

**Networks, Signals and Systems**  
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**Lecture -13**

**Driving Point Immittance Functions – Realisability Conditions**

Okay, Good afternoon friends we are discussing about poles and zeros location of poles and zeros and determination of residues. Suppose we have a function  $z(s)$ ,  $s$  plus  $a_1$  in to  $s$  plus  $a_2$  divided by  $s$  plus  $b_1$  in to  $s$  plus  $b_2$ . So this type of function or may be  $s$  plus  $b_3$  also when we make partial fraction of such functions it may be just the response function or when  $z(s)$  is there if you give an impulse input then this itself will give you the output function.

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POLES & ZEROS.

$$z(s) = \frac{(s+a_1)(s+a_2)}{(s+b_1)(s+b_2)(s+b_3)}$$

$$\frac{A_1}{s+b_1} + \frac{A_2}{s+b_2} + \frac{A_3}{s+b_3}$$

$$A_1 = z(s)(s+b_1) \Big|_{s=-b_1}$$

So any function of this kind when we want to measure then when we want to calculate the residues  $A_1, A_2$  etcetera. We computed say  $A_1$ , we computed like this this function  $z(s)$  multiplied by  $s$  plus  $b_1$  and then evaluate this at  $s$  equal to minus  $b_1$  is it not now this  $b_1, b_2, b_3$  they can be complex conjugate also if they are complex conjugate they should be in pair conjugate pairs okay. So  $z(s)$  in to  $s$  plus  $b_1$   $s$  evaluated minus  $b_1$  I mean by putting  $s$  equal to minus  $b_1$  evaluate  $a_1$  what does it mean if I multiply by  $s$  plus  $b_1$  this will go  $z(s)$  in to  $s$  plus  $b_1$  means this we are evaluating at  $s$  equal to minus  $b_1$ .

So suppose you are having  $b_1$  is here  $b_2$  may be here, okay  $a_1$  may be here,  $a_2$  may be its complex conjugate  $a_1, a_2$  both could have been real also similarly there can be  $a_3, a_4$  and so on. So  $z(s)$  in to  $s$  plus  $b_1$  when we are evaluating at this point that means I am placing  $s$  at this point

is it not s is minus b<sub>1</sub>. So from here last time we are talking about any location of s what was this vector representing at any s s plus a<sub>1</sub> if it is from the 0 it was representing one of these numerator factors s plus a<sub>1</sub> was this s plus a<sub>2</sub> was this so this is s plus a<sub>2</sub> similarly s plus b<sub>1</sub> is this s plus b<sub>2</sub> is this when I am taking s itself at minus b<sub>1</sub> and then measuring these distances so this will be s plus a<sub>1</sub> at s equal to minus b<sub>1</sub> similarly s plus a<sub>2</sub> and this is s plus b<sub>2</sub> okay had there been another point minus b<sub>3</sub> then from here to here that will be s plus b<sub>3</sub> so let us take a few concrete values of a<sub>1</sub> a<sub>2</sub> a<sub>3</sub> etcetera and then evaluate s

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The slide shows a handwritten derivation on a light blue background. On the left, a complex plane is sketched with poles at -1, -1-j, and -4 on the real axis. The derivation starts with the partial fraction decomposition of a rational function:

$$\frac{(s+a_1)(s+a_2)}{(s+b_1)(s+b_2)(s+b_3)}$$

with the following values for the poles and zeros:

$$a_1 = -1+j, \quad a_2 = -1-j, \quad b_1 = -1, \quad b_2 = -3, \quad b_3 = -4$$

The partial fraction expansion is given as:

$$= \frac{A_1}{s+1} + \dots$$

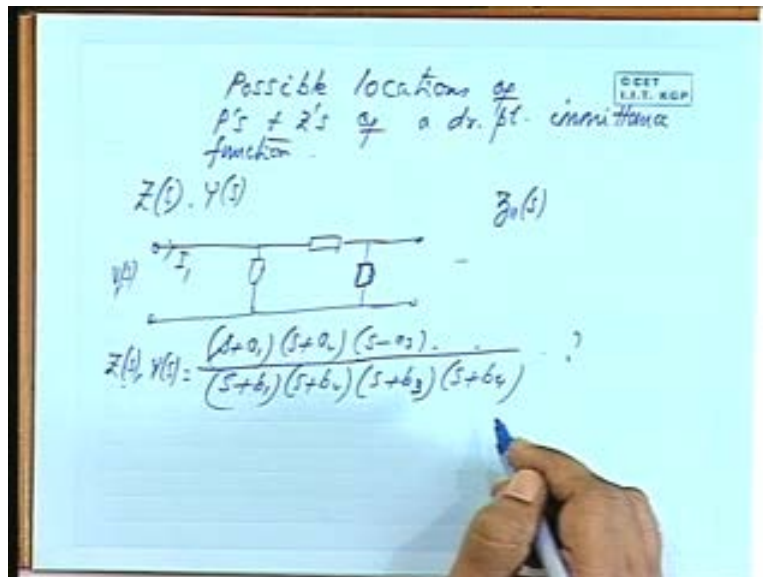
The residue A<sub>1</sub> is then calculated using the limit method:

$$= \frac{1 \angle -90^\circ \cdot 1 \angle 90^\circ}{2 \angle 0^\circ \cdot 3 \angle 0^\circ} = \frac{1 \angle 0^\circ}{6}$$

Therefore suppose you have sorry this is at minus 1 this is at minus 3 this is a<sub>0</sub> and this is a<sub>0</sub> I have got s plus a<sub>1</sub> in to s plus a<sub>2</sub> by s plus b<sub>1</sub> and say another 1 at minus 4 s plus b<sub>2</sub> in to s plus b<sub>3</sub> where a<sub>1</sub> is minus 1 plus j a<sub>2</sub> is minus 1 minus j b<sub>1</sub> is equal to minus 1 b<sub>2</sub> is minus 3 and b<sub>3</sub> is minus 4 so if I write this as equal to a<sub>1</sub> by s plus b<sub>1</sub> that is s plus 1, I am interested in calculating a<sub>1</sub> then this is s plus a<sub>1</sub> this is s plus a<sub>2</sub> this is s plus b<sub>2</sub> and this is from 4 to 1, s plus b<sub>3</sub> alright all evaluated after multiplying by b<sub>1</sub>. So this all this one will not come okay and then we are evaluating at s equal to minus b<sub>1</sub> this is what we are interested in, so how much is this the numerator distances are how much is this this is one in magnitude and angle of minus 90 degree.

So 1 minus 90 degree, what about this 1 plus 90 okay divided by this is magnitude 2 and an angle of 0 is horizontal and this 1 is 4 to 13 angle 0. So how much is this 1 by 6 and an angle of 0 okay so vectorially also you can calculate this residues a<sub>1</sub>, a<sub>2</sub>, a<sub>3</sub>, a<sub>4</sub> okay. **Sir this is going to be better method**, well geometrically you can always construct and see the residues okay if you can evaluate these vectors quickly from the position of the poles and zeros and then the calculations sometimes become very easy.

