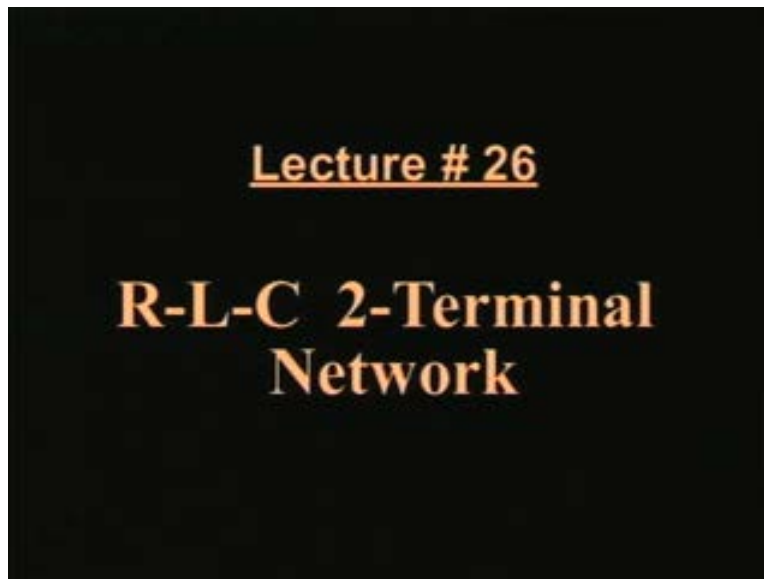


**Networks, Signals and Systems**  
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**Lecture - 26**  
**R-L-C 2-Terminal Network**

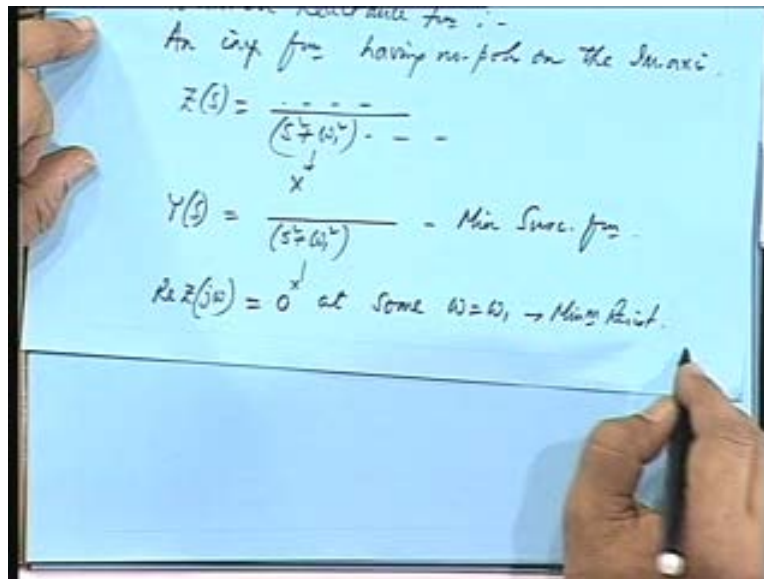
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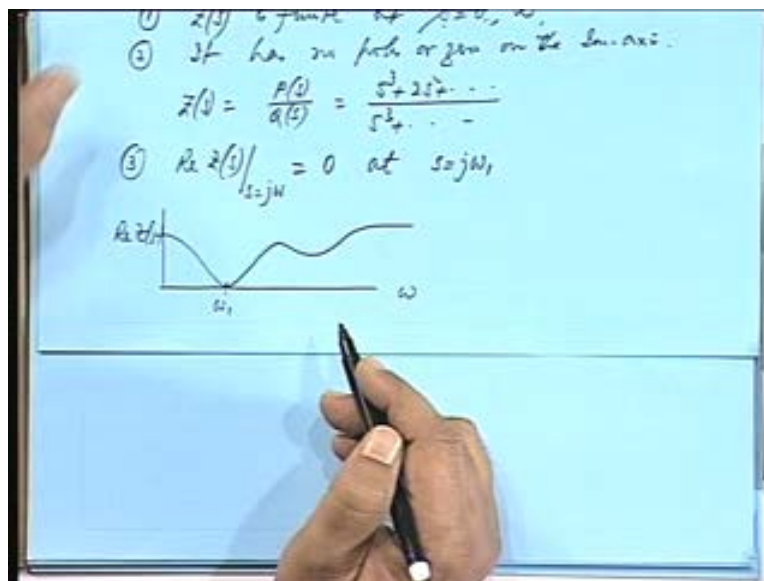
Good afternoon friends. Today we shall be discussing about synthesis of RLC network, when you say synthesis you are still restrict to ourselves to 2 terminal network that is 1 port network. Now before we go to the synthesis will define some functions special type of functions, minimum function, what is a minimum reactance function? First, we define a minimum reactance function. A minimum reactance function we get when an impedance function has got no poles on the imaginary axis that is an impedance function having no poles on the imaginary axis that means in  $Z(s)$  factors of this type will be ruled out, this type of factor there are other factors here this type of factor is not present then in it is a minimum reactance function.

Similarly, if you have in the admittance function factors of this type absent that is there are no poles on the imaginary axis for the admittance function then we call it minimum susceptance function okay. If we have the real part of  $Z_j$  omega equal to 0 at some frequency at some omega equal to omega 1 then we call it minimum resistance function okay, minimum resistance function.

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Similarly, if you have real part of  $Y_j$  omega equal to 0 at some omega equal to omega 1 we call it minimum conductance function, minimum conductance function. If a function is simultaneously, minimum reactance, minimum susceptance, minimum resistance minimum conductance, then we call it a minimum function.

Now there are certain specific properties of the minimum functions, we will just mention about the essential features of minimum function the first one is say  $Z(s)$  is finite at  $s$

equal to 0 and infinity at  $s$  equal to 0 and infinity, it has no poles or 0s on the imaginary axis, no poles or 0s on the imaginary axis because it is finite at 0 and infinity obviously if I write  $Z(s)$  equal to  $P(s)$  by  $Q(s)$  both of them will be of the same order, same degree, say this is  $S$  cubed plus twice  $S$  square plus something this also has to be  $S$  cubed if it is  $S$  to the power 4, if it is of higher degree then  $S$  tending to infinity at  $s$  equal to infinity this will be 0 and  $Y(s)$  will tend to infinity. So similarly if this is of higher order at  $s$  equal to 0 this will tend to 0 but at  $S$  equal to  $Y(s)$  at  $s$  equal to 0 will tend to infinity.

So it will be of the same degree these 2 polynomials then thirdly a real part of  $Z(s)$  is equal to 0 at some  $\omega$ , say  $\omega = 1$  that means if I sketch the real part may be like this. So at some  $\omega = 1$  this will touch 0 value what does it physically mean that means the real part shows the active power consumed I square into the real part is that active power consumed at some frequency that active power will be 0 for a minimum function.

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The image shows a hand holding a white marker writing on a whiteboard. The equations are as follows:

$$\begin{aligned}
 \operatorname{Re} z(j\omega) &= \frac{(2\omega^2+1)(2\omega^2+4) - (2\omega)^2}{(2\omega^2+4)^2 - (2\omega)^2} \\
 &= \frac{4\omega^4 - 8\omega^2 + 4}{4\omega^4 - 12\omega^2 + 16} = \frac{(\omega^2-1)^2}{(\omega^2-3\omega^2+4)} \\
 &= \frac{\omega^4 - 2\omega^2 + 1}{\omega^4 - 3\omega^2 + 4} \\
 \operatorname{Re} z(j\omega) &= 0 \text{ at } \omega^2 = 1, \omega = 1 \\
 z(j1) &= \frac{2(-1) + j1 + 1}{+2j^2 + 4} = \frac{-1 + j}{2 + 2j} = \frac{\sqrt{2} / 135^\circ}{2\sqrt{2} / 45^\circ} \\
 &= \frac{1}{2} j
 \end{aligned}$$

Let us take an example, let  $Z(s)$  be given by say twice  $s$  squared plus  $s$  plus 1 by twice  $s$  square plus  $2s$  plus 4 then what would be a real part of  $Z_j$   $\omega$  we take  $m_1, m_2$  minus  $n_1 n_2$  if you remember twice  $s$  squared plus 1 is the even part multiplied by the even part twice  $s$  squared plus 4 minus the odd parts product divided by even square, even part squared for the denominator minus odd part square okay.

