Networks Signals and Systems

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Lecture - 29
Tutorial

We will continue with the numerical problem that we are discussing in the last class.

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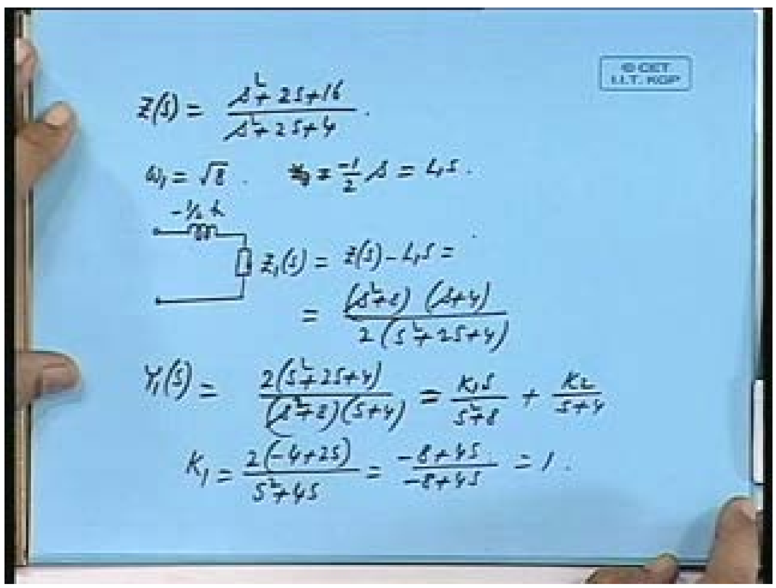

We took a function z(s) equal to s squared plus 2 s plus 16 divided by s squared plus 2 s plus 4 we started off with this function and we found that at omega 1 is equal to root 8 the real part vanishes and we also obtained the value of x_1 as half s rather corresponding impedance, this was L_1(s) all right and this was minus half s minus half Henry. This is z_1(s) which will be equal to z(s) minus L_1(s) and that give me L_1(s) is having a negative sign. So that give me a plus sign so we got s squared plus 8 into s plus 4 divided by 2 into s squared plus 2 s plus 4 okay. To realize a pole, to realize a pole we know we can make partial fractions, so this 0 is to be converted to a pole.

So we can very easily realize z_1 is in terms of admittance function which will be 2 into s squared plus 2 s plus 4 divided by s squared plus 8 into s plus 4, we write as k_1(s) by s squared plus 8 we know whenever the error routes on the imaginary axis it will be realized in terms of an LC network k_1(s) by s squared plus 8 plus k_2 by s plus 4 this is to be seen later.

Let us see what this means how much is k_1 multiplied by s squared by 8 divided by s make s squared plus 8 equal to 0. So this will give me 2 into 4 minus 8, so minus 4 plus 2 s divided by s into s squared plus 8, so s into s s plus 4, so s squared plus 4 s which gives me this is minus 8 plus 4 s divided by minus 8 plus 4 s. So that is equal to 1.

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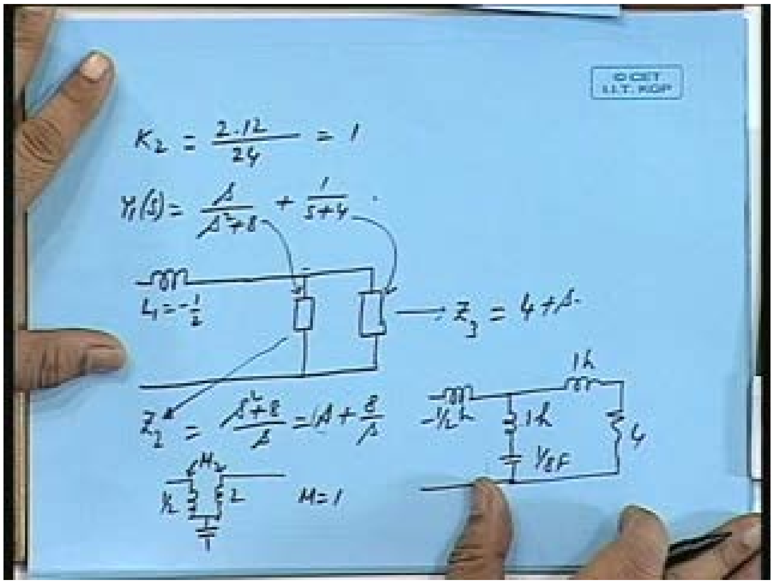

Similarly, k_2 if I multiplied by s plus 4 and make s plus 4 equal to 0 say s is equal to minus 4, so 16 plus 4, 20 minus 2 into 48, so 12, so 2 into 12 divided by this is 16 plus 8, 24, is not, 24, so that is equal to 1. So y_1 is turns out to be s by s squared plus 8 plus 1 by s plus 4. So what is it mean you have got l_1 equal to minus half Henry then we have got y_1(s) plus y_2(s) okay, this is y_2, this is y_1. So corresponding z_1 or this z is s squared plus 8 by s which is 1 Henry plus 1, 8h farad and this one is, this z, if I call it z_2 and z_3 then z_3 is 4 plus s all right.

So the network looks like this. This is 1 Henry, this is 18 farad, this is minus half Henry, this is 1 Henry and this is 4 ohms and this 1 Henry, 1 Henry and minus half Henry, this can be combined together in the form of a transformer 1 minus half. So this will be half Henry primary inductance, 2 Henry secondary inductance and 2 into half square root of that is 1, 1 Henry, m 100 percent coupling and then you have the capacitor, capacitor and resistor. So this is the network, what does a network sorry, Brune's network, is it all right?

Let us see, what it will look like if we go for Bott Duffin synthesis, if you go for Bott Duffin synthesis, our starting point will be this omega 1 all right, whether that reactance where the frequency omega 1 has been identified corresponding to real part 0 okay any question at this point if the real part is not 0 suppose it is having a minimum value but it is not 0 it is minimum at certain value then we will take out that minimum value. Suppose we had given function where the real part varies like this.

