Networks, Signals and Systems

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Lecture - 21

Graph Theory (Contd…)

Good morning friends, today we shall be discussing about computer generation of topological matrices.

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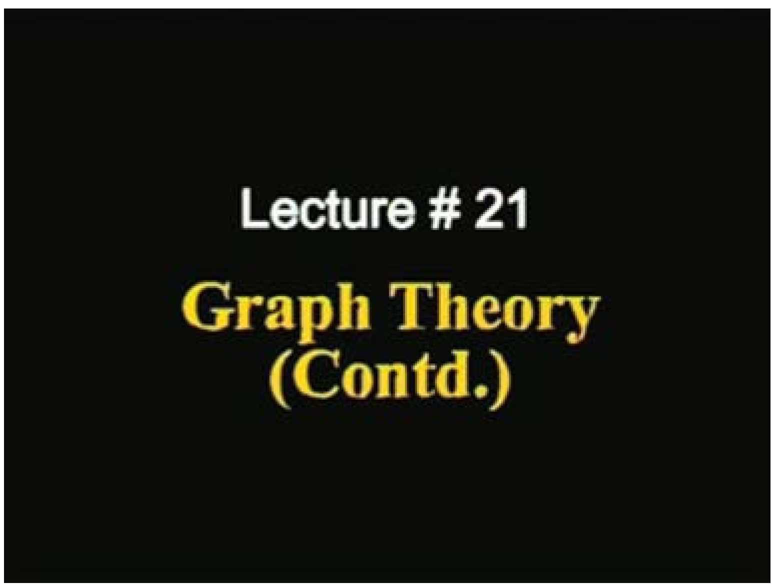

Now generation of the matrices b_f and b_f is quite a formidable task. Here see how to generate A, now A we started of with a 0 element matrix and then we filled it up with plus 1 or minus 1 depending on the interconnections between various elements. Now for q_f it is if you remember it is at inverse into a, a_t inverse into a and b_f was unity matrix and at inverse a_l okay sorry b_f transpose b_f transpose was minus of a_t inverse a_l and I unity matrix all right. So competition of this will require competition of at inverse. Now if you are having say 50 or 60 node system, 50 or 60 node system then it is it is not advisable to go for the inversion of such a matrix because a_t will have the dimension 50 by 50 or 60 by 60, all right. So if a_t equal to 50 by 50 then we can imagine the number of steps of computation and the time involved in the computation will be cohesive, computation time is very large.

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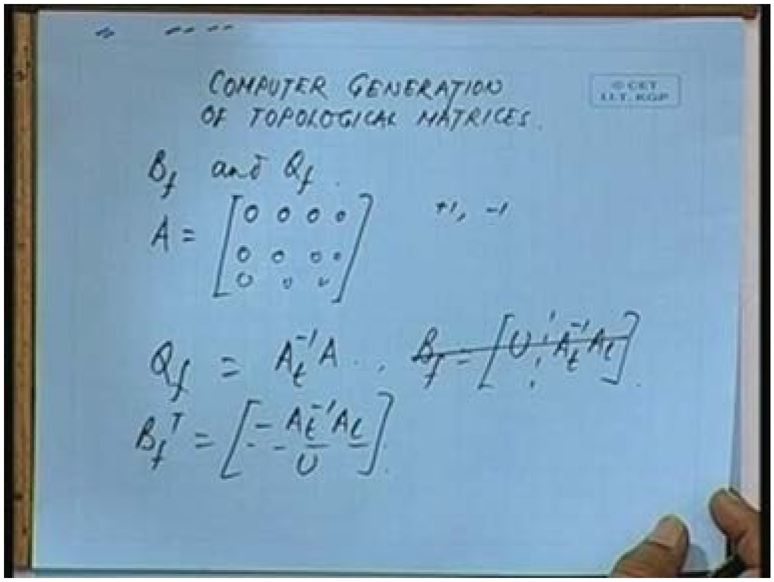

So we have already discussed about the 3 types of elementary matrices depending on the 3 types of operations, the operations are interchange of rows and then multiplication of a rows by minus 1 and then the third one is replacement of a row by the sum or difference with other rows. Now these 3 operations if you carry out, these 3 operations if you carry out on a unity matrix we get a elementary matrices.

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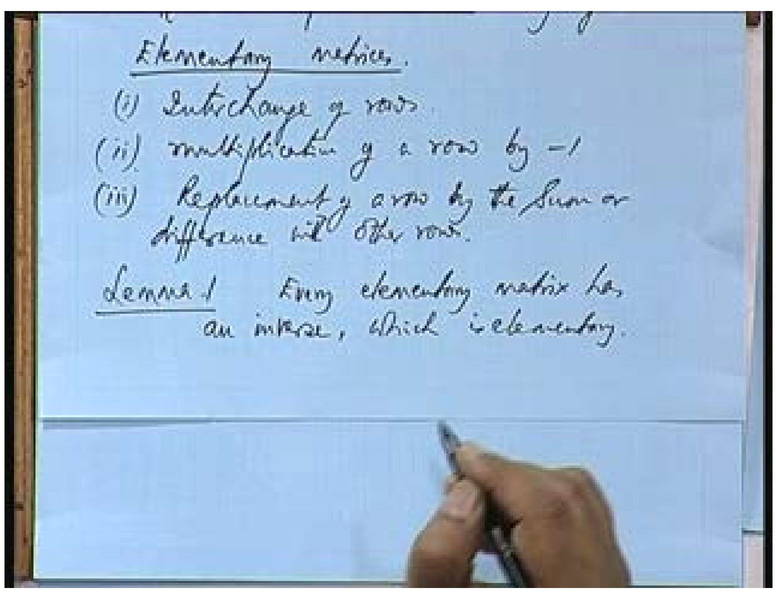

I will just make a mention of some of that standard properties lemma 1 of this elementary matrices that is every elementary has an inverse which is elementary then last time we discussed, I am just repeating the same point once again. The matrix obtained by performing an elementary operation on a matrix F is equal to the product E into F where E is obtained by performing, e is obtained by performing the same operation on the identity matrix that is U equal to 1, 0, 0, 0 like that. So if it is a 4 by 4 matrix then if we carry out a same operation on this unity matrix I will get an elementary matrix E, so if I multiply F by E, E into F will give me the same final product which I get by doing this operation on F directly. So we discussed about an example last time say F was taken as minus 1, minus 1, 0, 0, 1, minus 1, 1, 0, 0. If we take sorry, if we have sorry, F was taken as minus 1, minus 1, 0, 1, 0, 0, 0, 1and minus 1 and by na elementary operation that is if we interchange a row 2 and 3 we get this final product as F dashed.

