

Energy Resources and Technology
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Lecture - 6
Energy Economics: Input-output Analysis

In the last classes, you have learnt that energy has a very special role to play in **this** economy. All the other sectors of economy, say transportation, say steel, say plastic, say all these essentially use some raw materials and produce something that will be either used in other sectors of economy or directly useful to men; you directly use it. A plastic bag something that you directly use, but the plastic that goes into making other things, that goes into another type of industry. The point is that in all these you need energy. So, energy is a special sector, because in all the other sectors of economy, these things, say a steel industry needs the things that are made, that is necessary to make steel and its products are used only in those sectors that needs steel.

Similarly, plastic; the sectors that need plastic produce that and the sectors who do not need it will not use that. But, energy is a sector, energy is a particular component of the economy that is used everywhere. No production is possible without energy, right and it is most important, because whenever you are planning, a planning takes place that means what does the nation do? Nation plans for certain improvement, certain development over the next 5 years, next 10 years or so. Now, suppose a country has planned to increase the residential housing by say 10% over the next 10 years; very logical plan that will require an increased input of bricks, mortar, stone, cement, steel, everything.

So, the production of steel will have to be increased, production of cement will have to be increased. Now, the production of steel needs energy, production of cement needs energy. So, you see, if you make some plan that has a multiplier effect throughout the economy and many things needs to be changed, right. Most importantly, the energy inputs also need to be increased. If the nation does not make provision for the necessary amount of energy to produce the increased amount of steel, the increased amount of cement, the

increased amount of brick, increased amount of so, then the whole planning is completely useless, right.

This is not true for other sectors of the economy. Suppose you make a plan for increase of production of steel only, that does not require the increase of plastic, for example; but, in case of energy, it is not so. Unless the provision for the adequate amount of energy is made, every planning will be doomed to failure. So, one of the important things that an energy engineer has to address is questions like supposing I plan to increase the steel production by 10% in the next 5 years, how much additional provision for electricity and for petroleum products should I make? Now, this is not a qualitative statement, qualitative question. One needs a quantitative response, numbers, right. One needs a number; how much, how big a input and exactly that amounts provisions should be made.

So, what we are now going to address is a quantitative issue, a numerical issue, something that needs to be computed. The other aspect is that whenever we talk about a product, for example my pen, it has some bit of steel in it; it has some bit of plastic in it, it has some bit of ink in it. So, all that had been produced first in order to make this. Now, when these were produced, obviously these were produced in separate industries. So, what went into production of this pen? First, somebody mined iron ore. Then, it went to the steel plant, there the iron was extracted, then from the iron, steel was made. Then, from the steel these particular components were made and then it came to this - this particular pen industry where things were assembled. Parallely, in another place, somebody was either mining petroleum or importing petroleum and it was going to a petroleum, petrochemical industry and the petrochemical industry was producing the material that goes into making this plastic and then it went to some plastic industry where the plastic material was made, then it went to this pen industry where this particular shape was given.

All this went behind making this pen and all these steps needed energy; mining needed energy, transportation to the factory needed energy, then the factories production of steel needed energy, so all that components needed energy. So, what we have in our hand is

embodied energy, right. How much? How much energy is embodied in this? Much is not an answer, for at least technologists; you have to say numbers. How much, how many kilocalories **is** here. So, we should also be able to calculate that. It is necessary also in the sense that over the years how much are we improving that is often quantified by how efficient, energy efficient are our industries. If say 15 years back this pen would require something like 100 kilocalories of energy, just for the sake of some numbers, I am not saying that it is something like 100 kilocalories and now it is costing something like 80 kilocalories, then we have a quantification of the improvement we have done. We are having industries that are now running more energy efficiently.

So, we need to quantify. Otherwise, everything is fussy; everything is you know, in the air. Technologists cannot talk in terms of fussy things. We need to talk in terms of concrete stuff. So, the problem that I will deal with today is how to concretize it, how to quantify these things and how to calculate this? So, in that one advantage is that in economics, a method was developed quite long back in the 50's. This is, this was developed by the Nobel Laureate economist Wassily Leontief. The method is called input output technique in economics. That is very widely used in economics, in planning process and we will make use of that technique to answer the question that I just raised.

So, initially, first I need to introduce the techniques per say and then, we will see how to apply it for the energy sectors, its basic, specifically. So, what is happening in the economy? In the economy, just imagine the whole economy, whole economy of this country. It produces various things. It produces, it has agriculture, it has, you know, fertilizers, it has say, the tractors are made that go into the agriculture, that then these are manufacturing industries. There is steel industries, the plastic industries, petrochemical industry, huge number of industries. Now, all these industries can be sort of clubbed. There may be 5 petrochemical industries of more or less the same nature. So, they can be clubbed together as the petrochemical industry of the nation. Similarly, electricity production can be clubbed together to make one electricity sector of the economy.