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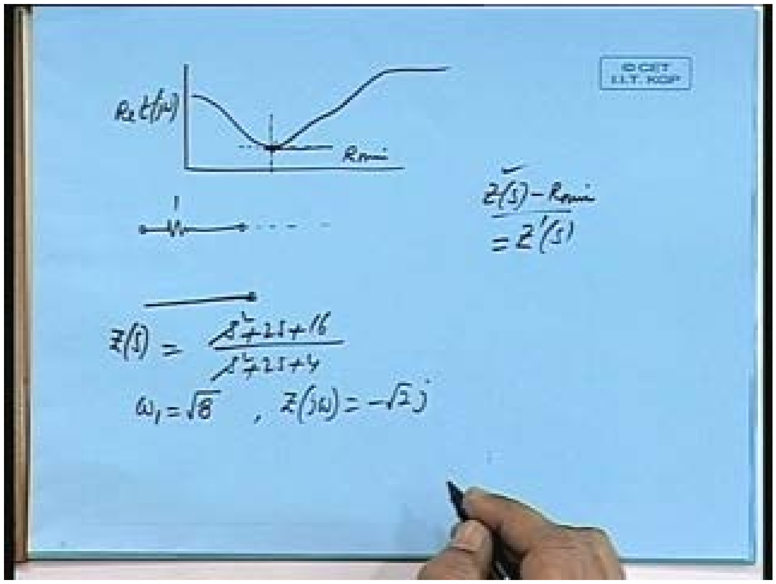

So we try to find out if it is not 0, if it is not vanishing at a particular frequency then we try to find out its minimum value then from the real part you separate out that. So we realize these are minimum separately okay and from this point onward our Brune’s synthesis starts. so separate out suppose that comes out to be 1 ohm then you subtract from z(s) that 1 ohm whatever is a balance that will have a minimum point equal to 0 at this frequency, is not.

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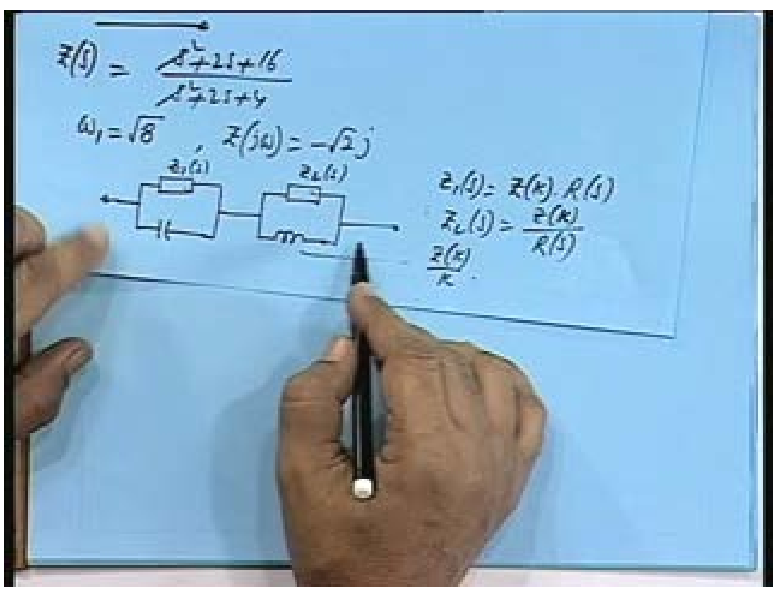

So this frequency has already been identified where this occurs and I have calculated the r minimum value separate it out subtract this r minimum from z(s) whatever is left over I call it z dash s, we will start the synthesis procedure from this point either by Bott Duffin or Brune's synthesis technique, is that point clear, because I want the real part should vanish at a particular frequency, so what is that frequency we have identified the frequency at which the value is minimum but it is not 0 say then if we extract out that resistance from the original z(s) then whatever is left over that will have a minimum equal to 0 at that frequency. So rest of the network now it becomes a minimum function earlier then function z(s) given may not be a minimum function all right. So we will take up this z(s) by Bott Duffin synthesis, so s squared plus 2 s plus 16 divided by s squared plus 2 s plus 4, we found at omega 1 is equal to root 8, zj omega, zj omega was how much minus root2 j, minus root 2 j is that all right minus root 2 j this will be our starting point.

Now what are the different functions that we took for Bott Duffin synthesis, the structure was a capacitor and then $z_1(s)$ and an inductor and $z_2(s)$ okay where, $z_1(s)$, $z_1(s)$ if you remember what did you write for $z_1(s)$ yesterday, $z_1(s)$ and $z_2(s)$ that we wrote if you k times z(k) times R(s), is it not and $z_2(s)$ was z(k) by R(s) that reach at function L was z(k) by k all right and this was 1 by kz(k). So if this is negative, this reactance is negative at that frequency, negative reactance comes only out of a capacitor. So at that frequency I want the current to should current should flow through this that means they this should be infinity and this should be a short if this is an infinite impedance this is a short because 1 is R(s) into something the other 1 is 1 by R(s) into that something all right.

So if this is infinity this will be 0 if this is infinity this will be 0 in this case what do you want if it is minus j_x then current should flow through this that means this is a reactance that should come into picture so it will be minus j_x plus 0, if it is plus j_x then this should be the path. So this should be short and current should flow like this this should be open is that all right. So you know when this will be infinity this will be 0 and this will be 0 this will be infinity if it is turning out to be plus j_x then it will be corresponding to j omega L.

So this will be infinity this will be 0, so current will be short and then this reactance. So the reactance seen is this when it is minus j_x then reactance seen is this that means this will be open and this will be short so the overall impedance is this. So follow that procedure it is very simple so how much is C, C_1, if I call it C okay or C_1 whatever we call it, will be 1 by omega 1 into z(c) which is 1 by omega 1 is root 8, 2 root 2 okay omega 1 is already identified and what was the magnitude of this reactance that was root 2, is it not, z at that frequency is root 2 magnitude is root 2. So this is the value of the capacitance, so that is equal to 1 by 4 farads. Now $z_c(s)$ is nothing but kz(k) by s okay, $z_c(s)$ is z(k) by k is L all right and C was 1 by kz(k), C is 1 by kz(k) and that has already been computed as 1 by 4.