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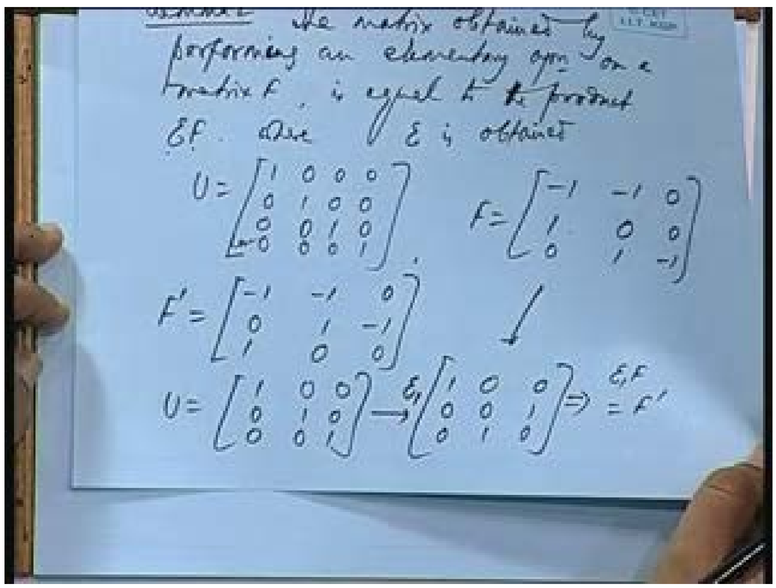

Now the same thing we can get if I do the same operation on the unity matrix, so a unity matrix is 1, 0, 0, 0, 1, 0, 0, 0, 1 if we interchange row 2 and 3, it will be 1, 0, 0, 0, 0, 1, 0, 1, 0. We can see if I multiply F by this elementary matrix E_1 then that will give me E_1 into F, you can verify yourself will give me the same F dashed. This property we shall be making use of when generating q_f and b_f from A. Let us have, let us have first of all a matrix A from the information’s given about the interconnections of networks. So we generate A and then by the equivalent transform we have identified say structure like this it may be 1, 2, 3, 4, 5 and so on. Suppose we find something like this the nodes, the elements which are connecting to the nodes 1, 2, 3, 4 and so on to form a tree will be corresponding to these elements that is 1 the it may be 3, it may be 4, so we may have a tree, a possible tree from these interconnections 1, 3, 4 and may be 5, there may be more number of nodes okay and so on.

So once you have identified the elements of a tree then we rearrange A such that we have 1, 3, 4, 5, these are the nodes, these are the elements corresponding to A_t and rest of it is put separately as a sub matrix A_l okay a means, small a means plus minus 1 or 0. So we get a non-singular matrix A_t which is a square matrix alright which is a square matrix and we will see from this square matrix, a square matrix which is having a structure like this 1, 1, 1, 1 like this these are all 0's and a, a, a, a, a, a and so on. So it is having all the diagonal elements as 1, 0's on the left and a's which may be plus minus 1 or 0 on the right.

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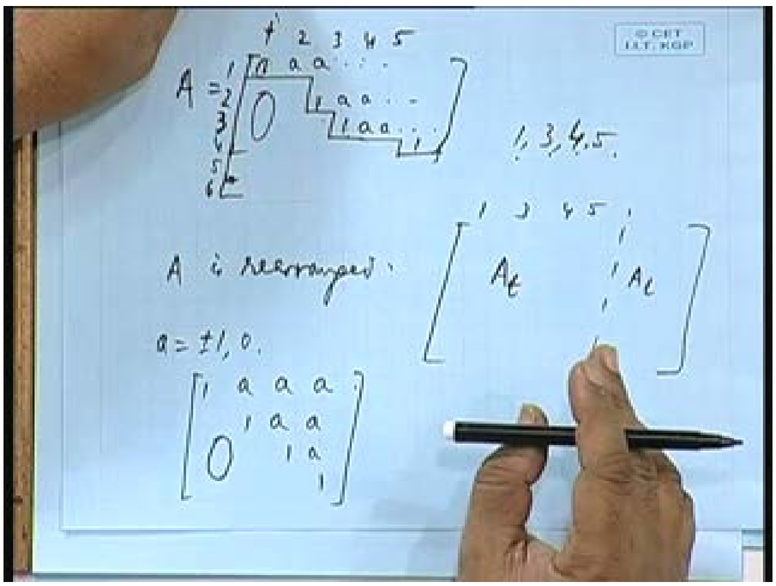

Now if this is our matrix A_t then we reduce, we reduce this matrix by unitary transform. Let us see we reduce it by an unitary transform to a unity matrix. You transform this to a unity matrix; let us see what we get. Let me start with an example first, let F be a matrix like this say minus 1, 1, 0, 1, 0, minus 1, 1, 0, minus 1, 0, 0, 0, 0, 0, 0, 1, is it possible to get an equivalent transform, is it possible to get a unity matrix here, let us see. We take the first one row 1 is negated then we get 1, minus 1, 0, minus 1, 0, minus 1, 1, 0, minus 1, 0, 0, 0, 0, 0, 0, 1, so R_1 replaced by minus R_1.

Then from here R_1 plus R_3 we substitute for R_3 then f_2 becomes R_1 plus R_3 will be replacing $R_{3,}$ minus 1, 0, minus 1, 0, minus 1, 1, 0 then this plus this is 0, this plus this is minus 1 then 0, minus 1 then 0, 0, 0, 1 okay then replace R_2, interchange R_2 and R_3, R_2 and R_3 are interchanged okay there will be a short cut. Let me negate R_2 first, R_2 is replaced by minus R_2 and then we replace R_3 by R_2 plus R_3 then what do you get.

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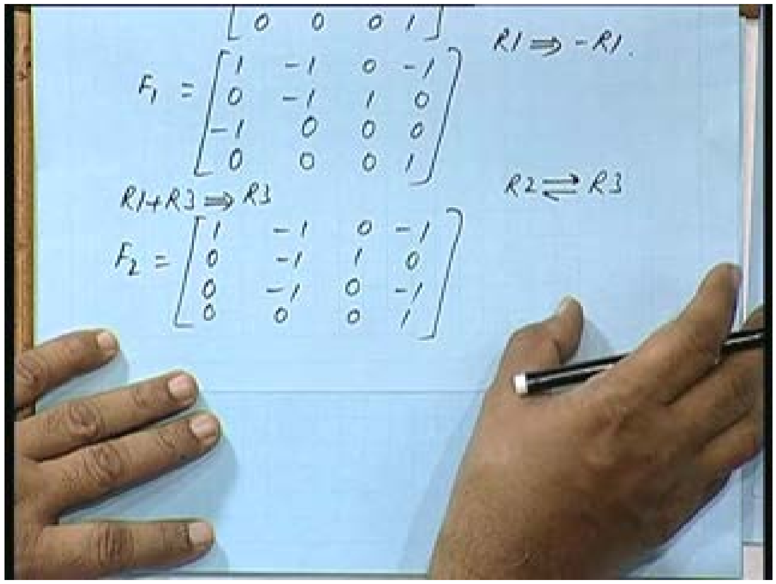