Unless you do that, it becomes huge. There are huge numbers of production units and if you consider each production unit, it becomes massive. So, that is one way of sort of organizing the information. So, the economy is then divided into a few sectors. The sectors essentially depend on how minutely you want to divide the economy. Normally for the planning process, the economy is divided into something like 100 to 130 sectors; then, each sector, say electricity sector, say steel sector. What does it do? It produces some amount of steel. Where does the steel go? Where does the steel go? Where does the electricity go?

Some part of the steel goes into other industries. Steel goes into making cars, steel goes into making pens, steel goes into other sectors of the economy. Electricity goes into producing electricity, because electrical power plants also need electricity. Electricity goes into agriculture, irrigation; electricity goes into industries, all industries and also, electricity goes into the final consumers, who light their bulbs at their homes or fans in their homes. So, essentially the production of every sector is consumed into two ways – one, either it goes to the other sectors of the economy or it goes directly into consumption, right.

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$X_1 \rightarrow$ total production of sector 1.

$$X_1 = x_{11} + x_{12} + x_{13} + \dots + x_{1n} + y_1$$

$$X_2 = x_{21} + x_{22} + \dots + x_{2n} + y_2$$

$$\vdots$$

$$X_n = x_{n1} + x_{n2} + \dots + x_{nn} + y_n$$

x_{ij} = sales from sector i to sector j

y_i = final demand for the products of sector i

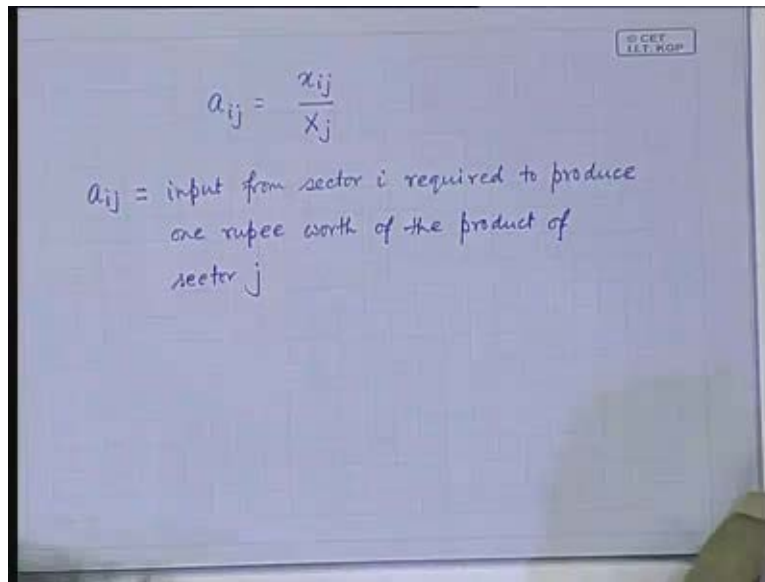
X_i = Total output of sector i

So, if for sector 1, the total production is say capital X_1 or the sector 1 could be anything; we are just indexing the different sectors of the economy as 1, 2, 3, 4, 5, 6 and all that, so capital X_1 is the total production of the sector 1. Then, this X_1 would be divided into components. Some part of it will be going to another sector, another sector; So, X_1 is actually small x_{11} plus small x_{12} plus small x_{13} and all that; means small x is the component of the production that goes from sector 1 to sector 1, sector 1 to sector 2, sector 1 to sector 3 and all that. Sector 1 to sector 1 - what does it mean? Say, electricity sector consuming electricity, steel sector consuming steel, which is obviously possible. It needs energy to produce energy. So, this x_{11} will be there and the component of production that goes from sector 1 to sector 2 is x_{12} , a component of production that goes from sector 1 to 3 is x_{13} and all that. So, it will be x_{1n} , provided there are n sectors plus there will be a component that goes to the final demand that is directly consumed by the people. Let that be called y_1 .

Similarly, you have, so X_1 is this; X_2 will be, you can write in the similar way, x_{21} plus x_{22} plus and all that x_{2n} plus y_2 , so on and so forth and finally you have X_n is equal to x_{n1} plus x_{n2} plus and all that x_{nn} plus y_n , right. You can write that for the whole economy. Note what these are. You have to carefully keep track of these quantities. So x_{ij} is, what is x_{ij} ? Sales from sector i to Now, ij goes from 1 to n . This was important, because I am talking about sales, because each one will be in different units. Steel is produced in say, metric tonnes, electricity is produced in megawatts, bricks are produced in number of bricks. So, obviously everything has different units and unless everything is brought into a same, same unit, you cannot really write equations like this.

So, the same unit means money units. So, all these will be in money units, in terms of how much monetary worth of the sales goes from sector 1 to sector 2, sector 1 to sector 3 and all that. Similarly, y_i is final demand, final demand for the products of sector i in monetary units again and capital X_i is total output sector I , alright.