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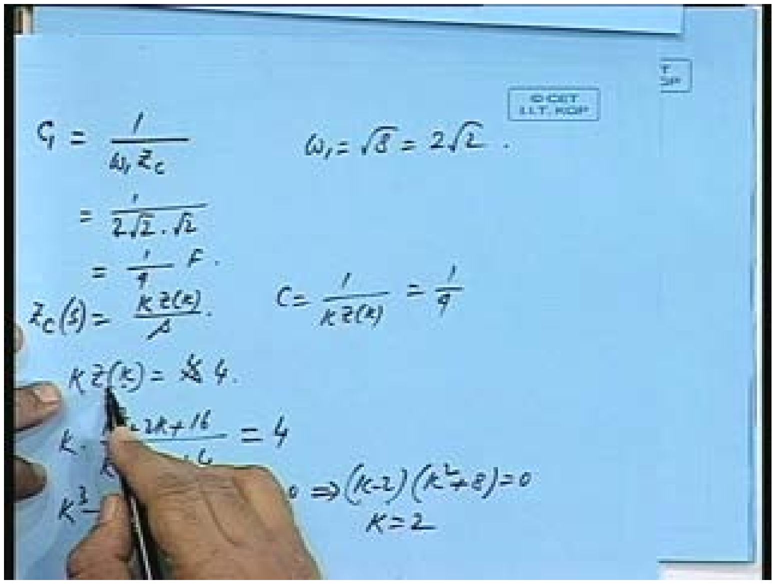

So kz(k) is 4 all right, so k into put z is equal to z(k) in place of s you put k so k squared plus twice k plus 16 divided by k squared plus twice k plus 4. So that is equal to 4 solve for this we get k cubed plus 2 k squared minus 4, 4 k squared so minus 2 k squared okay plus 16 k minus 8 k, so plus 8 k plus 16 comes to this side minus 16 equal to 0 or k minus 2 into k squared plus 8 equal to 0. So the real value of k is 2 okay, real value of k is 2.

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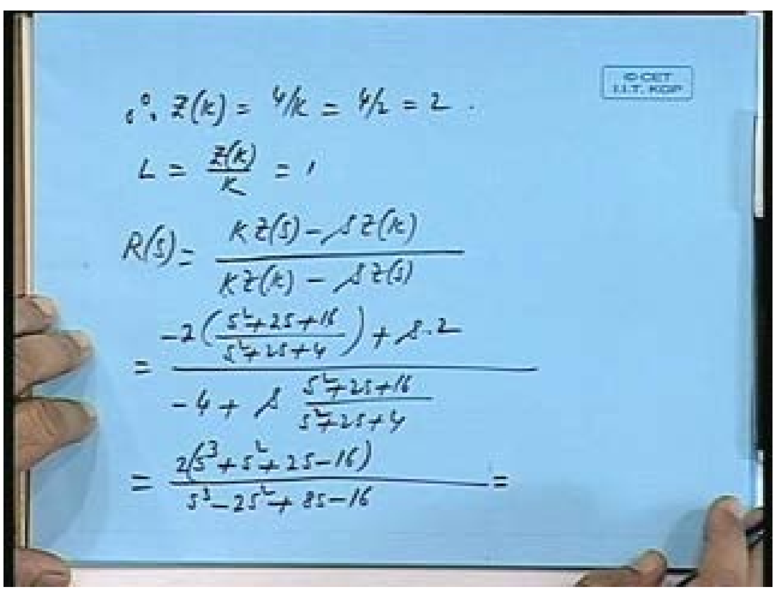

So how much is z(k), k into z(k) is 4k is 2, so z(k) is also 2, 4 by 2 is that okay therefore z(k) is 4 by k is equal to 4 by 2. So we have got z(k) and k so how much will be L, z(k) by k is it not, L is z(k) by k, C is 1 by kz(k), z_2(s) is this z_1 is this, so L is 1 okay L is 1, what is function R(s) we have to compute now R(s) kz(s) minus sz(k) sorry it is drying up divided by kz(k) minus sz(s) substitute the value of k.

So it is 2 into s squared plus twice s plus 16 divided by s squared plus 2 s plus 4 minus s into z(k) is 2 divided by k into z(k) is 4 minus s into z(s), s squared plus 2 s plus 16 by s squared plus 2 s plus 4 okay. If you multiplied by s squared plus 2 s plus 4 and simplify then you get twice s cubed and change the sign we can put this as plus, this as plus, this as minus, this as minus and then twice s cubed. You can even 2 take 2 common, so 2 into s cube plus 2, 2s are 4 minus 2, 2s are 4, thus check what you get my getting all right 2 into s squared plus z(s) just check the values I might have made a mistake 2 2s are 4 and 2 s into 2 s 4, I made a slip somewhere s squared twice s squared sorry, this is 2 s squared, this is 4 s squared. So s cube plus s squared plus 2 s minus 16 all right, if we take 2 common so 16 divided by we will get s cubed minus 2 s squared plus 8 s minus 16 okay s cube plus 2 s squared minus 4 s squared so minus 2 s squared plus 16 okay.