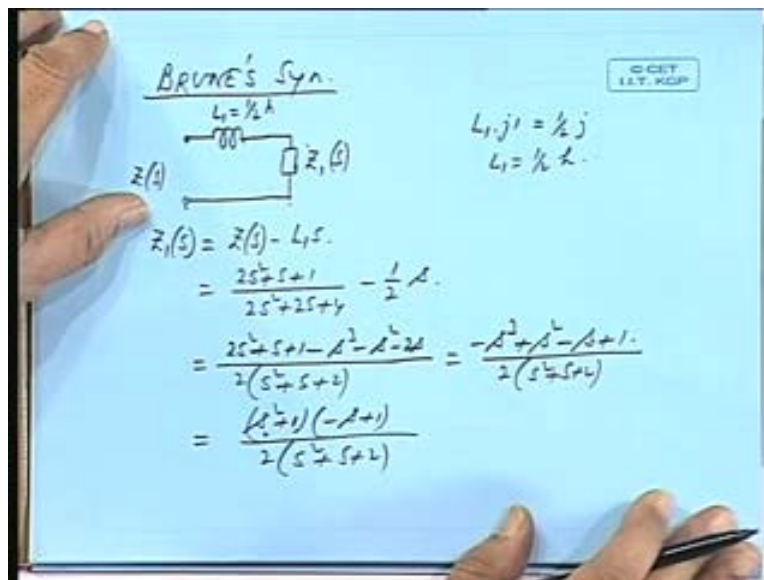
So that gives me twice  $s$  square plus into twice  $s$  square 4  $s$  to the power 4, so 4  $\omega$  to the power 4 okay 4 2's are 8 plus 2, 10 minus 2, so plus 8  $s$  squared means minus 8  $\omega$  squared plus 4 divided by 4  $\omega$  4 twice  $s$  squared so 4  $s$  to the power 4, 4  $\omega$  4, 4 2's are 8 minus 4 so sorry, 4 2's are 8, 2's are 16 minus 4, so plus 12  $s$  squared that gives me minus 12  $\omega$  squared plus 4 4's are 16.

So this can be written as  $4\omega^4 - 4\omega^2 + 1$  can be taken out so  $\omega^4 - 2\omega^2 + 1$  by  $\omega^4 - 4\omega^2 + 4$ . So clearly you see this is equal to  $(\omega^2 - 1)^2$ , so this will vanish at  $\omega^2 = 1$  okay.

So when  $\omega^2 = 1$  real part will be 0, real  $z_j$   $\omega = 1$  at  $\omega^2 = 1$  that is  $\omega = 1$ , what would be  $Z(s)$  like then then  $Z$  at  $\omega = 1$  how much is this. Let us see  $2 - 1 + j_1 + 1$  divided by  $-2 + 2j_1 + 1$  so  $2 - 1 + j_1 + 1$  is  $2 + j_1$  plus 4 correct if I am wrong so  $-2 + 2j_1 + 1$  is  $-1 + 2j_1$  that is magnitude of  $\sqrt{2}$  and angle of  $135^\circ$  is it not and  $2 + j_1$  is  $2\sqrt{2}$  and an angle of  $45^\circ$  so  $135^\circ - 45^\circ$  so that gives me  $90^\circ$ .

So  $j_1$ , so that gives me  $1 + 2j_1$ , so it is purely reactive the impedance function is purely reactive that was what we expected also the real part will vanish there is no real part at this frequency now this will be the starting point of a realization.

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Now we shall follow first the method suggested by Brune, Brune's synthesis starts from this point that means  $Z(s)$  I can conceive as at that particular frequency it is half  $j$  is that okay so this is plus half  $j$  at  $\omega = 1$ , so if this is the reactance can we suggest an inductor inductor value which will give me half  $j$  as the reactance value at  $\omega = 1$ . So let us call that as  $L_1$ ,  $L_1$  into  $j_1$  is equal to half  $j$   $\omega$  so  $L_1$  is half Henry. So let us take a half Henry inductor out of  $Z(s)$ ,  $Z(s)$  I am trying to realize as a reactance reactance which is inductance half of half Henry plus another reactance say  $Z_1(s)$  how

much is  $Z_1(s)$  therefore  $Z_1(s)$  will be  $Z(s)$  minus this  $L_1(s)$  which is  $Z(s)$  is twice  $S$  squared plus  $S$  plus 1 divided by twice  $S$  squared plus  $2S$  plus 4 minus half  $s$  is that okay

Now at this point you just see what would be the property of  $Z_1(s)$  what would be the value of  $Z_1(s)$  at  $s$  equal to  $j_1$  that is at  $\omega$  equal to 1 what would be the value of  $Z_1(s)$   $Z(s)$  is purely reactive half  $j$  and this is also half  $j$ . So this will be 0 at that frequency so  $Z_1(s)$  let us work it out let us see what it gives if you write it this is  $S$  squared plus  $S$  plus 2 okay simplify this twice  $S$  squared plus  $S$  plus 1 minus twice  $S$  cubed minus  $s$  cubed  $s$  into  $s$  cube minus  $s$  square minus twice  $s$  okay.

So that gives me twice  $s$  and  $s$  that will give me minus  $s$ , minus  $s$  cube plus  $s$  square minus  $s$  plus 1 which gives me finally you can factorize this  $s$  cubed and  $s$  if you take together that will give me  $s$  square plus 1 as a factor, so minus  $s$  plus 1 okay. So obviously  $s$  square plus 1 this factor at  $s$  equal to  $j$   $\omega$  at  $s$  equal to  $j$   $\omega$  [Noise]  $s$  square plus 1 will be equal to 0 okay so that was what we expected  $Z_1(s)$  will vanish and that particular frequency how do you remove this particular 0 then, you create a pole if you remember in a earlier synthesis you create a pole and then remove it this becomes a 0 at that particular frequency  $s$  equal to  $j$

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$L_1 = \frac{1}{2} s$   
 $Z(s) = \frac{2s^2 + s + 1}{2s^2 + 2s + 4}$   
 $Z_1(s) = Z(s) - L_1 s$   
 $= \frac{2s^2 + s + 1}{2s^2 + 2s + 4} - \frac{1}{2} s$   
 $= \frac{2s^2 + s + 1 - s^2 - s^2 - 2s}{2(s^2 + s + 2)} = \frac{-s^2 + s - 1}{2(s^2 + s + 2)}$   
 $= \frac{(s-1)(-s-1)}{2(s^2 + s + 2)} \quad Y_1(s) = \frac{2(s^2 + s + 2)}{(s-1)(1-s)}$   
 $L_1(j1) = \frac{1}{2} j$   
 $L_1 = \frac{1}{2} s$

So we take the corresponding admittance  $Y_1(s)$  as 2 into  $S$  squared plus  $S$  plus 2 divided by  $s$  square plus 1 into 1 minus  $s$  at this moment let us not bother about the sign it does not represent a positive real function at this moment but we will take care of that in course of time we will see what it means.