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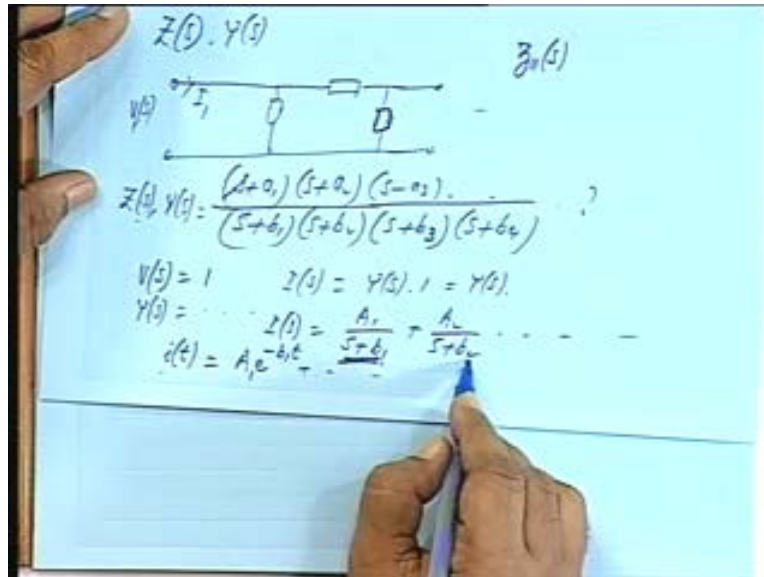


Now let us investigate, what are the possible location possible location of poles and 0s possible locations of poles and 0s of a driving point function, driving point immittance function, what you mean by immittance function? It is either admittance or impedance general  $g(s)$  or  $z(s)$  or  $y(s)$  function, there is some common properties both of them should have so what are those properties in terms of poles and zeros, locations of poles and 0s what could be the nature of poles and zeros now when we apply a voltage measure the current from the sending end then  $V_1$  by  $I_1$  it can be  $a_2$  port network also when this is kept open this is called  $z_{11}$  okay.

When I am keeping it open it is the impedance seen from this side alright the impedance seen from this side what would be the property of this, we are talking about network elements which are passive in nature RLC okay. So what would be the nature of  $z_{11}$  is, suppose  $z_{11}$  is  $s$  plus  $a_1$  in to  $s$  plus  $a_2$  divided by  $s$  plus  $b_1$  in to  $s$  plus  $b_2$ , may be  $s$  plus  $b_3$ , may be  $s$  plus  $b_4$  and so on can it be a driving point impedance or an admittance, is this possible? Any function of this type can it be driving point impedance or admittance function or is there any scope for a negative terms  $s$  minus  $a_3$  you would like to examine that.

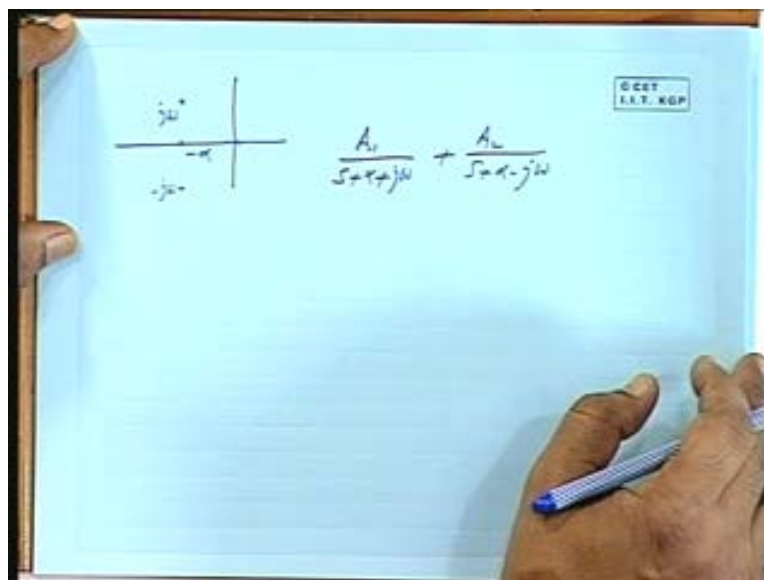
So before that would like to see the response suppose we excited by an impulse function of voltage, impulse function of voltage then  $V(s)$  equal to 1, suppose  $y(s)$  is given this function you say  $y(s)$  okay then what will be  $I(s)$  it will be  $y(s)$  itself  $y(s)$  in to 1, is it not? So if  $y(s)$  is written in a partial fraction form and so on then what should be the current  $i(t)$  what should be the current will be  $a_1$  in to  $e$  to the power minus  $b_1(t)$  and so on exponentially decaying terms had I had minus  $b_1$  term or minus  $b_2$  then it will be  $e$  to the power plus  $b_1(t)$  or  $b_2(t)$  that means the current will exponentially increase is it possible physically to get a current by applying just an impulse voltage.

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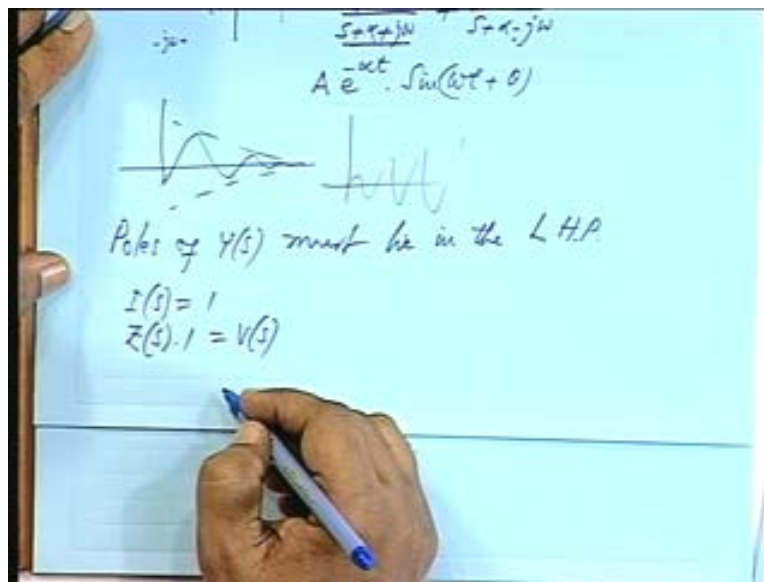
You just apply a voltage and withdraw it can you have a situation where the current will continuously flow through that and it will keep on increasing it goes against the principle of conservation of energy is it not so it cannot go up it can only die down alright so from the conservation of energy principle the factors will be of this type not  $s$  minus  $b_1$  or  $s$  minus  $b_2$  what does it mean  $s$  plus  $b_1$  means the roots  $b_1$   $b_2$  etcetera should be in the left half plane alright.

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Suppose you have a complex root, suppose  $b_1$  and  $b_2$  are say minus alpha plus j omega minus alpha minus j omega that is also possible. So you will get  $A_1$  by  $s$  plus alpha plus j omega plus  $a_2$  by  $s$  plus alpha minus j omega where  $A_1$  and  $A_2$  will be complex and they will be it can be shown they will be complex conjugate okay, otherwise if you add together the numerator will not be all real coefficients okay so if you are having a network function which is a polynomial in the numerator, a polynomial in the denominator which have been written in the factored form and if the polynomial coefficients are all positive then they must appear if they are complex they must appear in conjugate pairs alright number 1, number 2 if you make partial fractions then  $A_1, A_2$  will be complex conjugate it can be prove very easily.

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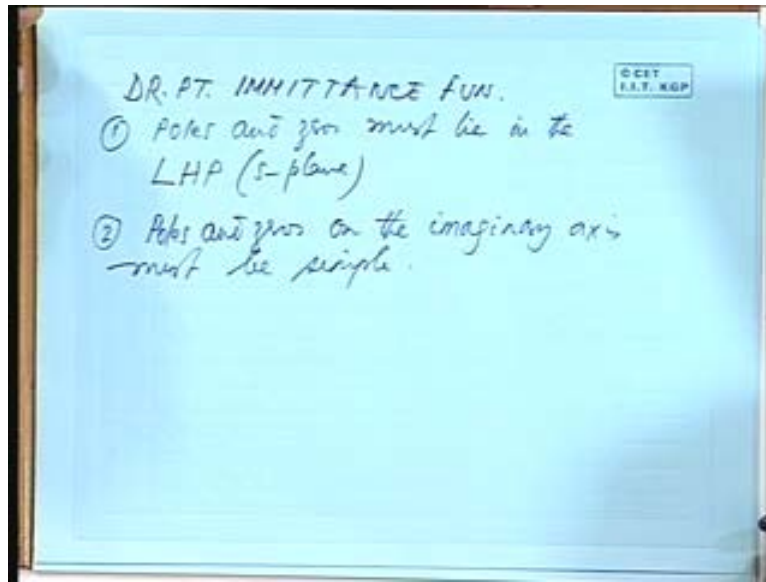


So if I combine them together they will be giving rise to some  $a e^{-\alpha t} \sin(\omega t + \theta)$ , so that will give me response like this which will be dying down if it is in this form had alpha been here that means if you have roots with real part on the right hand side then this will become plus alpha t and that will be causing a divergent oscillatory function alright. So it will keep on increasing can you have a situation when it will keep on increasing the current will keep on increasing and oscillatory when the voltage has been withdrawn again that is not possible it will be against the principle of conservation of energy.