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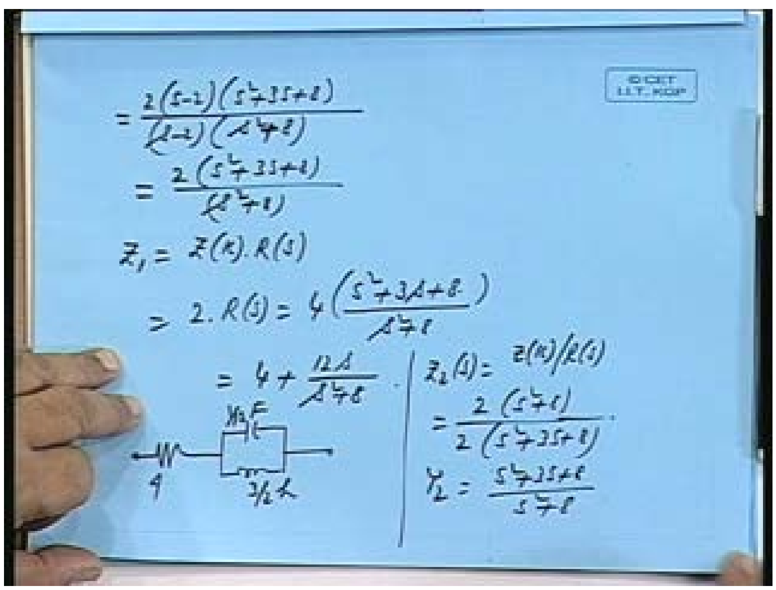

So I have got it will not come here equal to 2 into s minus 2 into s squared plus 3 s plus 8 divided by s minus 2 into s squared plus 8. So s minus 2 will get cancelled okay you get 2 into s squared plus 3 s plus 8 divided by s squared plus 8 okay. So z_1 which is z(k) into R(s) will be z(k) is 2, 2 into R(s) is equal to 4, 2 into this, so 4 into s squared plus 3 s plus 8 divided by s squared plus 8. You can see for yourself s squared plus z by s squared plus z is1, 4 plus 12 s by s squared plus 8 this is z_1.

So what is it 4 ohms and an LC parallel combination which is 12 Henry okay, 12 s by s squared 1 by 12 farad, 1 by 12 farad and 12 s by 8, 3 by 2 Henry, this is z_1 okay. Similarly, z_2 is z(k) by R(s) and that is equal to 2 by R(s) 2 and 2 will go, go so this is s squared plus 8 divided by s squared plus 3 s plus 8 okay. So z_2 I can realize as y_2 which is s squared plus 3 s plus 8 by s squared plus 8 and which will give me same factors that will be in the form of an admittance.

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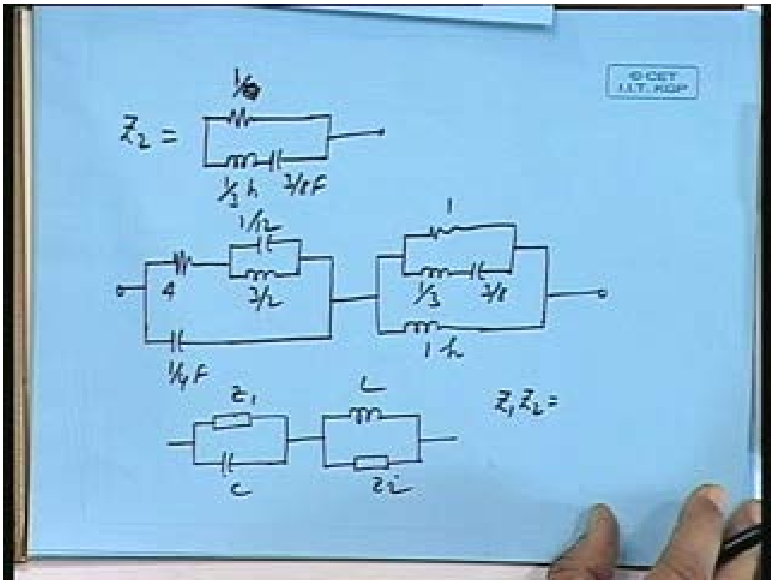

So 4 mho that z_2 part will be therefore one fourth, this is nothing but this same factor sorry 1 plus sorry 1 plus 3 s by s squared plus 8. Now this 4 constant is not there so 1 mho and on this side we will have s squared plus 8 by 3 s, so s by 3, 1 third Henry and 8 by 3 s so 3 by 8 farad. So this is z_2, so finally you got a capacitor C that was how much was C_1 by 4 farad and then with that we got z_1 and z_1 was 4 ohm and LC okay then L and how much was our L_1 Henry and on this side z_2 you have got this is a realization okay this is 4 this is 3 by 2 this is 1 by 12 farad, this is 1 ohm, this is 1 third and this is 3 by 8, so this is the Bott Duffin network one thing is very interesting. We have got c z_1 this is shorted I might as well write this since it is a parallel combination L and z_2 there is nothing wrong in it, is it not, is this all right.

Now you see this into this is equal to this into this, is it not how much is this z(k) by k is 1 by k into z(k). So take the products how much was z_1 and z_2, z_1 into z_2 how much is it okay let us see sorry I will write all these z_1 was z(k) into R(s) z_2 is z(k) by R(s), L_1 how much was L_n, L_1, z(k) by k and L_2 sorry C, C_1, how much is C_1, how much is $C_{1,}$ no I am talking in terms of z(k)'s in terms of z's, in terms of z's we have written 1 is z(k) by R(s), 1 by kz(k), 1 by kz(k) is that all right. So z_1, z(k) okay z_1 into z_2 is z(k) squared z_1 into sorry z_1 into L how much is it z(k) squared into R(s) by k, what is z_2 into this is C_1 sorry. So how much is this z_L, z(k) by k into s, how much is z_c, z_c is z(k) by k into s, is it not sorry 1 by C was 1 by kz(k), is it so 1 by cs, so kz(k) by s.