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Now, it is, that is the basic component of the theory of Leontief that we define coefficients of the, the transaction coefficients, the transaction coefficients like we define the a_{ij} as x_{ij} by capital X_j . So, these are the transaction coefficients. What does it mean physically? Can you, can you see what does it mean? Out of the total production of sector i , a bit went into sector j . Out of the total production of electricity, a bit went into the steel industry. So, steel industry consumed a bit of electricity and in order to produce an unit amount of steel, how much was the electricity needed that is contained in this. So, essentially these are, a_{ij} if I write in words, it will be input from sector i , the quantity of input from sector i required to produce one rupee worth of the product of sector j . That is what is this, clear?

Now, if you can define this way, then you have got this.

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$X_1 \rightarrow$ total production of sector 1.

$$X_1 = x_{11} + x_{12} + x_{13} + \dots + x_{1n} + y_1$$
$$X_2 = x_{21} + x_{22} + \dots + x_{2n} + y_2$$
$$\vdots$$
$$X_n = x_{n1} + x_{n2} + \dots + x_{nn} + y_n$$

x_{ij} = sales from sector i to sector j
 y_i = final demand for the products of sector i
 X_i = Total output of sector i

These can be reorganized in terms of a ij and x_j . So, how will you reorganize? Let us look at that.

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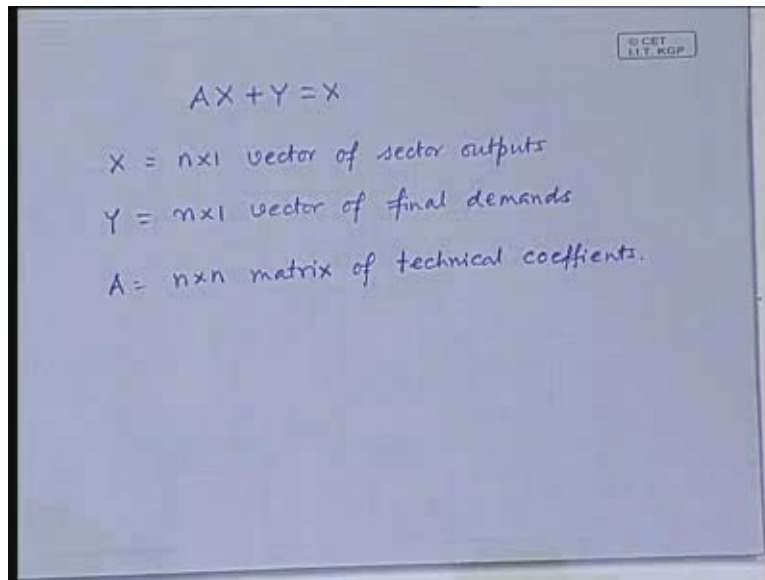
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$$a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + \dots + a_{1n}X_n + y_1 = X_1$$
$$a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + \dots + a_{2n}X_n + y_2 = X_2$$
$$\vdots$$
$$a_{n1}X_1 + a_{n2}X_2 + a_{n3}X_3 + \dots + a_{nn}X_n + y_n = X_n$$
$$AX + Y = X$$

Then it will be, it can be, that particular set of equations can be written as a 11 capital X_1 plus a 12 capital X_2 plus a 13 capital X_3 to a $1n$ X_n plus y_1 is equal to X_1 . The second line a 21 X_1 plus a 22 X_2 plus a 23 X_3 , so on and so forth, a $2n$ X_n plus y_2 is equal to

X_2 . Can you write this way, so on and so forth and finally, you have a $n_1 \times 1$ plus a $n_2 \times 2$ plus a $n_3 \times 3$ to a $n \times n$ plus y_n is equal to X_n , right. The moment we have written it in this form you see a matrix in it, right. Can you see the matrix? Can you see the, it is actually in the form of a matrix equation. The matrix equation is $AX + Y$ is equal to X , right, clear? So, let us take stock of the situation. We have got $AX + Y$ is equal to X . That is the basic equation we have obtained.

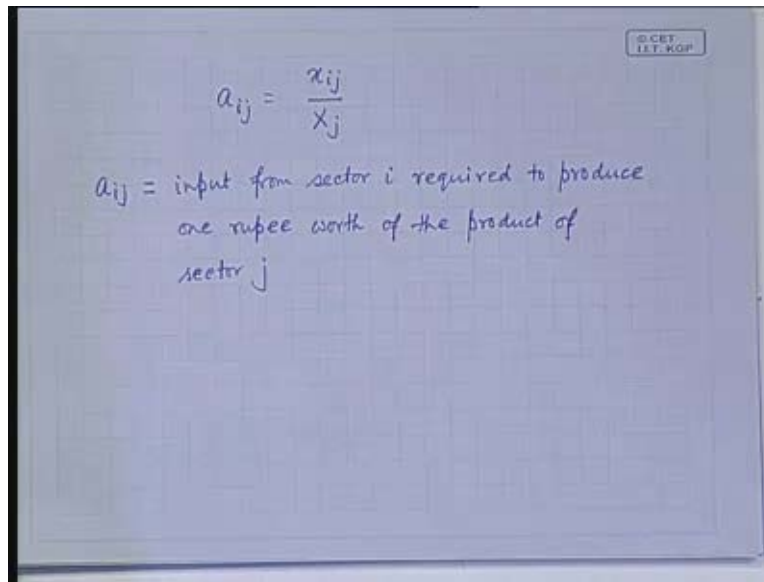
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$$AX + Y = X$$

$X = n \times 1$ vector of sector outputs
 $Y = n \times 1$ vector of final demands
 $A = n \times n$ matrix of technical coefficients.