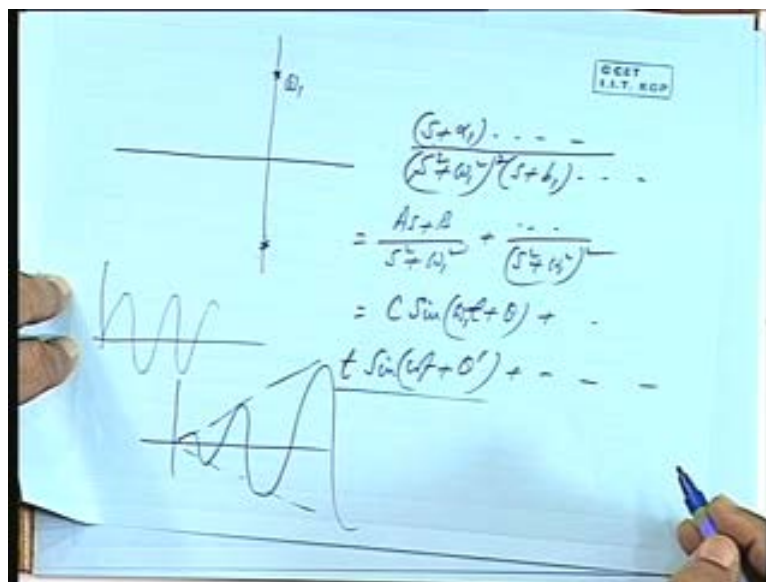
So the poles of  $y(s)$  must lie in the left half plane, left half of  $s$  plane will talk about the roots in the  $s$  plane by the same argument if I give an impulse of current and measure the voltage across the terminals if I can give an impulse of current that is  $I(s)$  is 1 and what will be the output  $z(s)$  in to 1 that will be  $v(s)$  okay. So  $Z(s)$  is nothing but 1 by  $y(s)$  so  $s$  plus  $b_1, s$  plus  $b_2$  will go in the numerator  $s$  plus  $a_1 a_2$  etcetera will come in the denominator and by the same logic if I apply an impulse of current I cannot expect the voltage to build up indefinitely okay. So the roots  $a_1 a_2$  these also should be in the left half plane so that means both poles and zeros of  $z(s)$  or  $y(s)$  should lie in the left half plane this is the first criteria for a driving point impedance or

admittance function. So 1 by 1 will list out the properties of the driving point immittance function so 1 is poles and zeros must lie in the LHP in the s plane okay, is it possible, is it possible to get roots on the imaginary axis poles and zeros on the imaginary axis if you have say at some omega 1 a pole like this that means y (s) or z (s) has roots like this s squared plus omega one squared in to s plus b\_1 and so on, one factor is say s square plus omega square omega 1 squared this will result in a term like some a (s) plus B by s squared plus omega 1 squared okay once again you apply an impulse.

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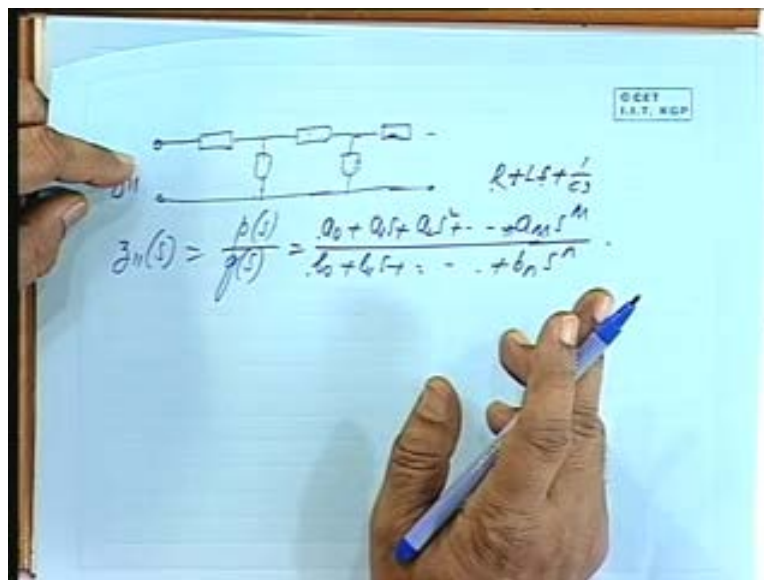
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Okay if you apply an impulse of voltage and if this is the admittance function voltage in to admittance that will be giving you the current so one of the current terms will be this will give me some  $C \sin \cos \omega t + \theta$  this kind of a term okay. So it will be giving you a current which may be oscillating it is not building up it remains a steady oscillation that is possible here theoretically if you have an LC network will have a sustained oscillation. So it can have a oscillation like this we call it a stable system state is it possible to have multiple roots on the imaginary axis if you have instead of  $s^2 + \omega^2$  if you have squared term then what happens? If you have a squared term then they will be a term like  $a s + b s^2 + \omega^2$  plus something in the numerator by  $s^2 + \omega^2$  whole square whose inverse will give you  $t \sin \omega t + \theta$  other terms are there okay.

So this term will give me  $t$  is function like this so it will go on increasing is that alright so such possibilities are ruled out by the same logic in the zeros also in the case of numerator also you cannot have multiple roots on the imaginary axis. So zeros and poles and zeros poles and zeros on the imaginary axis must be simple on the imaginary axis must be simple okay. Another thing when you talk about RLC elements, see both these admittance and driving point admittance and impedance functions they are after all obtained by some combinations of network elements like this. Suppose you want to measure  $Z_{11}$  driving point admi impedance then keep it open and then measure the impedance is it not while measuring the impedance you take this plus this and parallel combination of this and so on.