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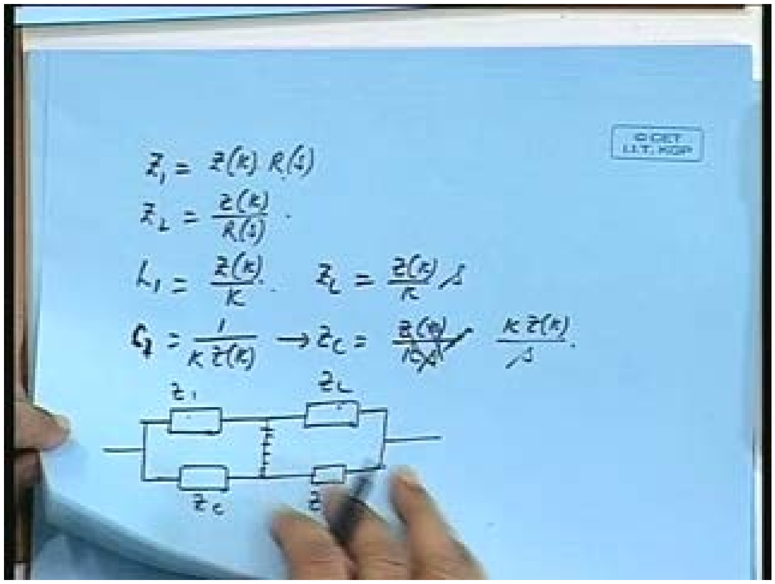

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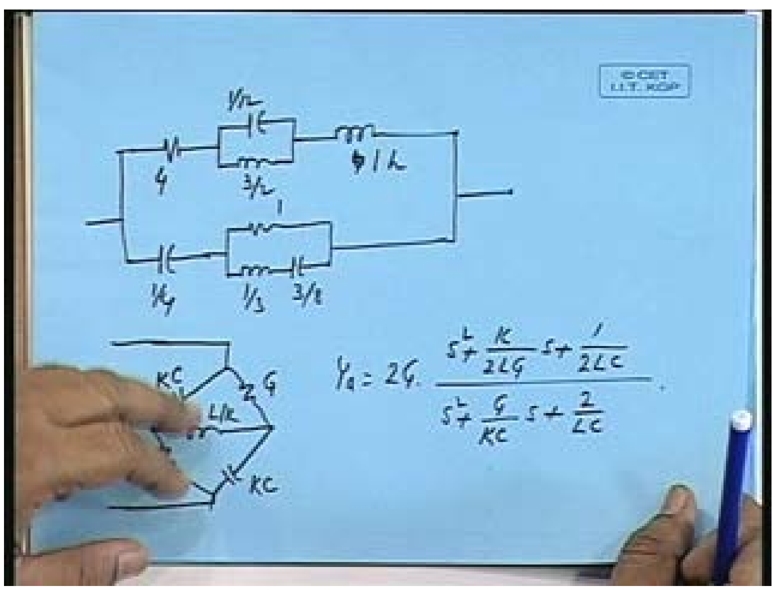

Now how much is z_1 into z_2, z_k squared, how much is z_L into z_c also z_2 squared. So if I write z_1, z_c, z_L and z_2 okay this is what we have written the same thing I am writing in in place of L and C I am just writing z_L and z_c take the product of these 2 is equal to this. So it is a balanced bridge so this can be eliminated this into this is equal to this into this. So it is a balanced bridge network, so this can be removed so I can put z_1 in series z_L, z_c in series z_2, impedance function is same is it not. So this same network one fourth farad in series with this and 1 Henry inductor in series with this both of them will be in parallel. So this network will be therefore reduced to 4 ohms, 3 by 2, 1 by 12, 1 Henry. This is 4 ohms 3 by 2, 1 by 12, 4 Henry, 1 Henry and this side will have 1 by 4, 1 ohm, 1 third and 3 by 8. So this is the overall realization you could have left at this point also is that 2 series drops here there are 2 parallel blocks okay.

It is an interesting question that has been asked here most of problems are taken from some of the very standard books you may come across in many places okay this interesting question is determine the admittance okay I will just draw this first. This is given as an admittance G, this is given kc, this is given as L by k, this is given as G, this is given as kc, the admittance seen from this side y_a show that y_a is twice G into s squared plus k by 2 LG, s plus 1 by 2 LC divided by s squared plus G by kc(s) plus 2 by LC. Obviously, the admittance is appearing to be quiet combustion, is it not one may go for start to delta conversion then again we will get parallel branches okay, we can do it. I thought that I go for a little short cut I mean it is easy to compute.

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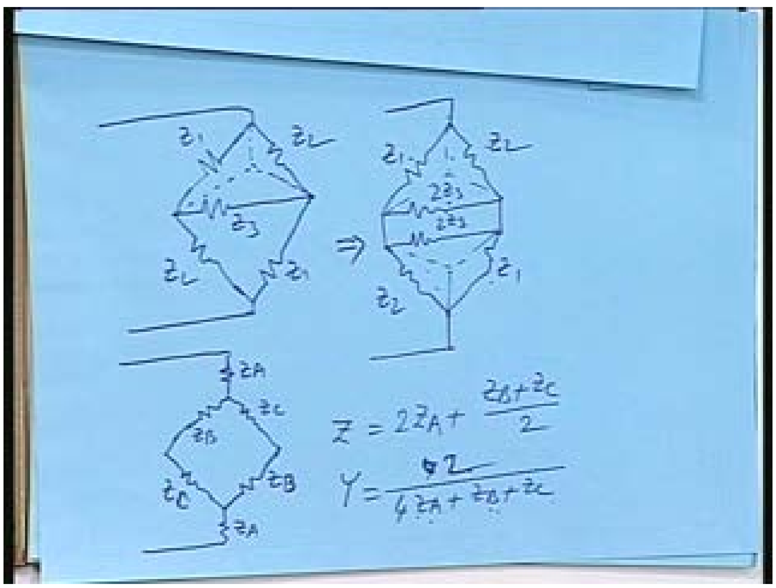

Okay it is not otherwise difficult you are you have been given say I will write in the very general form z_1, something like this z_2, z_3, z_2, z_1. So if I reduce this I am start to delta and then again add with this this add with the z_2 make the parallel combinations and then add with this series okay that because of this symmetric positions z_1 and z_2, suppose z_3 we replace by twice z_3 then I will have a delta here twice z_3 in parallel with that another twice z_3 okay.

Now this one will give me a star product okay, let me show it like this say z_a, z_b and z_c then what will be the products of this z_1, z_2 twice z_3 this one, this also z_1, z_2 twice z_3 it will be z_a I am sorry z_c, z_b just z_b and z_c will interchange their positions and this will remain as z_a, is it not. See this one and this one they are identical except that z_1, z_2 I have just rotated this, is it not. So the same element values will come in just change the order, so z_b and z_c will be interchanging their positions.