F_3 will be 1, minus 1, 0, minus 1, 0 see R_2 is replaced by minus R_2 so this will become 0, 1, minus 1, 0, 0, 1, minus 1, 0, and then this R_2, this R_2 plus this R_3 we replace this R_3 by this R_2 plus R_3. So this will become 0, 0, minus 1, minus 1 is it not 0 plus 0, 1 plus minus 1 is 0 then 0 and minus 1 is minus 1 and then minus 1 and 0 becomes minus 1 and the fourth one is 0, 0, 0, 1 this is F_3. Now I got almost the diagonal elements as unity if I just change the sign of this. So F_4 is 1, minus 1, 0, minus 1, 0, 1, minus 1, 0, 0, 0, 1, 1, then 0, 0, 0, 1. Now it is very simple if I add R1 plus R2 and replace this R1 by this, if I add these 2, I get F_5 as 1, 0, minus 1, minus 1 then 0, 1, minus 1, 0 then 0, 0, 1, 1, 0, 0, 0,1 okay then F_6 it is just a simple manipulation F_6 now if I add row 3 and row 4 so row 3 with row 1 so row 1 is replaced with row 1 plus row 3 then what we get 1, 0, 0, 0, then row 2 by row 2 plus row 3 then 0, 1, 0, 1 then 0, 0. 1, 1 then 0, 0, 0, 1 and then F_7 it is very simple R_2 minus R_4 if I take and replace R_2 by that then I will get this minus this, row 2 minus row 4 is going to replace this, so what we get 0, 1, 0, 0 similarly, row 3 minus row 4 is going to replace row 3 then what you get 0, 0, 1, 0 and then 0, 0, 0, 1, so I will get a unity matrix.

So through these equivalent transforms a square matrix which is a non-singular matrix, a square matrix can be reduced to a identity matrix, so these equivalent transforms. Therefore, we can write, it has write this as E_l, l minus 1, 1 minus 2 various steps of these transformation I will teach therefore, on F we have carried out these multiplications at different stages to obtain U and all these equivalent transforms they have their inverses. So F can be written as epsilon 1 inverse, epsilon 2 inverse, epsilon l inverse into U which means epsilon inverse, epsilon 2 inverse, epsilon l inverse.

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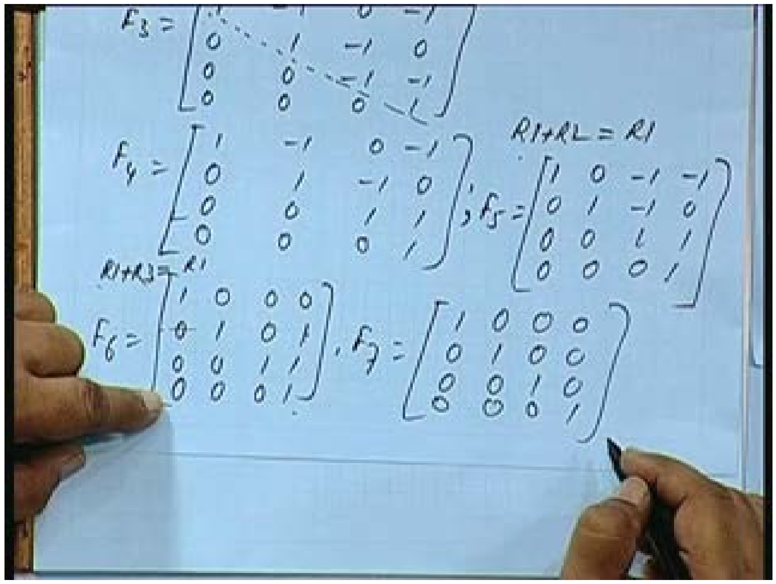

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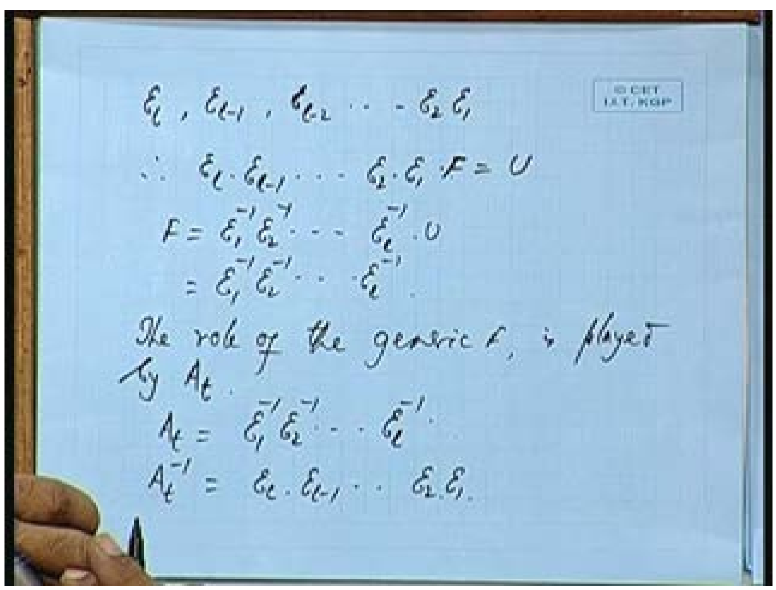

So the role of the generic F, the role of this generic F is to be played with A_t . Therefore, A_t will be like this or A_t inverse I can write as this, therefore what will be q, q_f is l, l inverse so on and so on E_2 and E_1 into A is it not because q_f we know At inverse A and A_t inverse is obtained by repeated operations like this elementary operations like this. So the repeated operations on A will give me q_f, A_t inverse can be considered as the product of such elementary operations, such elementary matrices okay.

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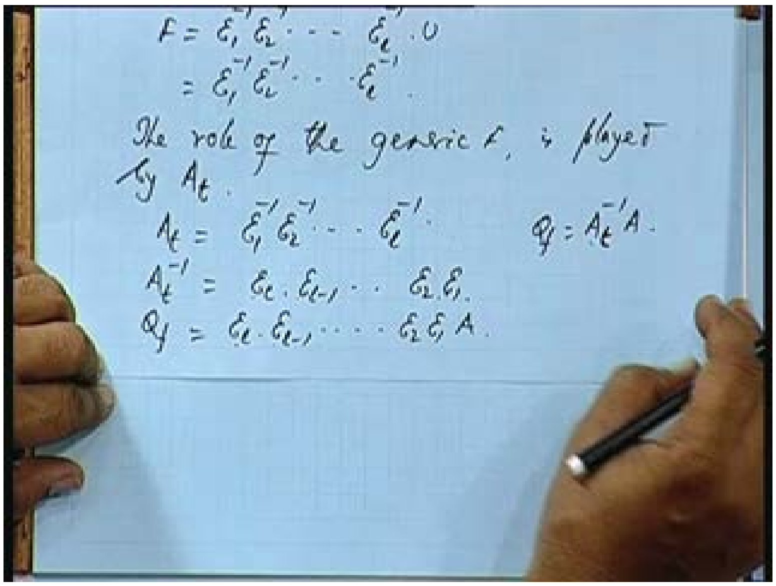