In it, let us write it $AX + Y$ equal to X ; in it, what is what? Capital X matrix is, what is size of this matrix? $n \times 1$, vector of sector outputs, right. Y is again $n \times 1$, vector of final demands and A is $n \times n$ matrix of, these are the technical coefficients, right. Now, notice the elements of A that we have already seen.

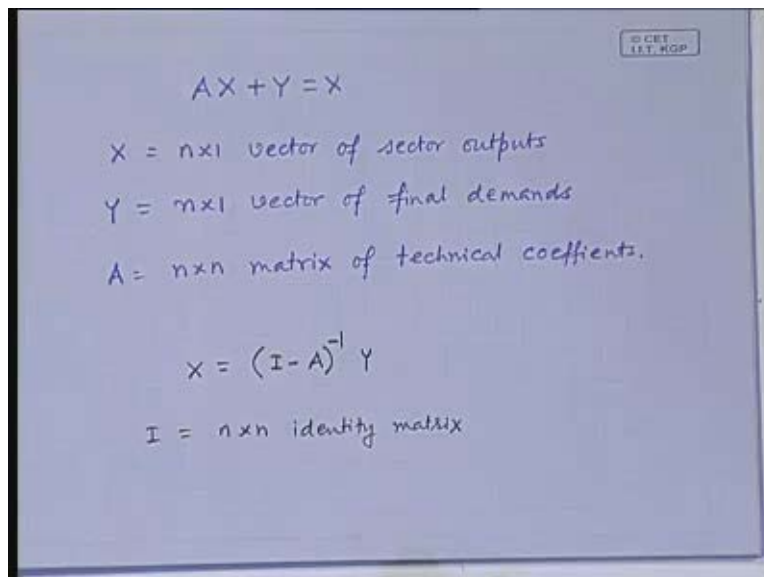
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A handwritten slide on a grid background. At the top right, there is a small box containing the text '© CET IIT KGP'. The main content consists of the equation $a_{ij} = \frac{x_{ij}}{x_j}$ followed by a definition: a_{ij} = input from sector i required to produce one rupee worth of the product of sector j.
$$a_{ij} = \frac{x_{ij}}{x_j}$$

a_{ij} = input from sector i required to produce one rupee worth of the product of sector j

These are the and these essentially tell how efficiently is that particular sector of economy performing, in terms of that particular input and that is why these are called the technical coefficients, depends on the status of technology.

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A handwritten slide on a grid background. At the top right, there is a small box containing the text '© CET IIT KGP'. The main content consists of the equation $AX + Y = X$ followed by definitions for X, Y, and A, and then the solution $X = (I - A)^{-1} Y$ and the definition of I.
$$AX + Y = X$$

X = $n \times 1$ vector of sector outputs
 Y = $n \times 1$ vector of final demands
 A = $n \times n$ matrix of technical coefficients.

$$X = (I - A)^{-1} Y$$

I = $n \times n$ identity matrix

So, A matrix is dependent on the status of technology; as technology improves, the A matrix changes. So, if you follow the evolution of the A matrix over the years in the

economy, you will be able to follow how are we improving in terms of technology, quantitatively; not those hand waving things that we are doing very well, no, in terms of numbers. So, we have come to this particular equation and immediately you can see that this can be easily reorganized, as I need to extract X, right. So, X is equal to what? From here, yes, so $I - A$ inverse times Y, fine, while I is a, this is also a $n \times n$.

Now, what does this equation tell us? Look at it carefully. Suppose what does that the economic planning do? Economic planning essentially tells that I want to make more of these particular things available to the people; more house to the people, more say, food to the people, 10% more food grains available to the people. So, essentially we make planning of Y. Y is set as a policy decision by the planning commission that I want this amount increased and then the economy needs to know that in order to make that amount available to the people, how much should be the multiplier effect in the rest of the economy.

In order to make 10% more housing, how much will be the steel needed, how much will be the cement needed, for that how much sand you have to mine, how much lime stone you have to mine and how much energy must go into the lime stone industry? So, all this there will be a multiplier effect immediately and naturally the planning has to take into account how much should be the increase in the production of each sector and that is here. So, you make a planning for Y, multiply it by this matrix and that gives, in terms of number how much should we increase X.