Now it is z_b plus z_c on this side this side also z_b plus z_c so parallel combination of these 2, so what will be the total z will be z_a plus z_a twice z_a plus z_b plus z_c divided by 2, is that all right. So make a tabulated conversion of these impedances z_1, z_2 and twice z_3 okay and then compute this. So this is quite a bit of reduction in level in the computational level once we know the z we can calculate y_1 over this, so this is 4 by 4 z_a plus z_b plus z_c. So I leave it as an exercise you can do this and derive this relation very easily there is no problem yes, and sorry this is 2, 2 by when you put the values of z_a, z_b, z_c you will get that in this form. Now that we have got the method of rlc synthesis ready you remember sometime back we checked s plus 2 into s plus 3, is it into this plus 4, is it a positive real function and if it is realizable can you realize one. So m_1, m_2 minus n_1, n_2 will check that time which is s squared plus 5 s plus 6 divided by s squared plus 5 s plus 4.

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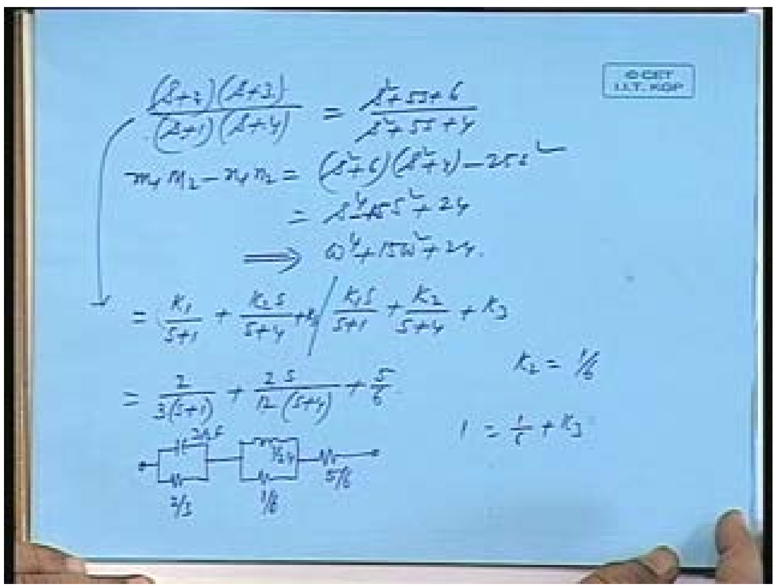

So s squared plus 6 into s squared plus 4 minus 25 s squared, so that give me s to the power 4 minus s squared plus minus 15 s squared 6 plus 4, 10 minus 25, 15 s squared plus 24, so if I put s equal to j omega that gives me omega to the power 4 plus 15 omega squared plus 24, this is always positive. So if I am asked to realize this function wholes and 0s are not coming alternately, so this is going to be an rlc combination, rlc combination, okay.

So one may try partial fractions. Let us try if it too if it works otherwise we will go for that Bott Duffin or Brune's synthesis, if it is possible to write this as k_1 by s plus 1 plus k_2 by s plus 4 or it can be k_1(s) by s plus 1 plus sorry k_2 by s plus 4 that means I put s with one of them one is RC the other one is RL plus if required a constant k_3, either of them. Let us see whether we can get in this form in the first one if I write in this form how much is k_1 put s plus 1 is equal to 0. So that gives you 1 into 2 by 3 so 2 by 3 s plus 1 okay k_2 if I multiply by s plus 4 divide by s and then make s plus 4 equal to 0 then it will be 2 minus 2 minus 1 divided by minus 4 minus 3.

So it will be positive so this will be 2 by 4 into 3, 12, s by s plus 4 it is 1 by 6 plus k_3 if I make s_{10} into infinity you get anything s_{10} into infinity will be k_2 plus k_3 okay k_2 is how much 1 by 6 if I make s_{10} into infinity this will become s_2 the part 2 by s_2 the part 2 so 1, 1 on the right hand side if I make s_{10} into infinity it will be k_2 plus k_3, k_2 is 1 by 6 plus k_3, so k_3 is 5 by 6 okay. So this will be an RC network all right. 2 by 3 s so 3 by 2 farads s_{10} into 0 means 2 by 3 ohms resistance this is an RL network s_{10} into infinity means 2 by 12, 1 by 6 ohm resistance when extending to 0 it is1 by 6 sorry when extend into infinity yes, s tending into 0 it will be 2 by 48 so 1 by 24 farads okay and then 5 by 6 ohm resistance. It is possible to realize simply by partial fractions only thing we will have to keep on trying possibilities either this or the other one is like this. It is an rlc you can also try by Brune's synthesis.

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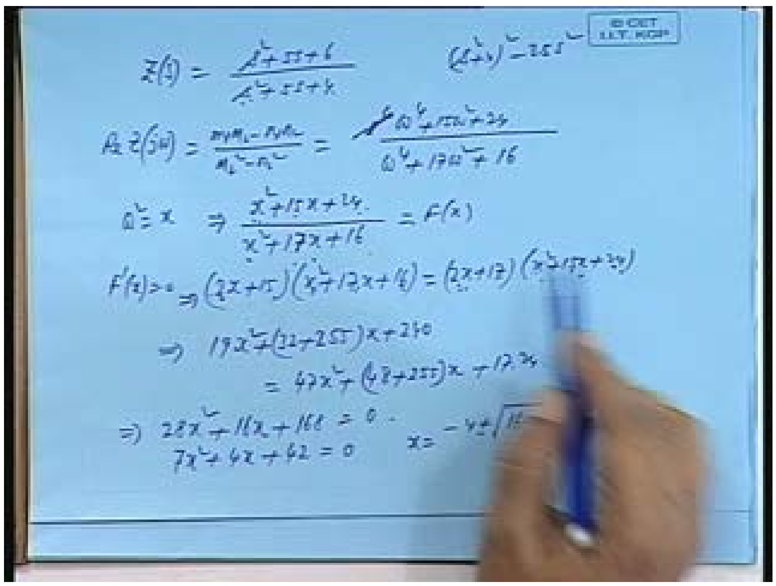

Let us see when is that real part vanishing, does the real part vanish if not what to do? So z(s) equal to s squared plus 5 s plus 6 I am just rewriting this problem s squared plus 5 s plus 4. So real z(s) real z j omega will be m_1, m_2 minus n_1, n_2 by m_2 squared minus n_2 squared and that is equal to we just now worked it out s to the power 4 sorry omega to the power 4 plus 15 omega squared plus 24 divided by how much is m_2 squared minus n_2 squared a squared plus 4 whole squared, a squared plus 4 whole squared minus 25 s squared, put s equal to j omega.