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$$Y_1(s) = \frac{K_1 s}{s^2+1} + \frac{K_2}{1-s}$$

$$K_1 = \frac{2(1-s)}{2(s^2+1)} \Big|_{s=1} = \frac{2(1-1)}{2(1+1)} = \frac{0}{4} = 0$$

$$K_2 = \frac{2}{2} = 1$$

$$Y_1(s) = \frac{2s}{s^2+1} + \frac{1}{1-s}$$

$$= Y_2 + Y_3$$

$$Z_2 = \frac{2(s^2+1)}{2s} = \frac{s^2+1}{s} = \frac{s^2}{s} + \frac{1}{s}$$

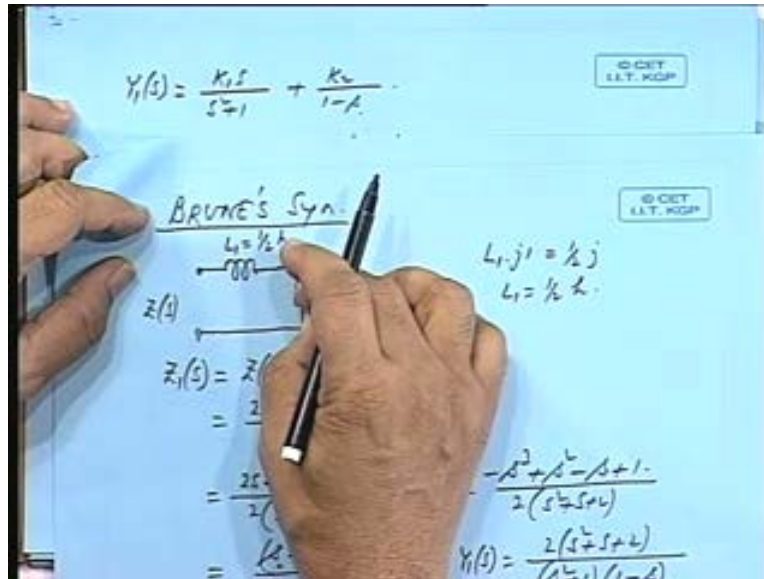
So  $Y_1(s)$  can be I can make partial fractions written as  $K_1(s)$  by  $S$  square plus 1 plus some  $K_2$  by 1 minus  $S$ , let us see what  $K_2$  will represent how much is  $K_1$ ,  $K_1$  comes out as let us calculate the value of  $K_1$  multiply by  $s$  square plus 1 make  $s$  square plus 1 equal to 0, so that give me 1 plus  $s$  and divided by  $s$  divided by  $s$ , so  $s$  minus  $s$  square so  $s$  minus  $s$  square and  $s$  square is equal to minus 1 so that gives me 1 plus  $s$  divided by  $s$  plus 1, so that is equal to 1, there is a 2 here know. So there will be a 2 here, so it is half.

So  $K_2$  similarly if you multiply by 1 minus  $s$  and then make 1 minus  $s$  equal to 0 that is  $s$  equal to 1 so this will be 1 plus 1, 2 plus 2, 4 into 2, 8 divided by 2 that is equal to 4. So you can write  $Y_1 S$  as  $s$  by 2 into  $s$  square plus 1 plus 4 by 1 minus  $s$ . So what does it mean I have got first  $L_1$  removed then I have got a  $Y_1$  and a  $Y_2$  sorry these I will call it  $Y_2$  plus  $Y_3$  this whole thing is  $Y_1$ , so this is  $Y_2$  this is  $Y_3$  okay.

So what is corresponding  $Z_2$  this shunt term will be just inverse of this 2 into  $S$  squared plus 1 by  $s$  that gives me twice  $s$ , check the value of  $K_1$ ,  $K_1$  I worked out 2 into  $s$  squared plus  $s$  plus 1 how much is this is  $Z_1$ , how much is  $Y_{12}$  into  $s$  square plus  $s$  plus 2 by  $s$  square plus 1 is that so I forgot to put the values correctly  $K_1$  is equal to if you remember  $K_1$  I multiplied by  $s$  square plus 1 make  $s$  square plus 1 equal to 0 so 2 should be in the numerator. So it will be sorry 2 I was by mistake I was looking at  $Z_1 S$  so this will be 2, so this will be 2  $s$  so  $S$  by 2 plus 1 by 2  $S$  that means this branch will be nothing but an inductor and a capacitor okay. So this is half Henry this is 2 Farads and this was also half Henry okay, this is what we got.

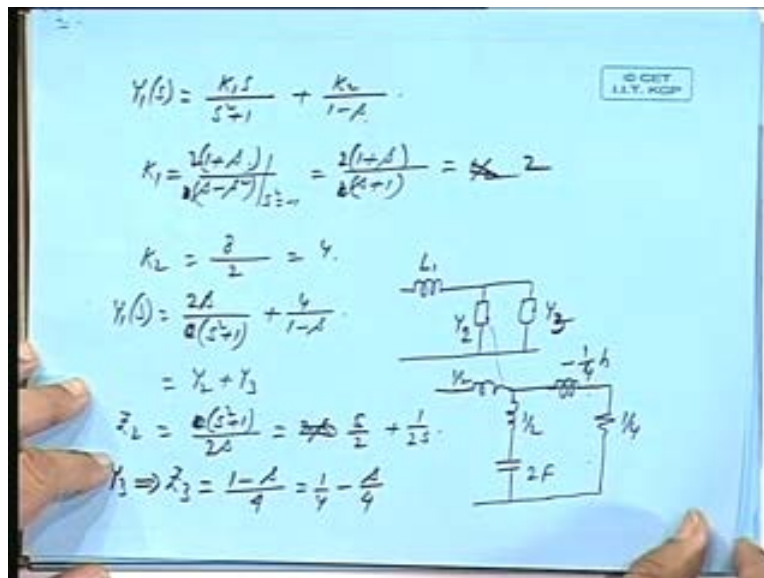


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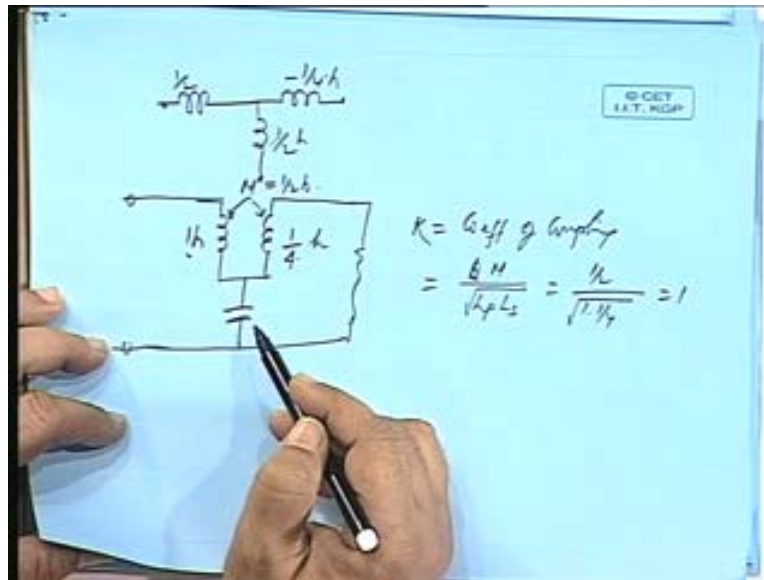
Now obviously whatever be the part here you see this will start resonating at that particular frequency  $s$  equal to  $j_1$  that is at  $\omega$  equal to 1 this will resonate, so that will be a short. So the entire impedance  $Z_1(s)$  that is what we are saying  $Z_1(s)$  will be 0, so  $Z(s)$  the impedance values of  $Z(s)$  which was half  $j$  is the impedance of this itself, this impedance is 0 that time because it is resonating at that frequency.