Let us do one example. Unfortunately, the computer here does not have MATLAB, otherwise I could have done the matrix inversion also; but, that we will leave to you to do. Let us say a particular country, indeed we need to have a problem that can be solved within this, you know, sheet of papers. So, I do not really, I cannot really take a big number of sectors. So, we will take 5 numbers, something that can be written and 5 by 5 matrix, I suppose you can invert, can you? Hey, you are scratching your head; no, no, you can always do that by some available routine. MATLAB has readymade routine with which you can do that or XL can also do that. So, you use whatever you are more

conversant with. I am for example, more conversant with MATLAB. But, in order to do that here I will need to install MATLAB on to the computer. So, let us leave that.

Suppose the industry, suppose the economy has five sectors – agriculture, manufacturing, manufacturing means that will take into account everything, every, all types of manufacturing, transport, electricity and other petroleum products. Let that be the division of the economy, sector wise; very artificial division, but let us take just for the sake of illustrations of the method.

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	Agri	Manu	Trans	Elect	Petro	FD	Total
Agriculture	10	20	0	0	5	55	90
Manufacture	20	30	20	10	10	40	130
Transport	10	10	0	10	10	20	60
Electricity	10	40	20	5	5	30	110
Petro products	20	20	30	5	5	10	90

So, you have agriculture, you have manufacturing. In one side, let me write the whole thing, otherwise you might forget what it means. It is not a computer menu or so agriculture, manufacturing, transport, then you have electricity and then finally, have petro products. Suppose, this is a very artificial division, but nevertheless suffices to illustrate the concept; then, there I will have to write agri, manufacturing, trans, elect and petro. So, we need to write down the matrix and we also have the final demand FD, Y vector and the total. Is everything visible on screen? Yes, okay.

So, suppose the, in million rupees, so all that we will write will be say, in crores of rupees. So, say 10 goes from agriculture to agriculture. Where does it go from agriculture to agriculture? Seeds, manure, everything, so organic manure; the agricultural product going to agricultural product, so it will have some component. 20 - agriculture to manufacture. Where does it go? Yeah, all the food producing industry; all the acharas and chutneys that you take, these are all food processing products. So, the things that you buy in the supermarket, they are all coming from here. Transport - agriculture to transport, does anything go? Practically none, so let us put a zero here. Later, well, when we have ethanol based transport then there will be some component here. That means the sugarcane or other things that go, that can produce ethanol that can be used as transport. Well, when that happens there will be component here.

To electricity - as it is there is practically none. It is possible to have the agricultural waste generate electricity, so presently put zero. Agriculture petroleum products or other kinds of energy, there can be of course, because this will have to include also other sources of energy like biogas and all, so let us put some small number and a large amount of agricultural products are actually eaten by us. So, there will be a relatively larger component say 55. So, how much is the total? 90. Now you have here manufacture to agriculture. Yes, all the agricultural implements are manufactured. They go into the fertilizers, so all that say let us put some number here. Manufacture to manufacture - yes, there will be large component say, 30. Manufacture to transport - yes, reasonably large component. To electricity - there will be, but not all that big. Petroleum products, yes, there will be, but not all that big. Manufactured products going to the final demand, obviously there will be a significant amount say, let that be slightly less than that for the agriculture, so 40. How much is the total? 130.

Transport to agriculture, yes, of course, agriculture products have to be transported, so there has to be some component. Transport to manufacture, yes, manufactured product has to be transported, so there will be some component. Transport to transport, no; transport to transport - do you transport, transport, no. That that will be, so that will be, that can be zero. It is possible to transport buses over buses, but let us, yeah, cars are

transported to their, but that is not the transport sector really. When car is transported to the shop, the car shop, it is essentially the manufacturing sector that goes to the **people needs**. So, transport sector means it is a service sector. Transport to electricity, yes, the coal has to be transported to the power plant, so you have some component. Transport to petroleum products, they also have to be transported and transport to the final demand, yes, people have to be transported. So, there will be some larger component. How much is that? 60.

Electricity to agriculture, yes; pumping is done by electricity, so but, that is yet in India, relatively smaller component, so 10. Electricity to manufacture, a large amount say 40; electricity to transport, yes, trains and trams and all these things, so let there be some component here and electricity to electricity, yes, you need to use electricity in the power plants in order to run the motors and stuff. So, there will some component, but that is small in comparison to that whole production of electricity, so let us put some number, but not very large. Electricity to petroleum products, yes; petroleum refineries do need electricity, but that will be relatively smaller compared to all the other sectors and electricity goes to people use, yes; reasonably large. How much is the total? 110.

Then, petroleum products to agriculture, yes. All the **shallow** pumps are now run with petroleum products that means, so you have 20. Petroleum products to manufacture, obviously a reasonably large amount, 20. Petroleum products to transport - transport actually runs on petroleum products, so Petroleum products to electricity, small amount is needed really; as I told you I will, I will tell where it is, later when we discuss the electricity production in detail, but let us put some number. Petroleum products to petroleum products – let us put some number. So, what do you have here and petroleum product going to directly to people use; well, people use means the transportation sector, public transportation sector is already taken care of, so individual use that is relatively smaller. Add them up, 90 so that is the matrix that we have sort of cooked up. From there we need to construct the A matrix.

