that as it flows and as you extract the power, effectively what you do? You slow it down; extract some power, power out of it means that you make it loose kinetic energy, so it looses. You have some amount here, some amount, some other amount there and it is something like, in case of any motor, any generator, suppose there is a generator, the kind of generator that you have heard of like the DC generator, like the industrial generator, those things that you heard of.

Imagine that as you run it in no load, that is you have not put in load, there will be some voltage induced and if you load it that voltage will drop, because there will be, when in no load the generator voltage is equal to the back EMF that maintains the balance. When you load it, the back EMF is there, but there is also a load, so due to which there will be a new balance. So, always there is a generated voltage and there is a back EMF. Now, here also there will be something like that. Here also there will be say a similar thing. So, you will consider, we will consider the E x E y and E z as the back EMF components. The generated voltage is U times B. U is the motion times B.

So, the generated voltage is U B. These are the back EMF. So, the resultants will be E, we will denote it as sx, E sy, E sz is the resultant. So, we can write the equations as E x in the x direction is equal to minus E sx, because there is no induced voltage in this direction, in the x direction. E y is equal to UB, U is the velocity of the plasma times the magnetic field, this is the voltage induced, minus E sy and E z is equal to minus E sz. So, these are the back EMF's which are expressed in terms of the generated voltage minus the resultant holder that we actually see that is E sy. So, all this voltage will be related like this and the loading factor K is the open circuit voltage oh, the closed circuit voltage by open circuit voltage.

So, if you have, there are electrodes here, you have not loaded it, whatever voltage induced is the denominator. If we load it, then whatever voltage induced is the numerator. That is the definition of the loading factor K, because you could put in different types of loads and you will soon find that for here also like in the photovoltaic, say like in the wind generator, you will always find that for a specific load it gives the maximum power. So, we need to find that. That is why this is important, loading factor K. So, if you, if you have seen all these, then you would realize these are the equations that we would be using. So, just keep them very clearly in your mind.

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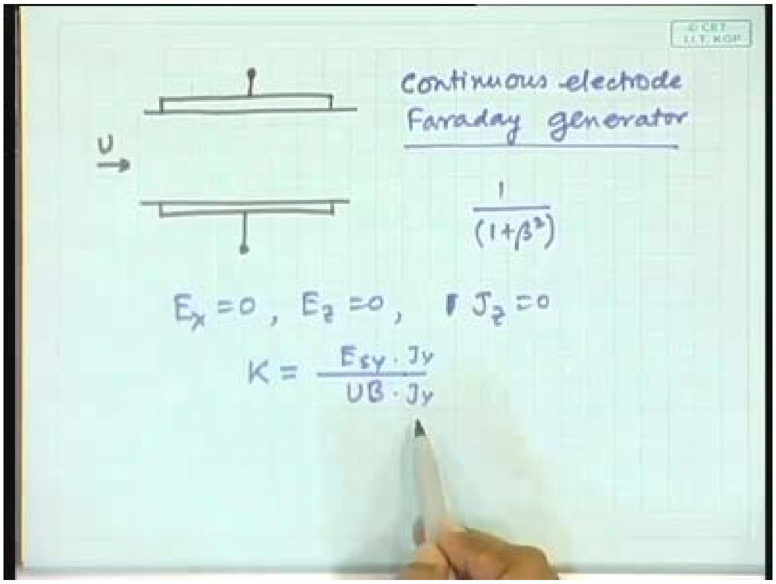

Continuous electrode Faraday generator

$$\frac{1}{(1+\beta^2)}$$

$E_x = 0$, $E_z = 0$, $J_z = 0$

$$K = \frac{E_{sy} \cdot J_y}{UB \cdot J_y}$$

So, out of all this we have concluded that if you have the channel like this and if you have the electrodes like that and if the plasma is flowing with a velocity U and you have this, you might connect a resistance to the external circuit which will lead to the voltage being changed to the s quantities, resultant quantities. If it is like this, this is called the continuous electrode Faraday generator, because these electrodes are continuous. Through the channel, there is one electrode up and one electrode down.

Now, the electric field is in this direction, but the current flows in this direction. As we have seen already, the current is not in the direction of electric field, as a result of which the effective power goes down. By how much? The current, the transverse current, the power is due to this voltage and this current, but the actual current is that direction. So, its component in this direction will be less by a factor 1 by 1 plus beta square; we have already seen that, see 1 plus beta square here. So, this will make the effective power somewhat less.

It would have been good if the current also were in the same direction, but the current is actually in this direction. That is why there will be a reduction in the power. In this case the boundary conditions are, boundary conditions are E x is zero, there is no voltage allowed in the x direction because we have continuous electrode, E x is equal to zero, E z is also is equal to zero and in the z direction we have not, no electrode and therefore. there is no current that can flow, J z is zero. But, there is current that can be flow in the x direction. Can you see? Through the electrode there is current that can flow in the x direction, so that is not zero. Current will flow in this direction that is not zero. So, these are the boundary conditions.

If you put that in these equations, what do you get? First, let us see in this case the K, the loading factor is E sy, the resultant voltage in that y direction divided by the generated voltage which is UB. So, this is the generated voltage and this is the, this is the generated voltage and this is the resultant voltage after the loading and that is that gives you the factor, loading factor K. It is not difficult to see that if you multiply this with J y and this with J y what does the numerator say? The actual power that is delivered to the load.