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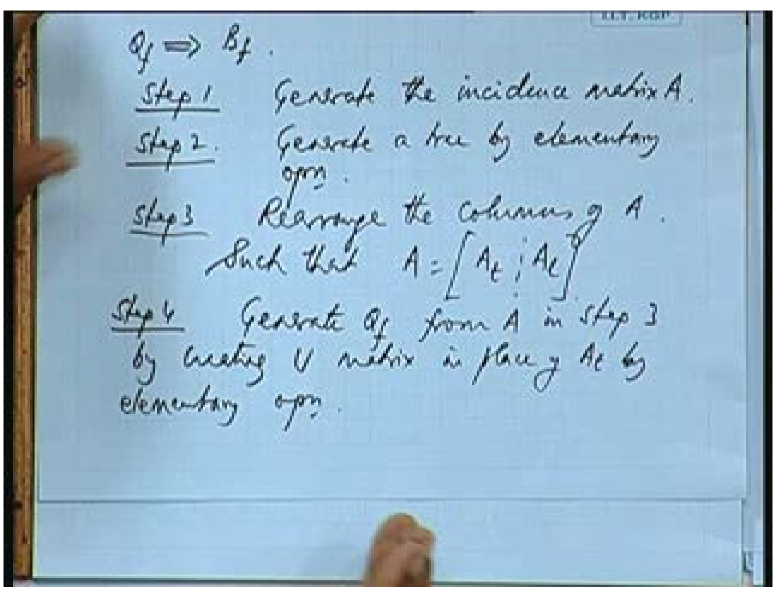

So let us use this and qf once qf is known you can calculate from their bf. So let us consolidate the steps, step 1 generate the incidence matrix A, generate any incidence matrix A from the given data then step 2, generate a key by elementary operations then step 3, rearrange a, rearrange the columns of A such that A equal to A_t portioned A_l up to this it is very simple and then on A_t we carry out this chain of elementary operation that is multiplication by various elementary operations. So step 4 generate qf from A in step 3, this A after rearrangement of the columns in step 3 by creating U, U matrix in this space. By creating U matrix in place of A_t by elementary operations okay I hope this point is clear. Once you have followed this q_f automatically becomes U, q_l, take this transform to q_f and once you have got q_f and next compute bf which is minus q_l transpose U.

So we shall take up an example, simple example and then see how to obtain q_f and b_f. Suppose the matrix A is given like this 1, 2, 3, 4, 5, 6, 7. 0, 1, 2, 3, 4, 0, 0, 1, 0, minus 1, 0, 1, 0, minus 1, minus 1, 1, 0, 0, 0, 0, 0, 0, minus 1, 0, minus 1, minus 1, minus 1, 1, 0, 0, 0, 1, 0. Suppose this is A, we interchange R_4 and R_1 and R_1 is multiplied by minus 1 what do you get A_1, remember you do not get the same matrix anymore it is a transformed matrix, once you go through these transformations A is gradually getting transformed to Q, so do not write A again. I have observed many of you while carrying out these operations you just put equal to this, this does not become equal to this. It is a new matrix that you obtain here so bear this in mind, so 1, 2, 3, 4, 1, 0, 0, 0, minus 1 because I have replaced 1 and 4 and then I have negated this 1, minus 1, 0, 0, 0, minus 1, 0 then 0, minus 1, minus 1, 1, 0, 0, 0, then 0, 0, 0, minus 1, 0, minus 1, minus 1 and then 0, 0, 1, 0, minus 1, 0, 1.

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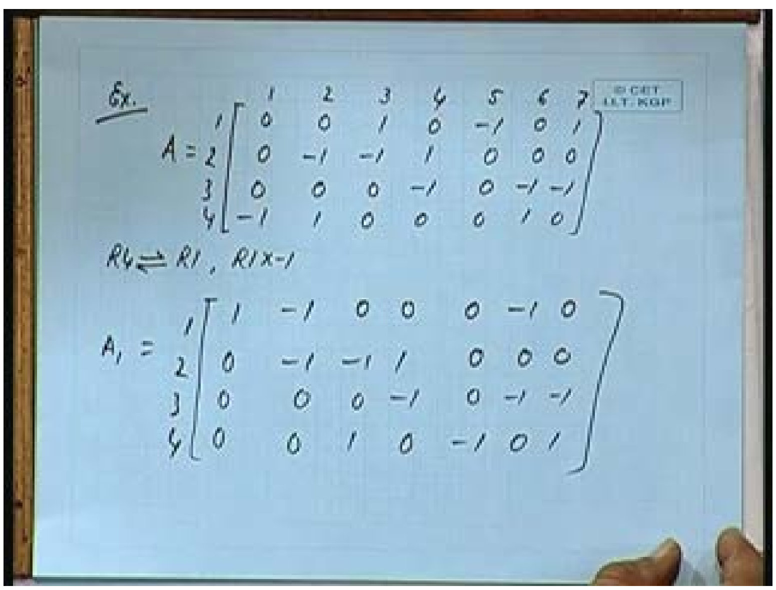

So this is the next matrix that you get then from here we multiply row 2 by minus 1, if I multiply row 2 by minus 1 this will become plus 1 okay this is, this will be plus 1, this will be minus 1, so I will get 1, 1 like this and then if we interchange R_3 and R_4 this one will come here and this will go here and then if I change the sign of this, so all these steps R_2 is multiplied by minus 1, R_3 and R_4 are interchanged and then in that new R_4 if I put a multiply minus 1 what will be the matrix like 1, minus 1, 0, 0, 0, minus 1, 0 then 0, 1, 1, minus 1, 0, 0, 0 then 0, 0, 1, 0,minus 1, 0, 1 and then 0, 0, 0, 1, 0, 1, 1. My interest was to first of all generate the unity elements here and then 0's to the left and then here it can be plus 1 or minus 1 or 0's this is my portioned part.

Now R_1 plus R_2, R_1 plus R_2 if we are adding this and put it in R_1, A_3 then becomes 1, 0, 1, minus 1, 0, minus 1, 0, o, 1, 1, minus 1, 0, 0, 0, 0, 0, 1, 0, minus 1, 0, 1, 0, 0, 0, 1, 0, 1, 1 okay then R_1 minus R_3 if I take R_1 minus R_3. So next step we replace R_1 minus R_3, R_1 minus R_3 is going to replace R_1 then R_2 minus R_3 is going to replace R_2 then what do I get, A_4 equal to this minus this it will be 1 this R_1 minus R_3 this minus this 0, 0, then minus 1 then 1, minus 1, minus 1 okay R_2 minus R_3, R_2 minus R_3 will gives me 0, 1, 0, minus 1, plus 1. 0, minus 1 then 0, 0, 1, 0, minus 1, 0, 1 and 0, 0, 0, 1, 0, 1, 1 okay, from here now you are gradually going to unity matrix only these 2 elements are to be changed. If I add row 4 with this 1 and minus 1 will make it 0, 1 and minus 1 will make it 0. So now next operation left is just to add R_4 to R_1 and R_2.