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The image shows a whiteboard with a handwritten matrix A and an equation. The matrix A is a 5x5 matrix with the following elements:

$$A = \begin{bmatrix} \frac{10}{90} & \frac{20}{130} & 0 & 0 & \frac{5}{90} \\ \frac{20}{90} & \frac{30}{130} & \frac{20}{60} & \frac{10}{110} & \frac{10}{90} \\ \frac{10}{90} & \frac{10}{130} & 0 & \frac{10}{110} & \frac{10}{90} \\ \frac{10}{90} & \frac{40}{130} & \frac{20}{60} & \frac{5}{110} & \frac{5}{90} \\ \frac{20}{90} & \frac{20}{130} & \frac{30}{60} & \frac{5}{110} & \frac{5}{90} \end{bmatrix}$$

Below the matrix, the equation is written as:

$$X = [I - A]^{-1} Y$$

Now, construct the A matrix, your job. Build the first one, 10, here is 10. So, x ii, x 11 divided by X 1, 90. Second one is 20 divided by, wrong; yes; because; yes; that is the point. It is not 90, not here; but, it is this, going to the manufacturing industry; how much agricultural product going to manufacturing industry divided by the total product of manufacturing industry, here. That is the technical intensity of the manufacturing sector. The other two, next two are 0, 0, therefore if you divide by something it will still remain zero. 5 by, 5 is going to petroleum products; petroleum products final production is 90, so by 90. Second here 20 divided by 90, 30 divided by 130, 20 divided by 60, 10 divided by 110, 40 divided by 90; am I writing right?

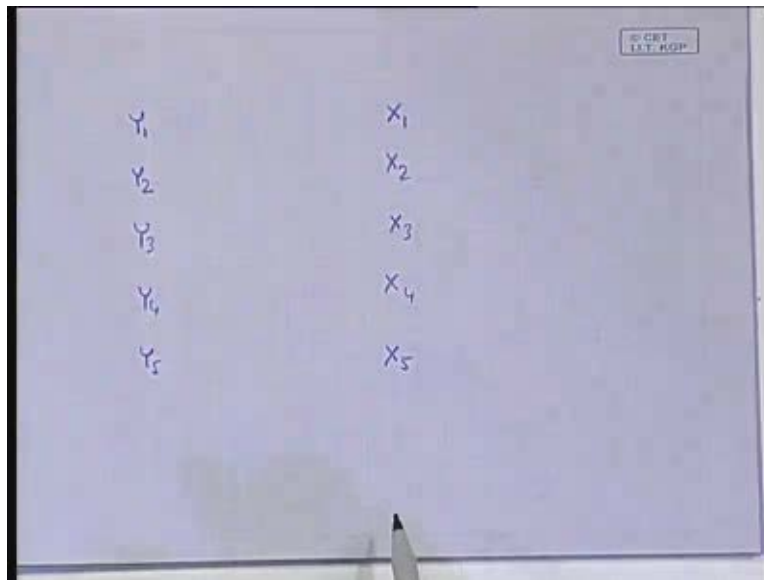
Student: Sir, 10.

Oh, 10, sorry, sorry; here 10 divided by 90, I have just moved one So, third one, transport, 10 divided by first sector – agriculture, whose production, total production is 90; 10 by 90. Second one, 10 by 130, 0, 10 by 110, 10 by 90. Please keep a check. We have come to the thing, fourth line; 10 by 90, 40 by 130, 20 by 60, 5 by 110, again 5 by 90. You have 20 by 90, 20 by 130, 30 by 60, 5 by 110, again 5 by 90. So, this is how you

construct the A matrix, clear? Then what will you do? Yes, then you will have to obtain I minus A, this matrix, invert it, multiply it by Y; you can do that, okay. That gives you X.

What does it tell you then? What does it tell you? Say, X will have, Y will have, Y 1, Y 2, Y 3, Y 4, Y 5 and you have obtained X 1, X 2, X 3, X 4, X 5. Now, if I ask you about the manufacturing sector, I am asking you about the meaning of terms now.

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We have Y 1, Y 2, Y 3, Y 4, Y 5. Tell me what is the meaning of Y 2 physically? Yes, how much is the final demand of sector 2, which is manufacturing sector. So, for that, if you obtain this, you have obtained X 1, X 2, X 3, X 4, X 5. If I ask you what the meaning of X 4 is, total output of the electric sector that is necessary to meet all these. Now, suppose the nation plans to increase manufacturing by say 20% in the next 10 years, what will you do? You have got this, this matrix already in hand. A component, a particular element of Y matrix will change, because of that.