So omega to the power 4 plus 4 into 28 minus 25 okay, 17 minus 17 omega squared means plus 17 omega squared, minus 17 s squared plus 16 all right. Now this is not vanishing at any frequency then what we do? what we do? Find out the frequency at which these value is minimum I can put omega square is equal to x, let me make it in a quantity form. So this will become x squared plus 15 x plus 24 divided by x square plus 17 x plus 16 then let us call it f(x). So f dash to x equal to 0, so that gives me how much twice x plus 15 into x squared plus 17 x plus 16 that will be equal to after equate to 0 twice x plus 17 into x squared plus 15 x plus 24, correct if I am wrong, is that all right, twice sorry twice x cube will get cancelled then 17 x square 34 x squared minus 15, 34 minus 15, 19 x squared okay plus 16 into 2, 32 plus 17 into 15 all right. So 255 into x plus 15 into 16, 240 on this side we get 2 x squared 2 x cube get cancelled then 15 into 2, 30 plus 17, 47 x squared all right plus 24 into 2, 48 plus 48 x ,48 x plus 255 x plus 17 into 24, is it all right 17 into 24. So that gives me 47 minus 19, 28 x squared 47 minus 19 255 will get cancelled 48 minus 32, 16 x plus 17 into 24 minus 24 into 10, so 7 into 24, is that all right, 168 if you divide by 47 x squared plus 4 x plus 42 is equal to 0 is that all right 7 x squared plus 42 plus 4 x, so you will get one real value, positive value of x.

So x is let us calculate minus 4 plus minus root over 16, we get imaginary values, sir both are imaginary, both are imaginary there is something something wrong I my might have made a mistake that must be some frequency where it is minimum omega squared is equal to x squared plus 15 x plus 24 x squared plus 17 x plus 16. I hope this is all right okay then 2 x plus 15into x squared plus 17 x plus 16, 2 x plus 17 into x squared plus 15 x plus 24, so 2 x cubed 2 x cubed they get cancelled 2x into okay.

Let me rewrite it, anyway what I want it to state is you will get a real value of x that is equal to omega square so calculate omega and substitute that omega here in the real part. So that will give you give you the minimum value that is when the real part, real part varies like this, it is this minimum value and after computing that R minimum subtract it from z(s), subtract it from z(s) whatever is left over you start realizing that z(s), the reminder z(s). There may be a small slips somewhere here we will discuss it in the next class, if time permits otherwise you work it out yourself and since there is not much of time okay, thank you very much. We will continue with this in the next class.

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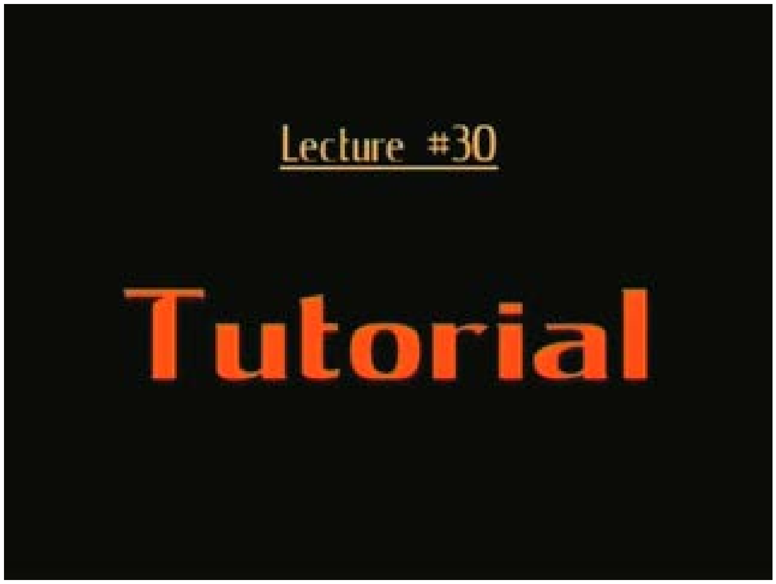

Good morning friends, today we will have a tutorial exercise on some of the topics that we have covered so far.

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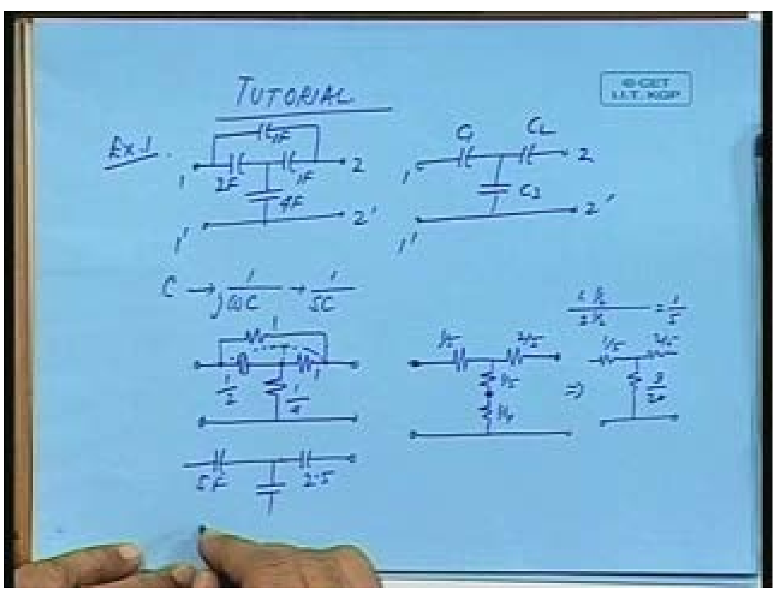

I will take up a few examples and discuss there is a problem, first example is a very simple one, you have bridged capacity circuit the values are 2 farads, 1 farad, 4 farads and 1 farad. What will be the equivalent capacitances C_1, C_2 and C_3 okay. Now in this case as you know a capacitor is having an impedance if you talk in terms of sinusoids, it will be 1 by omega C, it is j here if you talk in Laplace domain it will be 1 by sc that means the impedance is inversely proportional to C.