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So  $Z_1(s)$  is having a 0 at that point and that is been realized like this what is this  $Y_3$ , so from there let us see  $Z_3$  what will be  $Z_3$ ,  $1$  minus  $s$  by  $4$  so that is  $1$  by  $4$  ohm resistance minus  $s$  by  $4$  that is I am having an inductance of minus  $1$  by  $4$  Henry and a resistance of  $1$  by  $4$  ohms resistance okay, is that all right what is this minus  $1$  by  $4$  Henry, how do you realize minus  $1$  by  $4$  Henry?

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Now Brune suggested you have an inductor of half Henry another inductor in the form of a  $t$ , half Henry and this  $1$  is minus  $1$  by  $4$  Henry okay these  $3$  can be replaced by a transformer an ideal transformer where the primary inductance is these keep it open. So that will give me the primary inductance half plus half  $1$  Henry okay then this side half and minus  $1$  by  $4$  half minus  $1$  by  $4$  is  $1$  by  $4$  Henry. Add a mutually inductance of half Henry and then have the capacitor, resistor, how do you realize this what is the coefficient of coupling that is equal to  $m$  by root over of  $L_P, L_S$  or  $L_1, L_2$  and that is here half Henry divided by  $1$  into  $1$  by  $4$  that is equal to  $1$ .

So if you are having an ideal coupling,  $2$  transformer coils are ideally coupled and they are having the values  $1$  Henry and  $\frac{1}{4}$  Henry then you can realize this network. So connect the  $2$  transformer terminals here and then connect the capacitor to this end and the other ends are taken out like this. So this is the configuration of a Brune's network where a negative inductance has been incorporated in the coupled coils. So if one of the terms is negative do not get frightened it can be realized I could have got right in the beginning in instead of plus  $j_x$  at that frequency where the resistance part the resistive part was vanishing the real part was vanishing at that time the impedance of  $Z(s)$  where we got half  $j$  could have been negative.



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BRUNE'S Syn.

$L_1 = \frac{1}{2} s$

$Z_1(s) = Z(s) - L_1 s$

$$= \frac{2s^2 + s + 1}{2s^2 + 2s + 4} - \frac{1}{2} s$$

$$= \frac{2s^2 + s + 1 - s^2 - 2s - 2}{2(s^2 + s + 2)}$$

$$= \frac{(s^2 - s - 1)(-s + 1)}{2(s^2 + s + 2)}$$

$L_1 j\omega = \frac{1}{2} j$   
 $L_1 = \frac{1}{2} s$

$s^2 + s + 2 = (s + \beta)(s + \alpha)$   
 $\beta + \alpha = -1$   
 $\alpha\beta = 2$

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$$= \frac{4\omega^4 - 8\omega^2 + 4}{4\omega^4 - 12\omega^2 + 16}$$

$$= \frac{\omega^4 - 2\omega^2 + 1}{\omega^4 - 3\omega^2 + 4} = \frac{(\omega^2 - 1)^2}{\omega^4 - 3\omega^2 + 4}$$

$R_2 Z(j\omega) = 0$  at  $\omega^2 = 1$ ,  $\omega = 1$

$$Z(j\omega) = \frac{2(-1) + j + 1}{-2 + 2j + 4} = \frac{-1 + j}{2 + 2} = \frac{\sqrt{2} \angle 135^\circ}{2\sqrt{2} \angle 45^\circ}$$

$$= \frac{1}{2} j$$


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$$= Y_2 + Y_3$$

$$Z_2 = \frac{0(s^2 + 1)}{2s} = \frac{s}{2}$$

$$Y_3 \Rightarrow Z_3 = \frac{1 - s}{4} = \frac{1}{4} - \frac{s}{4}$$

Then we will start off with a negative inductor a minus half j it will not try to realize it by a capacitor you can try it at home if you try to realize it with a capacitor, what kind of difficulties you face later on, you can try with another example, we have many examples from in a many problems given in the book or we will take up in the tutorial class while realizing some of them we may find a negative sign here.

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$$= \frac{K_1 s}{s+1} + \frac{K_2}{1-s}$$

$$K_1 = \frac{2(1-s)}{2(1-s)} \Big|_{s=-1} = \frac{2(1-1)}{2(-1-1)} = \frac{0}{-4} = 0$$

$$K_2 = \frac{2}{2} = 1$$

$$Y(s) = \frac{2s}{s(s+1)} + \frac{1}{1-s}$$

$$= Y_2 + Y_3$$

$$Z_2 = \frac{s(s+1)}{2s} = \frac{s+1}{2} = \frac{s}{2} + \frac{1}{2}$$

$$Y_3 \Rightarrow Z_3 = \frac{1-s}{1} = 1-s$$

The circuit diagram shows an input terminal with voltage  $-100t$  connected to a network of components. A resistor  $Y_2$  is in parallel with a series combination of an inductor  $L_1$  and a resistor  $Y_3$ . This network is connected to a node with voltage  $-\frac{1}{4}t$ . From this node, a resistor  $Z_2$  is connected to ground, and a resistor  $Z_3$  is connected to a terminal with voltage  $\frac{1}{4}t$ .

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$$R(s) = \frac{KZ(s) - sZ(s)}{KZ(s) - sZ(s)} \text{ is also P.r.f.}$$
 Richard's fun.
 
$$Z(s) = \frac{KZ(s)R(s) + sZ(s)}{K + sR(s)}$$

$$= \frac{KZ(s)R(s)}{K + sR(s)} + \frac{sZ(s)}{K + sR(s)}$$

So there also the same logic we got. Now there is another method of realizing this that is Bott and Duffin synthesis we shall be discussing about the synthesis techniques and then we will take up a large number of problems in the tutorial class. So Bott and Duffin synthesis is a very interesting synthesis technique before we go to that we define another interesting positive real function Richard function which is that if  $Z(s)$  is a positive real function then for positive real values of  $K$  you take any value of  $K$  positive and real

values of K, K into Z(s) minus s into Z(k) divided by KZ(k) minus s Z(s) is also a p r f, R(s) is also a positive real function, this is called Richard function.

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$$\begin{aligned}
 Z(s) &= \frac{kz(k)R(s) + sZ(k)}{k + sR(s)} \\
 &= \frac{kz(k)R(s)}{k + sR(s)} + \frac{sZ(k)}{k + sR(s)} \\
 &= \frac{1}{\frac{k}{z(k)R(s)} + \frac{s}{kz(k)}} + \frac{1}{\frac{sR(s)}{k} + \frac{R(s)}{z(k)}}
 \end{aligned}$$

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$$\begin{aligned}
 Z_2(s) &= \frac{R(s)}{Z(k)} & Z_L &= \frac{sZ(s)}{k} \\
 \frac{Z(k)}{k} &= \frac{1 \times (0)}{0.1} = 0.1 = \frac{1}{10} & \frac{Z(k)}{k} &= \frac{2k^2 + k + 1}{(2k^2 + 2k + 4)k} = \frac{1}{2} \\
 \\ 
 \frac{Z(k)}{k} &= \frac{1}{2} & Z(k) &= \frac{1}{2} \cdot k = \frac{k}{2} \\
 \\ 
 R(s) &= \frac{k \cdot Z(s) - sZ(k)}{kZ(k) - sZ(s)} = \frac{1 \cdot Z(s) - s \cdot \frac{1}{2}}{\frac{1}{2} - sZ(s)} \\
 &= \frac{2s^2 + s + 1}{2s^2 + 2s + 4} - \frac{s}{2} \\
 &= \frac{1}{2} - \frac{s \cdot (2s^2 + s + 1)}{2s^2 + 2s + 4}
 \end{aligned}$$