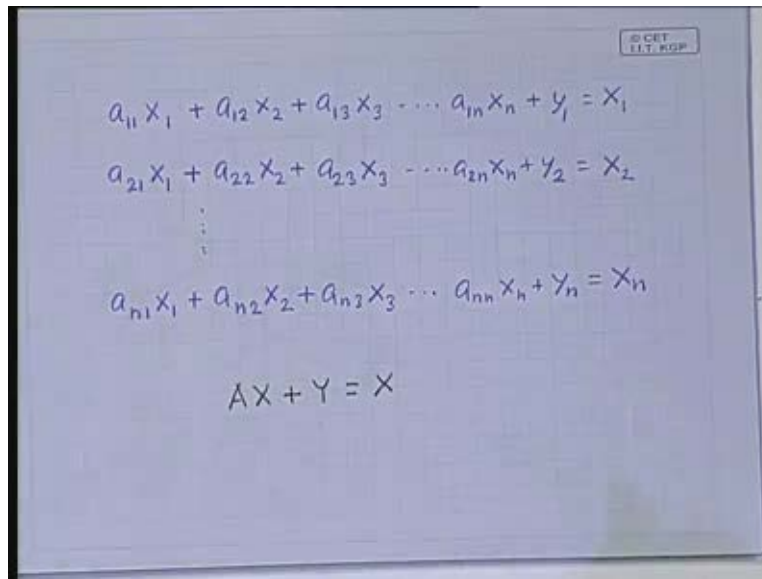
With that changed Y matrix, you can again multiply by this matrix and obtain the changed X matrix. You will find that it will change many places and as a result the changed, this X vector will tell you how much increased amount of petroleum products

you need to meet that, how much increased amount of electricity you need to meet that, even how much increased amount of agricultural products you need there, which is not immediately clear. From logic it is not immediately clear, but then because of the interdependence of various sectors of the economy, things that are not immediately intuitively clear that becomes clear only when you compute these matrices, clear.

So, this is how in the classical way, the energy sector or the economic planning is done in any country, including ours and that is why for all countries, either some kind of government agency keeps track of these numbers and publishes it. In India also these are published, but unfortunately the published matrix, it is published, available on the net, but you cannot download it, because you need to be a member. It is central statistical organization; you have to be a member of the site and the membership fee is more than 20,000 rupees and so, obviously very few people will be really, very few students will be able to become members of that. But, on the net you will find available the input output matrices of many countries.

If you simply do a Google search it will show up. For example, one of the first one that comes is Scotland's input output matrix. Take a look at that. That will sort of tell you what are the entries, how the sector is divided, the sector wise production is divided. For example, in most countries as I told you, this, the total number of sectors would be something like between 100 to 130; that is the average number in which people divide the sectors. In the next class, I will bring some of those matrices, so that on the computer I can show you what the sectoral divisions are.

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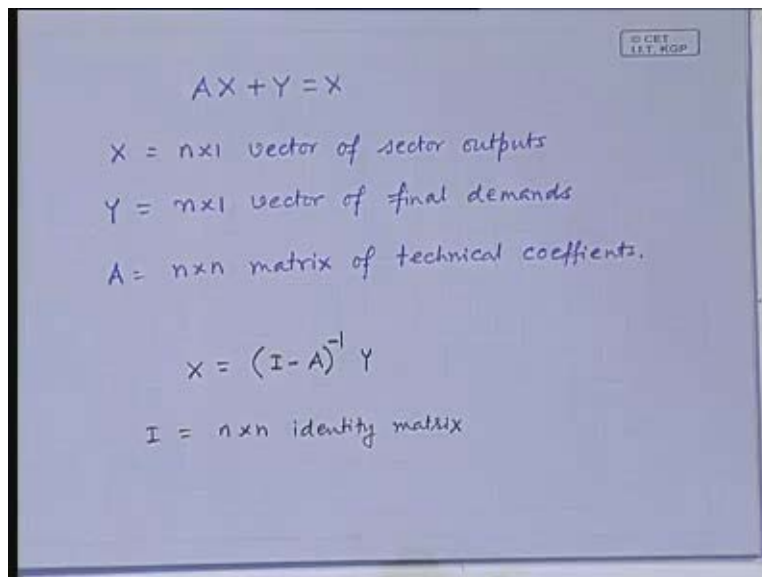


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$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n + y_1 &= x_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n + y_2 &= x_2 \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \dots + a_{nn}x_n + y_n &= x_n \end{aligned}$$
$$AX + Y = X$$

But, I have noticed one thing that in this, when we wrote it this way and wrote it as a matrix like this or finally we transform into the form here, some of these rows are energy rows.