What does the numerator say? Actual power that is delivered to the load. What does the denominator say? That power U B is the is the voltage generated, J y is the current flowing through that that is the sort of that will be related to the breaking that is produced in the gas. So, this is also effectively the efficiency that you have in the system.

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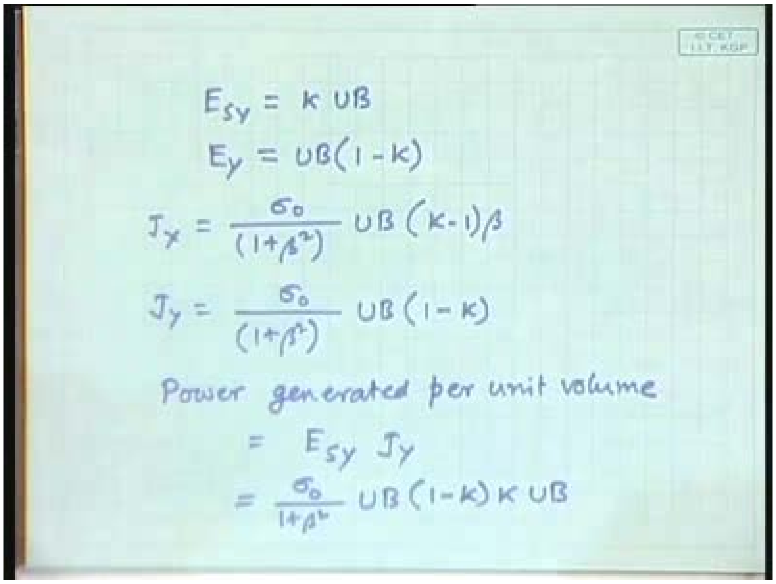

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$$E_{sy} = k\,UB$$

$$E_y = UB(1-k)$$

$$J_x = \frac{\sigma_0}{(1+\beta^2)}\,UB\,(k-1)\beta$$

$$J_y = \frac{\sigma_0}{(1+\beta^2)}\,UB\,(1-k)$$

Power generated per unit volume

$$= E_{sy}\,J_y$$

$$= \frac{\sigma_0}{1+\beta^2}\,UB\,(1-k)\,k\,UB$$

So, if you have that then you can write E sy is equal to K UB and E y, we already know as, here K UB you substitute, you get UB 1 minus K. So, now in that terms we can write J x is equal to sigma naught divided by 1 plus beta square UB K minus 1 beta and J y is sigma naught 1 plus beta square UB 1 minus K. Here there was a negative sign that is why it has become K minus 1. So, the power generated, power generated per unit volume will be E sy is the voltage times J y. So, if you substitute that you will get this will become sigma naught divided by 1 plus beta square UB 1 minus K times K UB. So, that is what you got.

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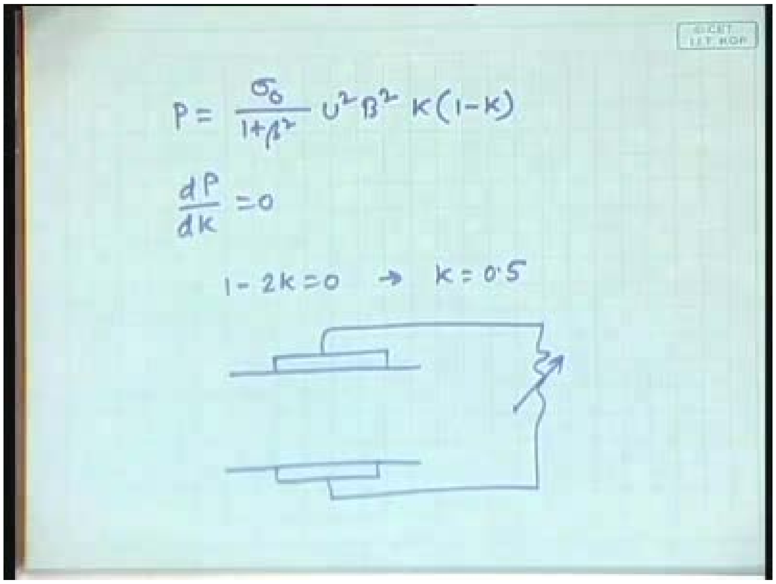

$$P = \frac{\sigma_0}{1+\beta^2} U^2 B^2 K(1-K)$$

$$\frac{dP}{dK} = 0$$

$$1 - 2k = 0 \rightarrow k = 0.5$$

We can write that as P is equal to sigma naught 1 plus beta square U square B square K 1 minus K. Interesting thing is that you would like to maximize the power. So, in order to obtain the maximization of power we will write, dP dK equal to zero. Just do that. This fellow is constant, only K 1 minus K, so you get …

Student: …

No, only this much will have to be differentiated with respect to K.

Student: K is equal to half.

Yes, K is equal to half. So, it will be K minus K square. So, 1 minus twice K is equal to zero or K is equal to …, which means that the power maximizes for K is equal to 0.5. So, you have the arrangement like this and there will be an external load. That external load has to be variable and it has to be so set that K becomes 0.5. K depends on the external load and this can be done by a power electronic controller in between. Today let us stop. We will continue with that in the next class.