See if R(s) is positive real then let us extract Z(s) from here write Z(s) in terms of R(s) you multiply the denominator by R(s) and then take Z(s) terms on one side so Z(s) comes out as KZ(k) into R(s) plus s Z(k) divided by s Z(s) R(s) and K Z(s) it will be K plus s R(s) okay. Now we can break it up into 2 parts KZ(k), R(s)divided by K plus s R(s) plus s

$Z(k)$  by  $K$  plus  $s$   $R(s)$  okay. If you divide the denominator by the numerator then  $K$  by  $KZ(K)$  so I can write this as  $1$  by  $Z(k)$  into  $R(s)$  plus  $s$  by  $KZ(k)$ ,  $R(s)$  will get cancelled plus similarly here  $K$  by  $SZ(k)$  plus  $s$  will get cancelled  $R(s)$  by  $Z(k)$ , correct me if I am wrong. Now let us see what will be the structure of this impedance function this is  $1$  by  $2$  admittances, so this is admittance, this is an admittance. So I can write like this what is  $1$  by  $KZ(k) R(s)$ , so the first  $1$  can be written as  $1$  by  $1$  by  $Z_1$ , so  $Z_1(s)$  plus what is this admittance see  $Z(S)$  is  $1$  by this admittance this is  $1$   $Z_1$ , this is  $Z_2$  and  $Z_1$  I am writing as some of  $2$  admittances inverse of that. So this is an admittance of what a capacitive element so a capacitor of value  $1$  by  $KZ(k)$  Farads is that all right.

Similarly, this one  $K$  by  $SZ(k)$  so this can be written as  $1$  by  $Z(l)$  plus  $1$  by  $Z_2(s)$  where  $Z(l)$  will be this inductance. So  $Z(k)$  by  $K$  Henry that will give me  $K$  by  $SZ(k)$  as the admittance, is it not and  $1$  by  $Z_2$  means this will be  $Z_2$  okay where,  $Z_2$  means I will write  $Z_1(s)$  means  $Z(k)$  into  $R(s)$ ,  $Z_2(s)$  equal to  $Z_2(s)$  is  $R(s)$  by  $Z(k)$  okay,  $Z_c$  we have already seen  $Z_c$  is  $KZ(k)$  by  $s$  and  $Z(l)$  is  $s$  into  $Z(k)$  by  $K$  okay  $s$  into  $Z(k)$  by  $K$ .

So let us start from this point where we stopped at we initiated for Brune's synthesis, we start from there for Bott Bott Duffin synthesis also that is  $Z(k)$  by  $K$  is that inductance value for  $ZL_2$ , so if it is positive, if it is positive then  $Z(k)$  by  $K$  we take as that value of  $L$ , what was that value  $j_x$  at  $\omega = 1$  by  $\omega = 1$  okay. If you remember we will not put  $j$  because that will be divided by  $j \omega = 1$ , this was taken as  $L_1$  is it not. So we chose that value of  $L_1$  as  $Z(k)$  by  $K$  because  $Z(k)$  by  $K$  will give me that inductance value okay, if it is positive, if it is positive then we will start realization from here otherwise we will start from here.

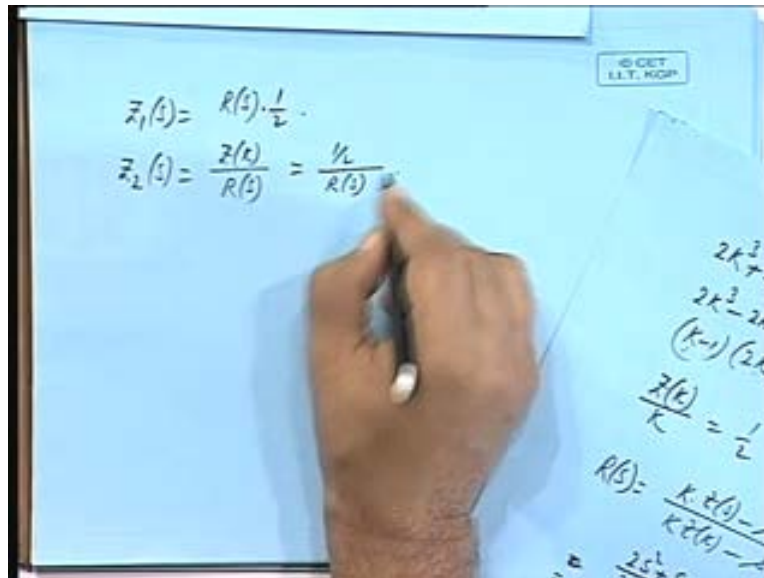
So  $x \omega = 1$  by  $\omega = 1$  is  $L_1$ , so that gives me  $K$  if you get say half  $L_1$  is equal to in the last example we got half then solve for  $K$ . So let us take up that same example we will try from there how much was  $L$  half that was equal to half, so how much is  $Z(k)$  by  $K$ ,  $Z(k)$  will be  $K$  twice  $K$  square the polynomial was twice  $s$  square plus  $s$  plus  $1$  divided by twice  $s$  squared see I am putting in place of  $s$ , I am putting  $K$ , so twice  $K$  squared plus twice  $K$  plus  $4$  all right this is  $Z(k)$ , so  $Z(k)$  by  $K$  so this divided by  $L$  is equal to half okay. So  $Z(k)$  by  $K$  equal to this divided by  $K$  and that has to be equated to half all right.

So let us solve twice  $K$  cubed plus twice  $K$  squared plus  $4$   $K$  equal to  $2$  into  $2$   $K$  square  $4$   $K$  square plus  $2$   $K$  plus  $2$  okay. So let us bring all of them to this side twice  $K$  cubed, twice  $K$  square minus  $4$   $K$  square, so minus twice  $K$  squared plus  $2$   $K$  minus  $2$  equal to  $0$ . So  $K$  minus  $1$  gets common to  $2$   $K$  squared plus  $2$  okay equal to  $0$ , one root will be imaginary, the other one will be  $K$  equal to  $1$  so we will take the real value of  $K$  that is  $K$  equal to  $1$  so once you know  $K$  equal to  $1$  how much is  $Z(k)$ ,  $Z(k)$  by  $K$  is equal to half, so  $Z(k)$   $K$  equal to  $1$ .

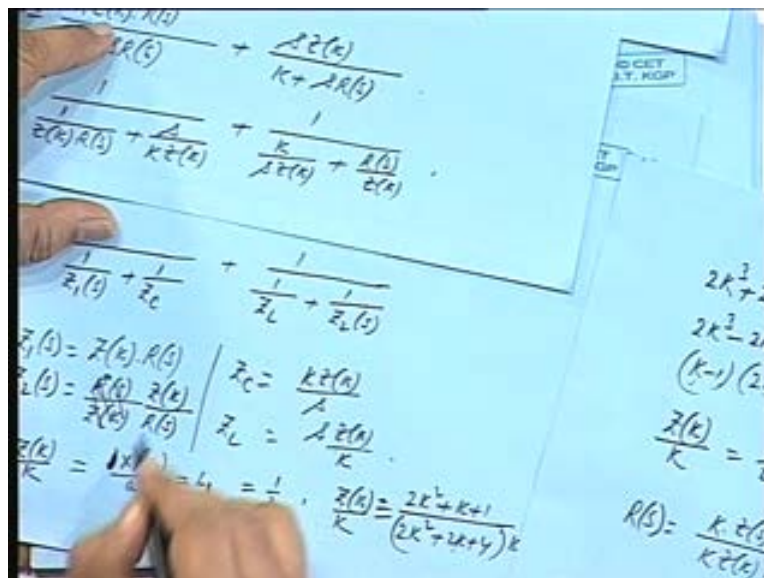
So  $Z(k)$  equal to half okay so if you know  $Z(k)$  is equal to half  $K$  is equal to  $1$  calculate the Richard function because we need for calculation of  $Z_1$  and  $Z_2$  this  $R(s)$  function all right. So let us calculate  $R(s)$  what was Richard function?  $R(s)$  was given as  $K$  into  $Z(s)$

minus s into Z(k) divided by KZ(k) minus SZ(s) okay K equal to 1, so 1 into Z(s) we will substitute Z(s) afterwards minus s into Z(k) is half divided by K into ZK<sub>1</sub> into half minus s into Z(s) okay, you have to expand put Z(s) and expand you will get R(s).