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$$AX + Y = X$$

$X = n \times 1$ vector of sector outputs
 $Y = n \times 1$ vector of final demands
 $A = n \times n$ matrix of technical coefficients.

$$X = (I - A)^{-1} Y$$

$I = n \times n$ identity matrix

For example, in the example that we have taken the last two rows are energy rows. This row belongs to electricity sector and this row belongs to the other petroleum sectors,

right. So, these two are energy rows and so the information necessary in order to infer the energy requirement for the production are all ingrained here, but often people do some further manipulation of these matrices, so that these energy impact of the industry, energy efficiency of the industry, energy efficiency of particular products these are silently clear. So, that manipulation of these matrices will come to Let us start today; we will continue in the next class, because we do not have as much time to complete that part.

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A photograph of a whiteboard showing a handwritten matrix A and an equation. The matrix A is a 5x5 matrix with the following elements:

$$A = \begin{bmatrix} \frac{10}{90} & \frac{20}{130} & 0 & 0 & \frac{5}{90} \\ \frac{20}{90} & \frac{30}{130} & \frac{20}{60} & \frac{10}{110} & \frac{10}{90} \\ \frac{10}{90} & \frac{10}{130} & 0 & \frac{10}{110} & \frac{10}{90} \\ \frac{10}{90} & \frac{40}{130} & \frac{20}{60} & \frac{5}{110} & \frac{5}{90} \\ \frac{20}{90} & \frac{20}{130} & \frac{30}{60} & \frac{5}{110} & \frac{5}{90} \end{bmatrix}$$

Below the matrix, the equation $X = [I - A]^{-1} Y$ is written. In the top right corner of the whiteboard, there is a small logo that reads "© CET I.T. KOP".

So, we have a matrix something like this from which we start, in order to extract the information regarding the energy, because when you actually look at the total matrix for a country, there will be some rows scattered here and there that are actually belonging to energy rows, right and you need to extract that information. So, there has to be some consistent methodology to extract that information about the energy rows. That is what we will do now.

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The image shows a whiteboard with handwritten definitions and an equation. In the top right corner, there is a small logo for 'SCEP I.T. RGP'. The definitions are as follows:

- E_i = total output of energy sector i
- E_{ik} = Intersectoral transaction from energy sector i to sector k
- E_{iy} = Sale of energy of type i to final demand.

The equation is:

$$E_i = \sum_{k=1}^n E_{ik} + E_{iy} \quad \dots \quad \text{Eqn for sector } i$$

The word "Energy" is written above the summation symbol, and a small triangle is drawn under the letter 'i' in "sector i".

So, for the energy rows, if E_i is the total output of energy sector i , E_{ik} be the intersectoral transaction from energy sector i to another sector, any other sector, sector k . Apart from that there will be the finally demand. So, E_{iy} is the sale of; it is the sale of energy of type i to final demand. Then, we can write that as in the form E_i is summation of k is equal to 1 to n of E_{ik} plus the final demand of component. So, that is the equation for energy sector i . Now, remember when we talk about the energy sectors, there are various ways of representing this.

In what unit do you, do you represent this energy? You can represent that in terms of the kilowatt equivalent or energy, energy not kilowatt, kilowatt hour equivalent. You can also represent these in terms of British thermal unit. Also, you can represent these in terms of, because all these are energy sectors, petroleum products, how much is the heat content of that, you can express in that term. So, that will be British thermal unit or if you consider its electricity equivalent, then it will be kilowatt hour equivalent. Also, it is possible to express this in terms of monetary units.

Now, in literature we will find all these in practice. In some literature we will find they are expressed in terms of the BTU's, in some literature you will find they are expressed in

the kilowatt, KLOE - kiloliters of oil equivalent, you can express in that form. Coal, its heat content can be equivalent to the equivalent amount of oil or it can also be expressed in terms of the monetary value; monetary value fluctuates, the coal is there. So, there is a disadvantage of using the monetary value though. The advantage is that if you use this in terms of the monetary value, then that can be simply read from the published tables that are, as I told you, the whole A matrix is published you can simply read the values from there. So, here you have the basic equation for the energy sector, which can be expressed in various units. Remember, do not be confused if you see in literature that this is expressed in terms of the kiloliters of oil equivalent, KLOE; do not be confused.

We will, however, since I would like you to use the input output matrices published, I would continue with the monetary terms. In the next class, when we continue, we will continue with the monetary terms, fine. So, as I told you, the assignment for the next week is to obtain the best fit curve, the parameters for the best fit curve for the world production of oil, whose graph I have given you. Unfortunately, right now the eni.in is down. Therefore, you will not be able to access it; it is down, we cannot help it, but you will be able to, where do you give, where, how do I give you?

Student:

Yeah, it is there; it is there. I downloaded it from there, therefore it is there. So, all you need to do is to find the particular article in which the graph is there and if you go to the site map and go to the articles that are in the repository you will find it. You can do that, else after the eni.in comes, you will have it. So, fine, that is all; by the next week it should be in. In the mean time, I will put it in my web page, but you may not be able to access it, there is a problem. Till the eni.in machine comes back, its hard disk has crashed, so it is only a transient problem.

Thank you very much.