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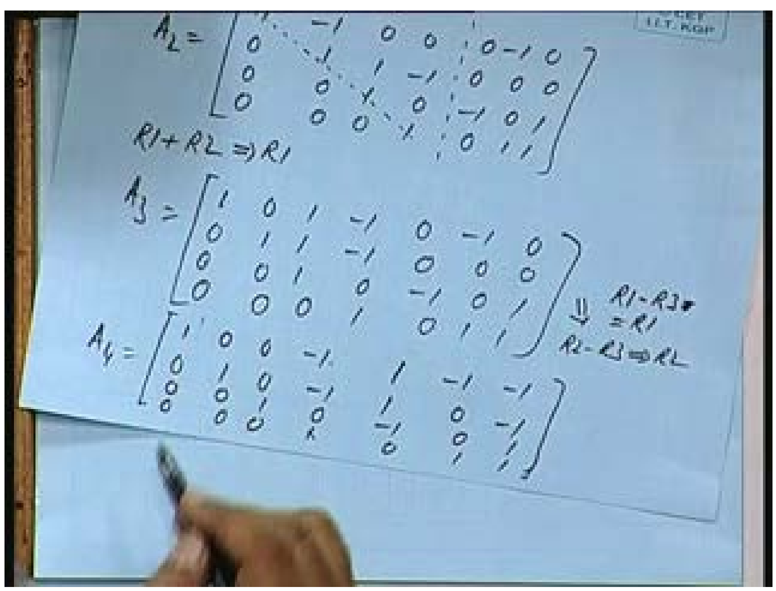

So R_1 plus R_4 replace R_1 and R_2 plus R_4 is going to replace R_2 then we get A_5 equal to 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, on this side we have got 1, 0, 0, 1, 1, 0, minus 1, 0, 1and 0, 1, 1 and what is this? This is q_f see you have obtained qf which has nothing but U and this side you have got q link and this is q link. So what will be b_f very simple minus transpose of this first and then U that means if I take transpose of this it will be 1, minus 1, plus 1 sorry 1, 1, minus 1, 0 and with a negative sign it will become minus 1, minus 1, 1, 0, then 0, minus 1, 0, minus 1, 0, minus 1, 0, minus 1, then 0, 0, minus 1, minus 1, 0, 0, minus 1, minus 1 okay and then unity matrix 1, 0, 0, 0, 1, 0, 0, 0, 1, so this is b_f . So it is so simple from starting from the A matrix we have got bf and qf by this elementary transformations. So if I given an A matrix first of all through an equivalent form you have to find out the possible tree if you find out a possible tree then again rearrange the columns get A. So that you can partition the A into A_t and A_l and A_t is a square matrix which has to be transformed to a unity matrix and automatically the righten part that is the part associated with A_l which also get transformed which will be generating q_l. So U partitioned q_l this will be the total matrix q_f okay.

We are given a problem, an interesting problem take up now for which of the following At or At inverse which of the following A_t or A_t inverse sub matrices, sub matrices correspond to a tree of a network okay tree of a graph with 4 nodes? Do you understand the question which of the following A_t or A_t inverse sub matrices, which of the following A_t or A_t inverse sub matrices correspond to a tree of graph with 4 nodes and the sub matrices are I will take up 1 by 1, you have to identify which can be a possible case, possible sub matrix of a tree 1, minus 1, 0, 1 next 1, 0, 0, 1, 1, 0, 0, 0, 1. Let us take up 1 at 2 at a time, is this a possible sub matrix obviously, the 4 node system should have At of dimension 3 by 3, so since this is of dimension 2 by 2 this is ruled out.

Again this one, let us take up this one this is a 3 by 3, see it matrix could have been a possible case but here you have find both the signs are plus in this column. Now if you remember in the A matrix in each column you will have 1 plus 1 and 1 minus 1 at the most you cannot have 2 plus 1 or 2 minus 1, so this is also ruled out.

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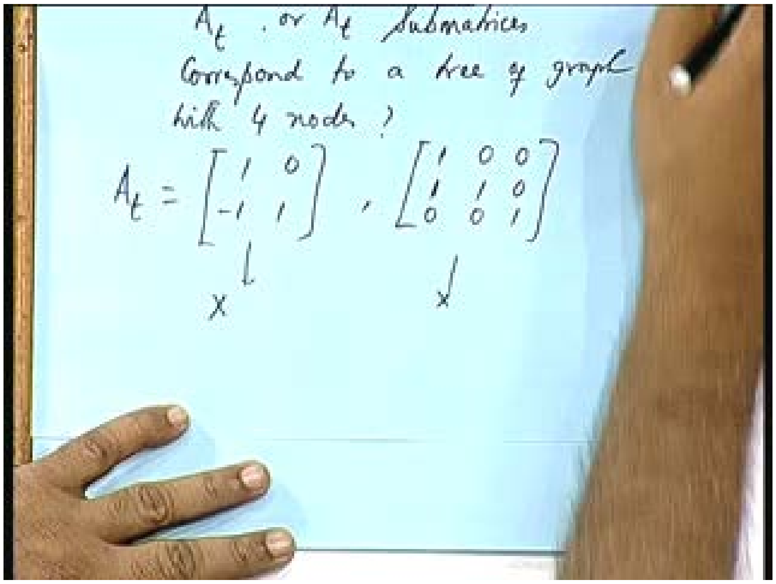

Let us take up one or two more cases, A_t equal to minus sorry, 1, 0, 0, minus 1, 1, 0, 0, 0, 1, is it possible to have, is it possible to have A_t as a sub matrix corresponding to this values, see the determinant is 1, determinant of A_t should be 1 or minus 1, so here that is satisfied it is 3 by 3 and you see that is no presence of 2 plus ones and 2 minus ones for all zeros. So this is a possible case this is a candidate. Let us take another example A_t minus 1 is equal to 1, 0, 0, minus 1, 1, 1, 0, 0, 0 is this possible for a sub matrix, what would be the determinant of this. There are 3 0's, so the determinant is 0, so the inverse of this that is A_t does not exist, A_t does not exist. So this cannot be a candidate okay. Let us take another matrix At equal to 1, 0, minus 1, 0, 0, 1, minus 1, 0, 0, 0, 1, 0, and 0, 0, 0, 1, can this form a 4 node, sub matrix of the 4 node system, these are 4 by 4 matrix and for a 4 node system it can be a 3 by 3 matrix, therefore this is also ruled out.