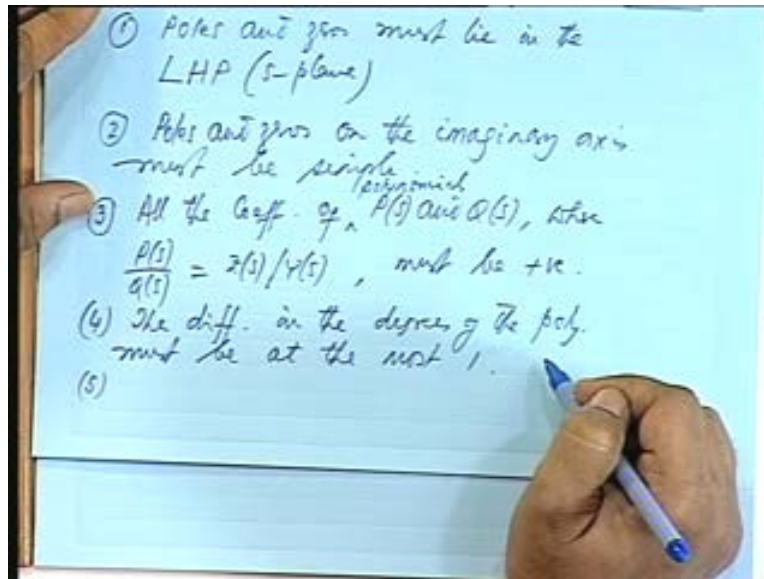
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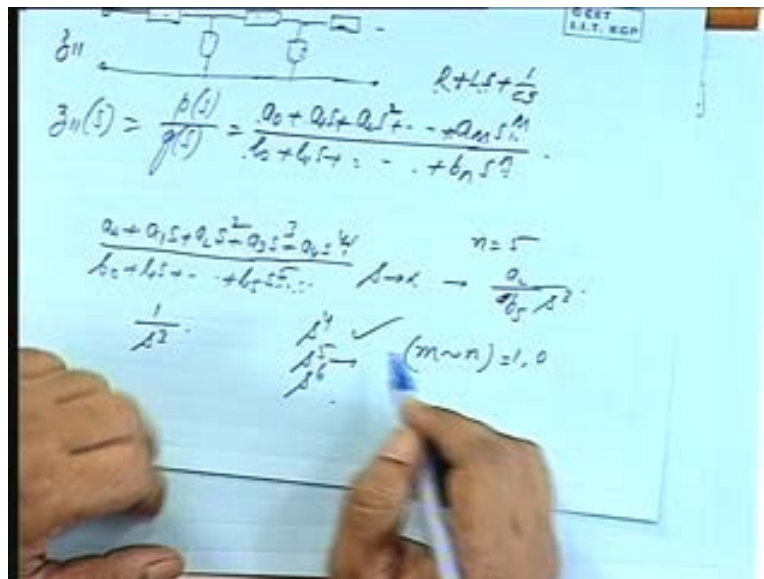
Now these elements are R plus some LS plus some 1 by CS this kind of additive elements or series parallel combinations of these, so when you make repeated conversions say from star to delta or making parallel combinations of such elements it is all positive terms, positive terms like this and their say parallel combination that is products and then again you divide by some sum in

no case there is any scope for a negative sign, did you get my point? So after making all this combinations the numerator and the denominator will always have positive coefficients. So if I write  $Z_{11}(s)$  as some  $p(s)$  by  $q(s)$  then this will be if I write in a polynomial form some  $a_0, a_1(s)$  plus  $a_2(s)$  squared and so on divided by  $b_0$  plus  $b_1(s)$  and so on say  $a_m s$  to the power  $M$  plus  $b_n s$  to the power  $n$  okay. So all the coefficients  $a_0, a_1, a_2$  etcetera and  $b_0, b_1, b_2$  etcetera must be positive is that alright.

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So all the coefficients of  $p(s)$  and  $q(s)$  where coefficients of polynomials  $p(s)$  and  $q(s)$  where  $p(s)$  by  $q(s)$  is the driving point admittance for impedance function must be positive okay. Can you have by any states of imagination by combining  $1/s$  by  $cs$   $R$  etcetera can you have polynomial where you get a fractional part. So it has to be a rational polynomial  $s$ ,  $s$  to the power 1,  $s$  to the power 2 you cannot have  $s$  to the power 2.5 alright. Similarly, for this okay, this also another point this polynomials must be in integral powers of  $s$ . Can you have an impedance say what can be the possible difference between  $m$  and  $n$ . Suppose  $n$  is 5,  $n$  is 5 then what could be the possible value of  $m$ . So suppose  $s$  to the power 2 is the last term, I get  $a_2/s^2$ ,  $a_0$  plus  $a_1/s$  plus  $a_2/s^2$  squared divided by  $b_0$  plus  $b_1/s$  plus up to  $b_5/s^5$  is it possible to have this type of network function driving point impedance. Let us make  $s$  tending to infinity  $s$  tending to infinity means these will be negligible it will tend to  $a_2/s^2$  and this will tend to  $b_5/s^5$  to the power 5.

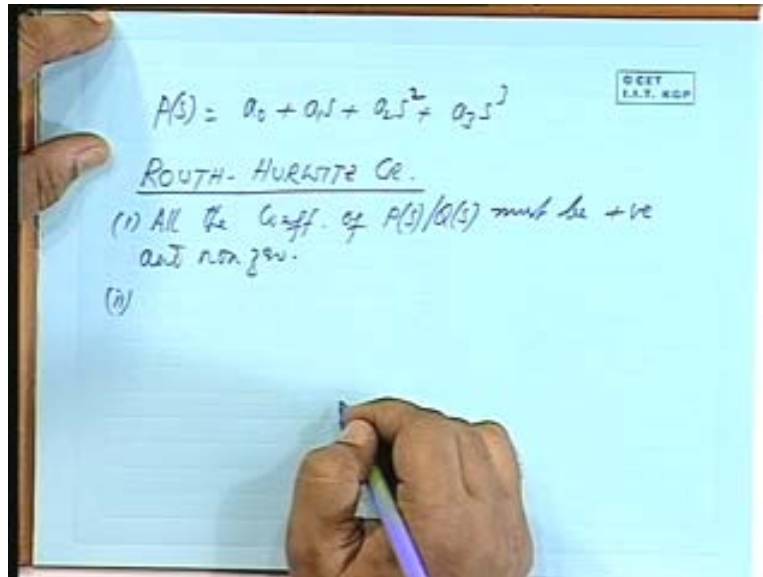
So when  $s$  tends to infinity this tends to  $a_2/b_5$   $(s)$  to the power 3. Now can you have an impedance which is represented by one by  $s$  to the power 3 when you take a very large value of inductance a sorry large value of  $s$  then all the inductive elements will be offering highest impedance and all the capacity elements will be shorts. So by any states of imagination say short you take capacity values as shorts, so you do not ignore the capacity elements alright and you are left with only inductances. So what is the all these elements will be consisting of either a short circuit or an inductance alright. Now you will have suppose they are having all of them are having LC, LC, LRC etcetera. So this is an inductance this an inductance this an inductance this an inductance so overall combination will tend to be another inductance and that cannot be  $1/s^3$  it can be only  $1/s$  okay.

So the difference should be such that it cannot be, it cannot terminated in a square, can it be  $s^3$  cubed then also it will be ending in  $1/s^2$  there is no element which can give you finally a value like  $1/s^2$ , is it possible to get  $a_4/s^4$  then it will tend to  $1/s^4$ ,  $s$  that means the entire network is tending towards the capacity parameter okay. So it can be  $s$  to the power 4 can it be  $s$  to the power 5. So  $s$  to the power 4 is possible can it be  $s$  to the power 5 it can be ratio constant that means it will tend to a resistive element, can it be  $s$  to the power 6 yes, then it will tend to an inductance alright. So  $s$  to the power 4  $s$  to the power 5  $s$  to the power 6 these are possible that means if I have the maximum power of this as  $s^5$  this can be either 4 or 6 that means difference in the maximum **maximum** power of the polynomials the difference can be at the most one okay that is  $m$  minus  $n$  or  $n$  minus  $m$  that is  $m$  difference  $n$  can be 1 or 0 okay. So if this is  $s$  to the power 5 it can be one less or one more or equal the difference cannot be more than that is that alright.

So next property is the difference in the degrees of the polynomial **polynomial** must be restricted to one must be, at the most 1 poles and zeros must be simple on the imaginary axis the coefficients must be positive difference in the degrees in the polynomial must be restricted to one. Suppose we have one term missing that is, is it possible that  $a_1$  is 0 can there be a missing term in the polynomial, so suppose  $p(s)$  is  $a_0$  plus  $a_1/s$  plus  $a_2/s^2$  squared plus  $a_3/s^3$  cubed and so on if one of the terms is missing then what happens does it ensure all the roots lie in the left half plane, no so to check to check whether all the poles and zeros lie in the left half plane we have a criteria Routh Hurwitz criteria it states that all the coefficients must be pre present, all the

coefficients of  $p(s)$  any of the polynomials because we want to ensure both  $p(s)$  and  $q(s)$  should have roots in the left half plane.