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So let us put those values Z(s) twice S squared twice S squared plus S plus 1 by twice S squared plus 2S plus 4 minus s by 2 divided by half minus s into twice S squared plus S plus 1 by twice S square plus twice S plus 4. So this is R(s), once you have got R(s) Z(k)

into R(s) is Z<sub>1</sub> SZ(k) into R(s) is Z<sub>1</sub>(s) and Z<sub>2</sub>(s) is R(s) by Z(k), so how much is Z<sub>1</sub>(s), Z<sub>1</sub>(s) is R(s) into Z(k) and Z(k) we have got already the value of Z(k) as half, so into half Z<sub>2</sub>(s) R(s) by Z(k) am I correct Z<sub>2</sub>(s), Z<sub>2</sub>(s). Let us see Z<sub>2</sub>(s) I hope I have not made any slip one upon R(s) by Z(k) there was a small slip Z<sub>1</sub>(s), Z<sub>1</sub> is Z(k) into R(s) and Z<sub>2</sub>(s) should be actually this is 1 by Z<sub>2</sub>(s), so Z(k) by R(s), is it not Z(k) by R(s). Now this is all right, so Z<sub>2</sub>(s) is is Z(k) by R(s) and that is equal to half divided by R(s).

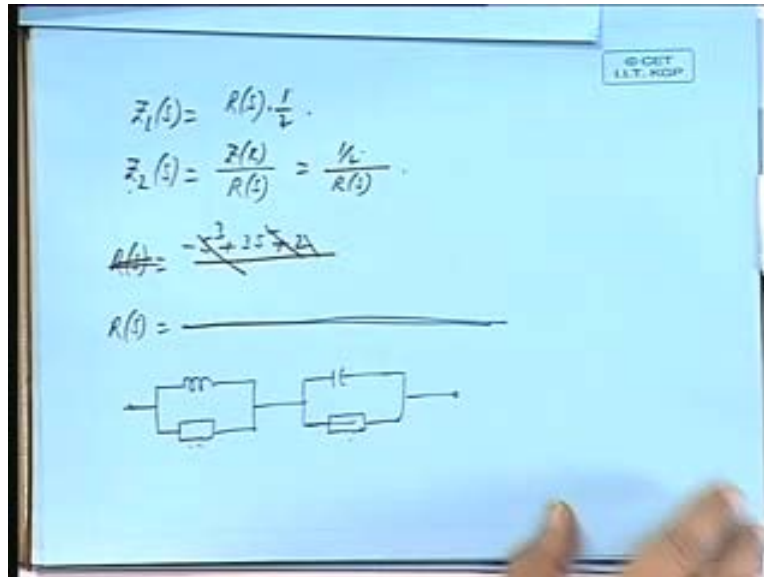
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The image shows handwritten mathematical work on a blue background. At the top, there is a quadratic equation:  $2k^2 - 2k + 2k - 2 = 0$ . Below it, the equation is factored as  $(k-1)(2k+2) = 0$  with the solution  $k=1$ . Then,  $\frac{Z(k)}{k} = \frac{1}{2}$  and  $Z(k) = \frac{1}{2} \cdot k = \frac{k}{2}$  are written. The main derivation for  $R(s)$  is shown as:  $R(s) = \frac{k Z(s) - 1 Z(k)}{k Z(k) - 1 Z(s)} = \frac{1 Z(s) - 1 \frac{1}{2}}{\frac{1}{2} - 1 Z(s)}$ . This is further simplified to  $\frac{2s^2 + s + 1}{2s^2 + 2s + 4} - \frac{1}{2}$ . The final result is  $\frac{1}{2} - \frac{2s^2 + s + 1}{2s^2 + 2s + 4} = \frac{4s^2 + 2s + 2 - 2s^2 - s - 2}{2s^2 + 2s + 4} = \frac{2s^2 + s}{2s^2 + 2s + 4}$ .

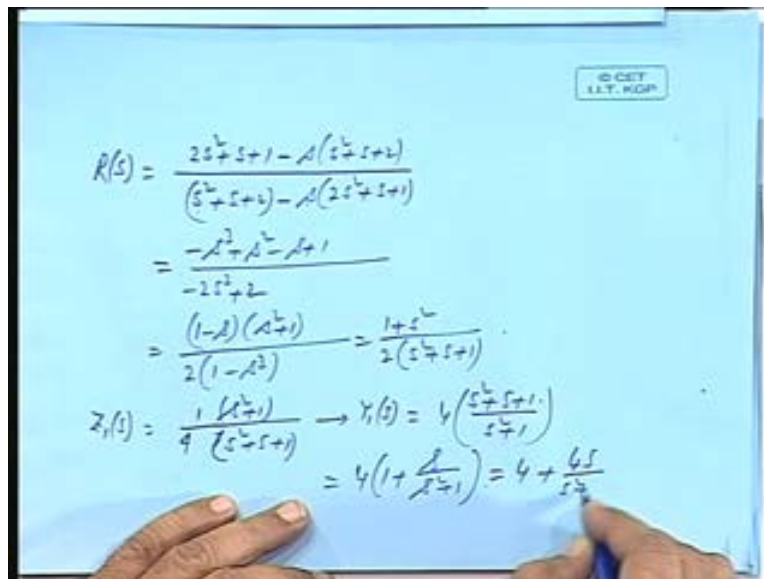
So R(s) let us complete this 2 into S squared, so 4 S squared plus 2S plus 2 minus twice S cubed minus 2S sorry, it should be 2 goes so S cubed minus S squared minus 2S divided by 2 into this and sorry, divided by this and this will also get cancelled. So let me work out here, here it will be S, S squared plus S plus 2 minus twice S cubed minus S squared minus S if you simplify this R(s) will come out as minus S cubed minus S square plus 4 S square. So minus 3S squared sorry, plus 3S squared 2S and minus 2S will go is that so plus 2 I am getting some complicated figures please check there should be 2S square plus, I have made a slip here while computing this it should be 2S squared because 2 is a common factor 2S squared plus S plus 1 in a, you complete this you can complete this any way R(s) will be whatever is the simplified form I will just work out separately there is a lot of mix up, what I wanted to stress is once you have found out R(s) you have already got Z<sub>1</sub>(s), Z<sub>1</sub>(s) will be R(s) by 2 and Z<sub>2</sub>(s) will be half R(s). So the impedance function here and the admittance function here will look alike okay.