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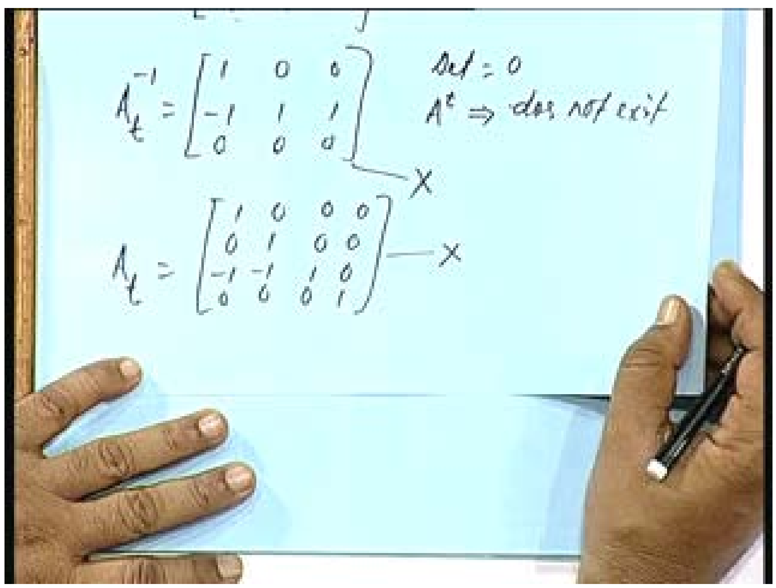

Next A_t inverse equal to minus 1, 0, 0, 1, minus 1, minus 1, 0, 0, 1, now here the determinant is 1, it is a 3 by 3 matrix all the elements are either plus 1 minus 1 or only minus 1 and so on, so this could be a possible case. Let us see what A_t will be like, A_t will be, determinant is plus 1, so we will replace this by the corresponding minus how much is that, this will be minus 1 this will be replaced by minus 1 this will be replaced by okay, this will be replaced by 0 then this will be replaced by 0, this will be replaced by 1 then this will be replaced by 0, this will be replaced by 0, this will be replaced by 1 and this will be replaced by 1, is that so and then the transpose of this. So that gives me minus 1, minus 1, 0, 0, 1, 0 then 0, 1, 1. Now once again you will find there are 2 similar signs in each column, so this is also ruled out okay.

Let us take another example A_t equal to 0, 0, minus 1, minus 1, 0, 0, 0, 1, 1, here is the determinant non-zero, yes minus 1, minus 1, 1, so the determinant is positive 1, it is a 3 by 3 matrix and all other conditions are satisfied, so this is also a candidate. So there are 2 situations one is this and the other one is this where A_t given by this can form a part of sub matrix, can form a part of a tree.

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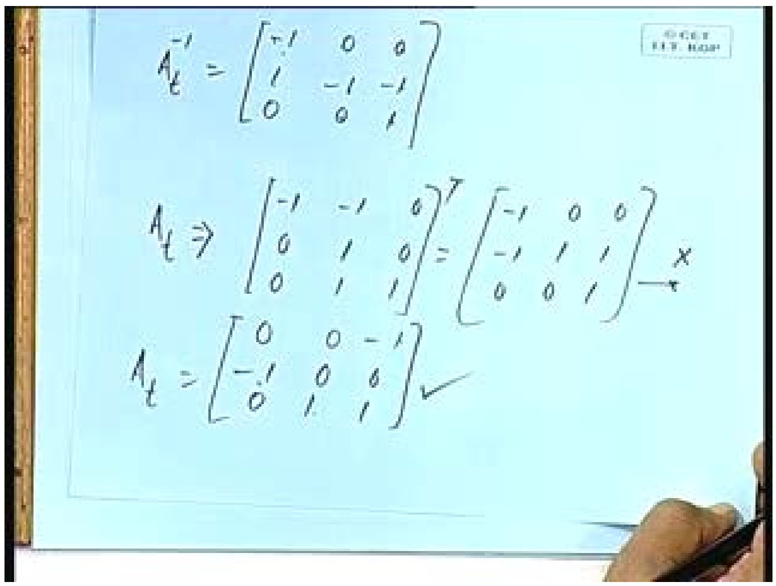

Another question, we will take up another example, the elements of a graph suppose this is given to you in a tabular form the branches I will just write down the problem, starting node and the finishing node. Branch 1 starting node 0, finishing node 1, simultaneously I will draw the graph; you are asked to draw the graph. So branch number 1, let us take this as the starting node 0 to 1, so put the arrow to the finishing node then branch number 2 is between 0 and 2, so between 0 and 2 so let us this is node 2, direction is like this then third one 0 and 5, so let us take 0 and 5, this is the third node 1, 2, 3, then fourth node is 1 to 2 fourth node is fourth branch is between 1 and 2. So the third is 0 and 5, the fourth element is between 1 and 2, the fifth element, the fifth element is between 1 and 3.

So fifth element is between 1 and 3, let us take node 3 here then 1 and 3, this is the fifth element then the sixth element is between 3 and 4, between 3 and 4, so between 3 and 4. So let us take this as the sixth element then seventh element is between 2 and 5, so between 2 and 5, seventh element, eighth element is between 4 and 5, between node 4 and 5, eighth element and the ninth element is between 3 and 5, between 3 and 5 okay so this is the graph taking 0 th node as the reference, taking 0 th node as the reference write A matrix in the ascending order that is you take the elements and the nodes in the proper order and then form a tree.

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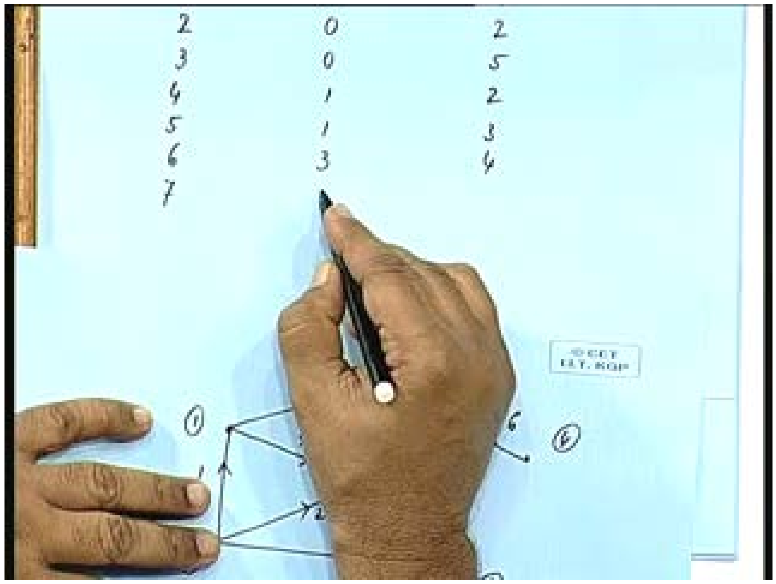

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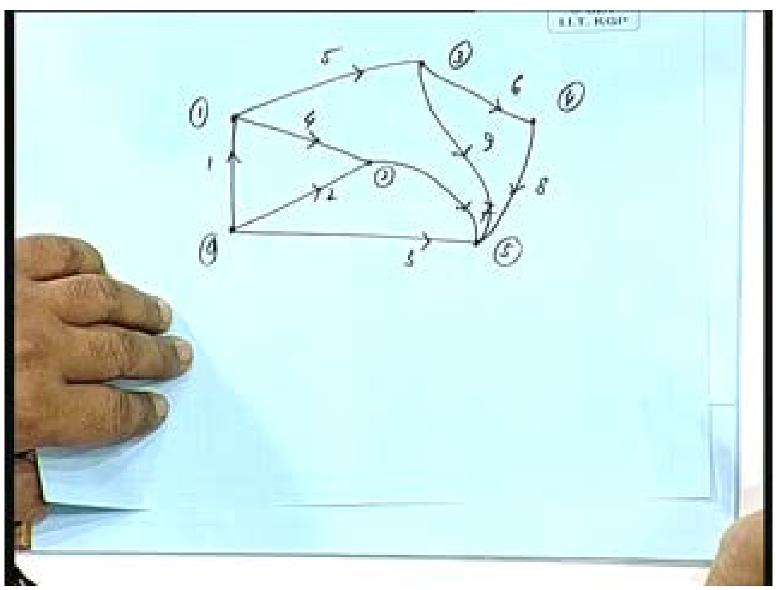