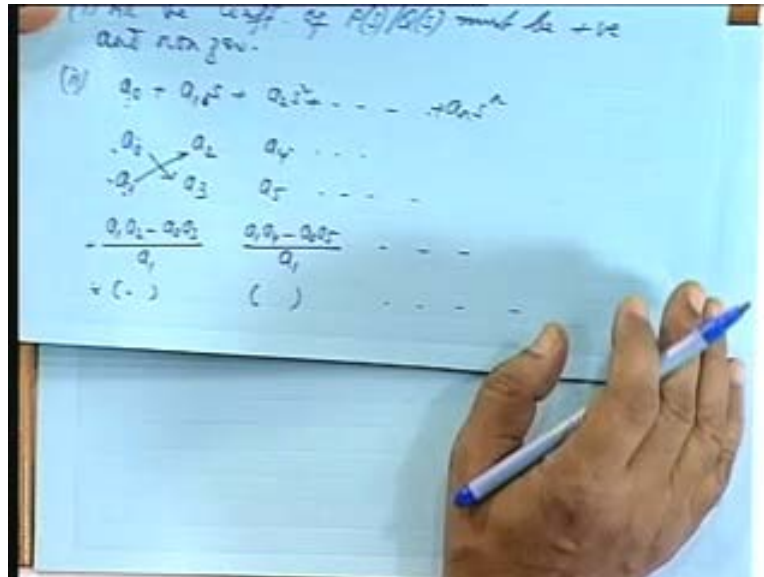
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So all the coefficients of  $p(s)$  and  $q(s)$  must be positive and non-zero any question there can be an exception will come to that exception later on. Can you suggest where it will not be necessarily I mean it is not necessary to have all the coefficients present no suppose alternate terms are absent that means either it is a an even function or an odd function see  $s$  square plus omega square if it is a factor that means what it is an even function so for such cases it is possible and that will come to that will give you an oscillatory term that is for LC network for example.

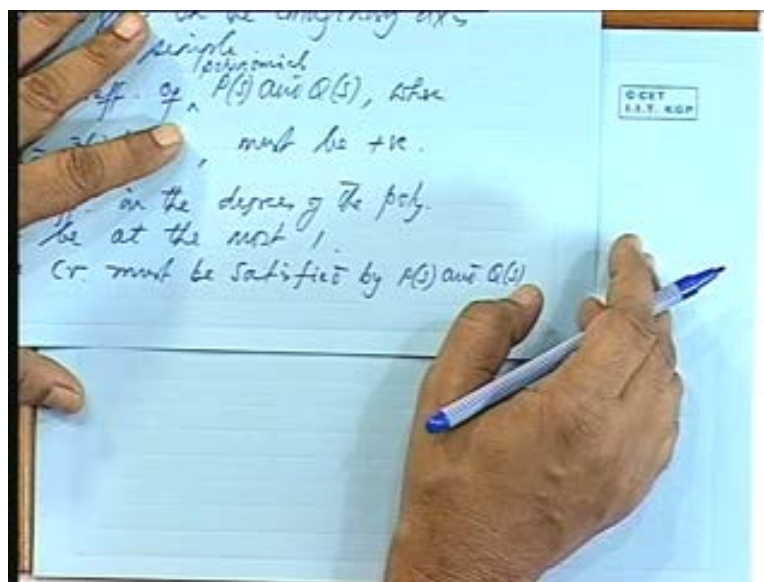
You will have functions  $z(s)$  or  $y(s)$  where you will get  $s$  square plus omega one square  $s$  square plus omega 2 square and so on and one will be even, the other will be odd that is possible that is an exception that is alternate terms are absent. In that case you can have this will be evaluated. So for the general case all the coefficients must be positive and non-zero and the second criteria is you make a table, you make a Routh Hurwitz array and from the table of that from the column you can see the positiveness of those terms and decide whether you can decide whether it is a stable system or unstable system that is whether all the roots are in the left half plane or right half plane. So let us write the table suppose you are having  $a_0, a_1(s)$  plus  $a_2(s)$  squared etcetera as the polynomial  $n s$  to the power  $n$  you start with the coefficients  $a_0$  alternate coefficients  $a_2, a_4$  and so on. Then next make the next row with  $a_1, a_3, a_5$  and so on next  $a_1, a_2$  minus  $a_0, a_3$  sorry  $a_1, a_2$  minus  $a_0, a_3$  divided by  $a_1$  similarly  $a_1, a_4$  minus  $a_0, a_5$  divided by once again  $a_1$  and so on you make another row.

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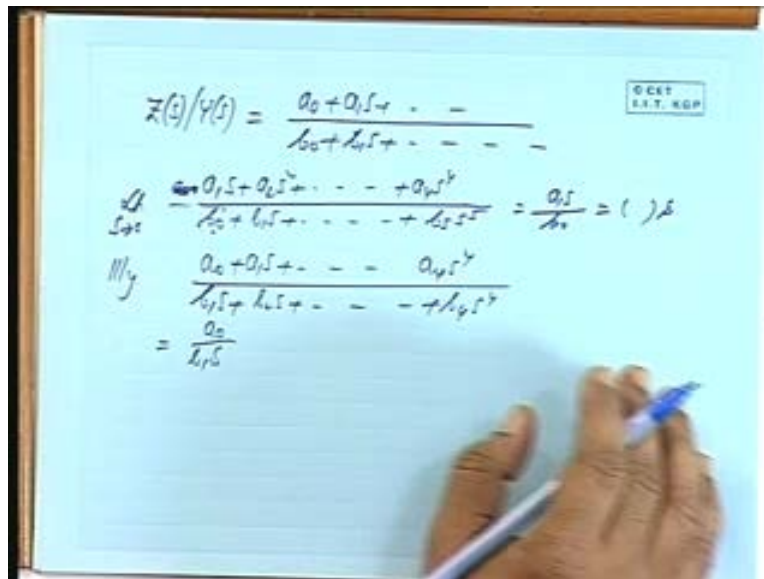


So these synthetic rows you generate again you take this and  $a_3$  minus  $a_1$  in to this quantity divided by this quantity itself that would be the new element in this row. Similarly, another element and so on after completing this you check the signs of the first column this is positive, this is positive, this is positive, this is positive, if all of them are positive then the roots are in the left half plane all the roots are in the left half plane if you find there is a sign change the number of times the sign is changing here will give you the number of roots lying in the right half plane.

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So RH criteria Routh Hurwitz criteria must be satisfied must be satisfied by p (s) and q (s), another point you I just forgot Z (s) or y (s) if it is written like this a<sub>0</sub> plus a<sub>1</sub> (s) and so on, b<sub>0</sub> plus b<sub>1</sub> (s), can the last term be absent say either a<sub>0</sub> or b<sub>0</sub>, is it possible, can we have a situation like this a<sub>1</sub> s plus a<sub>2</sub> (s) squared suppose, we have a<sub>4</sub> s<sub>4</sub> divide by b<sub>0</sub> plus b<sub>1</sub> s plus say b<sub>5</sub> s to the power 5 or may be b<sub>4</sub> (s) to the power 4, is it possible to have the first term absent? Is it possible? That means the first coefficients a<sub>0</sub> absent okay let us see when s tends to 0 s is very very small. So it will tend to the a lowest order term mean s and that will be equal to so a limit if s tends to 0 this will be a<sub>1</sub> s by b<sub>0</sub> it will be something like an inductor.

So it will tend to be an inductor it is possible similarly in the denominator can b<sub>0</sub> be absent a<sub>0</sub> is present say but then can b<sub>0</sub> be absent similarly, a<sub>0</sub> plus a<sub>1</sub> (s) a<sub>4</sub> (s) to the power 4 divided by b<sub>1</sub> (s) plus b<sub>2</sub> (s) square and say b<sub>4</sub> s<sub>4</sub> is it possible to have this term b<sub>0</sub> absent once again you put s tending to 0 it will be tending to a<sub>0</sub> by b<sub>1</sub> (s) it will be tending towards a capacitive element that is also possible so the last term either of the numerator or of the denominator may be present may be absent if both are absent a<sub>0</sub> and b<sub>0</sub> then you will be able you are you will be able to take 1 s common here 1 s common here, so those 2 common s will get cancelled.