So I can take these capacitances to be equivalent impedances or even resistances with the values which will be just inverse of this something like half ohm, one fourth ohm, 1 ohm and 1 ohm. I can replace it by an equivalent impedance circuit. Now after this we can get equivalent 1 by a star delta conversion see if this is a delta then I can have a star like this. So these 3 nodes are these 3, so what will be the value of this 1 into half divided by 1 plus half plus 1. So 1 into half divided by 2 and a half, so that gives me 1 by 5, 1 by 5.

Similarly, this one will be 1 into 1 by 2 and a half, so 2 by 5 and similarly this is 1 into 1 and this one will be again 1 and a half, so this one will be 1 by 5, this is 1 by 4 ,so you get 1 by 5, 2 by 5 and 1 by 5 plus 1 by 4, 9 by 20. So take the inverse of these that will give you the equivalent capacitance 5 farads, 2.5 farads and 20 by 9 farads okay. So these are the 3 equivalent capacitances, what will be the expression for this type of periodic function? It is pretty simple 0, this is 5, 2, 4, 6, 8, 10 and so on, this is minus 5.

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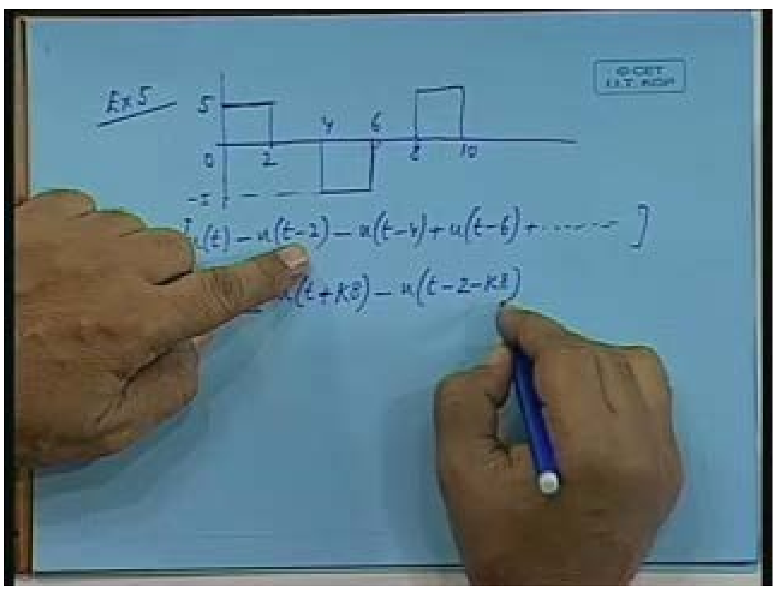

Obviously the first block can be written as 5 into u(t) and then at 2 seconds I apply a negative step at u(t) minus 2. So that gives me the first block again at 4 it is negative so minus u(t) minus 4 and at 6 I apply u(t) minus 6 a positive step and that completes 1 period up to 8 seconds and then it keeps on repeating. So show 1 period if I show 1 period and then show it as a repetition of the same function this will be written as 5, I can write this as summation u(t) minus 2 okay u(t) so plus after 8 seconds again it is plus u(t) okay.

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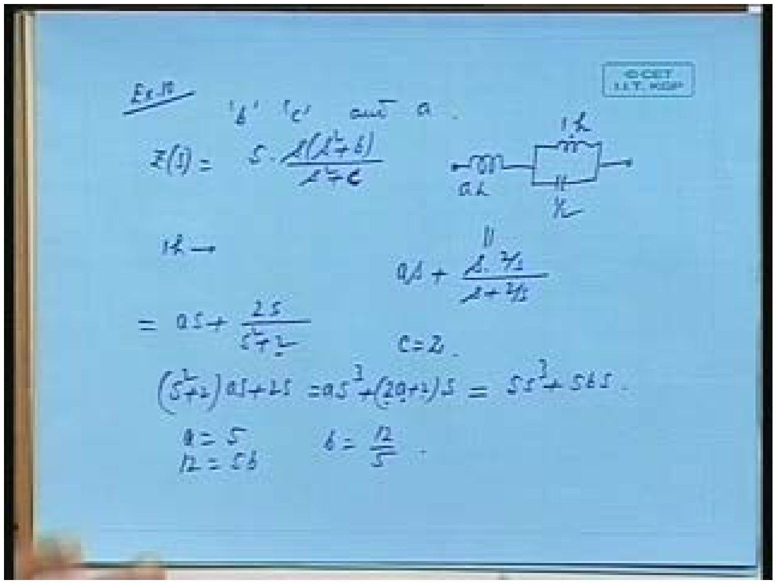

So u(t) plus k_8 minus u(t) minus 2 minus k_8, C has to be equal to 2 okay, C has to be equal to 2. If I add this s squared plus 2 into as plus2 s that gives me s cubed plus twice a plus 2 into s and that is equal to 5 s cubed plus 5 s cubed plus 5 bs. So here a is cubed sorry, a is equal to 5 and if I put equal to 5, 5 into 2, 10 plus 2, 12 is equal to 5 b, therefore b is equal to 12 by 5 okay, this is a very simple example. Well, before we take up any other problem I think we will stop here today, we will continue with the next class because there is not much of time.