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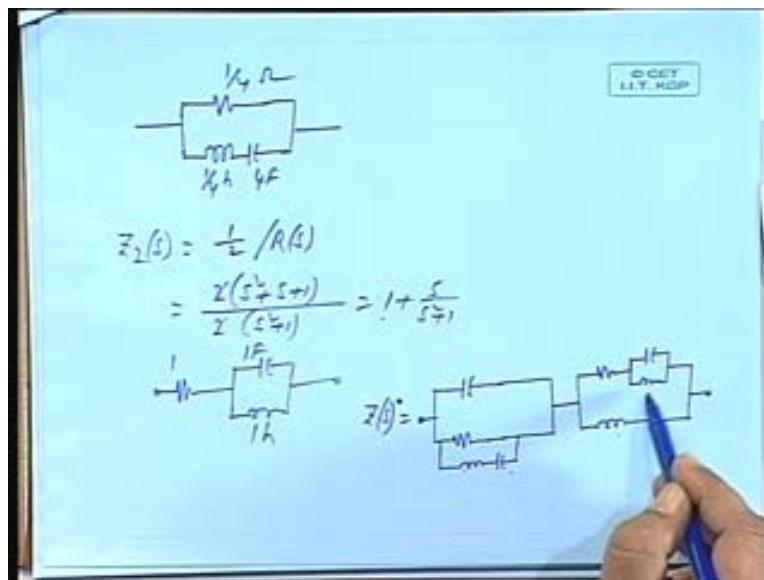


We will complete this problem in the next class time is running out the other 2 parts are there is a inductor here and is a capacitor here. So this is a kind of network you are getting these 2 are to be realized let me work out the value of R(s) here. R(s) twice S squared plus S plus 1 minus s into S square plus S plus 2 s into S square plus S plus 2 divided by so S squared plus S plus 2 minus s into twice S squared plus S plus 1 is that all right. Check twice minus s cubed minus s squared plus twice s square plus s squared minus 2 s plus s minus s plus 1 divided by minus twice S cubed S squared and minus S

squared gets canceled plus S and minus S that will also get canceled, so plus 2. So that gives me in the numerator 1 minus s into s square plus 1, check numerator and this is, this is 2 into 1 minus s cubed which is nothing but 1 minus s will get cancelled. So it will be 1 plus S squared divided by 2 into S square plus S plus 1 okay so this is R(s) all right.

What is therefore, what is our  $Z_1$  R(s) by 2 if you see  $Z_1$  is R(s) by 2, so  $Z_1(s)$  is 1 by 4 s square plus 1 divided by S square plus S plus 1 if I invert it corresponding  $Y_1(s)$  it will be S square plus S plus 1 by S square plus 1 into 4 this is nothing but S square plus 1 by S square plus 1 will give me 1, so 4 into 1 plus s by s square plus 1. So that is 4 mho plus 4 S by S square plus 1.

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So this is our familiar elements  $Y_1$  plus  $Y_2$ ,  $Y_1$  is 4 means 1 by 4 ohm resistance and this is  $Y_2$ . So if I invert it how much is this, if I invert it 1 by 4 Henry and 4 Farads, 1 by 4S so 4 Farads what about  $Z_2(s)$   $Z_2(s)$  was R(s) half  $Z(k)$  by R(s) that is half divided by R(s) so half divided by RS so we can write R(s) was 1 plus S squared, so it will be inverted 2 into S square plus S plus 1 divided by 2 into S square plus 1 so 2 will get cancelled it will be 1 plus S by S square plus 1,  $Z_2$  is this so  $z_1$  plus  $Z_2$  I mean  $Z_2$  can be written as 2 sum sum of 2 components 1 is 1 ohm, 1 ohm resistance other 1 is 1 Henry and 1 Farad, capacitor, so this is  $Z_2$  okay.

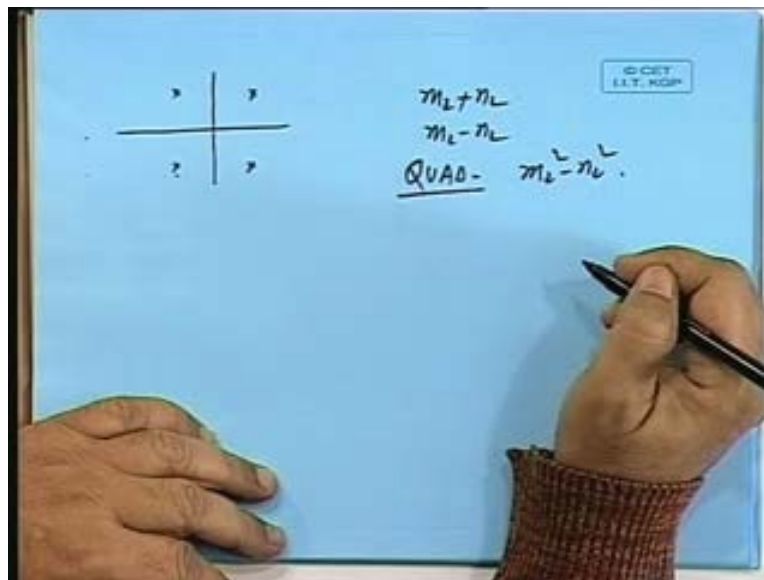
So now you come to the final realization  $Z_1(s)$  was in parallel with the capacitor okay. So you have final  $Z(s)$  has a capacitor in parallel with that  $Z_1(s)$  that is one fourth ohm resistance and network like this and then an inductor and this one is resistance. So this is the structure of  $Z(s)$  the number of elements required here will be more you see it is no more canonic in that sense because we are getting a negative inductor in case of Brune's synthesis we coupled it with a transformer the number of elements there we got much

less here it will be 1, 2, 3, 4, 5, 6, 7, 8 never the less this is easier to implement there is no coupling involved okay. So thank you very much we shall continue with this with a few more examples in the next class.

Good afternoon friends. Today we shall be discussing about parts of network functions. Now before we go to that I will just briefly summarize what we did yesterday in the Bott Duffin synthesis then we will go to parts of network functions. So the Bott Duffin synthesis we defined the Richard function which is a positive real function in terms of  $Z(s)$  as  $KZ(k)$  minus  $SZ(s)$  if  $Z(s)$  is a positive real function then all for all positive values of  $K$ ,  $R(s)$  will also be a positive real function from there we wrote  $Z(s)$  in terms of  $R(s)$  as  $KZ(k)$ ,  $R(s)$  plus  $SZ(k)$  divided by  $K$  plus  $SR(s)$  and this we broke up in this form if you remember in the form of 2 parallel elements put in series with another set of this type  $K$  by  $SZ(k)$  plus  $R(s)$  by  $Z(k)$  okay. So this was shown to be like this this is the admittance of a capacitor is that all right. So this is this admittance of a capacitor so  $1$  by  $KZ(k)$  Farads will be the value of this capacitance this is say this is some  $Z_1$  then in parallel with this capacitor this impedance  $Z_1$  will come.

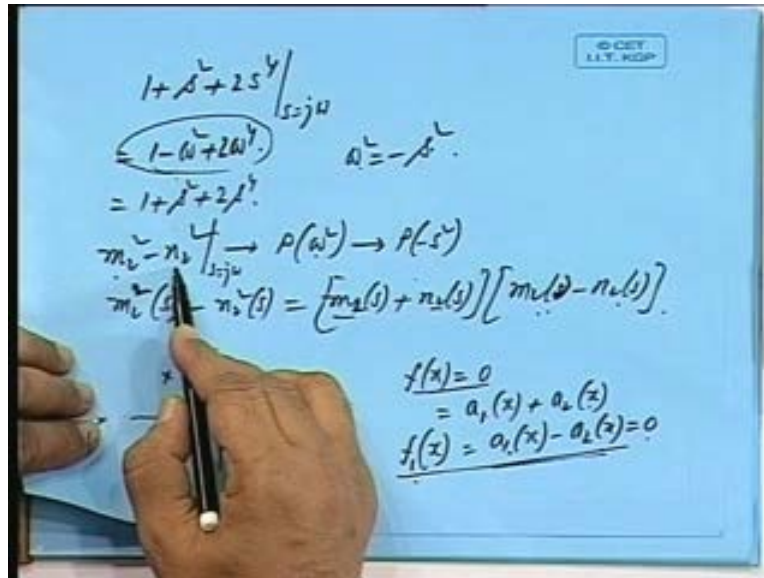
Similarly this one is the admittance of a reactor whose value is  $Z(k)$  by  $K$  so many inductance in parallel with that we have  $Z_2$  which means  $Z(k)$  by  $R(s)$  is that all right. So this is what we wrote yesterday now  $Z_1$  to be calculated  $Z_1$  is  $Z(k)$  into  $R(s)$ ,  $R(s)$  being a positive real function  $Z(k)$  is having a constant value for a particular value of  $K$ , real value so this is an impedance function, realizably impedance function. Similarly,  $Z(k)$  by  $R(s)$  is the other impedance function you can see one is  $Z(k)$  into  $R(s)$  the other  $1$  is  $Z(k)$  by  $R(s)$ .