So this is the next property that is the polynomials can have either the numerator or the denominator polynomial event, can have the last term or the here it is the first term absent given this what would be the next condition. Suppose we have all the poles and zeros in the left half plane is it a realizable function, can you realize it. So if I have a function s plus 1 in to s plus 10 divided by s plus 2 in to s plus 3, can you realize this, is it possible to realize both the poles and 0s are in the left half plane. Let us see what it will look like A by s plus 2 plus B by s plus 3 plus C by plus C may be okay a constant.

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The whiteboard shows the following work:

$$z(s) = \frac{(s+1)(s+13)}{(s+2)(s+3)}$$

$$= \frac{A}{s+2} + \frac{B}{s+3} + \frac{C}{s}$$

$$A = \frac{-1 \times 2}{1} = -2, \quad B = \frac{-2 \times 3}{-1} = 6$$

$$C = 1$$

$$z(s) = \left( \frac{-2}{s+2} \right) + \frac{6}{s+3} + 1$$

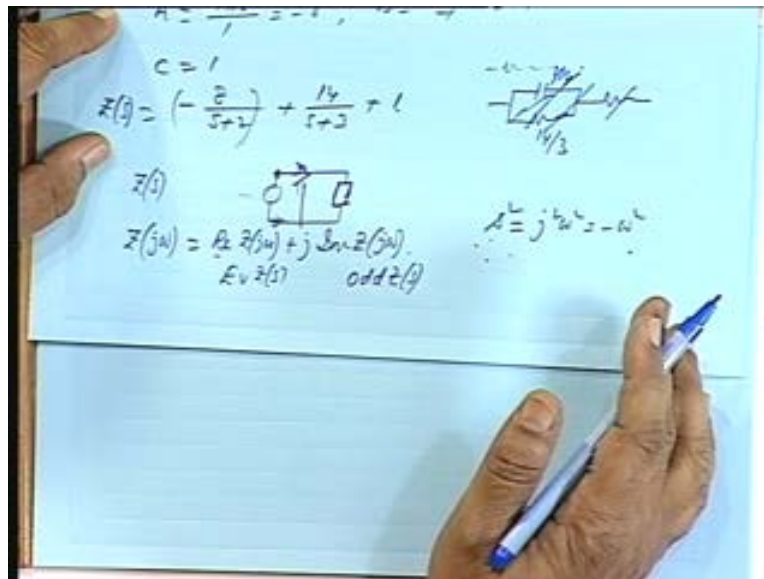
There is also a small diagram on the right side of the whiteboard showing a circuit with a voltage source  $V$ , a resistor  $R$ , and a capacitor  $C$  in series, with a current  $i$  flowing through it.

So how much is A if I make  $s$  plus 2 equal to 0 so this is minus 1 in to 8 divided by plus 1 so minus 8 okay B by  $s$  plus 3 if I multiply by  $s$  plus 3 this will be minus 2 minus 1. So it is 14 okay C if I make  $s$  10 in to infinity this will be 0 this will be 0 so it will be  $s$  squared by  $s$  square 1. So this thing this thing can be written as minus 8 by suppose this is  $z(s)$  can I break it up in to form like this  $s$  plus 2 plus 14 by  $s$  plus 3 plus 1 this is  $z(s)$  is it realizable this is having a negative coefficient I have seen we have seen earlier all RC combinations will give me positive terms here it is minus 8 divided by  $s$  plus 2.

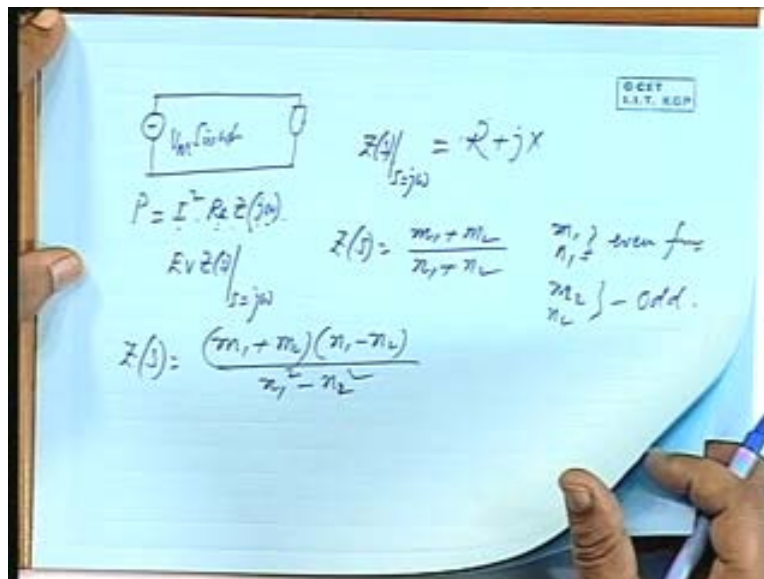
So this is not realizable I am bringing them to very small simple form okay, 14 by  $s$  plus 3 plus 1 this one I can realize as 1 ohm resistances 14 by  $s$  plus 3 I can realize by RC combination you put  $s$  tending to 0, so  $s$  tending to 0 will be blocking this capacitor so 14 by 3 is the resistance make  $s$  10 in to infinity it is 14 by  $s$  that means 1 by 14 farad so this is an RC combination this is a 1 ohm resistance but I cannot realize minus 8 by  $s$  plus 2, it is not possible to realize by RC elements okay. So I would like to know whether it is possible to realize this okay so  $z(s)$  if it is given to you it can be  $y/s$  also the general property we want to extract for  $z/s$  and  $y/s$  from a given function so that it is realizable I give a voltage an impulse voltage and then I withdraw it that means the source is returned here okay and impulse of currents say and then the current through this is allow to flow okay I have a an impulse of current and then it is shorted.

So that the current is maintained, how much will be the energy how much will be the energy of this if I apply if I apply any sinusoidal quanti sinusoidal input then what will be  $z/j\omega$  at any frequency I can put  $s$  equal to  $j\omega$   $z/j\omega$  will have a real part and an imaginary part okay. Now if I put  $s$  equal to  $j\omega$  if there are  $s$  squared terms it will be  $j$  squared  $\omega$  square that will be minus  $\omega$  square it will be real. So all the  $s$  squared  $s^4$  terms will be real and  $s$   $s$  cube terms will be imaginary.

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So real part will correspond to the even part even part of  $z(s)$  and the odd part will be if I put  $s$  equal to  $j\omega$  in the odd part I will straight away get the imaginary part, is that alright is that okay. Now I excite  $z(s)$  by a sinusoidal source  $v_m \sin \omega t$  any frequency then how much will be the power active power lost  $I^2$  in to real part of  $Z$  is it not that means  $z(s)$  at  $s$  equal to  $j\omega$  I can write as some  $R$  plus  $jX$  okay, so it is  $I^2$  in to this  $R$  which will be the power and that has to be always positive okay  $I^2$  is a positive quantity that means the

real part of this for any omega must be positive is that alright real part of  $z(s)$  at  $s = j\omega$  is nothing but even part of  $z(s)$  evaluated at  $s = j\omega$  is that okay.

So let us evaluate what is the even part so if you have  $z(s)$  equal to  $m_1$  plus  $m_2$  by  $n_1$  plus  $n_2$  where  $m_1$  and  $n_1$  are the even parts, even functions and  $n_1, n_2, m_2$  and  $n_2$  are the odd parts okay then  $z(s)$  can be written as  $m_1$  plus  $m_2$  I will multiply by  $n_1$  minus  $n_2$  both sides. So that denominator will become  $n_1$  square minus  $n_2$  square okay is that alright.