So let us write matrix A, we will take a first element 1, element 1 is between 0 and 1 so it is nodes and these are the elements and branches. This is minus 1 and the reference node is 0, so I am not showing that 0, 0, how many nodes are there 1, 2, 3, 4, 5 so 1, 2, 3, 4, 5 excluding 0 there are 5 nodes. Element 2, element 2 is between 0 and 2, so 0, minus 1, 0, 0, 0, is that all right. Next element 3 between 0 and 5, so element 3 is between 0 and 5, 0, 0, 0, 0, minus 1. Then element 4 if you see element 4 we have taken out 1, 2, and 3, element 4 if I put that forms a loop, so 4 has to be put in the link element. I am trying to form A_t going by order of the elements as they appear without forming a tree. So 4, element 4 will appear here then element 5 if I take, so the elements or the links I will put across, elements 5 comes next that is between 1 and 3, between 1 and 3, 0, 0, 0 okay so element 6, can I have element 6 now, yes element 6 that is between 3 and 4, 0, 0, 1, minus 1, 0 okay element 7, element 7 we can have between 3 and 5 but then if I put that then 1, 5 sorry element 7 if I put then 2, 3 and 7 will form a loop. So element 7 you cannot have here, it has to be put here.

Similarly, element 8 if I put that will also will form a loop, so element 8, 9, 7 they will all have to go there, so you will see 1, 2, 3, 4, 5, 1, 2, 3, 4, 5 cannot be more than 5 by 5. So 4, 7, 8, 9 I will put in this order, number 4 element number 4 was between 1 and 2, so 1 and 2 0, 0, 0, element 7, element 7 was between 2 and 5, so 0, 1, 0, 0, minus 1, element 8 was 4 and 5, 0, 0, 0, 1, minus 1 and element 9 is 0, 0, 1, 0, minus 1, so this is the matrix A. Now by transformations of these 2 unity matrices you can generate q and finally b matrix okay.

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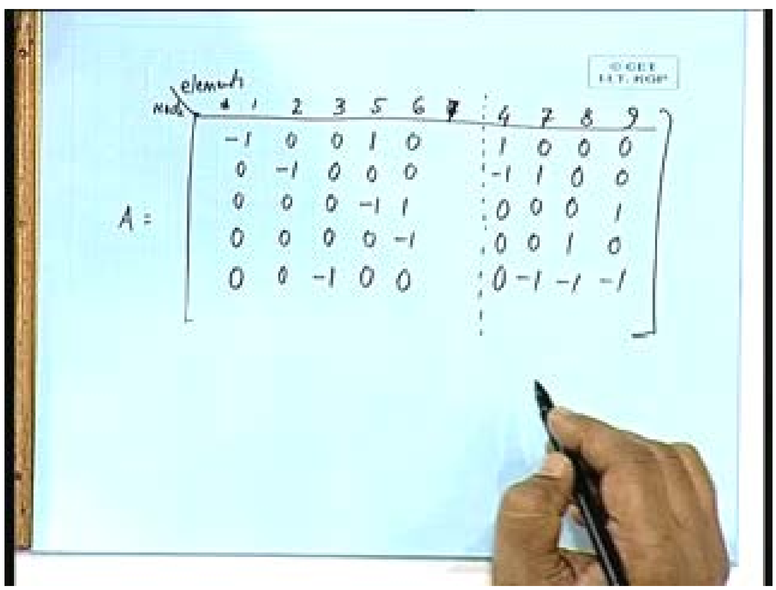

Next time we shall discuss about image impedence and iterative impedence and from there we will go to the characteristic impedence of symmetric network okay this will be in the introduction to the classical filters. So 2 port network whatever we have studied earlier you please brush up whatever you have learned. So that you can follow this, thank you very much for a patient hearing.

Preview of Next Lecture

Lecture - 22

Image Impedance, Iterative Impedance and Characteristic Impedance

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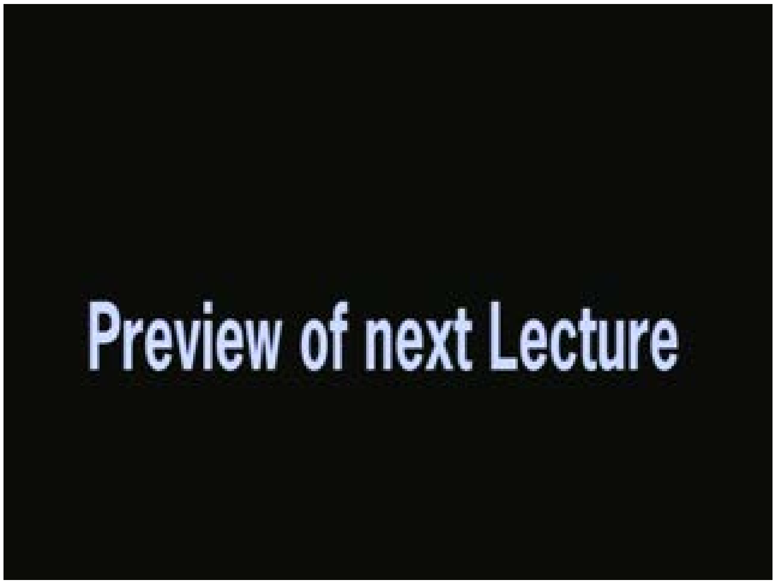

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Good morning friends, today we shall be discussing about image impedance, iterative impedance and finally the characteristic impedance of a 2 port network, characteristic impedance of a 2 port network. Now let us consider a general network having impedances Z_A, Z_B and Z_C the ports are 1 and 2. Now image impedance we define, we define image image impedance as this if I load this side by an impedance Z_{i2} the impedance in from this side is Z_{i1} and if I load the same network, I show the network just by a block if on this side if I put Z_{i1} then impedance in from this side is Z_{i2} then Z_{i1} and Z_{i2} will be the image impedances for this network. So for a general network where Z_A, Z_B, Z_C are any certain values Z_{i1} and Z_{i2} will be different so there are 2 image impedances you look into the circuit from this end if I load on that side Z_{i2} then the impedance in is Z_{i1}, if I look at the network from this end and if I load it with Z_{i1} the impedance in is Z_{i2} mind you you cannot have these unique values with any set, you cannot have any combination so they are dependent on these values are dependent on Z_A, Z_B and Z_C.