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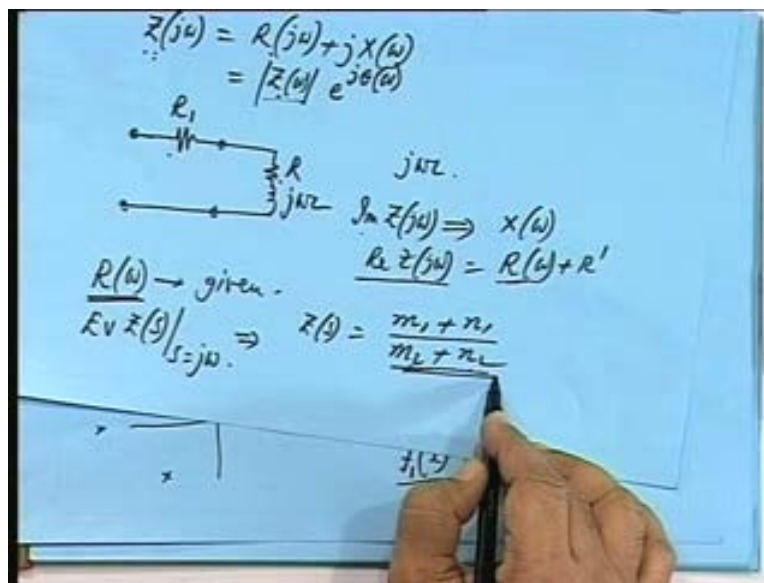


For example, if you have a complex pair like this for  $m_1$  plus sorry  $m_2$  plus  $n_2$ , 1 pair of roots is like this then for  $m_2$  minus  $n_2$  correspondingly you have roots here mirror images. So you get quad of points for each complex root set you get actually quad of points for the function into  $m_2$  square minus  $n_2$  square okay.

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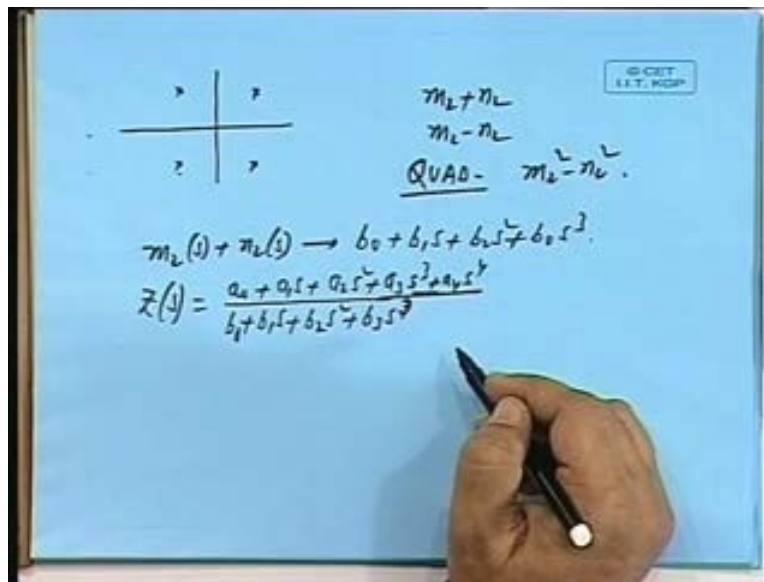
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So problem is simplified you are given  $m_2$  squared minus  $n_2$  squared in terms of  $\omega$ . So make a substitution  $\omega$  square is equal to minus  $S$  squared then factorize it then

chose the factors corresponding to the left of plain roots that will give you  $m_2(s)$  plus  $n_2(s)$  why left of plain roots because original function  $Z(s)$  is a positive real function which should satisfy Rowther witz criteria that means all the roots of the numerator and the denominator must lie in the left of plain. So  $m_2(s)$  plus  $n_2(s)$  corresponds to the function  $Z(s)$  it is the denominator of  $Z(s)$  is it not which is a realizable function, so the roots are here are laying in the left of plain so you select out the roots corresponding to the left up plain.

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Once you have done that then  $m_2(s)$  plus  $n_2(s)$  is defined  $m_2(s)$  plus  $n_2(s)$  is selected. Now you chose  $Z(s)$  equal to suppose this equal to some  $b_0$  plus  $b_1(s)$  plus  $b_2(s)$  square suppose it is like this may be  $b_0(s)$  to the power 3  $b_0$  plus  $b_1(s)$  plus  $b_2(s)$  square plus  $b_3(s)$  to the power 3 then what will be the numerator like it can have a 0 plus  $a_1(s)$  plus  $a_2(s)$  squared plus  $a_3(s)$  cubed okay. Any question? The condition for a positive real function is the power of  $S$  should differ at the most by 1, can I take  $a_4 S$  to the power 4 see by division there will be a free  $S$  term that means some reactance element, additive reactance element will come, it can be anything that becomes indeterminate, is it not. As I was telling you right in the beginning I can identify from that only  $x$  corresponding to the minimum value any additional  $x$  will also give me the same real part, real part remains unchanged if you keep on adding reactances.

So we will consider only that reactance function which is just just required to realize that  $S$ ,  $n_1$   $m_2$  okay this part divided by  $m_2$  square minus  $n_2$  square,  $m_2$  square minus  $n_2$  square is already known that is 1 plus omega to the power 6 and so on like that. So it is this part which has to be computed okay from that  $a_0$ ,  $a_1$  all right in terms of  $a_0$ ,  $a_1$ ,  $a_2$  we can calculate this and substitute once you have calculated this co-efficient substitute there we will get that all part.

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Handwritten notes on a blue background showing the derivation of a partial fraction expansion and a corresponding circuit diagram. The text includes:

$$a_1 = \frac{1}{3}$$
$$Z(s) = \frac{\frac{1}{3}s}{2 + 3s + s^2}$$
$$\frac{1}{Z(s)} = Y(s) = \frac{(2 + 3s + s^2) \frac{2}{3}}{s}$$
$$= \frac{6}{s} + 9 + 3s$$

Next to the equations, there is a circled expression:  $m_1, m_2 - m_1, m_2$ . Above it, the word "X(w)" is written with a double arrow pointing to the right.

Below the equations is a circuit diagram consisting of two parallel branches connected between two terminals. The left branch contains a resistor with a value of  $\frac{1}{6} \Omega$ . The right branch contains a resistor with a value of  $\frac{1}{3} \Omega$  in series with a capacitor with a value of  $3F$ .

Otherwise, in  $Z(s)$  put  $S$  equal to  $j\omega$  take out the real part and the imaginary part that will give you  $x$  okay. So we will stop here for today, we will take up a few more problems in the next class along with problems of RLC synthesis, thank you very much.