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$$P = I^2 R_e Z(s)$$

$$Ev Z(s) \Big|_{s=j\omega}$$

$$z(s) = \frac{m_1 + m_2}{n_1 + n_2}$$

$$z(s) = \frac{(m_1 + m_2)(n_1 - n_2)}{n_1^2 - n_2^2}$$

$$= \frac{(m_1 n_1 - m_2 n_2)}{n_1^2 - n_2^2} + \frac{m_2 n_1 - m_1 n_2}{n_1^2 - n_2^2}$$

$n_1$  } even f-  
 $n_2$  } - odd.

EV.  
 Odd.

so the numerator is  $m_1 n_1$  minus  $m_2 n_2$  this is even in to even minus odd in to odd is even, even in to even is even this is an even function odd square even square minus odd square will also be an even function so even function divided by another even function plus  $m_2 n_1$  minus  $m_1 n_2$  divided by  $n_1$  square minus  $n_2$  square. So this is the odd part this is the even part okay and these has to be tested for  $s$  equal to  $j\omega$  if I put here it must be always positive because that represents the resistive part that represents the power consume part is it not now  $n_1$  squared minus  $n_2$  square.

Let us see  $n_1$  plus  $n_2$  in to  $n_1$  minus  $n_2$ , so  $n_1$  plus  $n_2$  you say  $P$  plus  $JQ$  or some  $A$  plus  $JB$  capital  $A$  plus capital  $JB$  then  $n_1$  minus  $n_2$  will be  $A$  minus  $JB$  because it is the odd part which is becoming negative. So it will become minus  $JB$ , so it is only the imaginary part which will become minus is it not so  $A$  plus  $JB$  in to  $A$  minus  $JB$  is what  $s$  square plus  $b$  square that is always positive so the denominator is always positive, so I have to test the positiveness only of the numerator is it not the denominator is always positive is that alright. So let us take an example it will be clear the same example  $s$  plus 1 by  $s$  plus 10 divided by  $s$  plus 2 in to  $s$  plus 3 so this is  $z(s)$  I will write this as  $s$  squared plus 11  $s$  plus 10 divided by  $s$  square plus 5  $s$  plus 6.

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Handwritten derivation on a whiteboard:

$$z(s) = \frac{(s+1)(s+10)}{(s+2)(s+3)}$$

$$= \frac{s^2 + 11s + 10}{s^2 + 5s + 6} = \frac{(s^2 + 10) + 11s}{(s^2 + 6) + 5s}$$

$$m_1 = s^2 + 10, \quad n_1 = s^2 + 6$$

$$m_2 = 11s, \quad n_2 = 5s$$

$$m_1 n_2 - m_2 n_1$$

$$= (s^2 + 10)(5s + 6) - 55s^2$$

$$= s^4 + 16s^2 - 55s^2 + 60$$

$$= s^4 - 39s^2 + 60 \Rightarrow \omega^4 + 39\omega^2 + 60$$

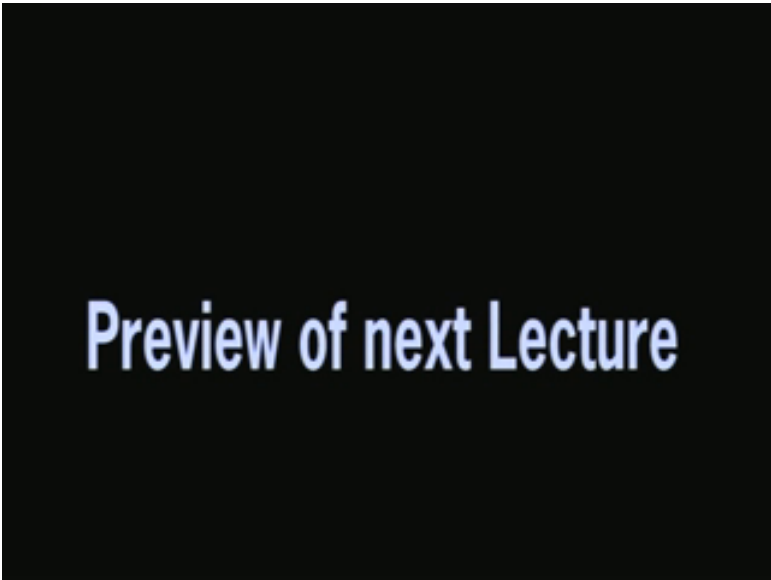
So what will be the even and odd part  $s^2 + 10$  is the even part plus  $11s$  this is the odd part similarly  $s^2 + 6 + 5s$  okay so  $m_1 n_2 - m_2 n_1$  is to be tested for positiveness because the denominator is always positive that is what we have ensure this square minus this square is always positive so  $m_1$  is  $s^2 + 10$   $n_1$  is  $s^2 + 6$   $m_2$  is  $11s$   $n_2$  is  $5s$ . So  $m_1 n_2 - m_2 n_1$  will become  $s^4 + 10s^2 + 60 - 55s^2$  okay. So that is  $s^4 + 10s^2 + 60 - 55s^2$  okay.

So that is  $s^4 - 39s^2 + 60$ . Now if you evaluate at  $s = j\omega$  what was this give you if I evaluate this at  $s = j\omega$   $\omega^4 + 39\omega^2 + 60$  is it always positive for all values of  $\omega$  yes, so it is realizable because power dissipated if it is negative what is it physically mean if the real part is negative that means if I pass the current instead of getting heated up it will be cooling down, is it possible  $I^2 R$  is negative means what it is cooling down that is not physically possible so the real part of the impedance function  $z(s)$  must be always positive so even though we have seen earlier for this example their residues are coming negative it may not be possible to realize by RC combinations. You may try with LC it may fail it may be possible with RLC alright.

So but it is a realizable function, so if you find that this test fails it is not positive throughout for all values of  $\omega$  somewhere it is becoming negative then you can say it is not realizable by passive elements this is not a network function is that alright. So will stop here for today will continue with this and a few simple realizations will see with LC, RC and so on.