Let us what will be the relation between Z_{i1}, Z_{i2} and these element values Z_A, Z_B and Z_C. Now by definition you have got Z_{i1} the impedance in from this side is how much Z_A plus parallel combination of Z_C and Z_B plus Z_{i2}, Z_C in parallel with Z_B plus Z_{i2} agreed. So that gives me Z_A plus Z_C into Z_B plus Z_{i2} divided by Z_C plus Z_B plus Z_{i2} agreed. Similarly, Z_{i2} will be equal to Z_B plus Z_C into just replace a by b interchange a and b, Z_C will be Z_A plus Z_{i1} divided by Z_C plus Z_A plus Z_{i1} agreed.

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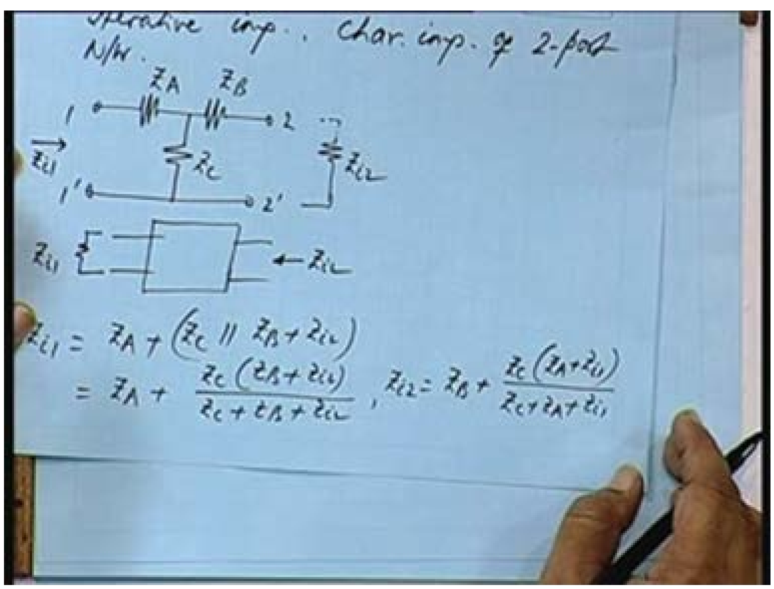

So let us take once again from definition Z_{t1} impedence in Z_{t1}, so Z_{t1} will be equal to Z_a plus parallel combination of Z_b plus Z_{t1} with Zc okay so Zt1 is equal to Za plus parallel combination Z_c with Z_b plus Z_{t1} which means Z_a plus Z_c into Z_b plus Z_{t1} divided by Z_c plus Z_b plus Z_{t1}. If multiply again cross multiplication will give me Z_{t1} squared plus Z_{t1} into Z_c plus Z_b from this side I have got Z_a, Z_c plus Z_b plus Z_a, Z_{t1} plus Z_c, Z_{t1} plus Z_c, Z_b. If I transform everything, if I transfer everything on this side it will get Z_{t1} squared plus Z_{t1} into Z_c plus Z_b minus Z_c minus Z_a, so it becomes Z_b , Z_c will go, so Z_b minus Z_a, correct me if I am wrong okay minus Z_a, Z_c, Z_a, Z_c minus Z_a, Z_b minus Z_c, Z_b equal to 0 or I can put all of them as plus okay that is Z_{t1} squared plus Z_{t1} into Z_b minus Z_a minus sigma Z_a, Z_b equal to 0.

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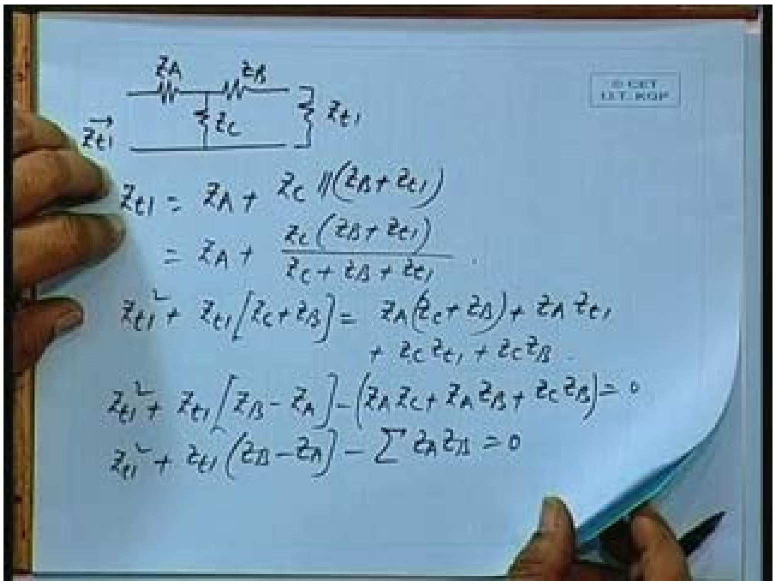

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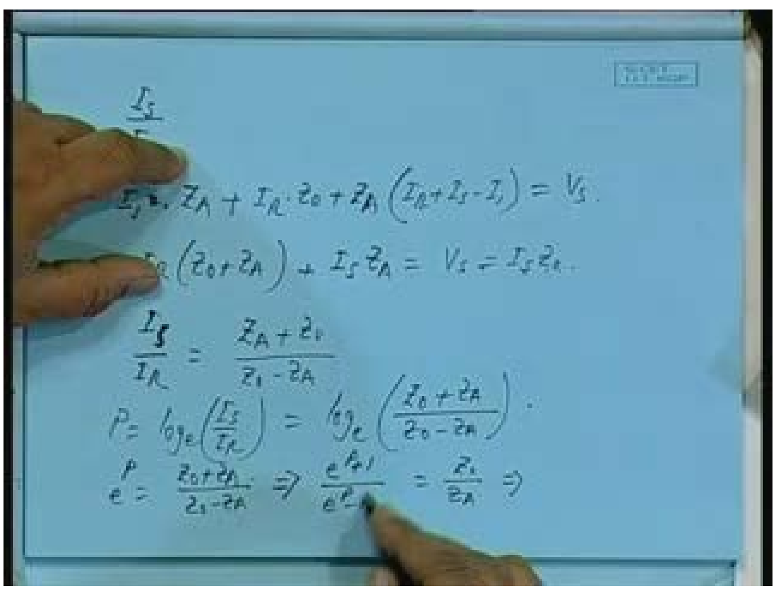

So how much is Z_{t1}, if I invert it so that will be giving me tan hyperbolic p by 2 is equal to Z_a by Z_0 or tan hyperbolic p by 2 is Z_a by Z_0, Z_0 is equal to root over of Z_a into Z_b so that gives me root over of Z_a by Z_b okay so we get Z_a is equal to Z_0 tan hyperbolic p by 2 on my right like this and Z_b is equal to cot hyperbolic p by 2 either you can write alright. Okay we will stop here for today, we will take up some problems in next class, thank you very much.