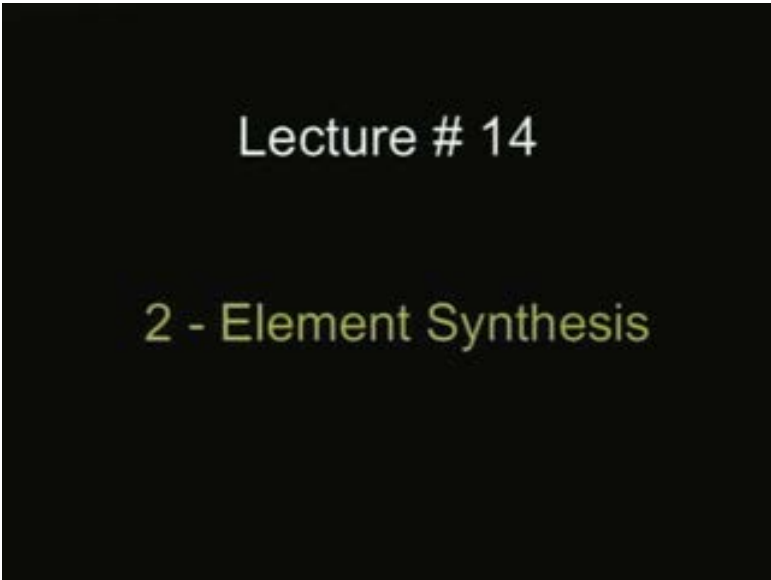


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Preview of next Lecture

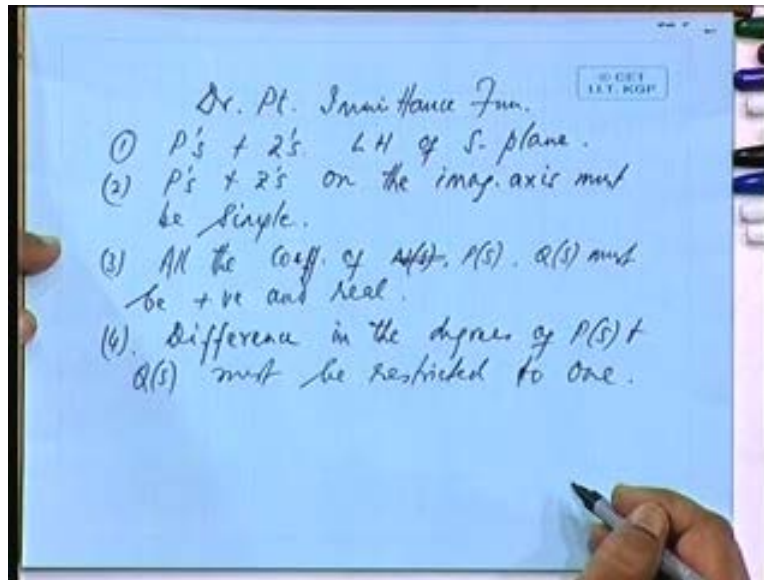
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Lecture # 14

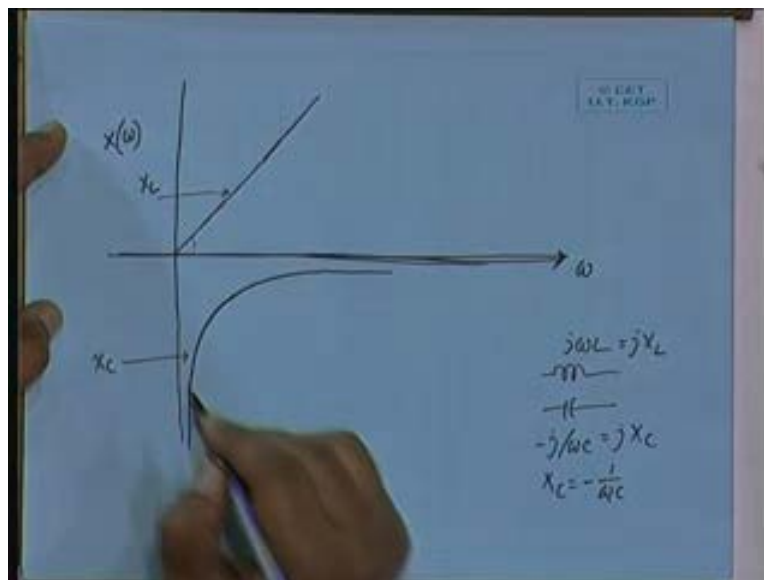
2 - Element Synthesis

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Good morning friends, we will be continue with driving point impedance function or immittance function. We have seen the properties to be satisfied are poles and zeros of the immittance function must lie in the left half plane, left half of s plane then poles and zeros on the imaginary axis must be simple on the imaginary axis must be simple.

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These are 2 very important properties and then to ensure that poles and zeros lie in the left half of s plane actually all the coefficients of the polynomials all the coefficients of numerator and denominator polynomials we were using p (s) and q (s) okay p (s) and q (s) must be positive all the coefficients of p (s) and q (s) must be positive okay and real then the difference in the degree difference in the degree of p (s) and q (s) the degrees of p (s) and q (s) must be restricted to one alright difference in the degrees must be restricted to one will draw a diagram. So this is omega because now we can take the frequency as the axis and corresponding value of x omega x omega how will x omega vary with omega. Let us go back to the first problem suppose, we have just an inductance what would be its x j omega L.

So if I write this as j x<sub>1</sub> j x<sub>1</sub> then x<sub>1</sub> will be varying linearly with frequency is that alright with a slope of L, with a slope of L. So this is x<sub>1</sub> what about x<sub>c</sub> if I have a capacitor then x<sub>c</sub> will be minus j by omega C is j x c so x c is minus one by omega C is that alright x<sub>c</sub> is always negative so it is inversely proportional to frequency it will be a rectangular hyperbola varying like this. So this is x<sub>c</sub> s square plus 1 suppose it is 2 s by s square plus 1 what does this mean if I make s tending to 0 it will be going to 2 s so when is the admittance becoming 2 s admittance is c s in case of admittances is c s is the admittance.

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$$Y(s) = \frac{s(s^2+2)(s^2+4)}{(s^2+1)(s^2+3)(s^2+5)}$$

$$= \frac{A_1 s}{s^2+1} + \frac{A_2 s}{s^2+3} + \frac{A_3 s}{s^2+5} + \cancel{\frac{A_4}{s}} + \cancel{\frac{A_5}{s}}$$

$$= Y_1 + Y_2 + Y_3$$

$$Y_1 = \frac{2s}{s^2+1}$$

$$Z_1 = \frac{s^2+1}{2s}$$

$$= \frac{1}{2} + \frac{1}{2s}$$

So it is two farads so it will be a two farad capacitance at that time at a very low frequency inductance is 0 it is a series combination you see you if Y<sub>1</sub> is this corresponding Z<sub>1</sub> is how much s squared plus 1 by 2 s that means s by 2 plus 1 by 2 s. So half Henry inductor and 2 farads capacitor will give me this impedance whose admittance is this so each admittance you just invert and then separate you will get the corresponding LC values is that alright.

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$$Z(s) = \frac{(s^2+1)(s^2+3)(s^2+5)}{s(s^2+2)(s^2+4)}$$

$$= \frac{K_1 s}{s^2+2} + \frac{K_2 s}{s^2+4} + \frac{K_3 s}{s} + \frac{K_4}{s}$$

The diagram shows a Foster-I realization of the transfer function. It consists of three parallel branches connected in series:

- Branch 1: A capacitor with value  $\frac{1}{K_4} F$  in series with an inductor with value  $K_3$ .
- Branch 2: A parallel combination of an inductor with value  $\frac{K_2}{4} H$  and a capacitor with value  $\frac{1}{K_2} F$ .
- Branch 3: A parallel combination of an inductor with value  $\frac{K_1}{L} H$  and a capacitor with value  $\frac{1}{K_1} F$ .

Below the main circuit, there are two alternative circuit diagrams showing different ways to connect the components.

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$$Y(s) = \frac{s(s^2+2)(s^2+4)}{(s^2+1)(s^2+3)(s^2+5)}$$

$$= \frac{A_1 s}{s^2+1} + \frac{A_2 s}{s^2+3} + \frac{A_3 s}{s^2+5} + \cancel{\frac{A_4}{s}} + \cancel{\frac{A_5}{s}}$$

$$= Y_1 + Y_2 + Y_3$$

The diagram shows a Foster-II realization of the transfer function. It consists of three parallel branches connected in series:

- Branch 1: A capacitor with value  $\frac{1}{2A}$  in series with an inductor with value  $\frac{1}{2}$ .
- Branch 2: A parallel combination of an inductor with value  $\frac{1}{2}$  and a capacitor with value  $\frac{1}{2}$ .
- Branch 3: A parallel combination of an inductor with value  $\frac{1}{2}$  and a capacitor with value  $\frac{1}{2}$ .

Below the main circuit, there are two alternative circuit diagrams showing different ways to connect the components.

$$Y_1 = \frac{2A}{s^2+1}$$

$$Y_2 = \frac{s^2+1}{s^2+3}$$

$$= \frac{s}{3} + \frac{1}{2s}$$

FOSTER-II

So similarly you get another LC combination another LC combination okay interesting it these values will be different from the foster realization, foster one realization but how many inductances were there 1, 2, 3 and 3 capacitances you go for this realization you will get the same number of elements type of elements will be of the same number 3 inductance is

3capacitances so if you go from impedance it is foster 1 if we start from admittance function this is foster 2 realization okay, we will stop here now we will continue with this in the next class.